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Cruise itineraries optimal scheduling

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Abstract

A cruise company faces three decision problems: at a *strategic level*, to decide in which maritime area and in which season window to locate each ship of its fleet; at a *tactical level*, given a ship in a maritime area and in a season window, to decide which cruises to offer to the customers; at an *operational level*, to determine the day-by-day itinerary, in terms of transit ports, arrival and departure times and so on. This paper focuses on the tactical level, namely on the Cruise Itineraries Optimal Scheduling (CIOS), aiming at determining a scheduling of cruises with the objective to maximize the revenue provided by a given ship placed in a specified maritime area, in a selected season window, taking into account a number of constraints. In particular, we refer to luxury cruises, implying several additional considerations to be taken into account. We propose an Integer Linear Programming (ILP) model for such a CIOS problem. This model has been experimented by a major luxury cruise company to schedule the itineraries of its fleet in many geographical areas all over the world. A commercial solver has been used to solve the ILP problem. Here we report, as illustrative examples, the results obtained on some of these real instances to show the computational viability of the proposed approach.

Keywords Cruises optimal scheduling · Integer Linear Programming model

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1 Introduction

The annual reports issued by CLIA (Cruise Lines International Association) show how the cruising industry has become the fastest growing sector of leisure and tourist business. In particular the 2019 Cruise Industry Outlook (see [5]) points out that the expected number of passengers in 2019 amounts to the order of 30 millions, with an increase of 69% in the last 10 years. Also in terms of capacity, an extraordinary increase is recorded, because the companies make multimillion investments to have more innovative and ever-bigger ships. While at December 2015, CLIA's fleet was composed of 448 ships, between 2016 and 2020, 65 new ships will sail (for more details on the trends of the sector, see [6]).

As regards the current challenges for cruises management, in [16] the most important ones are surveyed. In particular, in the cruising industry business, ships deployment strategies and itineraries planning play a fundamental role. They are usually designed according to economic and operational considerations, but they are also strongly affected by the target market. Hence, the cruise companies can be divided into two broad classes: the mass market class and the luxury market class. The mass market cruise companies usually offer a catalogue of cruises where a same basic itinerary is repeated many times, often week-by-week, aiming at satisfying customers at a first, and probably unique, cruise experience, not requiring so much in terms of originality of the itinerary. On the contrary, a luxury market company usually deals with returning customers, which require new itineraries not previously experienced. Therefore a luxury market cruise company is compelled to publish yearly a catalogue of cruises different among them and from the year before. The different demands between first-time cruisers and more experienced cruisers are clearly stated in [3] and in the references reported therein.

In its decision making process, a cruise company faces the following three levels of management decisions (see, e.g. [15]):

- at a *strategic level*, the deployment of the fleet, that is to decide in which maritime area and in which season window to locate each ship of its fleet;
- at a *tactical level*, given a ship in a maritime area, in a season window, to establish which cruises to offer to the customers, where each cruise is characterized by the embarkation port, the disembarkation port and cruise length (in days);
- at an *operational level*, given a cruise, namely embarkation port, disembarkation port and number of days, to determine the day-by-day itinerary, in terms of transit ports, arrival and departure times in transit ports, and so on.

The problem of the *Cruise Itineraries Optimal Scheduling* (CIOS) concerns the tactical level. It consists in determining the scheduling of the cruises offered to customers in a given maritime area in a determined season period, with the objective to maximize the company revenue, taking into account several constraints on the scheduling.

Although the cruise sector is of great and growing economical importance, it appears that quantitative models in the decision processes are up to now rarely adopted to tackle CIOS problem. Most papers on maritime transport regards the management of cargo ships (see e.g. [4,9]); other papers deal with the management of passenger ferries (see e.g. [17]). Only few papers deal with cruise ships management and they mainly focus on the Cruise Itinerary Design Problem (CIDP), namely the design of day-by-day itineraries, that is they only consider the problem at the operational level [2,11–13,18]. In particular, see the recent paper [2] and the references reported therein for an update literature review on the CIDP problem. Other papers are limited to economic, social and environmental issues (see e.g. [14]). We have not been able to find papers on the CIOS problem, i.e. which consider the tactical level, and it seems to be new in the Operations Research literature.

In this paper, we consider the CIOS problem referring to a luxury market cruise company and, as already observed, this implies several additional considerations to be taken into account, hence an increased difficulty. In particular, three different types of cruises must be considered:

– *standard cruise*: a cruise defined by an embarkation port, a disembarkation port and a duration;
– *cruise with a milestone*: a cruise that must depart and/or arrive in a specific port in a specific day of the season; its presence is usually due to some particular event that a cruise must meet in some port;
– *charter cruise*: a cruise with both embarkation port/departure day and disembarkation port/arrival day given. They are usually offered as corporate cruises.

The cruises with a milestone and the charter cruises are typically proposed only by luxury cruises companies, being usually very expensive.

We propose an Integer Linear Programming (ILP) model for this problem. We coded it by using AMPL language [8] and solved the resulting ILP problem by using the commercial solver GUROBI [10]. Several real instances have been solved in order to provide the cruise company with the itineraries schedule of its fleet in many geographical areas all over the world. It is important to note that the adoption of an exact solution approach, rather than the use of some metaheuristic, is allowed since the cruises scheduling is developed years in advance, before becoming operative, so that a long computing time (even of some hours) for an instance is admissible. Of course, if the computing time becomes too large, the run can be untimely truncated, getting an approximate solution along with the optimality gap that gives a measure of the quality of the current feasible solution.

This work has been developed within a joint project (named *Magellano Project*), between ACTOR SRL[1] and a major luxury cruise company (which we do not mention for the sake of privacy), aiming at providing cruise companies with a decision support system able to tackle all three levels of the decision making process. As regards the third level of the decision making process, the day-by-day itinerary optimization with the objective to minimize costs and to maximize some attractiveness index of the cruise, has been dealt with in the talk [7] and will be the subject of a subsequent paper.

The paper is organized as follows: in Sect. 2 the description of the CIOS is reported. In Sect. 3 we describe in detail the mathematical model developed, and in Sect. 4 we report some experimental results on real problem instances. Finally, some concluding remarks are drawn in Sect. 5.

[1] ACTOR SRL is a Spin-Off of SAPIENZA University of Rome (www.actorventure.com).

2 Problem description

The CIOS problem consist in selecting a subset of cruises among all the available cruises in a given maritime area, in a determined season period, aiming at maximizing the overall company revenue. Such revenue is estimated by using information on past records of cruises in the same maritime area and by the same or similar ships, as detailed in the sequel.

Now we report all the elements that characterize the CIOS problem. A ship, a maritime area, a season window (defined with a starting and ending date) and a set of *turnaround ports* of the area are given. These ports can be used as the embarkation or disembarkation port of a cruise and their set is denoted by $\mathcal{P}$. We highlight that turnaround ports have some specific characteristics so that not all ports presented in the maritime area con be considered. In particular, turnaround ports must be easily reachable by passengers, e.g. being close to an international airport, and they could also be attractive destinations where cruisers may spend a few days before or after enjoying the cruise. Turnaround port can also be transit port but, of course, these latter are of interest in the day-by-day itinerary planning and not in the CIOS problem.

The *first cruise* of the season must start at a given embarkation port $p^{first} \in \mathcal{P}$, where the ship must be docked at the beginning of the season; the *last cruise* must end at a given disembarkation port $p^{last} \in \mathcal{P}$, where the ship will be docked at the end of the season. In the sequel, for the sake of shortness we denote them p^{fi} and p^{la}, respectively.

