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Stability Analysis of Parallel Fuzzy P + Fuzzy I + Fuzzy D Control Systems

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Abstract: The study presented in this paper is in continuation with the paper published by the authors on parallel fuzzy proportional plus fuzzy integral plus fuzzy derivative (FP + FI + FD) controller. It addresses the stability analysis of parallel FP + FI + FD controller. The famous "small gain theorem" is used to study the bounded-input and bounded-output (BIBO) stability of the fuzzy controller. Sufficient BIBO-stability conditions are developed for parallel FP + FI + FD controller. FP + FI + FD controller is derived from the conventional parallel proportional plus integral plus derivative (PID) controller. The parallel FP + FI + FD controller is actually a nonlinear controller with variable gains. It shows much better set-point tracking, disturbance rejection and noise suppression for nonlinear processes as compared to conventional PID controller.

Keywords: Proportional plus integral plus derivative (PID), fuzzy proportional plus fuzzy integral plus fuzzy derivative (FP + FI + FD) controller, fuzzy control, bounded-input and bounded-output (BIBO) stability, nonlinear process.

1 Introduction

Proportional plus integral plus derivative (PID) controller is extensively used in industry for more than 50 years in many forms, such as, pneumatic, hydraulic, analog and digital, etc., because of its effectiveness, simplicity of implementation, inexpensive cost, ease of design and broad applicability. It has been reported that in process industries, more than 95% of the controllers are from the PID controller′s family[1−4].

Driankov et al. reported that conventional PID controllers are generally incapable to control processes with additional complexities such as time delays, significant oscillatory behavior (complex poles with small damping), parameter variations, nonlinearities, and multiple input and multiple output (MIMO) plants[1, 2, 5]. Therefore, scientists and researchers were trying to use intelligent techniques, such as fuzzy logic, to enhance the capabilities of the classical PID controller and its family. They were trying to combine fuzzy logic control technology with the conventional PID controller to obtain a behavior similar to that of a regular PID controller[1, 2, 5−11]. It is believed that by combining these two techniques, a better control system can be achieved.

Lack of stability assurance can limit the applications of fuzzy controllers[12]. There exist many stability theories for different fuzzy control systems. Some of the stability theories are the small gain theorem, the energetic method[13], the fuzzy transfer function and phase plane analysis[14], Lyapunov function method[5, 15], etc.

Chen et al. have proposed fuzzy PI/PD/PID controllers, and performed comparison with the conventional controllers. They have derived the structure of fuzzy controllers, with simple analytical formulas as the final results by considering two fuzzy sets on each input variables and three fuzzy sets on output variable in the fuzzification process, rule base with four control rules, intersection T-norm, Lukasiewicz OR T-conorm, drastic product inference method, and center of area (COA) defuzzification method. They studied the bounded-input and bounded-output (BIBO) stability of fuzzy controllers using small gain theorem[1, 2, 12, 16, 17]. Malki et al.[18] proposed a new design and stability analysis of fuzzy proportional derivative controller. Further, Chen et al.[2, 19] have developed fuzzy PID controller, which is a combination of fuzzy PI and fuzzy D controller having same structure as mentioned earlier. Here, derivative function is performed on controlled variable rather than error signal. Stability analysis is performed using small gain theorem. Kim et al.[20] have proposed another configuration of fuzzy PID controller (fuzzy PI + fuzzy ID) with the same structure as discussed above.

Mohan et al.[21−25] introduced an analytical structure and analyzed the simplest fuzzy PI/PD/PID controllers. Sufficient conditions for BIBO stability of fuzzy PI/PD/PID control systems are established using the small gain theorem. Kumar et al.[26] presented the design, performance and stability analysis of formula-based fuzzy PI (FPI) controller. They use a large number of fuzzy sets for input and output variables to obtain more formulae for corrective action. Further, Kumar et al.[27] presented a review on classical and fuzzy PID controllers. They presented the history of the development of classical PID controllers and their enhancement using fuzzy logic theory.

