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# NRAP: Nearest reliable anchors-based wireless positioning for irregular multi-hop networks

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**Abstract**

The position of nodes is the critical foundation for information transfer between "person to things" and "things to things" at any time and from any location in wireless multi-hop networks. The hop-based wireless positioning technology has received widespread attention because it does not requiring additional ranging devices. However, most existing hop-based positioning algorithms ignore the network topology irregularities issues, which are frequently observed in multi-hop networks and may lead to poor positioning performance. In this paper, we present a novel wireless positioning algorithm, named NRAP, for irregular networks based on the nearest reliable anchors to mitigate the impact of topology irregularities. Specifically, a more accurate per-hop distance estimation model is firstly adopted. Then, NRAP divides the entire network into multiple sub-networks with the node to be positioned and its nearest four neighbor anchor nodes. Moreover, a hybrid particle swarm optimization and natural selection algorithm are employed to search in each sub-network to find the optimal estimated position of the node to be positioned. We evaluate and analyze the performance of NRAP under various network topologies and parameters in comparison with the many state-of-the-art works, and the results further demonstrated the superior performance than these benchmarks.

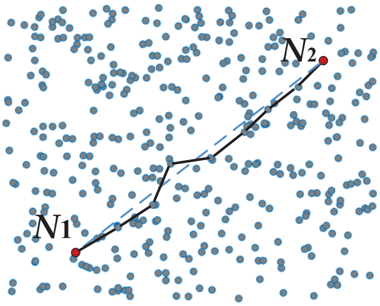
**Keywords:** Nearest reliable anchors, Wireless positioning, Irregular multi-hop networks, Hybrid PSO

# 1 Introduction

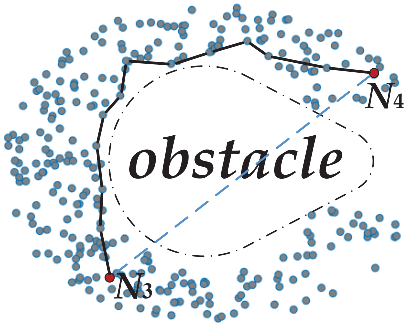
With the popularity of satellite navigation systems, such as Global Positioning System (GPS) and BeiDou Navigation Satellite System (BDS), numerous position-based apps have become increasingly common in people's daily lives. However, in practical applications, the majority of people's productive activities and life activities take place in restricted spaces, such as high buildings and tunnels, where wireless nodes struggle to catch satellite signals, resulting in a massive number of blind spots for position-based services[1–3]. To address this issue, researchers have developed a series of range-free positioning solutions. These solutions employ a small number of nodes with known locations (called *anchor* nodes), which can be manually placed nodes or just selected nodes equipped with independent satellite positioning devices. Once a normal node wants to localize its location, it will determine and estimate its position by exchanging data with these anchor nodes in a multi-hop manner [4].

Generally, wireless nodes are treated as if they are randomly deployed in a regular region by means of random casts in many previous solutions [5–8]. In such a network, data exchange between pairwise nodes is possible if the distance between them is less than or equal to their communication radius; however, there is no direct communication for pairwise nodes far apart from each other, and thus their data exchange must rely on other relay nodes in the network. The hop counts between pairwise nodes are the number of relay nodes plus one. Based on this observation, researchers have designed several hop-count-based positioning algorithms (abbreviated as hop-based positioning algorithms) [9]. Hop-based positioning algorithms are typical low-cost positioning approaches that do not require expensive hardware support because they rely solely on network connectivity [4, 9, 10].

Basic hop-based algorithms also assume that nodes are uniformly and densely distributed in a regular region (as shown in Fig. 1(a)), where data packets can be exchanged in a nearly straight-line manner based on some routing protocols such as the distance vector routing protocol [11, 12], so that hop count is proportional to the Euclidean distance between pairwise nodes.




(a) Regular network




(b) Irregular network

**Fig. 1**: Hop count measurement.

Unfortunately, in practice, the distribution of nodes is influenced by obstacles and the shape of the deployment region itself, resulting in irregular network topologies (as shown in Fig. 1(b)).

As the example shown in Fig. 1, the node pairs $N_1 - N_2$ and $N_3 - N_4$ have the same Euclidean distance length, *i.e.*, $d_{N_1 \leftrightarrow N_2} = d_{N_3 \leftrightarrow N_4}$, while only the former routing path is almost unaffected by external elements and closer to its Euclidean distance. Routing paths are heavily influenced by obstacles between $N_3 - N_4$ and more hop counts are needed to realize data exchange between them, thus causing the routing distance to largely deviate far from its Euclidean distance. Given per-hop estimated errors are unavoidable for hop-based measurement techniques, hence, using multi-hop measured distance as the estimated distance between pairwise nodes will inevitably introduce substantial cumulative errors, especially for the pairwise nodes with a large number of hop counts. Furthermore, we also find a serious issue in irregular networks, such as the one shown in Fig. 1(b), where nodes in the network are frequently squeezed into narrow and long areas by obstacles. In the case of poor nodes' geometry distribution, the possibility of anchor nodes' collinearity is significantly increased, which will directly lead to an inaccurate positioning service.

In this paper, we propose a high-performance node positioning algorithm based on the nearest reliable anchors, called NRAP, to eliminate issues caused by irregular network topologies in multi-hop networks. NRAP excludes those measured or estimated distances with significant accumulated errors or outlier errors from the positioning process. Simultaneously, a position estimation algorithm with region constraints is designed to mitigate the impact of the anchor nodes' collinearity issue. The major contributions of this work can be summarized as follows.