The basic requests which must be taken into account in the selection of the cruises are the following:

- C1: the disembarkation port of a cruise is the embarkation port of the next cruise, except for the first (p^{fi}) and the last (p^{la}) port visited during the season;
- C2: in absence of milestones and charter cruises, disembarkation and embarkation in a port $p \in \mathcal{P}$ (with $p \neq p^{fi}$ and $p \neq p^{la}$) occur in the same day;
- C3: for each port $p \in \mathcal{P}$, the minimum and the maximum number of allowed visits of the port p is given. We denote them $nv_p^{\min}$ and $nv_p^{\max}$, respectively;
- C4: for each port $p \in \mathcal{P}$, the minimum number of days between two consecutive visit of the port p is given and denoted by $nd_p^{\min}$;
- C5: a set $\mathcal{L}$ of the cruise durations (in days) allowed is assigned and for each $\ell \in \mathcal{L}$ the minimum and the maximum number of cruises of duration ℓ which are allowed in the schedule is given. They are denoted by $nc_\ell^{\min}$ and $nc_\ell^{\max}$, respectively.

While constraints C1 and C2 are *basic constraints*, i.e. usual continuity constraints, constraints C3, C4 and C5 are relevant to luxury cruises. Indeed, they clearly aim at offering a catalogue of cruises differentiated enough among them. We call constraints C3–C5 *operational constraints*.

To complete the description of the notations used, we indicate the days of the season period as elements of the ordered set $\mathcal{D} = \{0, 1, 2, \ldots, N\}$ ($d = 0$ and $d = N$ are the first and the last day of the season, respectively). The cruises can start at any day $d \in \mathcal{D}$ and d^e and d^d denote the day of the embarkation and disembarkation of a cruise. Similarly, d^{fi} and d^{la} indicate the day of embarkation at p^{fi} and the day of disembarkation at p^{la}. This distinction is necessary since, both d^{fi} and d^{la} will be determined by the optimization process and, as described in the sequel, it could be $d^{fi} \neq 0$ and $d^{la} \neq N$.

Therefore, if we denote by $\mathcal{C}$ the set of all possible cruises that can be operated by the ship in the given area and in the assigned season segment, any cruise $C \in \mathcal{C}$ is characterized by the embarkation port $p^e \in \mathcal{P}$, the disembarkation port $p^d \in \mathcal{P}$, the day of embarkation d^e and the duration $\ell \in \mathcal{L}$. Hence, it results

$$\mathcal{C} = \{(p^e, p^d, d^e, \ell) \in \mathcal{P} \times \mathcal{P} \times \mathcal{D} \times \mathcal{L}\}$$

and each cruise is identified by a quadruple $C = (p^e, p^d, d^e, \ell)$.

In the case of *cruise with milestones*, we can have a milestone on embarkation, i.e. a request for a cruise which must embark from a prefixed port $\bar{p}^e$ on given day $\bar{d}^e$ and/or a milestone on disembarkation, i.e. a request for a cruise which must disembark in a prefixed port $\bar{p}^d$ on given day $\bar{d}^d$. We denote by $\mathcal{M}^e$ the set of the pairs $\mathcal{M}^e = \{(\bar{p}^e, \bar{d}^e) \in \mathcal{P} \times \mathcal{D}\}$ indicating milestones on embarkation and by $\mathcal{M}^d$ the set of the pairs $\mathcal{M}^d = \{(\bar{p}^d, \bar{d}^d) \in \mathcal{P} \times \mathcal{D}\}$ indicating milestones on disembarkation. A *charter cruise* is a cruise with embarkation and disembarkation ports, duration and embarkation day assigned. We denote by $\mathcal{C}^{ch}$ the set of the charter cruises, i.e. the set of particular quadruples $\mathcal{C}^{ch} = \{(\bar{p}^e, \bar{p}^d, \bar{d}^e, \bar{\ell}) \in \mathcal{C}\}$.

Finally, observe that the season segment could not be exactly covered by cruises of given durations $\ell \in \mathcal{L}$. Therefore we must allow for some days tolerance ndt^{fi} on the day d^{fi} of embarkation in p^{fi}, so that, actually $d^{fi} \in \{0, \dots, ndt^{fi}\}$. Similarly, some days tolerance ndt^{la} on the day d^{la} of disembarkation in p^{la} are allowed, so that $d^{la} \in \{(N - ndt^{la}), \dots, N\}$ (see also Fig. 1 in the sequel).

In the absence of milestones or charter cruises, a feasible solution for the CIOS problem, i.e. a *feasible cruise schedule*, is a sequence of cruises $\{C_i\}_{i \geq 1}$, with $C_i = (p_i^e, p_i^d, d_i^e, \ell_i) \in \mathcal{C}$, ordered according to d_i^e such that

(i) there exists

$$(p^{fi}, p^d, d^{fi}, \ell) \equiv (p_1^e, p_1^d, d_1^e, \ell_1) \in \mathcal{C}, \text{ with } d^{fi} \in \{0, \dots, ndt^{fi}\};$$

(ii) there exists $M \geq 2$ such that

$$\begin{aligned}&(p_M^e, p_M^d, d_M^e, \ell_M) \equiv (p_M^e, p^{la}, d^{la} - \ell_M, \ell_M) \in \mathcal{C},\\ &\quad \text{with } d^{la} \in \{(N - ndt^{la}), \dots, N\};\end{aligned}$$

(iii) for all $i = 1, \dots, M-1$,

$$\begin{aligned} p_{i+1}^e &= p_i^d \\ d_{i+1}^e &= d_i^e + \ell_i; \end{aligned} \tag{2.1}$$

(iv) it results

$$\sum_{i=1}^{M} \ell_i \leq N; \tag{2.2}$$

(v) the operational constraints C3, C4 and C5 are satisfied.

The cruise defined in item *(i)* is the first cruise (C_1) of the sequence. The cruise defined in item *(ii)* is the last cruise (C_M) of the sequence which embarks in the day $d^{la} - \ell_M$. Of course, the value of the integer M (the number of cruises scheduled in the season) is not known in advance, but it will be an output of the optimization process.

In the presence of milestones or charter cruises some days off in port, nd_i^{off}, can be expected between the end of cruise C_i and the beginning of the next one C_{i+1}, $i = 1, \dots M - 1$. So that, in this case, (2.1) and (2.2) are replaced by

$$d_{i+1}^e = d_i^e + \ell_i + nd_i^{off} \tag{2.3}$$

$$\sum_{i=1}^{M} (\ell_i + nd_i^{off}) \leq N. \tag{2.4}$$

The maximum value allowed for nd_i^{off}, $i = 1, \dots M - 1$, is given and indicated by $nd_{\max}^{off}$.

Finally, we denote by $\mathcal{S}$ the set of feasible cruise schedules, i.e. the set of sequences $S = \{C_1, \dots, C_M\}$ which are feasible solution for the CIOS problem.

For the sake of clarity, we summarize in Table 1, the fundamental elements of the CIOS problem up to now introduced.

Now we turn to the computation of the company revenue. For each cruise $C_i \in \mathcal{C}$ the net revenue $r(C_i)$ is given by the company. It is estimated from past records of cruises by the same or a similar ship deployed in the same maritime area. Forecasts on the cruise market trend can be also taken into account to possibly upgrade this revenue. Therefore, to any feasible schedule $S \in \mathcal{S}$ it is associated a total net revenue given by

$$R(S) = \sum_{\substack{1 \leq i \leq M \\ C_i \in S}} r(C_i). \tag{2.5}$$

Moreover, for any $S \in \mathcal{S}$, a payoff is due for the number of days of the schedule off in ports, i.e. for the not sailing days of the ship. This payoff can be computed as

$$PO(S) = w\Big(d^{fi} + (N - d^{la}) + \sum_{\substack{1 \leq i \leq M-1 \\ C_i \in S}} nd_i^{off}\Big), \tag{2.6}$$

where w is a suitable parameter which accounts for the loss of revenue due to the days of the schedule off in ports. Of course, $PO(S) \neq 0$ if there exist days tolerance at the beginning ($d^{fi} \neq 0$), at the end ($d^{la} \neq N$) of the schedule, or between the cruises C_i and C_{i+1} ($nd_i^{off} \neq 0$ for some $i = 1, \dots, M - 1$).