In this paper, BIBO stability analysis is performed for a parallel FP + FI + FD controller proposed in [28]. The well known small gain theorem is used for the stability analysis of nonlinear process controlled by the parallel FP + FI + FD controller developed in [28]. Sufficient BIBO-stability conditions are derived for the various regions of parallel FP + FI + FD controller. Also, the set-point tracking, disturbance rejection and noise suppression capabilities are studied in the case of FP + FI + FD controller for nonlinear processes.

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2 Review of the FP + FI + FD controller

The design and performance analysis of parallel FP + FI + FD controllers have been discussed in [28]. Same notations are used to describe and analyze this fuzzy controller as presented in [28]. The configuration of the closed loop of the parallel FP + FI + FD controller is shown in Fig. 1, where $T > 0$ is the sampling time, $y_{\rm SP}(nT)$ is the reference set-point, $y(nT)$ is the process variable, $e(nT) = y_{\rm SP}(nT) - y(nT)$ is the error signal, $r(nT)$ is the rate of change of error signal, and $u_{P+I+D}(nT)$ is the output of the parallel FP + FI + FD controller.

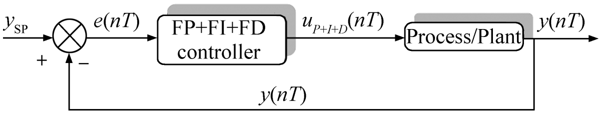

Fig. 1 The closed loop of parallel FP + FI + FD control system

As presented in [28], the overall control action of parallel FP + FI + FD controller can be obtained by algebraically summing fuzzy P control action, fuzzy I control action and fuzzy D control action simultaneously. The resultant parallel FP + FI + FD control action is

$$\begin{aligned} &u_{P+I+D}(nT) = u'_P(nT) + u_I(nT) + u_D(nT)\\ &u_{P+I+D}(nT) = K_{UP}u_P(nT) + u_I(nT-T) +\\ &K_{UI}\Delta u_I(nT) - u_D(nT-T) + K_{UD}\Delta u_D(nT) \end{aligned} \qquad (1)$$

where $u_P(nT)$ is the fuzzy P control action, and K_{UP} is the fuzzy P controller gain, $\Delta u_I(nT)$ is the incremental control action of fuzzy I controller, K_{UI} is the fuzzy I controller gain, $\Delta u_D(nT)$ is the incremental control action of fuzzy D controller, and K_{UD} is the fuzzy D controller gain.

As presented in [28], two triangular membership functions are considered for input variables and three singleton membership functions are considered for output variable for the fuzzy P, fuzzy I and fuzzy D control components. Also, rule based with the four control rules, max-min inference mechanism and center of mass for defuzzification method are considered. The regions of the fuzzy P, fuzzy I and fuzzy D controllers′ input combination (IC) values are graphically shown in Fig. 2. The regions for each fuzzy control components are divided into 12 different ICs regions. The results of $u_P(nT)$, $\Delta u_I(nT)$ and $\Delta u_D(nT)$ are obtained by applying defuzzification algorithm to each membership area, shown as

$$\begin{cases} u_P(nT) = \dfrac{\sum\limits_{r=1}^{4}\mu_{P_r}u_{P_r}}{\sum\limits_{r=1}^{4}\mu_{P_r}}\\ \Delta u_I(nT) = \dfrac{\sum\limits_{r=1}^{4}\mu_{I_r}\Delta u_{I_r}}{\sum\limits_{r=1}^{4}\mu_{I_r}}\\ \Delta u_D(nT) = \dfrac{\sum\limits_{r=1}^{4}\mu_{D_r}\Delta u_{D_r}}{\sum\limits_{r=1}^{4}\mu_{D_r}} \end{cases} \qquad (2)$$

where μ_{P_r}, μ_{I_r} and μ_{D_r} are the membership values at the r-th rule, u_{P_r}, Δu_{I_r} and Δu_{D_r} are the singleton outputs at the r-th rule.

The expressions of $u_P(nT)$, $\Delta u_I(nT)$ and $\Delta u_D(nT)$ in IC I to IC XII regions are shown in Table 1, respectively.