- We model and derive an accuracy per-hop distance estimation function on the minimum error criterion that can be used to characterize the measured distances between adjacent nodes. This function can assist in calculating the

measured distance between multi-hop nodes while exchanging hop-by-hop packets to construct a positioning topology network.
- We introduce a constrained region construction function based on the four nearest anchor nodes to make the routing path between the source and destination node appropriate to a straight line. The constrained region has two potential advantages. One advantage is that the constrained region is convex, so the measured distance between a normal node and the four nearest anchors is less affected by the obstacles. Two, these constrained regions only involve the local network, implying that communication is highly efficient.
- We employ a hybrid strategy that uses natural selection to improve the performance of the original particle swarm optimization (PSO) algorithm [13, 14]. Furthermore, the hybrid PSO is used to search for the positions of nodes to be positioned by introducing a constrained region based on the four nearest anchor nodes, which helps to mitigate the effects of the collinearity issue.

The rest of the paper is structured as follows. Section 2 reviews related work about hop-based positioning algorithms. Section 3 presents the network model and discusses positioning. Section 4 provides a thorough explanation of our proposed NRAP algorithm. Section 5 displays the evaluated results, and Section 6 concludes.

## 2 Related Work

In the early 2000s, Niculescu and Nath first proposed the famous hop-based positioning algorithm based on the distance vector routing protocol, named DV-hop [15], for wireless multi-hop networks. Firstly, DV-hop computes the least hop counts between two nodes in network by using the Bellman-Ford algorithm [16], and estimates the average per-hop distance between anchor nodes based on the distance-vector protocol [11, 12]. Secondly, the estimated distances are calculated by multiplying the average per-hop distance and the hop counts between the normal nodes and the anchor nodes. Finally, the position of a normal node can be determined by using multilateration on multiple anchor nodes (at least 3 or 4 anchor nodes in the 2-dimensional or 3-dimensional scene, respectively). Although the DV-hop is simple and inexpensive in network deployment, it could only provide an accurate position estimate when the nodes were distributed in regular networks. As described in the introduction section, if nodes are deployed in irregular networks, the accuracy of DV-hop suffers greatly in overall positioning errors when the routing path was not close to straight lines. Moreover, the multilateration-based method may produce incorrect positioning results because of the potential of the anchor nodes' collinearity issue [17, 18].

In [19], the PSO-DV-hop algorithm was developed and applied to cases in which anchor nodes were collinear. The main idea of the PSO-DV-hop algorithm is to determine the position of the normal node using the PSO [13, 14] rather than the multilateral measuring approach. The typical PSO, originally developed by simulating the feeding activity of a flock of birds, is a stochastic search algorithm based on group collaboration. It generally initiates with a population of random particles (random solutions), and a near-optimal solution could be found through a finite number of iterations. In every round of iteration, each particle updates itself by tracking two "extreme values." The first extreme value was the particle's ideal solution, known as the individual extremum $\boldsymbol{p}_{best}$ ; the second was the population's optimal solution, known as the global extremum $\boldsymbol{g}_{best}$ . Alternatively, rather than the entire population, it is possible to use only the neighbors of some of the ideal particles, in which case the local extremum was the extremum among all neighbors. We could see that the PSO search range is predetermined. Thus, the PSO solution must be within this range; otherwise, the PSO solution would be affected by the collinearity of anchor nodes. PSO, on the other hand, is merely a solution method with no mechanism for filtering out distance measurement errors; therefore, PSO-DV-hop is not immune to outlier distance estimate errors caused by irregular networks, which makes PSO-DV-hop a low solution accuracy for the irregular networks.

Other approaches mainly select the appropriate distance estimations from anchor nodes before calculating node positions in an irregular network. In [20], Shang *et al.* proposed using only the four nearest anchor nodes instead of all in the positioning process. The motivation was that the region made up of these five nodes might be intuitively less affected by obstacles, so the router paths were close to straight lines between them, and the corresponding measured distances would contain fewer outlier errors. To facilitate our presentation, we denote this algorithm as the Nearest-4 algorithm. However, the Nearest-4 algorithm was also unable to objectively immune to the degradation of positioning performance caused by the collinearity issue. Recently, an improved DV-hop algorithm called DV-maxHop has been introduced in[21]. DV-maxHop adopts a hop count threshold to limit the range of data exchange between pair-wise nodes. Specifically, it employed a multi-objective optimization approach with the position estimation error and number of packets sent as constraints to obtain a hop count range threshold. Because during the positioning process a node to be positioned only needs to use the anchor nodes within the maximum hop range, the positioning process is simply and practically. However, the positioning process still based on the multilateration approach, making DV-maxHop unable to avoid collinearity issues. A follow-up improved algorithm called OW DV-hop [10] had also been proposed for multi-hop irregular networks. The problem of normal nodes are generally located far away from anchor nodes was considered. OW DV-hop adopts an optimal weight function to eliminate the adverse effects of accumulated errors, but in irregular networks, but the collinearity issue is still exist.

We can find that previous works, including [21] and [10], lack tailored mechanisms to mitigate the impact of topology irregularities. For this reason, we propose NRAP, which is, to the best of our knowledge, the first wireless positioning algorithm for irregular networks based on the nearest reliable anchor nodes.