Now we can formally state the CIOS problem as the problem of determining $S^* \in \mathcal{S}$ which maximize the objective function $R(S) - PO(S)$ for all $S \in \mathcal{S}$, namely the problem

$$\begin{cases} \max R(S) - PO(S) \\ S \in \mathcal{S}. \end{cases} \tag{2.7}$$

Table 1 Summary of the elements which characterize a CIOS problem

$\mathcal{P}$	The set of turnaround ports
$p^e, p^d \in \mathcal{P}$	Embarkation/disembarkation port of a cruise
$p^{fi}, p^{la} \in \mathcal{P}$	Embarkation/disembarkation port of the first/last cruise of the season
$nv_p^{\min}, nv_p^{\max}$	Minimum/maximum number of allowed visits of port p
$nd_p^{\min}$	Minimum number of days between two consecutive visit of port p
$\mathcal{L}$	Set of allowed cruise durations
ℓ	An allowed cruise duration $\ell \in \mathcal{L}$
$nc_\ell^{\min}, nc_\ell^{\max}$	Minimum/maximum number of cruises of duration ℓ allowed
$\mathcal{D} = \{0, 1, 2, \ldots, N\}$	Ordered set of the days of the season
$d^e, d^d \in \mathcal{D}$	Embarkation/disembarkation day of a cruise
$d^{fi}, d^{la} \in \mathcal{D}$	Embarkation day at p^{fi}/disembarkation day at p^{la} of the first/last cruise of the season
ndt^{fi}	Number of days tolerance allowed on the day d^{fi} of embarkation in p^{fi}
ndt^{la}	Number of days tolerance allowed on the day d^{la} of disembarkation in p^{la}
nd_i^{off}	Number of days off in port between cruise C_i and C_{i+1}
$nd_{\max}^{off}$	Maximum value allowed for nd_i^{off}, $i = 1, \ldots M - 1$
$\mathcal{C}$	Set of all possible cruises in the given area
$C = (p^e, p^d, d^e, \ell)$	A cruise $C \in \mathcal{C}$
$\mathcal{M}^e$	Set of milestones on embarking
$\mathcal{M}^d$	Set of milestones on disembarking
$\mathcal{C}^{ch}$	Set of charter cruises
$\mathcal{S}$	Set of all feasible cruise schedules
$S = \{C_1, \ldots, C_M\}$	A feasible cruise schedule $S \in \mathcal{S}$

3 The mathematical model

In this section we describe the mathematical model we propose for solving the CIOS problem. First we report the problem data which constitute the data parameters of the model. Then we introduce the decision variables of the model.

3.1 The model parameters

The data parameters are:

- the sets $\mathcal{P}, \mathcal{C}, \mathcal{M}^e, \mathcal{M}^d, \mathcal{C}^{ch}, \mathcal{L}$;

- the ordered set $\mathcal{D} = \{0, \ldots, N\}$;
- the ports $p^{fi}, p^{la} \in \mathcal{P}$;
- for each $p \in \mathcal{P}$, the values $nv_p^{\min}, nv_p^{\max}, nd_p^{\min} \in \mathbb{N}$, with $nv_p^{\min} \leq nv_p^{\max} \leq N$ and $nd_p^{\min} \leq N$;
- for each $\ell \in \mathcal{L}$, the values $nc_\ell^{\min}, nc_\ell^{\max} \in \mathbb{N}$, with $nc_\ell^{\min} \leq nc_\ell^{\max}$;
- the number of days tolerance allowed $ndt^{fi}, ndt^{la} \in \mathbb{N}$, with $ndt^{fi} \leq N$ and $ndt^{la} \leq N$;
- the number of days off in port allowed $nd_{\max}^{off} \in \mathbb{N}$, with $nd_{\max}^{off} \leq N$;
- for each $C \in \mathcal{C}$ the value of the revenue $r(C) \in \mathbb{R}$;
- the parameter $w \in \mathbb{R}$.

All these parameters have been already described in Sect. 2. They are summarized in Table 1; the parameter w is introduced in (2.6) and its meaning is expounded immediately after.

In the model implementation we also need a technical parameter, denoted by $BigM$, adopted to allow binary variables to turn constraints on or off.

3.2 The decision variables

Preliminarily, for the sake of brevity of the notation, for each $p \in \mathcal{P}$, we introduce a set $\mathcal{V}(p) \subseteq \mathbb{N}$ which is the set of the number of allowed visits at the port p during the whole season, namely

$$\mathcal{V}(p) = \{n \in \mathbb{N} \mid nv_p^{\min} \leq n \leq nv_p^{\max}\}.$$

Now we can define the decision variables of the model we propose. First we introduce the following binary variables used for identifying the sequence of the cruises in the schedule:

$$x(p^e, p^d, \ell, np^e, np^d) = \begin{cases} 1 & \text{if a cruise of duration } \ell \text{ starting from port } p^e, \\ & \text{visited for the } np^e\text{-th time, and arriving to port} \\ & p^d, \text{ visited for the } np^d\text{-th time, is in the schedule} \\ 0 & \text{otherwise,} \end{cases}$$

where $p^e \in \mathcal{P}$, $p^d \in \mathcal{P}$, $\ell \in \mathcal{L}$ and $np^e \in \mathcal{V}(p^e)$, $np^d \in \mathcal{V}(p^d)$ are integer variables used to take into account the request on the minimum/maximum number allowed of visits of ports p^e and p^d, respectively.

Then we introduce other binary variables for coupling a cruise with its embarkation and disembarkation days:

$$y^e(p^e, p^d, \ell, np^e, np^d, d^e) = \begin{cases} 1 & \text{if a cruise of duration } \ell \text{ starting on day } d^e \\ & \text{from port } p^e, \text{ visited for the } np^e\text{-th time,} \\ & \text{arriving to port } p^d, \text{ visited for the } np^d\text{-th time,} \\ & \text{is in the schedule} \\ 0 & \text{otherwise,} \end{cases}$$

$$y^d(p^e, p^d, \ell, np^e, np^d, d^d) = \begin{cases} 1 & \text{if a cruise of duration } \ell \text{ ending on day } d^d \\ & \text{in port } p^d \text{, visited for the } np^d\text{-th time,} \\ & \text{originating from port } p^e\text{, visited for the } np^e\text{-th} \\ & \text{time, is in the schedule} \\ 0 & \text{otherwise,} \end{cases}$$

where $p^e \in \mathcal{P}$, $p^d \in \mathcal{P}$, $\ell \in \mathcal{L}$, $np^e \in \mathcal{V}(p^e)$ and $np^d \in \mathcal{V}(p^d)$. Of course, if it results $x(\bar{p}^e, \bar{p}^d, \bar{\ell}, \bar{np}^e, \bar{np}^d) = 1$, then both $y^e(\bar{p}^e, \bar{p}^d, \bar{\ell}, \bar{np}^e, \bar{np}^d, d^e) = 1$ and $y^d(\bar{p}^e, \bar{p}^d, \bar{\ell}, \bar{np}^e, \bar{np}^d, d^d) = 1$, for some $d^e, d^d \in \mathcal{D}$.

Then, in order to identify the embarkation day (from p^e) and the disembarkation day (in p^d) of a cruise, taking into account the number of times a cruise visits a port, we need to specify these days also on the basis of the counters np^e and np^d. Hence, we introduce the following integer variables:

$g^e(p^e, p^d, \ell, np^e, np^d) \in \mathcal{D}$: the embarkation day from the port p^e, visited for the np^e-th time by a cruise of duration ℓ which disembarks in the port p^d, visited for the np^d-th time,
$g^d(p^e, p^d, \ell, np^e, np^d) \in \mathcal{D}$: the disembarkation day in the port p^d, visited for the np^d-th time by a cruise of duration ℓ, originated from the port p^e, visited for the np^e-th time,

where $p^e \in \mathcal{P}$, $p^d \in \mathcal{P}$, $\ell \in \mathcal{L}$, $np^e \in \mathcal{V}(p^e)$ and $np^d \in \mathcal{V}(p^d)$.