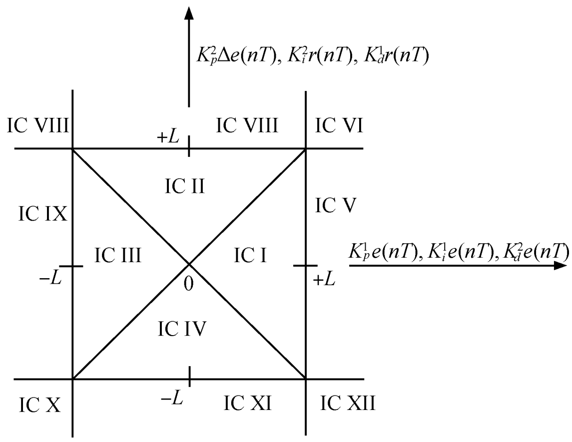

Fig. 2 Regions of the parallel FP + FI + FD controller values

Table 1 Analytical formulas for the 12 IC regions for fuzzy P, fuzzy I and fuzzy D controller[28]

IC#	Fuzzy P controller output "$u_P(nT)$"	Fuzzy I controller output "$\Delta u_I(nT)$"	Fuzzy D controller output "$\Delta u_D(nT)$"
IC I & IC III	$\dfrac{L[K_p^2\Delta e(nT) - K_p^1 e(nT)]}{2[2L - \lvert K_p^1 e(nT)\rvert]}$	$\dfrac{L[K_i^1 e(nT) + K_i^2 r(nT)]}{2[2L - \lvert K_i^1 e(nT)\rvert]}$	$\dfrac{L[K_d^2 e(nT) - K_d^1 r(nT)]}{2[2L - \lvert K_d^2 e(nT)\rvert]}$
IC II & IC IV	$\dfrac{L[K_p^2\Delta e(nT) - K_p^1 e(nT)]}{2[2L - \lvert K_p^2\Delta e(nT)\rvert]}$	$\dfrac{L[K_i^1 e(nT) + K_i^2 r(nT)]}{2[2L - \lvert K_i^2 r(nT)\rvert]}$	$\dfrac{L[K_d^2 e(nT) - K_d^1 r(nT)]}{2[2L - \lvert K_d^1 r(nT)\rvert]}$
IC V	$\frac{1}{2}[-L + K_p^2\Delta e(nT)]$	$\frac{1}{2}[L + K_i^2 r(nT)]$	$\frac{1}{2}[L - K_d^1 r(nT)]$
IC VI	0	L	0
IC VII	$\frac{1}{2}[L - K_p^1 e(nT)]$	$\frac{1}{2}[L + K_i^1 e(nT)]$	$\frac{1}{2}[-L + K_d^2 e(nT)]$
IC VIII	L	0	$-L$
IC IX	$\frac{1}{2}[L + K_p^2\Delta e(nT)]$	$\frac{1}{2}[-L + K_i^2 r(nT)]$	$\frac{1}{2}[-L - K_d^1 r(nT)]$
IC X	0	$-L$	0
IC XI	$\frac{1}{2}[-L - K_p^1 e(nT)]$	$\frac{1}{2}[-L + K_i^1 e(nT)]$	$\frac{1}{2}[L + K_d^2 e(nT)]$
IC XII	$-L$	0	L

It may be noted that the fuzzy P/I/D controller may be switched automatically from one control algorithm to another from time to time, depending on the input signals to the respective fuzzy P/I/D controllers. However, such switching is always continuous in time and smooth on boundaries of any two adjacent regions[12].

From Table 1, it can be observed that the coefficients of input signals of fuzzy P controller, fuzzy I controller, and fuzzy D controller are variable gain that change with $e(nT)$, $\Delta e(nT)$ and $r(nT)$, respectively. Therefore, parallel FP + FI + FD controller is a nonlinear controller with variable gains.

3 Stability analysis

In this section, BIBO stability analysis of FP + FI + FD control system is done using the well known small gain theorem[2,12,16−19,23−27,29−32], as shown in Fig. 3.