# 3 Network Model and Problem Statement

Consider that there are $n$ wireless nodes uniformly and randomly distributed in a plane, with the first $m$ nodes knowing their position (*i.e.*, anchor nodes). Given that there are obstacles in the node distribution region, some data exchange paths must go around obstacles and cannot be close to a straight line. Let $\boldsymbol{p}_a = (x_a, y_a), a = 1, \cdots, m$ be the known coordinates of the $a$-th anchor node and $\boldsymbol{p}_t = (x_t, y_t), t = m+1, \cdots, n$ be the position of the node to be positioned (also called normal node). We assumed that all nodes in the network are equipped with isotropic antennas and have the same communication range $r$, allowing them to communicate directly with any node within their transmission range. If the distance between the source and the destination node is less than or equal to $r$, they can interact directly; if else, data packets will be delivered in a multi-hop manner.

A multi-hop positioning algorithm generally consists of two phases: the distance measurement phase and the position determination phase. After the network initialization, node $t$ receives a packet flooded from an anchor node $a$, which contains the position and identification (ID) of the anchor node $a$, as well as the hop count from $a$ to $t$. Then, the measured distance between any pair of nodes is often inferred by approximating the length of the data exchange path from the average per-hop distance, which can be sampled by anchor nodes. After the average per-hop distance is available, the length of the data exchange path from node $t$ to the $a$-th anchor can be measured by

$$\hat{d}_{t\to a} = h_{t\to a} \times ph_a, \tag{1}$$

where $\hat{d}_{t\to a}$ and $h_{t\to a}$ respectively denote the measured distance and the number of hops of the router path from node $t$ to $a$-th anchor node, and $ph_a$ represents the average distance per-hop to the $a$-th anchor.

Node $t$ can keep track of the current number of anchor nodes in the network by maintaining a counter. The event of the counter exceeding a particular value will trigger the position determination phase to estimate the position of node $t$. Given that distance measurement errors are inevitable, to alleviate these errors, the position determination phase needs to minimize the squared error between the known relationship and measured distances from the node $t$ to all available anchor nodes, *i.e.*,

$$\hat{\boldsymbol{p}}_t = \arg\min_{\boldsymbol{p}_t} \sum \left( |\boldsymbol{p}_t - \boldsymbol{p}_a| - \hat{d}_{t\to a} \right)^2, \tag{2}$$

where $\hat{\boldsymbol{p}}_t$ is node $t$'s position to be estimated, $|\boldsymbol{p}_t - \boldsymbol{p}_a|$ is the Euclidean distance from $t$ to $a$.

However, it should be noted that since Eq. 2 requires the measured distance $\hat{d}_{t\to a}$ to be close to the Euclidean distance from $t$ to $a$, the basic positioning approaches may fail due to non-straight-line data packets routing paths and poor nodes' geometry distribution.

As we pointed out before, the irregular networks and nodes' geometry distribution may have a significant impact on the distance measurement of multi-hop-based positioning approaches. In this paper, we aim to design a high-performance positioning algorithm for irregular multi-hop networks, which can determine the positions of the nodes to be positioned based on the anchors and the pairwise distance from nodes to anchors. Therefore, our proposed positioning model can be defined as

$$\hat{\boldsymbol{p}}_t = \arg\min_{\boldsymbol{p}_t} \sum \left( |\boldsymbol{p}_t - \boldsymbol{p}_a| - \hat{d}_{t\to a} \right)^2, \tag{3}$$

$$subject\ to \quad \hat{d}_{t\to a} \in [R_L, R_U], \tag{3a}$$

where $[R_L, R_U]$ respectively denotes the measurement distances' ceiling and floor range.

# 4 NRAP Algorithm

NRAP firstly attempts to model and derive an accuracy per-hop distance formulation based on a minimum error criterion. Then NRAP constructs a restricted region containing nodes to be positioned inspired by the data exchange path from the positioning node to the four nearest anchor nodes, which may be less affected by irregular networks. And finally, NRAP adopts a hybrid PSO algorithm to search for the optimal estimated position.

## 4.1 Model of quantifying the accurate per-hop distance and hop counts in irregular networks

It is critical to quantify the relationship between measured distances and hop counts. The measured distance is the path length from a regular node to an anchor node with the fewest hop counts among all such router paths. Obviously, if any pair of nodes are directly connected, the number of hops between them is just one; otherwise, the number of relay nodes in their routing path plus one is the number of hops between them.

Given that nodes are deployed into the implementation region in the same manner, thus normal and anchor nodes have the same distribution character in a single network. In other words, the average per-hop distance between the anchor nodes can be used to characterize the measured distance between any pair of nodes. Assuming that any anchor node $a$ is selected out from the entire anchor node group, the corresponding Euclidean distance to any other anchor node $i$ is

$$d_{a\to i} = \sqrt{(x_a - x_i)^2 + (y_a - y_i)^2}. \tag{4}$$

After collecting hop counts from the remaining $m$-1 anchor nodes on the anchor node $a$, we can get the functional relationship

$$f=\sqrt{\frac{\sum_{i=1,a\neq i}^{M}\left(d_{a\rightarrow i}-ph_a\times h_{a\rightarrow i}\right)^2}{M-1}},\tag{5}$$

where $ph_a$ is the average forward distance per hop from anchor $a$ to anchor $i$ and $h_{a\rightarrow i}$ is the number of hops from $a$ to $i$.