Of course, the decision variables up to now introduced are not indipendent. Constraints (3.5) and (3.6) reported in the sequel, show the relationship among them. Actually, some of them could be eliminated, but we prefer to keep them in the description of the model, since their removal would make the expression of the constraints much less readable.

Finally, we need to identify the starting day d^{fi} of the first cruise and the ending day d^{la} of the last cruise of the sequence, hence we introduce the following pair of integer variables:

$nd^{fi} \in \{0, 1, \ldots, ndt^{fi}\}$: the number of days that elapse from the first day of the season (day 0) to the embarkation day of the first cruise of the sequence (d^{fi});
$nd^{la} \in \{0, 1, \ldots, ndt^{la}\}$: the number of days that elapse from the disembarkation day of last cruise in the sequence (d^{la}) to the last day of the season (day N).

The value of these variables enables to determine the days d^{fi} and d^{la}; in fact, it results $d^{fi} = nd^{fi}$ and $d^{la} = N - nd^{la}$.

Finally, for each port $p \in \mathcal{P}$ we introduce the following integer variable:

$nd^{off}(p, np) \in \{0, \ldots, nd^{off}_{\max}\}$: the number of days that the ship spends in the port p visited for the np-th time, if there are days off in port p between the disembarkation day in p and the embarkation day from p of two consecutive cruises.

Figure 1 provides an illustrative example of the variables nd^{fi}, nd^{la} along with the parameters ndt^{la} and ndt^{fi}.

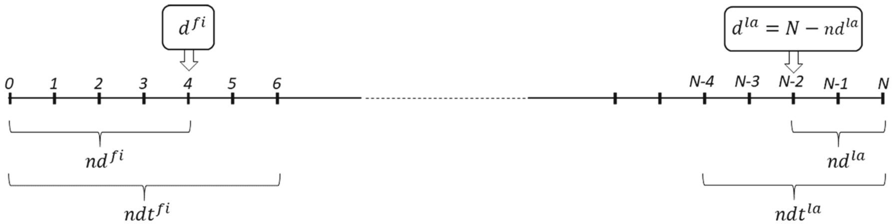

Fig. 1 Scheme for a season segment $\{0, \ldots, N\}$. Example of $nd^{fi} = 4$, $nd^{la} = 2$ and days tolerance $ndt^{fi} = 6$, $ndt^{la} = 4$

3.3 The objective function

In terms of the decision variables now introduced, the objective function of the CIOS problem defined in (2.7) can be expressed as follows. Since the net revenue of a cruise $C = (p^e, p^d, d^e, \ell)$ for all $C \in \mathcal{C}$ is a given issue denoted by $r(p^e, p^d, d^e, \ell)$, for each feasible cruise schedule $S \in \mathcal{S}$, the *total net revenue* (2.5) can be written

$$R(S) = \sum_{p\in\mathcal{P}} \sum_{q\in\mathcal{P}} \sum_{d\in\mathcal{D}} \sum_{\ell\in\mathcal{L}} \left(r(p, q, d, \ell) \sum_{n\in\mathcal{V}(p)} \sum_{m\in\mathcal{V}(q)} y^e(p, q, \ell, n, m, d) \right). \tag{3.1}$$

Similarly, the *payoff* (2.6) can be expressed as

$$PO(S) = w\left(nd^{fi} + nd^{la} + \sum_{p\in\mathcal{P}} \sum_{n\in\mathcal{V}(p)} nd^{off}(p, n) \right). \tag{3.2}$$

Then we define the objective function to be maximized as

$$R(S) - \psi \cdot PO(S), \tag{3.3}$$

where $\psi \geq 0$ is a real parameter enabling us to give different weights to the term $PO(S)$ in the objective function.

3.4 The constraints

In this section we describe the set of constraints which define the feasible set $\mathcal{S}$ in (2.7). They are *structural constraints* and *operational constraints*.

3.4.1 Structural constraints

- Constraints forcing *at most one cruise at each day*:

$$\begin{aligned} &\sum_{p\in\mathcal{P}} \sum_{q\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{n\in\mathcal{V}(p)} \sum_{m\in\mathcal{V}(q)} y^e(p, q, \ell, n, m, d) \leq 1, \\ &\sum_{p\in\mathcal{P}} \sum_{q\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{n\in\mathcal{V}(p)} \sum_{m\in\mathcal{V}(q)} y^d(p, q, \ell, n, m, d) \leq 1 \end{aligned} \tag{3.4}$$

for all $d \in \mathcal{D}$.
The first constraint in (3.4), impose that at most one cruise can embark each day. The second one is the analogous but corresponding to disembarkation days.

- Constraints that define the *embarkation days* and *disembarkation days*:

$$\sum_{d \in \mathcal{D}} d \cdot y^e(p, q, \ell, np, nq, d) = g^e(p, q, \ell, np, nq),$$
$$\sum_{d \in \mathcal{D}} y^e(p, q, \ell, np, nq, d) = x(p, q, \ell, np, nq) \tag{3.5}$$
$$\sum_{d \in \mathcal{D}} d \cdot y^d(p, q, \ell, np, nq, d) = g^d(p, q, \ell, np, nq),$$
$$\sum_{d \in \mathcal{D}} y^d(p, q, \ell, np, nq, d) = x(p, q, \ell, np, nq), \tag{3.6}$$

for all $p, q \in \mathcal{P}$, for all $\ell \in \mathcal{L}$ and for all $np \in \mathcal{V}(p)$, $nq \in \mathcal{V}(q)$.
Constraints (3.5) can be easily interpreted as definitions of departure days, since they couple the decision variables x, y, g^e with the cruise departure days d. Constraints in (3.6) are the analogous but corresponding to disembarkation days.

- Constraints which impose that *the first cruise in the schedule must embark from* p^{fi}, that is p^{fi} must be visited for the first time by the first cruise of the sequence:

$$\begin{aligned} &\sum_{\substack{q \in \mathcal{P} \\ q \neq p^{fi}}} \sum_{\ell \in \mathcal{L}} x(p^{fi}, q, \ell, 1, 1) + \sum_{\ell \in \mathcal{L}} x(p^{fi}, p^{fi}, \ell, 1, 2) = 1; \\ &\sum_{\substack{p \in \mathcal{P} \\ p \neq p^{fi}}} \sum_{\ell \in \mathcal{L}} \sum_{n \in \mathcal{V}(p)} x(p, p^{fi}, \ell, n, 1) = 0. \end{aligned} \tag{3.7}$$

More precisely, the first constraint in (3.7) assures that either one cruise embarking from p^{fi} and disembarking in any other port, visited for the first time is selected, or a cruise departing from p^{fi} and disembarking in the same port, visited for the second time, is selected. The second one, completes the first one, by imposing that no cruise, departing from whatever port (except than p^{fi}) can end in p^{fi} visiting p^{fi} for the first time. Indeed, the first time of visiting p^{fi} has to be the first departure, not an arrival.