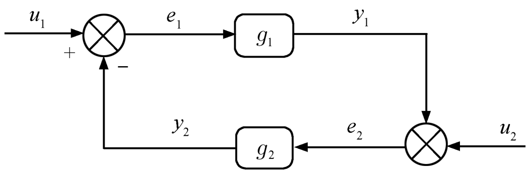

Fig. 3 Closed loop control system

The subsystem g_1 represents the FP + FI + FD controller and the subsystem g_2 represents the process/plant/system to be controlled. Here e_1 and e_2 are errors; u_1 and u_2 are the inputs to the system, and y_1 and y_2 are the outputs from the respective subsystems. The closed loop equations are

$$\begin{cases} e_1 = u_1 - y_2 \\ e_2 = u_2 + y_1 \\ y_1 = g_1(e_1) \\ y_2 = g_2(e_2). \end{cases} \tag{3}$$

It is assumed that g_1 and g_2 are BIBO stable so that

$$\|y_1\| \leqslant \alpha_1 \|e_1\| + \delta_1 \tag{4}$$

$$\|y_2\| \leqslant \alpha_2 \|e_2\| + \delta_2 \tag{5}$$

where $\alpha_1 = \alpha(g_1)$, is the gain of subsystem g_1, and $\alpha_2 = \alpha(g_2)$, gain of subsystem g_2; δ_1 and δ_2 are constants, and α_1, $\alpha_2 \geqslant 0$.

Under the above two conditions, (4) and (5), the system is BIBO stable if $\alpha_1, \alpha_2 > 1$[2,12,17−19,23−26].

For the BIBO stability analysis of overall fuzzy control system, Fig. 4 is the equivalent block diagram of FP + FI + FD control system. Here, process/plant is denoted by g_2, and the FP + FI + FD controller is denoted by g_1, which is represented by dashed box in Fig. 4. Now, based upon the "small gain theorem", the sufficient conditions for the BIBO stability of the closed loop control system can be found from the two conditions given below and if $\alpha_1\alpha_2 > 1$[2, 17, 19].

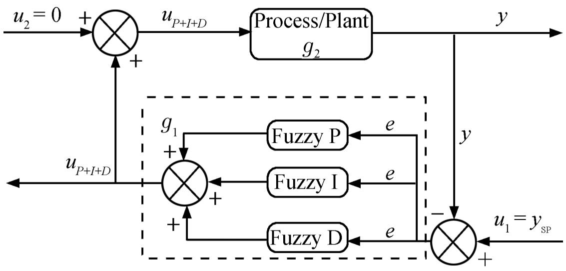

Fig. 4 Equivalent closed loop control system

$$\|y_1\| = \left\| g_1(\begin{bmatrix} e \\ e \\ e \end{bmatrix}) \right\| \leqslant \alpha_1 \left\| \begin{bmatrix} e \\ e \\ e \end{bmatrix} \right\| + \delta_1 \tag{6a}$$

$$\|y_2\| = \|g_2(u_{P+I+D})\| \leqslant \alpha_2 \|u_{P+I+D}\| + \delta_2 \tag{6b}$$

where $\alpha_1, \alpha_2, \delta_1$ and δ_2 are constants.

Expanding (6a) for region IC I and IC III,

$$\begin{aligned} \|y_1(nT)\| = \ & \|K_{UP}u_P(nT) + \\ & K_{UI}\Delta u_I(nT) + K_{UD}\Delta u_D(nT)\| \end{aligned}$$

or

$$\begin{aligned} &\|y_1(nT)\| = \\ &\left\| K_{UP}\frac{L[K_p^2\Delta e(nT) - K_p^1 e(nT)]}{2[2L - |K_p^1 e(nT)|]} + \right. \\ &K_{UI}\frac{L[K_i^1 e(nT) + K_i^2 r(nT)]}{2[2L - |K_i^1 e(nT)|]} + \\ &\left. K_{UD}\frac{L[K_d^2 e(nT) - K_d^1 r(nT)]}{2[2L - |K_d^2 e(nT)|]} \right\|. \end{aligned}$$

But $r(nT) = \frac{e(nT) - e(nT-T)}{T}$, and $\Delta e(nT) = e(nT) - e(nT-T)$.