Thus, per-hop distance formulation can be obtained by minimizing the mean square errors (Eq. 5). Set the partial derivative of Eq. 5 to be 0, we can get equations

$$ph_a=\begin{cases}\frac{\sum_{a\neq i}d_{a\leftrightarrow i}h_{a\leftrightarrow i}}{\sum_{a\neq i}h_{a\leftrightarrow i}^2}, & f\neq 0;\\ \frac{d_{a\leftrightarrow i}}{h_{a\leftrightarrow i}}, & f=0.\end{cases}\tag{6}$$

In general, the likelihood of the Euclidean distance between nodes matching the measured distance is extremely low. In other words, $f=0$ is extremely rare. As a result, in practice, Eq. 6 should be expressed in a condensed form as

$$ph_a=\frac{\sum_{a\neq i}d_{a\leftrightarrow i}h_{a\leftrightarrow i}}{\sum_{a\neq i}h_{a\leftrightarrow i}^2}.\tag{7}$$

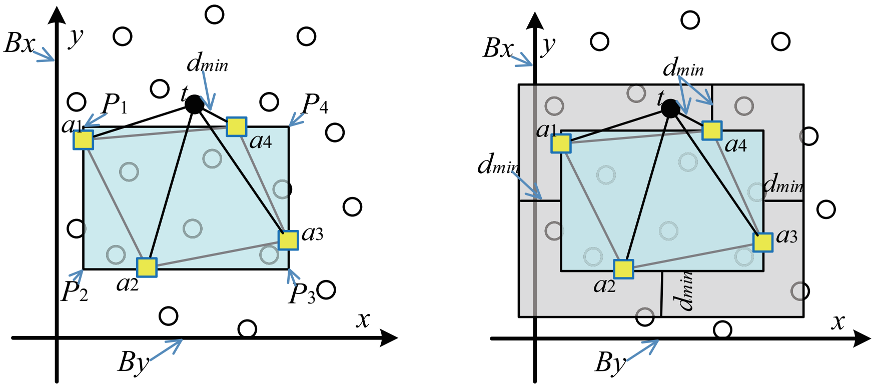
## 4.2 Constraint Region Selection

As previously stated, irregular network topologies have a significant impact on the distance measurement of multi-hop approaches. This is because the exchange paths may be distorted by obstacles and thus cannot be close to a straight line. The previous study [20] has pointed out that a normal node can only use the four nearest anchors to determine its position, rather than all of them. In light of this idea, we propose to construct a restricted region with the distance between each normal node to be positioned and its four nearest anchor nodes, as well as the positions of these anchor nodes.

Fig. 2 shows an example of the constraint region selection with the normal node $t$ and its four nearest anchor nodes $a1$, $a2$, $a3$, and $a4$. The distance between node $t$ and the four nearest anchor nodes can be expressed as

$$\begin{cases}\hat{d}_{t\leftrightarrow a1}=ph_{a1}\times h_{t\leftrightarrow a1},\\ \hat{d}_{t\leftrightarrow a2}=ph_{a2}\times h_{t\leftrightarrow a2},\\ \hat{d}_{t\leftrightarrow a3}=ph_{a3}\times h_{t\leftrightarrow a3},\\ \hat{d}_{t\leftrightarrow a4}=ph_{a4}\times h_{t\leftrightarrow a4},\end{cases}\tag{8}$$

where $ph_{a1}$, $ph_{a2}$, $ph_{a3}$, $ph_{a4}$ denote the average per-hop distance of the four nearest anchors, and $h_{t\leftrightarrow a1}$, $h_{t\leftrightarrow a2}$, $h_{t\leftrightarrow a3}$, $h_{t\leftrightarrow a4}$ denote the number of hops from node $t$ to the four nearest anchor nodes.




(a) Constraint Sub-region

(b) Constraint Region

**Fig. 2**: Constraint region selection.

To facilitate our observation, we first construct a region (Fig.2(a)) based on the four nearest anchors, the vertices of which are positioned as follows.

$$\begin{cases} P_1 = & [min\left(x_{ai}\right), max\left(y_{ai}\right)], \\ P_2 = & [min\left(x_{ai}\right), min\left(y_{ai}\right)], \\ P_3 = & [max\left(x_{ai}\right), min\left(y_{ai}\right)], \\ P_4 = & [max\left(x_{ai}\right), max\left(y_{ai}\right)], \end{cases} \quad \forall i \in [1,2,3,4], \tag{9}$$

where $(x_{ai}, y_{ai})$ are the coordinates of the four anchors, $min()$ and $max()$ represent the minimum and maximum functions. It should be noted that node $t$ may be inside or outside the region defined by Eq. 9, and that the distance from node $t$ to the region's specific boundary is less than or equal to the minimum value in Eq. 8. As shown in Fig. 2(b), to necessarily include node $t$, we expand the constraint region based on Eq. 9, while ensuring that location of node $t$ does not exceed the node distribution region. Therefore, we can get the following formula for the constraint region:

$$\begin{aligned} (x_t, y_t) \in & \left[min\left(max\left(x_{ai}\right) + d_{min}, Bx\right), max\left(min\left(x_{ai}\right) - d_{min}, Bx\right)\right] \\ & \times \left[min\left(max\left(y_{ai}\right) + d_{min}, By\right), max\left(min\left(y_{ai}\right) - d_{min}, By\right)\right], \end{aligned} \tag{10}$$

where $Bx$ and $By$ are the maxima of the x- and y-axis of the network boundary,respectively; and $d_{min}$ is the maximum measured distance from $t$ to the four anchor nodes. In fact, it is easy to notice that Eq.10 is a concrete representation of the bounded range constraint of Eq.3$a$.