- Constraints which assure that *the last cruise in the schedule must disembark at* p^{la}:

$$\begin{aligned} &\left| \sum_{q \in \mathcal{P}} \sum_{\ell \in \mathcal{L}} \sum_{m \in \mathcal{V}(q)} x(q, p^{la}, \ell, m, np) - \sum_{q \in \mathcal{P}} \sum_{\ell \in \mathcal{L}} \sum_{m \in \mathcal{V}(q)} x(p^{la}, q, \ell, np, m) \right| \\ &\quad \leq BigM \left(1 - \sum_{q \in \mathcal{P}} \sum_{\ell \in \mathcal{L}} \sum_{m \in \mathcal{V}(q)} x(q, p^{la}, \ell, m, np + 1) \right), \end{aligned} \tag{3.8}$$

for all np, $(np+1) \in \mathcal{V}(p^{la})$, with $p^{la} \neq p^{fi}$ or $np \neq 1$.

$$\sum_{p\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{n\in\mathcal{V}(p)} \sum_{m\in\mathcal{V}(p^{la})} x(p, p^{la}, \ell, n, m)$$
$$-\sum_{q\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{n\in\mathcal{V}(p^{la})} \sum_{m\in\mathcal{V}(q)} x(p^{la}, q, \ell, n, m) = 1, \text{ if } p^{la} \neq p^{fi},$$
$$\sum_{p\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{n\in\mathcal{V}(p)} \sum_{m\in\mathcal{V}(p^{la})} x(p, p^{la}, \ell, n, m)$$
$$-\sum_{q\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{n\in\mathcal{V}(p^{la})} \sum_{m\in\mathcal{V}(q)} x(p^{la}, q, \ell, n, m) = 0, \text{ if } p^{la} = p^{fi}. \quad (3.9)$$

Constraints in (3.8) express the fact that, if the cruise disembarking in p^{la}, visiting p^{la} for the $(np+1)$-th time is selected, then the difference within the absolute value must be zero, i.e. the number of all the cruises that disembark in p^{la}, visiting p^{la} for the np-th time and the number of all the cruises that embark from p^{la}, visited for the np-th time must be the same.
Constraints in (3.9) impose that there must be one additional cruise disembarking in p^{la} compared with all the cruises that embark from it. The only case in which there can be the same number of cruises embarking and disembarking in p^{la}, is $p^{la} = p^{fi}$.

- The *continuity constraints* given by (2.3):

$$g^d(p, q, \ell, np, nq) = g^e(p, q, \ell, np, nq) + \ell \cdot x(p, q, \ell, np, nq) + nd^{off}(q, nq)$$

for all $p, q \in \mathcal{P}$, for all $\ell \in \mathcal{L}$ and for all $np \in \mathcal{V}(p)$, $nq \in \mathcal{V}(q)$.

- The constraint on the *number of days off* in port:

$$nd^{off}(q, nq) \leq nd^{off}_{\max} \sum_{p\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{m\in\mathcal{V}(p)} x(p, q, \ell, m, nq), \quad (3.10)$$

for all $q \in P$, for all $nq \in \mathcal{V}(q)$
Constraint (3.10) allows days off in a port $q \in \mathcal{P}$ in the sequence of cruises if $nd^{off}_{\max} > 0$ and set $nd^{off}(q, nq) = 0$ if no cruise disembarking in the port q is selected.

- Constraints which ensure that for any port $p \neq p^{fi}$, $p \neq p^{la}$, if the ship disembarks np times at port p, then it must also embark np times from p:

$$\sum_{q\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{n\in\mathcal{V}(q)} x(q, p, \ell, n, np) = \sum_{q\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{m\in\mathcal{V}(q)} x(p, q, \ell, np, m) \quad (3.11)$$

for all $p \in \mathcal{P}$, $p \neq p^{la}$, for all $np \in \mathcal{V}(p)$, with ($p \neq p^{fi}$ or $np \neq 1$).

- Constraints that *sort the number of visits* in the same port:

$$\sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{n\in\mathcal{V}(q)} x(q,p,\ell,n,np+1) \le \sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{n\in\mathcal{V}(q)} x(q,p,\ell,n,np),$$
$$\sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} x(p,q,\ell,np+1,m) \le \sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} x(p,q,\ell,np,m),$$
$$\sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} x(p,q,\ell,np+1,m) \le \sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} x(q,p,\ell,m,np), \tag{3.12}$$

for all $p \in \mathcal{P}$ and for all np, $(np+1) \in \mathcal{V}(p)$, with $p \neq p^{fi}$ or $np \neq 1$.
Constraints in (3.12) guarantee that the number of visits at the same port is increasing. Therefore, a port p cannot be visited for the $(np+1)$-th time unless it has been already visited for np-th time.

- Constraints that *sort the days of visits* in the same port:

$$BigM\left(1-\sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} x(q,p,\ell,m,np+1)\right)$$
$$+\sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} g^e(q,p,\ell,m,np+1) \ge \sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} g^e(q,p,\ell,m,np),$$
$$BigM\left(1-\sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} x(p,q,\ell,np+1,m)\right)$$
$$+\sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} g^d(p,q,\ell,np+1,m) \ge \sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} g^d(p,q,\ell,np,m) \tag{3.13}$$

for all $p \in \mathcal{P}$ and for all np, $(np+1) \in \mathcal{V}(p)$.
The first constraint in (3.13) imposes that, if the cruise going from port q to port p, visited for the $(np+1)$-th time, is selected, than the day of embarkation of a cruise going from any port q and disembarking at port p, visited for the $(np+1)$-th time, must be greater than or equal to the day of embarkation of a cruise going from any port q and disembarking at port p, visited for the np-th time.
Similarly, the second constraint in (3.13) requires that, if the cruise embarking from port p, visited for the $(np+1)$-th time, and disembarking at port q is selected, then the day of disembarkation of a cruise embarking from port p, visited for the $(np+1)$-th time, and disembarking at any port q must be greater than or equal to the day of disembarkation of a cruise embarking from port p, visited for the np-th time, and disembarking at any port q.

- Constraints that *sort the days of disembarkation and embarkation* in the same port:

$$\sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} g^d(q,p,\ell,m,np) \le \sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} g^e(p,q,\ell,np+1,m) + BigM\,(1-x(p,q,\ell,np+1,nq)), \tag{3.14}$$

for all $p \in \mathcal{P}$ and for all np, $(np+1) \in \mathcal{V}(p)$.
Constraint in (3.14) requires that the day of disembarkation of a cruise that disembarks at port p for the np-th time must be smaller than or equal to the day of embarkation of a cruise that embarks from port p for the $(np+1)$-th time, if such a cruise is selected.

- Constraint which define the *first embarkation day* from p^{fi}:

$$\sum_{\substack{q\in\mathcal{P}\\ q\neq p^{fi}}}\sum_{\ell} g^e(p^{fi},q,\ell,1,1) = nd^{fi}.$$

This constraint states that the embarkation day of the first cruise of the season must be equal to the variable nd^{fi}.

- Constraints which define the *last disembarkation day* in p^{la}:

$$\sum_{p\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{n\in\mathcal{V}(p)} g^d(p,p^{la},\ell,n,nq) \le N - nd^{la}, \tag{3.15}$$

for all $nq \in \mathcal{V}(p^{la})$;

$$\begin{aligned} N - nd^{la} &- \sum_{p\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{n\in\mathcal{V}(p)} g^d(p,p^{la},\ell,n,np^{la}) \\ &\le BigM\left(1-\sum_{p\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{n\in\mathcal{V}(p)} x(p,p^{la},\ell,n,np^{la})\right. \\ &\quad\left. + \sum_{p\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{n\in\mathcal{V}(p)} x(p,p^{la},\ell,n,np^{la}+1)\right), \end{aligned} \tag{3.16}$$

for all $np^{la} \in \{nv^{\min}_{p^{la}},\ldots,nv^{\max}_{p^{la}}-1\}$;

$$\begin{aligned} N - nd^{la} &- \sum_{p\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{n\in\mathcal{V}(p)} g^d\big(p,p^{la},\ell,n,nv^{\max}_{p^{la}}\big) \\ &\le BigM\left(1-\sum_{p\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{n\in\mathcal{V}(p)} x\big(p,p^{la},\ell,n,nv^{\max}_{p^{la}}\big)\right). \end{aligned} \tag{3.17}$$

Constraint in (3.15) states that the disembarkation day in p^{la} of the last cruise in the season (as well as of any other cruise disembarking in p^{la}) is selected respecting the defined days tolerance.
As regards the constraints in (3.16), the right hand side of (3.16) is zero only if port p^{la} is visited for the np^{la}-th time and not visited for the $(np^{la}+1)$-th time, that is if the np^{la}-th visit is the last visit of port p^{la} in the season window. In this case (3.16) states that the last disembarking day in p^{la} must be greater than or equal to the day defined by $N - nd^{la}$. Constraint (3.15) along with (3.16) implies that the disembarking day of the last cruise of the season, namely d^{la}, is equal to $N - nd^{la}$.
Constraint (3.17) is analogous to constraint (3.16), but it applies to the case that np^{la} reaches the value $nv^{\max}_{p^{la}}$.