Putting $r(nT)$ and $\Delta e(nT)$, and rewriting the above equation

$$\begin{aligned} &\|y_1(nT)\| \leqslant \\ &\left| \frac{LK_{UP}(K_p^2 - K_p^1)}{2(2L - K_p^1 M_e)} + \frac{LK_{UI}(TK_i^1 + K_i^2)}{2T(2L - K_i^1 M_e)} + \right. \\ &\left. \frac{LK_{UD}(TK_d^2 - K_d^1)}{2T(2L - K_d^2 M_e)} \right| |e_1(nT)| + \\ &\left| \frac{LK_{UP}K_p^2 M_e}{2(2L - K_p^1 M_e)} + \frac{LK_{UI}K_i^2 M_e}{2T(2L - K_i^1 M_e)} - \right. \\ &\left. \frac{LK_{UD}K_d^1 M_e}{2T(2L - K_d^2 M_e)} \right| \end{aligned} \tag{7}$$

where $M_e := \sup_{n\geqslant 0} |e(nT)| = \sup_{n\geqslant 1} |e(nT-T)|$ is the maximum magnitude of error signal.

Further, a general case is considered where the process under control is nonlinear, denoted by R. Define various signal as shown in Fig. 4. Since control action is different in different IC regions, stability for every IC region may be discussed. First, consider the location of error signal and the change rate of error signal in region IC I & IC III. Rewrite (6a) in the region IC I and IC III and rearrange it.

However, (7) is in the form of (4) with

$$\alpha_1 = \left| \frac{LK_{UP}(K_p^2 - K_p^1)}{2(2L - K_p^1 M_e)} + \frac{LK_{UI}(TK_i^1 + K_i^2)}{2T(2L - K_i^1 M_e)} + \frac{LK_{UD}(TK_d^2 - K_d^1)}{2T(2L - K_d^2 M_e)} \right|$$

gain of subsystem g_1 (FP + FI + FD controller) for the region IC I & IC III and

$$\delta_1 = \left| \frac{LK_{UP}K_p^2 M_e}{2(2L - K_p^1 M_e)} + \frac{LK_{UI}K_i^2 M_e}{2T(2L - K_i^1 M_e)} - \frac{LK_{UD}K_d^1 M_e}{2T(2L - K_d^2 M_e)} \right| \tag{8}$$

are the constant. $\|y_2(nT)\| = \|R(e_2(nT))\|$, or $\|y_2(nT)\| \leqslant \|R\| \, |(e_2(nT))|$, which is in the form of (5) with

$$y_2 = \|R\| < \infty \tag{9}$$

where $\|R\| := \sup_{v_1 \neq v_2, n \geqslant 0} \frac{|R(v_1(nT)) - R(v_2(nT))|}{|v_1(nT) - v_2(nT)|}$ is the operator norm for the given R. This is the gain of the given nonlinear process.

Using (7) and (9), and the small gain theorem produces the following sufficient condition for the BIBO stability of the nonlinear FP + FI + FD controller in the region IC I & IC III:

1) $\|R\| < \infty$;

2) $\left| \frac{LK_{UP}(K_p^2 - K_p^1)}{2(2L - K_p^1 M_e)} + \frac{LK_{UI}(TK_i^1 + K_i^2)}{2T(2L - K_i^1 M_e)} + \frac{LK_{UD}(TK_d^2 - K_d^1)}{2T(2L - K_d^2 M_e)} \right| \times \|R\| < 1$.

Similarly, the BIBO stability conditions for other input combination regions, i.e., for IC I to ICX II may be obtained. It has been observed that expression α_1 and gain of subsystem g_1 (PP+FI+FD controller) are different indifferent IC regions, as shown in Table 2. Therefore, the above results are in the form of Theorem 1.

Theorem 1. The sufficient conditions for nonlinear FP + FI + FD control system to be stable are:

1) The nonlinear process under control has a bounded norm (gain) i.e., $\|R\| < \infty$;

2) The parameters of FP + FI + FD controller satisfy

$$\alpha_1 \|R\| < \infty \tag{10}$$

where α_1 is given in Table 2.