### 4.3 Position Determination based on Hybrid PSO

Most algorithms utilize multilateration or the maximum likelihood estimation method to determine the positions of normal nodes. Although they have good computing performance, they are more sensitive to the measured error and node geometry, which results in estimation results that are far from the true position.

To solve the above problem, we combine the PSO with natural selection to search for the optimal estimated position of the node to be positioned. Combining the measured distance from normal nodes to anchor nodes and the four nearest anchors in the constraints region, hybrid PSO is used to replace the multilateration or the maximum likelihood estimation method to search for the positioning process of each normal node. Thus, the collinearity issues are constrained, and the computational overheads are decreased. In this paper, the fitness function of hybrid PSO adopts the distance estimation error function, i.e., Eq. 3. By combining PSO with natural selection, we can obtain an optimal estimated position of a normal node with iterative calculation, which is composed of the following steps.

1. Initialize the position $\boldsymbol{P} = (\boldsymbol{p}_1, \cdots, \boldsymbol{p}_K)$ and velocity $\boldsymbol{V} = (\boldsymbol{v}_1, \cdots, \boldsymbol{v}_K)$ of $K$ particles with a pseudo-random generator in the constraints region (obtained by Eq. 9) of possible occurrences of the node to be positioned.
2. Store the initialized position of each particle in the variable $\boldsymbol{p}_{best}$, and select the optimal individual position by the minimum fitness value to store in $\boldsymbol{g}_{best}$, which comes from the fitness function of all $\boldsymbol{p}_{best}$, *i.e.*,
$$\boldsymbol{P}_{best} = \boldsymbol{P}, \tag{11}$$
and
$$\boldsymbol{g}_{best} = \boldsymbol{p}_{index} | \left[ f_{min}, index \right] = min \left( fitness \left( \boldsymbol{P} \right) \right), \tag{12}$$
where $f_{min}$ is the minimum fitness function value for the $K$ particles, and *index* represents the index of the corresponding particle.
3. Update the velocity and position of each particle as
$$\begin{cases} \boldsymbol{v}_i^{t+1} = \boldsymbol{v}_i^t + c_1 r_1 \left( \boldsymbol{p}_{best\,i}^{\ \ t} - \boldsymbol{p}_i^t \right) + c_2 r_2 \left( \boldsymbol{g}_{best}{}^t - \boldsymbol{p}_i^t \right), \\ \boldsymbol{p}_i^{t+1} = \boldsymbol{p}_i^t + \boldsymbol{v}_i^{t+1}, \end{cases} \tag{13}$$
where the subscript $i$ denotes the particle's index, superscript $t$ is the current iteration's number, and $\boldsymbol{p}_i^t$ is the $t$-th iteration position of the $i$-th particle, $\boldsymbol{v}$ is the velocity vector, $r_1$ and $r_2$ are uniformly distributed within the range [0,1], and $c_1$ and $c_2$ are the "learning factors", which are positive constants.
4. Compare the fitness value of any particle to the best position it has experienced, and if the former is greater than the latter, take the former as the current best position.
5. Compare all current $\boldsymbol{p}_{best}$ and $\boldsymbol{g}_{best}$ values and update $\boldsymbol{g}_{best}$.

6. Sort the entire swarm of particles by fitness value, replacing the position and velocity of the worst half with the position and velocity of the best half of the population, while keeping the $\boldsymbol{p}_{best}$ and $\boldsymbol{g}_{best}$ unchanged.
7. Stop the search process when the iteration conditions are met and output the estimated results.

To sum up, the overall procedure of NRAP in determining normal nodes' optimal position is exhibited in Algorithm 1.

**Algorithm 1** **N**earest **R**eliable **A**nchors-based wireless **P**ositioning (**NRAP**).

**Input:** $\boldsymbol{p}_a = (x_a, y_a)$, $a = 1, \cdots, m$: the position of anchors. r: the communication radius of all nodes.

**Output:** $\hat{\boldsymbol{p}}_t = (\hat{x}_t, \hat{y}_t)$, $t = m+1, \cdots, n$: the estimated optimal position of the nodes to be positioned.

1: **Per-hop distance derivation**. To begin with, NRAP employs the Bellman-Ford algorithm to ensure that all nodes in the irregular network get router paths. Then, anchors cooperatively calculate the per-hop distance by Eq. 7 in a distributed manner.

2: **Constraint region selection**. Any node to be positioned calculates its measured distance from it to the nearest four anchor nodes by Eq. 8. Then, the constraint region is constructed based on the corresponding measured distance and the positions of the four anchors using Eq. 10.

3: **Position determination**. Any node to be positioned employs the proposed hybrid PSO to search for its optimal estimated position within a constraint region.

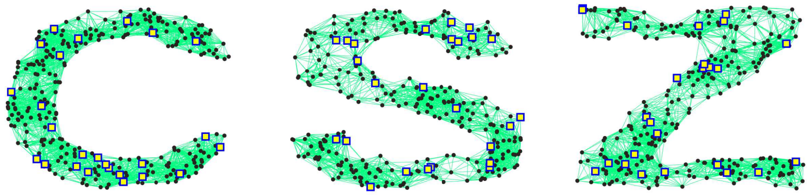
4: **return** $\hat{\boldsymbol{p}}_t = (\hat{x}_t, \hat{y}_t)$.