- Sequencing constraints:

$$\left|\sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} g^e(p,q,\ell,np,m) - \sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} g^d(q,p,\ell,m,np)\right| \\ \leq BigM\left(1-\sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} x(q,p,\ell,m,np)\right) + nd^{off}(p,np), \tag{3.18}$$

for all $p \in \mathcal{P}$, $p \neq p^{la}$, for all $np \in \mathcal{V}(p)$, with ($p \neq p^{fi}$ or $np \neq 1$);

$$\left|\sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} g^e(p^{la},q,\ell,np,m) - \sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} g^d(q,p^{la},\ell,m,np)\right| \\ \leq BigM\left(1-\sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} x(q,p^{la},\ell,m,np)\right) \\ + BigM\left(1-\sum_{q\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{V}(q)} x(q,p^{la},\ell,m,np+1)\right) + nd^{off}(p^{la},np), \tag{3.19}$$

for all np, $(np+1) \in \mathcal{V}(p^{la})$, with ($p \neq p^{fi}$ or $np \neq 1$).
Constraints in (3.18) express the requirement that if some cruise disembarked in p, with $p \neq p^{la}$, visiting p for the np-th time, with $p \neq p^{fi}$ or $np \neq 1$, then the difference between the embarkation day and the previous disembarkation day at the port p, visited for the np-th time, must be equal to the number of days off $nd^{off}(p,np)$ of staying moored in p. Constraints in (3.19) express the analogous requirement for $p = p^{la}$.

3.4.2 Operational constraints

A second set of constraints must be considered for taking into account the additional requirements on the schedule given by constraints C3–C5. They are reported in the following.

- Constraints on *minimum and maximum number of cruises of duration* ℓ allowed:

$$nc_{\ell}^{\min} \leq \sum_{p\in\mathcal{P}} \sum_{q\in\mathcal{P}} \sum_{n\in\mathcal{V}(p)} \sum_{m\in\mathcal{V}(q)} x(p,q,\ell,n,m) \leq nc_{\ell}^{\max}$$

  for all $\ell \in \mathcal{L}$.
- Constraint on *minimum number of days between two consecutive visits* at the same port:

$$\begin{aligned}\sum_{q\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{m\in\mathcal{V}(q)} g^{d}(q,p,\ell,m,np+1) - \sum_{q\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{m\in\mathcal{V}(q)} g^{e}(p,q,\ell,np,m) \\ \geq nd_{p}^{\min} - BigM \left(1 - \sum_{q\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{m\in\mathcal{V}(q)} x(q,p,\ell,m,np+1)\right)\end{aligned} \tag{3.20}$$

  for all $p \in P$ and for all $np, np+1 \in \mathcal{V}(p)$.
  Constraint (3.20) makes sure that, if a port p has been visited for the np-th time, and there is a subsequent visit in the same port p, a minimun number of days must elapse between these two visits.
- Constraint on *minimum and maximum number of visits* in each port:

$$nv_{p}^{\min} \leq \sum_{p\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{n\in\mathcal{V}(p)} \sum_{m\in\mathcal{V}(q)} x(p,q,\ell,n,m) \leq nv_{p}^{\max}$$

  for all $p \in \mathcal{P}$, $p \neq p^{la}$;

$$nv_{p^{la}}^{\min} \leq \sum_{q\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{n\in\mathcal{V}(p^{la})} \sum_{m\in\mathcal{V}(q)} x(p^{la},q,\ell,n,m) + 1 \leq nv_{p^{la}}^{\max} + 1. \tag{3.21}$$

  Constraint in (3.21) allows one additional visit in p^{la}, besides the maximum number of visits on that port, in order to assure that p^{la} is always be reached.
- *Milestones on embarkation*, i.e. ports to be visited on embarkation in specified days:

$$\sum_{q\in\mathcal{P}} \sum_{\ell\in\mathcal{L}} \sum_{n\in\mathcal{V}(p^{e})} \sum_{m\in\mathcal{V}(q)} y^{e}(p^{e},q,\ell,n,m,d^{e}) = 1, \quad \text{for all} \quad (p^{e},d^{e}) \in \mathcal{M}^{e}.$$

- *Milestones on disembarkation*, i.e. ports to be visited on disembarkation in specified days:

$$\sum_{p\in\mathcal{P}}\sum_{\ell\in\mathcal{L}}\sum_{n\in\mathcal{V}(p)}\sum_{m\in\mathcal{V}(p^d)} y^d(p, p^d, \ell, n, m, d^d) = 1, \quad \text{for all} \quad (p^d, d^d) \in \mathcal{M}^d.$$

- Constraints that define *charter cruises*:

$$\sum_{n\in\mathcal{V}(p^e)}\sum_{m\in\mathcal{V}(p^q)} y^e(p^e, p^d, \ell, n, m, d^e) = 1, \quad \text{for all} \quad (p^e, p^d, \ell, d^e) \in \mathcal{C}^{ch}.$$

4 Some experimental results

As already mentioned, the main aim of this paper is to propose an ILP model for the class of CIOS problems, with a particular focus on luxury cruises. The overall goal is to provide a luxury cruise company with a decision support system able to face the three levels of decisions reported in the Introduction, where the CIOS problem represent the important intermediate level. The CIOS problem is dealt with well in advance, usually two years before the interested season, hence it represents an off-line decision stage. In other words, in practice, the computing time for solving the CIOS problem, actually is not a critical issue. Therefore we decided to use a standard ILP solver, rather than to develop some metaheuristic.

The availablity of a large database including all the main ports of the world usually visited by cruises enabled to perform an extensive experimentation of our model on several different maritime areas, with season segments of different lenghts, for different ships and taking into account many different operational constraints. Of course there is no room to report here the results of such large experimentation. In order to give evidence of the viability and the reliability of the proposed approach, we limit to describe in the sequel some results obtained on real instances as illustrative examples.

Following the indication of the cruise company, for the particular ship and maritime area considered in this experimentation, the value of the parameter w which appears in the objective function corresponding to the payoff in (3.2) has been set to $w = 188{,}008.6$ US\$. Moreover, in the experimentation reported in the sequel, we choose $\psi = 10$ in (3.3) and we set $BigM = 10^6$. Bigger values of ψ could be adopted in order to further "penalize" the introduction of days off in port in the optimal schedule, since these are are usually not desirable. Note that, according to this last approach, in the experimentation reported here, we also set $nd^{off}_{\max} = 0$, hence allowing days off in port only in the first (p^{fi}) and in the last (p^{la}) port of the cruise schedule.

We coded the ILP model described in Sect. 3 by using AMPL language [8] and we used the GUROBI 8.1 solver [10] for solving all the problem instances considered. The runs have been performed on a PC with an Intel Core i7-2600 3.40 GHz Processor and 16 GB RAM. Moreover, we set the maximum CPU elapsed time to a prefixed *time_limit*. The quality of the solution obtained is measured by mean of the relative optimality gap (*rel_opt_gap*).

Now we describe an example of a real instance of the CIOS problem. We consider a given ship located in the Mediterranean maritime area in the season segment April 01–June 30, 2019.

The *problem data* of this example are the following:[2]

- $\mathcal{P} = \{$PTLIS, ESBCN MCMCM, ITCVV, ITVCE, GRPIR$\}$;
- $\mathcal{D} = \{0, 1, 2, \ldots, 91\}$;
- $p^{fi} = p^{la} =$ PTLIS;
- $\mathcal{L} = \{7, 10, 12\}$;
- $\mathcal{C} = \{(p^e, p^d, d^e, \ell) \in \mathcal{P} \times \mathcal{P} \times \mathcal{D} \times \mathcal{L}\}$;
- the revenues $r(C)$ in US\$ for each cruise $C \in \mathcal{C}$.