4 Illustrative example 1

Generally, processes are nonlinear in nature. Therefore, a nonlinear process is considered as

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = 0.0001\,|y(t)| + u(t) \tag{11}$$

Table 2 Gain α_1 of subsystem g_1 (FP + FI + FD controller) in the IC regions

IC#	Value of α_1
IC I & IC III	$\left\lvert \frac{LK_{UP}(K_p^2 - K_p^1)}{2(2L - K_p^1 M_e)} + \frac{LK_{UI}(TK_i^1 + K_i^2)}{2T(2L - K_i^1 M_e)} + \frac{LK_{UD}(TK_d^2 - K_d^1)}{2T(2L - K_d^2 M_e)} \right\rvert$
IC II & IC IV	$\left\lvert \frac{LK_{UP}(K_p^2 - K_p^1)}{2(2L - K_p^2 M_{\Delta e})} + \frac{LK_{UI}(TK_i^1 + K_i^2)}{2T(2L - K_i^2 M_r)} + \frac{LK_{UD}(TK_d^2 - K_d^1)}{2T(2L - K_d^1 M_r)} \right\rvert$
IC V	$\left\lvert \frac{K_{UP}K_p^2}{2} + \frac{K_{UI}K_i^2}{2T} - \frac{K_{UD}K_d^1}{2T} \right\rvert$
IC VI	0
IC VII	$\left\lvert \frac{K_{UI}K_i^1}{2} + \frac{K_{UD}K_d^2}{2} - \frac{K_{UP}K_p^1}{2} \right\rvert$
IC VIII	0
IC IX	$\left\lvert \frac{K_{UP}K_p^2}{2} + \frac{K_{UI}K_i^2}{2T} - \frac{K_{UD}K_d^1}{2T} \right\rvert$
IC X	0
IC XI	$\left\lvert \frac{K_{UI}K_i^1}{2} + \frac{K_{UD}K_d^2}{2} - \frac{K_{UP}K_p^1}{2} \right\rvert$
IC XII	0

where $M_r = \sup_{n\geqslant 1} |r(nT)| = \sup_{n\geqslant 1} \frac{1}{T} |e(nT) - e(nT-T)| \leqslant \frac{2}{T} M_e$, $M_{\Delta e} := \sup_{n\geqslant 1} |\Delta e(nT)| = \sup_{n\geqslant 1} |e(nT) - e(nT-T)| \leqslant 2M_e$.

Table 3 Values of tuned fuzzy parameters for nonlinear process

Nonlinear process	Fuzzy P controller			Fuzzy I controller			Fuzzy D controller		
	K_p^1	K_p^2	K_{UP}	K_i^1	K_i^2	K_{UI}	K_d^1	K_d^2	K_{UD}
$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = 0.0001\,\lvert y(t)\rvert + u(t)$	0.3	1	0.1	0.1	0.2	0.1	2	0.1	40#

For simulation, adjustable parameter $L = 700$.

Table 4 Transient-response specifications for step change in set-point for nonlinear process

Process	Type of controller	Settling time t_s (s) 2%	Overshoot (%)	Rise time # t_r(s)	ISE*	IAE*
$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = 0.0001\,\lvert y(t)\rvert + u(t)$	Conv. PID	2.5	16.980	0.3	0.174076	0.420647
	FP + FI + FD	0.1	–	0.1	0.10027	0.458072

*ISE and IAE values are calculated for time $t = 0$ to $t = 500$ s with $\Delta t = 0.1$ s. # t_r is 90% of its final value.

The preferences of performance criteria, for the present work, are minimizing the settling time, overshoot, integrated absolute error (IAE), integrated squared error (ISE) and rise time.

The FP + FI + FD controller and conventional PID controller are tuned manually to obtain the optimum values of the parameters so that the performance criteria are met. The attributes of FP + FI + FD controller are listed in Table 3. In the simulation loop, Runge-Kutta 1 (Euler), ordinary differential equation (ODE) is used with a loop time of 0.1 s.

4.1 Transient response

For a nonlinear process, a unit step signal is considered as a reference signal. The set-point tracking response is shown in Fig. 5. Transient-response specifications for step change in set-point are compared in Table 4. It can be observed that the performance of FP + FI + FD controller is excellent compared to conventional PID controller for the nonlinear processes.