# 5 Performance Evaluation

In this section, we designed a series of simulations to make quantitative comparisons of the proposed NRAP with the other three state-of-the-art algorithms: (1) PSO-DV-Hop presented in [19]; (2) Nearest-4 proposed in [20]; and (3) DV-maxHop presented in [21]. We ran simulations with the Matlab platform. In the simulations, we focus on the algorithm's complexity, the adaptability to irregular networks, and the influence of the number of anchor nodes on the positioning accuracy.

Note that the letter-shaped network topology [7, 8, 21–24] is most commonly employed to compare and verify the localization performance of irregular networks. We adopt similar network topologies in the simulation. Fig. 3 depicts three irregular networks: C-, S-, and Z-shaped networks.

In Fig. 3, 400 nodes are randomly distributed in these letter-shapped irregular networks, where 25 squares representing anchor nodes, 375 solid circles representing normal nodes to be positioned, and green link lines between two



(a) C-shaped network (b) S-shaped network (c) Z-shaped network

**Fig. 3**: The letter-shaped irregular networks.

**Table 1**: Simulation parameters

| Parameters | Value |
|---|---|
| Nodes distribution region size | $300 \times 300$ ($m^2$) |
| Radius | 43 ($m$) |
| Network shaped | C-,S-,Z-shaped |
| Nodes number | 300 |
| Anchor nodes number | 25 or 15 to 30 by interval 3 |
| Signal Propagation Model | Regular model |
| Number of particles | 20 |
| Maximum iterations | 20 |
| Learning factors $c_1$, $c_2$ | $c_1 = c_2 = 2.05$ |
| Maximum velocity | 2 |

nodes representing they Euclidean distance is less than the communication radius.

Some of the main simulation parameters related to the operation of four algorithms are listed in Table 1. And, to quantitatively analyze the estimation errors of NRAP and the remaining three benchmark positioning algorithms, we consider employing the root mean square ($RMS$) as the metric to evaluate their performance, $RMS$ is the value of the arithmetic mean of the squares of the location estimated errors, and thus $RMS$ can be defined as

$$RMS = \sqrt{\frac{\sum_{t=m+1}^{n} \left( (\hat{x}_t - x_t)^2 + (\hat{y}_t - y_t)^2 \right)}{N - M}}, \tag{14}$$

where $(\hat{x}_t, \hat{y}_t)$ and $(x_t, y_t)$ are the estimated and real positions of the node $t$ to be positioned, respectively; $N$-$M$ is the number of nodes to be located and $m$ is the number of anchor nodes.

## 5.1 Complexity Analysis

In all four algorithms, each node uses the Bellman-Ford algorithm to determine the routing path (least hop count) to other nodes in a flooding manner, thus their communication costs are $\mathcal{O}\left(n^2\right)$, where $n$ is the number of nodes in the network. PSO-DV-Hop for position estimation using PSO search methods based on DV-hop; Nearest-4 calculated positions based on the four closest anchor nodes; DV-maxHop, on the other hand, uses multi-objective optimization techniques [25] to obtain the range of possible normal node occurrences with energy consumption as a constraint, and divides the network into multiple sub-networks. Then, each normal node uses a few anchors in the sub-network to achieve position estimation. Thus, the computational complexity of Nearest-4, PSO-DV-Hop, and DV-maxHop is approximately $\mathcal{O}\left(n \cdot 4^3\right)$, $\mathcal{O}\left(n \cdot N_{iter} \cdot m \cdot N_p\right)$, and $\mathcal{O}\left(K^2 \cdot A\right) + \mathcal{O}\left(n\right)$, where $N_{iter}$ is iterations of the PSO; $N_p$ is the number of populations; $A$ is the number of solutions in the archive, $m$ is the number of anchor nodes, and $K$ is the number of objectives.

The proposed NRAP algorithm combines the benefits of Nearest-4 and PSO-DV-Hop by enclosing the nodes to be located into the corresponding constraint region with the nearest four anchor nodes, resulting in a computation time of around $\mathcal{O}\left(n \cdot N_{iter} \cdot 4 \cdot N_p\right)$. Table 2 compares the complexity of NRAP with the three state-of-the-art multi-hop algorithms.

**Table 2**: Complexity Comparison

| Algorithm | Communication | Computing |
|---|---|---|
| Nearest-4 [20] | $\mathcal{O}\left(n^2\right)$ | $\mathcal{O}\left(n \cdot 4^3\right)$ |
| PSO-DV-Hop [19] | $\mathcal{O}\left(n^2\right)$ | $\mathcal{O}\left(n \cdot N_{iter} \cdot m \cdot N_p\right)$ |
| DV-maxHop [21] | $\mathcal{O}\left(n^2\right)$ | $\mathcal{O}\left(K^2 A\right) + \mathcal{O}\left(n\right)$ |
| NRAP | $\mathcal{O}\left(n^2\right)$ | $\mathcal{O}\left(n \cdot N_{iter} \cdot 4 \cdot N_p\right)$ |

## 5.2 Impact of Irregular Networks

In this subsection, we compare the positioning results of the proposed algorithm and the other three algorithms, taking the nodes distribution in Fig. 3 as a sample. The comparison result is shown in Fig. 4, where "+" indicates the estimated position of the normal node by the positioning algorithm, and the line connects the true position of the node and its estimation, with longer lines indicating larger estimation errors and vice versa.