The cruises which can be considered in this season, for the assigned maritime area, taking into account all the possible durations are 108 different cruises; moreover, since the embarkation day of each cruise can vary within the season segment, on the overall we have an upper bound on the number of different possible cruises equal to 9, 828 (we do not report the complete list).

Moreover, we have the following *operational requests*:

- Minimum and maximum number of visits for each port $nv_p^{\min}$, $nv_p^{\max}$ along with minimum number of days between two consecutive visit of a port $nd_p^{\min}$:

	PTLIS	ESBCN	MCMCM	ITCVV	ITVCE	GRPIR
$nv_p^{\min}$	2	1	1	1	1	1
$nv_p^{\max}$	2	3	3	4	3	3
$nd_p^{\min}$	10	10	7	7	10	7

- Number of days tolerance and days off in ports allowed:

$$ndt^{fi} = 3, ndt^{la} = 4, nd_{max}^{off} = 0.$$

- Minimum and maximum number of cruises of duration $\ell \in \mathcal{L}$ allowed $nc_\ell^{\min}$ and $nc_\ell^{\max}$:

- A milestone on disembarkation in MCMCM (Monaco Monte Carlo port) on May 14, 2019 (May 14–25, 2019, Cannes Festival) and no milestone on embarking:

$$\mathcal{M}^d = \{(\text{MCMCM}, 48)\}, \qquad \mathcal{M}^e = \emptyset. \tag{4.1}$$

[2] The name of the ports are reported according to the *United Nations Code for Trade and Transport Locations* (UN/LOCODE Code List) which is a combination of a 2-character country code and a 3-character location code (e.g., PTLIS denotes Lisbon in Portugal, ITCVV indicates Civitavecchia, the port of Rome, in Italy, GRPIR Piraeus, port of Athen, in Greece). The complete list can be found at www.unece.org/cefact/locode/service/location.html.

	$\ell = 7$	$\ell = 10$	$\ell = 12$
$nc_\ell^{\min}$	3	1	1
$nc_\ell^{\max}$	10	3	2

Table 2 Optimal cruise schedule for *Scenario 1*

Seq.	p^e	p^d	ℓ	np^e	np^d	d^e	d^d	*revenue*
1	PTLIS	ESBCN	7	1	1	0	7	1,188,247
2	ESBCN	GRPIR	10	1	1	7	17	1,784,382
3	GRPIR	ITVCE	10	1	1	17	27	2,822,643
4	ITVCE	MCMCM	12	1	1	27	39	3,261,511
5	MCMCM	MCMCM	7	1	2	39	46	1,557,281
6	MCMCM	ITVCE	7	2	2	46	53	1,666,549
7	ITVCE	GRPIR	7	2	2	53	60	1,534,232
8	GRPIR	ITVCE	7	2	3	60	67	1,428,613
9	ITVCE	ITCVV	10	3	1	67	77	2,216,900
10	ITCVV	MCMCM	7	1	3	77	84	1,462,973
11	MCMCM	PTLIS	7	3	2	84	91	1,557,281

tot_revenue = 20,480,612, $nd^{fi} = nd^{la} = 0$, *rel_opt_gap* = 0.16

- A charter cruise with $p^e = p^d =$ ITCVV (Civitavecchia port) in the period June 01–08, 2019 (Corporate convention at sea):

$$\mathcal{C}^{ch} = \{(\text{ITCVV}, \text{ITCVV}, 62, 7)\}. \tag{4.2}$$

In particular, we consider 3 different scenarios of this instance:

Scenario 1: the problem instance without milestones and without charter cruises, i.e. $\mathcal{M}^d = \mathcal{M}^e = \emptyset$ and $\mathcal{C}^{ch} = \emptyset$, which correspond to standard cruises;
Scenario 2: the problem instance with the milestone on disembarkation (4.1) and no charter cruise, i.e. $\mathcal{C}^{ch} = \emptyset$;
Scenario 3: the problem instance without milestones, i.e. $\mathcal{M}^d = \mathcal{M}^e = \emptyset$ and with the charter cruise in (4.2).

In solving the ILP problems arising from these scenarios we adopt the CPU *time_limit* = 3600 seconds.

Now we report the output schedule obtained, i.e. the sequence of cruises in terms of their embarkation (p^e) and disembarkation (p^d) ports, duration ℓ (in days), number of visits of the ports p^e, p^d (np^e and np^d), embarking (d^e) and disembarking (d^d) days, net revenue (in US\$) of each selected cruise, overall net revenue (*tot_revenue*), possible days gap at the first and at the last port, namely nd^{fi} and nd^{la}, and the relative optimality gap (*rel_opt_gap*).

In particular, Table 2, Table 3 and Table 4 report the optimal schedule for *Scenario 1*, *Scenario 2* and *Scenario 3*, respectively.

Table 3 Optimal cruise schedule for *Scenario 2*

Seq.	p^e	p^d	ℓ	np^e	np^d	d^e	d^d	*revenue*
1	PTLIS	ITCVV	7	1	1	0	7	1,188,247
2	ITCVV	GRPIR	10	1	1	7	17	1,784,382
3	GRPIR	ITCVV	10	1	2	17	27	2,822,643
4	ITCVV	ITCVV	7	2	3	27	34	1,975,850
5	ITCVV	ITVCE	7	3	1	34	41	1,464,977
6	ITVCE	MCMCM	7	1	1	41	48	1,462,973
7	MCMCM	ITVCE	12	1	2	48	60	2,291,561
8	ITVCE	MCMCM	7	2	2	60	67	1,462,973
9	MCMCM	MCMCM	7	2	3	67	74	1,557,281
10	MCMCM	ESBCN	7	3	1	74	81	1,666,549
11	ESBCN	PTLIS	10	1	2	81	91	2,053,218

tot_revenue = 19,730,654, $nd^{fi} = nd^{la} = 0$, *rel_opt_gap* = 0.19

Table 4 Optimal cruise schedule for *Scenario 3*

Seq.	p^e	p^d	ℓ	np^e	np^d	d^e	d^d	*revenue*
1	PTLIS	ESBCN	7	1	1	0	7	1,188,247
2	ESBCN	GRPIR	10	1	1	7	17	1,784,382
3	GRPIR	ESBCN	10	1	2	17	27	2,822,643
4	ESBCN	MCMCM	7	2	1	27	34	1,975,850
5	MCMCM	ITVCE	7	1	1	34	41	1,666,549
6	ITVCE	GRPIR	7	1	2	41	48	1,534,232
7	GRPIR	GRPIR	7	2	3	48	55	1,379,656
8	GRPIR	ITCVV	7	3	1	55	62	1,428,613
9	ITCVV	ITCVV	7	1	2	62	69	1,464,977
10	ITCVV	ESBCN	12	2	3	69	81	2,431,270
11	ESBCN	PTLIS	10	3	2	81	91	2,053,218

tot_revenue = 19,729,637, $nd^{fi} = nd^{la} = 0$, *rel_opt_gap* = 0.17

Even if the output schedules obtained in these three scenarios are only illustrative examples, they clearly highlight that the approach proposed in this paper is viable in solving both standard CIOS problems, i.e. without operational constraints on milestones or charters cruise (*Scenario 1*) and problems involving specific operational requests (*Scenarios 2–3*). Moreover, it is worthwhile observing that in all the three cases a relative optimality gap lower that 0.20 is obtained within the CPU time limit. Moreover, note that in the three cases both nd^{fi} and nd^{la} turns out to be zero.

We report in the following figures the maps corresponding to the optimal solution of the *Scenario 1*, i.e. the 11 cruises whose schedule is detailed in Table 2. We splitted the maps into three figures for clarity. In particular, Figure 2 shows cruises 1–4, Figure 3 shows cruises 5–7 and Figure 4 shows cruises 8–11.