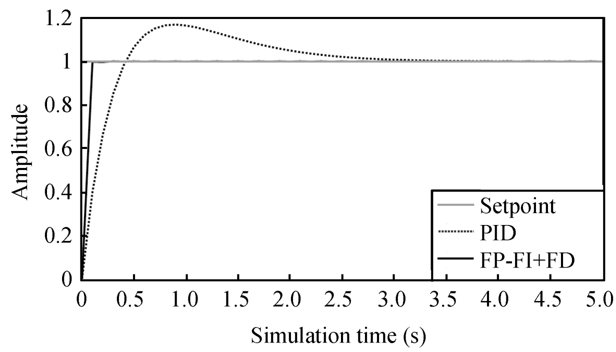

Fig. 5 Setpoint tracking response of nonlinear process

4.2 Disturbance rejection

To study the disturbance rejection behavior of controllers, disturbance is introduced at the input and output of the nonlinear process. A ramp input of magnitude 3.2 for nonlinear process for a time period of 0.5 s is suddenly introduced at the input to the process, when the system is already in steady state situation, shown in Fig. 6. The disturbance rejection responses are shown in Fig. 6. It has been observed that FP + FI + FD controller shows much superior disturbance rejection as compared to conventional controller. FP + FI + FD controller takes 0.1 s compared to 2.3 s taken by PID controller to reach in the 2 % band of the final value.

Also, a unit step input is introduced at the output of the nonlinear process at a time of 12 s from the starting point. The disturbance rejection responses are shown in Fig. 7. It can be observed that the response of the FP + FI + FD controller is remarkable against the disturbance at output to the process. FP + FI + FD controller takes 0.1 s compared to 2.5 s for PID controller to reach the 2% band of the final value.

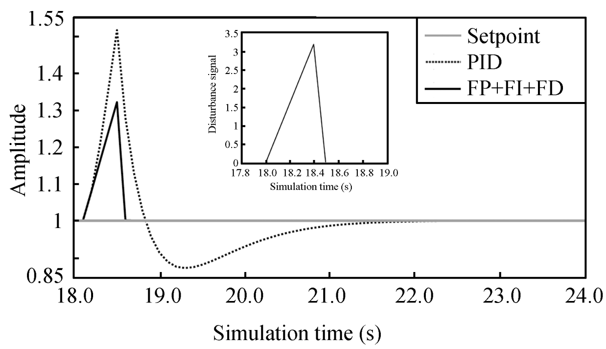

Fig. 6 Controllers′ responses against the disturbance at input to the nonlinear process

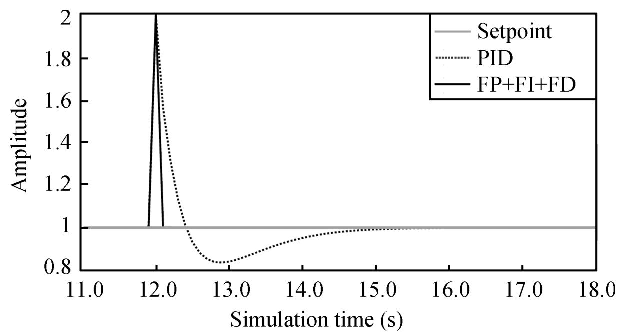

Fig. 7 Controllers′ responses against the disturbance at output to the nonlinear process

4.3 Noise suppression

For noise suppression, a random signal varying in the range of 5%, 10% and 15% of the reference signal is added in the process variable. During this study, the parameters of the conventional and fuzzy controllers are not changed. The step response for 10% random noise are shown in Fig. 8. The changes in the integral square of error (ISE) and integral of absolute error (IAE) due to the noise are compared in Table 5. It can be observed that parallel FP + FI + FD controller suppresses noise much better compared to conventional controller. From Table 5, it has been observed that for nonlinear process, ISE and IAE increase as the percentage of random noise added in the process variable increases.