As shown in Fig. 4, the proposed NRAP outperforms other algorithms for various irregular networks in terms of positioning accuracy. According to the quantitative value RMS comparison of the final result, NRAP improves positioning accuracy over PSO-DV-Hop, Nearest 4, and DV-maxHop in the C-shape network by 135.35%, 47.64%, and 63.52%. The RMS boost rates for the S-and Z-shape networks are 79.74%, 68.18%, 37.16% and 57.04%, 146.62%, 62.01%, respectively.

### 5.3 Impact of Number of Anchor Nodes

In this subsection, we investigated the impact of the number of anchor nodes on the localization accuracy for C-, S-, and Z-shaped networks. We varied the number of anchor nodes from 15 to 50 with an interval of 3, and all results were recorded for 90 trials by using Monte-Carlo simulations. Fig. 5 shows the distribution of recorded RMS obtained by Nearest-4, PSO-DV-hop, DV-maxHop, and the proposed NRAP using the boxplot for different numbers of anchor nodes.

Table 3 to Table 5 compare the statistical parameters of four algorithms for various anchor numbers under different irregular networks. From the tables, it is apparent that the proposed NRAP outperforms the previous methods not only in the median but also in the best, worst, and distribution range (stability) for different anchor numbers and irregular networks.

**Table 3**: Positioning accuracy of different algorithms with different numbers of anchor nodes

| C-shaped | Median | | Variance | | Inter-quartile range | |
|---|---|---|---|---|---|---|
| | MAX/AN[1] | MIN/AN[1] | MAX/AN | MIN/AN | MAX/AN | MIN/AN |
| Nearest-4[20] | 52.08/15 | 36.07/30 | 473.14/15 | 206.21/24 | 31.11/15 | 14.14/27 |
| PSO-DV-hop[19] | 57.93/21 | 53.91/30 | 204.30/15 | 85.83/30 | 20.12/15 | 12.39/27 |
| DV-maxHop[21] | 47.46/15 | 31.57/30 | 66.03/24 | 53.05/21 | 13.41/24 | 7.55/15 |
| NRAP | 32.94/15 | 23.27/30 | 35.77/15 | 9.56/30 | 7.71/15 | 3.40/30 |

**Table 4**: Positioning accuracy of different algorithms with different numbers of anchor nodes

| S-shaped | Median | | Variance | | Inter-quartile range | |
|---|---|---|---|---|---|---|
| | MAX/AN[1] | MIN/AN[1] | MAX/AN | MIN/AN | MAX/AN | MIN/AN |
| Nearest-4[20] | 62.229/15 | 48.42/30 | 545.27/15 | 279.65/21 | 23.89/21 | 19.77/30 |
| PSO-DV-hop[19] | 66.36/15 | 57.75/24 | 189.40/18 | 51.67/30 | 16.29/18 | 8.24/30 |
| DV-maxHop[21] | 60.57/15 | 40.72/30 | 256.72/15 | 87.49/30 | 20.12/15 | 13.26/30 |
| NRAP | 46.00/15 | 28.70/30 | 38.04/15 | 15.53/30 | 9.75/15 | 4.82/30 |

[1]The MAX/AN and MIN/AN respectively denote the maximum and minimum values when the number of anchor nodes is AN.

**Table 5**: Positioning accuracy of different algorithms with different numbers of anchor nodes

| Z-shaped | Median | | Variance | | Inter-quartile range | |
|---|---|---|---|---|---|---|
| | MAX/AN[1] | MIN/AN[1] | MAX/AN | MIN/AN | MAX/AN | MIN/AN |
| Nearest-4[20] | 45.71/15 | 32.73/30 | 340.94/15 | 127.36/21 | 18.67/30 | 11.29/30 |
| PSO-DV-hop[19] | 52.05/15 | 46.75/30 | 257.56/15 | 90.46/30 | 17.02/15 | 10.22/24 |
| DV-maxHop[21] | 42.88/15 | 27.74/30 | 80.69/15 | 21.64/30 | 12.03/15 | 5.58/30 |
| NRAP | 32.51/15 | 23.48/30 | 41.26/15 | 7.07/30 | 9.02/15 | 3.01/30 |

# 6 Conclusions

In this paper, a novel wireless hop-based range-free positioning algorithm, named NRAP, is proposed for irregular multi-hop networks. To mitigate the impact of the network topology irregularities issues, a more accurate per-hop distance estimation model is proposed, and only four nearest reliable anchor nodes are selected in NRAP to determine the position of a normal node. Moreover, a hybrid PSO is designed in NRAP to alleviate the effects of the anchor nodes' collinearity issue. Theoretical analysis shows the distributed solution process of NRAP is completed efficiently and quickly. Moreover, the performance of NRAP has been compared to pre-existing algorithms in various conditions, such as various irregular networks and different numbers of anchor nodes. Experimental results further show that NRAP outperforms the existing algorithms in terms of algorithmic complexity and positioning accuracy.

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**Data availability** The datasets generated during the current study are available from the corresponding author.