Fig. 2 *Scenario 1*, days {0, . . . , 39} of the cruises detailed in Table 2. Cruise 1: a 7-days cruise (in black). Cruises 2 and 3: two 10-days cruises (in green). Cruise 4: a 12-days cruise (in red) (color figure online)

Fig. 3 *Scenario 1*, days {39, . . . , 60} of the cruises detailed in Table 2. Cruise 5, 6 and 7: three 7-days cruises

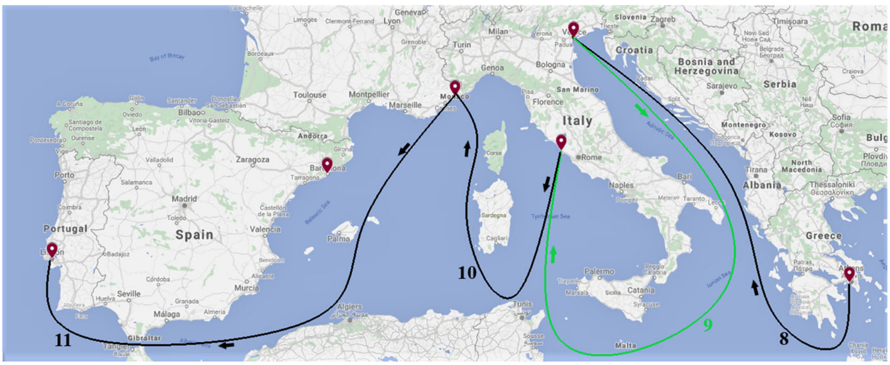

Fig. 4 *Scenario 1*, days {60, . . . , 91} of the cruises detailed in Table 2. Cruise 8: a 7-days cruise (in black). Cruise 9: a 10-days cruise (in green). Cruises 10 and 11: two 7-days cruises (color figure online)

To assess whether a better accuracy can be achieved, we performed another experimentation on the *Scenario 1*: we set the required relative optimality gap *rel_opt_gap*=0.10 aiming at evaluating the CPU time needed to obtain such accuracy. Starting from a relative optimality gap of 1.98 (at the first feasible solution) it turned out that a CPU time of 25 hours is necessary. It is also worth noting that the corresponding optimal value is *tot_revenue* = 20,662,249 and hence only an improvement smaller than 1% is get with respect to the optimal value obtained by one hour CPU time, with *rel_opt_gap* = 0.16 (see Table 2).

Although the main focus of this paper is not on computational aspects of the CIOS problem, we report in the sequel some results obtained on other problems instances. The aim is to give an idea on the performance of the solver used in tackling CIOS problems of increasing dimension. Therefore we considered two variations of the *Scenario 1* obtained by lengthening the season segment. The rationale behind this choice is that, from the extensive testing performed (not reported here), we noticed that an elongation of the season length, strongly affects the performance of the solver adopted. Therefore we considered two further standard cruise scenarios (i.e. without milestones and charter cruises) denoted by *Scenario 1B* and *Scenario 1C*. Of course, we needed to change accordingly the operational requests on minimum and maximum number of visits for each port and on minimum and maximum number of cruises of duration ℓ allowed as follows:

- *Scenario 1B*—Season segment April 1–July 31, 2019:

	PTLIS	ESBCN	MCMCM	ITCVV	ITVCE	GRPIR
$nv_p^{\min}$	2	1	1	1	1	1
$nv_p^{\max}$	2	4	4	4	4	4

	$\ell = 7$	$\ell = 10$	$\ell = 12$
$nc_\ell^{\min}$	5	1	1
$nc_\ell^{\max}$	15	4	3

- *Scenario 1C*—Season segment April 1–August 31, 2019:

	PTLIS	ESBCN	MCMCM	ITCVV	ITVCE	GRPIR
$nv_p^{\min}$	2	1	1	1	1	1
$nv_p^{\max}$	2	5	5	5	5	5

	$\ell = 7$	$\ell = 10$	$\ell = 12$
$nc_\ell^{\min}$	10	1	1
$nc_\ell^{\max}$	20	7	5

Note that the size of the latter *Scenario 1C*, actually goes far beyond a realistic scenario, since the season segment therein considered exceeds four months which is the maximum length of season segment usually adopted by cruise companies when dealing with CIOS problem.

Without reporting the complete output schedules obtained for the new scenarios, we now summarize in the following Table 5 the characteristics of the three scenarios (the new ones, along with *Scenario 1*) and the optimality gap reached within a prefixed CPU solving time. In particular, we report:

- length of the season segment (in days);
- size of the corresponding problem instance in terms of number of variables and number of constraints of the ILP model;
- CPU *time_limit* (in hours) and the corresponding value of the relative optimality gap (*rel_opt_gap*) obtained within such limit.

It is important to highlight that the number of variables and the number of constraints reported in Table 5 refer to the "adjusted problem" after presolving. As well known, presolve phase aims at reducing the model size and tightening the formulation. We only recall that presolve remove redundant information (in terms of variables and constraints) by performing single/multi-row and columns reduction and we refer to [1] for any detail on the presolve phase. In our experimentation, we adopted the standard presolve provided by GUROBI 8.1 solver with $presolve\_level = 1$. Of course, the elapsed time due to presolving is included in the total solution time.

First, Table 5 clearly points out how the problem size increases as the length of the season segment grows. As concerns the performance of the solver, if we refer to a relative optimality gap of 0.25, on both the real scenarios (*Scenario 1* and *Scenario 1B*) this value is reached within a reasonable time. Indeed an elapsed time of 3 hours can be considered feasible, taking into account that, as already observed, the CIOS problem represents an off-line step of the decision process, and cruise companies usually deals with it many months (even years) in advance. As regards *Scenario 1C* (which we artificially created) the 5 months long season segment leads to a larger dimension problem and 18 hours of computational time are not enough to reach the prefixed

Table 5 Summary of the characteristics of the three scenarios, CPU time limit adopted and relative optimality gap obtained

	Season segment (days)	Variables	Constraints	CPU *time_limit* (h)	*rel_opt_gap*
Scenario 1	91	135,796	5459	1	0.16
Scenario 1B	122	306,037	9087	3	0.24
Scenario 1C	153	535,860	12,655	18	0.26

relative optimality gap of 0.25. Even if this gap can be considered, in general, not a very good performance, we highlight that it is achieved starting from an initial gap (corresponding to the first feasible solution) which is usually greater than 1.90.

Of course, in our extensive experimentation, we also observed the solver performance on several instances obtained by increasing the number of ports in the set $\mathcal{P}$ and the number of cruise durations $\ell \in \mathcal{L}$, but there is no room to discuss these results in this paper.

5 Conclusions

In this paper we considered the CIOS problem, focusing on luxury cruises, which is an emerging market segment. We described in detail the ILP model we propose for determining an optimal schedule of cruises, aiming at maximizing the overall net revenue of a ship operating in a given maritime area, in a selected season segment. This problem corresponds to the tactical level, the second one, of the three levels decision making process of a cruise company. We showed that the model we proposed allows the user to obtain a complete cruises schedule of a ship for a whole season segment, by using a commercial ILP solver. Considerations about the performance of the solver on some instances of the problem are briefly reported, too.

The model we proposed has been experimented by a luxury cruise company to schedule the cruises of its fleet. This experimentation was possible, since we complemented the development of the model with a friendly graphical user interfaces to enter data corresponding to different scenarios and to display results, as well as to perform "what-if" analyses.

This work has been developed within an innovative project (named the *Magellano Project*) covering all three levels of the decision making process, as well as related issues of interest for the sector of the cruise companies, like e.g., the cruise prices optimization, the emergency evacuation of a ship, the on board food supply chain. The approach adopted is based on models and methods of Operations Research and we believe that the cruise sector could greatly benefit by adopting these models and methods to an extent larger than the present one, which is definitely very small.

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