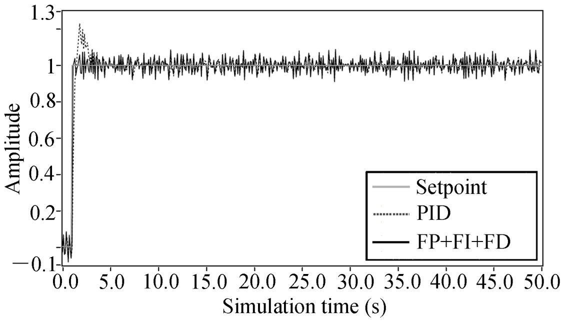

Fig. 8 The step response for 10% random noise for nonlinear process

Table 5 Comparison of ISE and IAE for random noise for nonlinear system

Controller	ISE	IAE
PID (without noise)	0.174076	0.420647
PID 5% noise	0.300134	7.19294
PID 10% noise	0.735062	14.346
PID 15% noise	1.39997	21.1175
FP + FI + FD (without noise)	0.10027	0.458072
FP + FI + FD 5% noise	0.310333	8.34284
FP + FI + FD 10% noise	0.975355	17.1174
FP + FI + FD 15% noise	1.99227	25.2831

*ISE & IAE values are calculated for time $t = 0$ to $t = 500\,\mathrm{s}$ with $\Delta t = 0.1\,\mathrm{s}$.

5 Illustrative example 2

Further, to critically check the set-point tracking capability of parallel FP + FI + FD controller, a single-link robot arm is considered for study, as shown in Fig. 9. where the mass of the rod "m" is 1 kg, the length of the rod "l" is 1 m and g is $10\,\mathrm{m}^2/\mathrm{s}$. The equation of motion of the single-link robot arm is

$$\frac{\mathrm{d}^2\theta(t)}{\mathrm{d}t^2} = -10\sin\theta(t) - 2\frac{\mathrm{d}\theta(t)}{\mathrm{d}t} + u(t) \qquad (12)$$

where $\theta(t)$ is the angle of the arm, and $u(t)$ is the torque supplied by the DC motor[33]. The FP + FI + FD controller is tuned manually to obtain the optimum values of the parameters to get perfect set-point tracking response. The attributes of FP + FI + FD controller are listed in Table 6. Also, in the simulation loop, Runge-Kutta 1 (Euler) ODE is used with a loop time of 0.1 s.

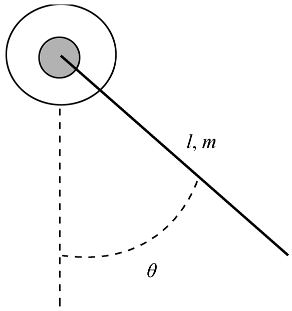

Fig. 9 A single-link robot arm[33]

Table 6 Values of tuned fuzzy parameters for single-link robot arm

Fuzzy P controller			Fuzzy I controller			Fuzzy D controller		
K_p^1	K_p^2	K_{UP}	K_i^1	K_i^2	K_{UI}	K_d^1	K_d^2	K_{UD}
0.01	4	4	0.2	0.03	8.4	1	0.1	2#

#For simulation, adjustable parameter $L = 700$.

It has been observed that parallel FP + FI + FD controller successfully tracks the reference trajectory for single-link robot arm as shown in Fig. 10.

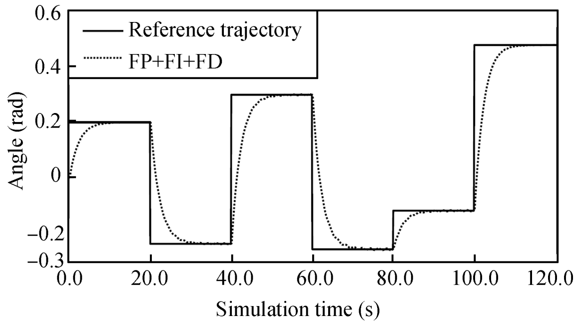

Fig. 10 Tracking control of a single-link robot arm

6 Conclusions

In this paper, BIBO stability of a nonlinear parallel FP + FI + FD controller is analyzed. The parallel FP + FI + FD controller is a nonlinear controller and has variable gains. Small gain theorem is used to study the BIBO stability of parallel FP + FI + FD controller. Sufficient conditions for BIBO stability of nonlinear parallel FP + FI + FD control system are derived for each input combination region. Also, for nonlinear process, FP + FI + FD controller shows much better set-point tracking, disturbance rejection and noise suppression capabilities.

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