**Code availability** The custom code is available from the corresponding author.

# Declarations

**Confict of interest** The authors have not disclosed any competing interests.

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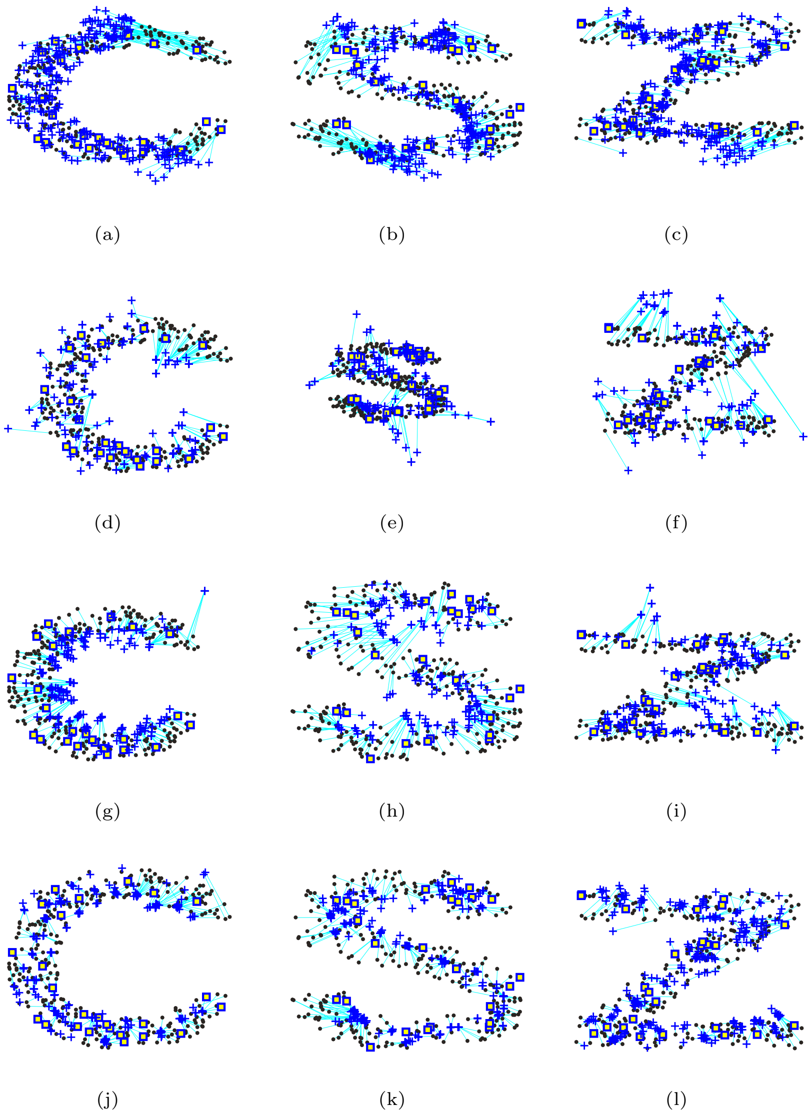
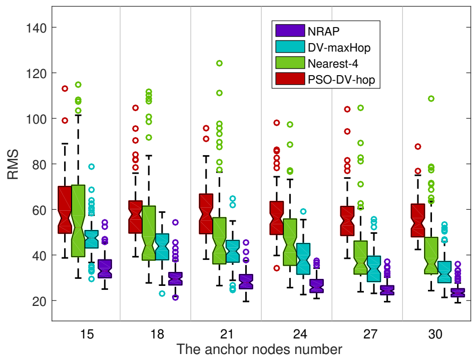
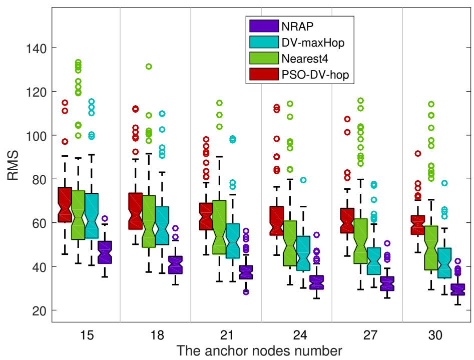


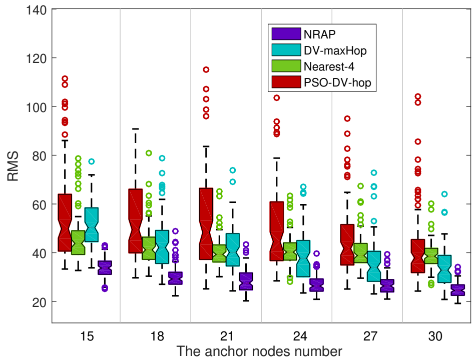

**Fig. 4**: Positioning results of four algorithms in different irregular networks. (a) PSO-DV-Hop, RMS=58.39. (b) PSO-DV-Hop, RMS=56.15. (c) PSO-DV-Hop, RMS=40.39. (d) Nearest-4, RMS= 36.63. (e)Nearest-4, RMS=52.54. (f) Nearest-4, RMS=63.43. (g) DV-maxHop, RMS=40.57. (h) DV-maxHop, RMS=42.85. (i) DV-maxHop, RMS=41.67. (j) NRAP, RMS=24.81. (k) NRAP, RMS=31.24. (l) NRAP, RMS=25.72.




(a) C-shaped network.




(b) S-shaped network.




(c) Z-shaped network.

**Fig. 5**: Simulation Results of four algorithms with different numbers of anchors for various irregular networks.