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A Stackelberg Game for Spectrum Leasing in Cooperative Cognitive Radio Networks

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Abstract: According to the property-rights model of cognitive radio, primary users (PUs) who own the spectrum resource have the right to lease part of spectrum to secondary users (SUs) in exchange for appropriate profit. In this paper, we propose a pricing-based spectrum leasing framework between one PU and multiple SUs. In this scenario, the PU attempts to maximize its utility by setting the price of spectrum. Then, the selected SUs have the right to decide their power levels to help PU′s transmission, aiming to obtain corresponding access time. The spectrum leasing problem can be cast into a stackelberg game, where the PU plays the seller-level game and the selected SUs play the buyer-level game. Through analysis based on the backward induction, we prove that there exists a unique equilibrium in the stackelberg game with certain constraints. Numerical results show that the proposed pricing-based spectrum leasing framework is effective, and the performance of both PU and SUs is improved, compared to the traditional mechanism without cooperation.

Keywords: Cognitive radio, spectrum leasing, power control, cooperative transmission, pricing, stackelberg game.

1 Introduction

In recent years, the scarcity of radio spectrum is becoming a serious problem, and it is mainly due to the inefficiency of traditional fixed spectrum allocation policies[1]. Cognitive radio has been recognized as a promising technology for dynamic spectrum usage. It can intelligently learn from the real-time environment and flexibly adapt to the transmission parameters. In order to improve the spectrum utilization, dynamic spectrum access is proposed as a promising approach, which allows the secondary users (SUs) to dynamically access the licensed bands from the primary users (PUs) in an opportunistic or a negotiated manner. Among the different debated positions, two main approaches to cognitive radio have emerged[2−4]:

1) Commons model: According to this framework, PUs are oblivious to the presence of SUs, thus behaving as if no secondary activity is present. SUs, instead, sense the radio environment in search for spectrum holes (portions of the bandwidth where PUs are not active) and then exploit the detected transmission opportunities.

2) Property-rights model: PUs own the spectral resource and decide to lease part of it to SUs in exchange for appropriate remuneration.

The existing works on dynamic spectrum access mainly deal with the commons model. However, the property-rights model based on cooperative transmission[5] has seldom been analyzed. The performance of cooperative communication depends on careful resource allocation such as relay placement, relay selection, and power control. Li et al.[6] investigated joint relay selection and power allocation to maximize the system throughput with limited interference to PUs in cognitive radio networks. Qiang and Zarakovitis[7] proposed a joint channel and power allocation algorithm for spectrum sharing in an orthogonal frequency division multiple access (OFDMA) system. Ibrahim et al.[8] investigated the relay selection problem focusing on when to cooperate and which relay to cooperate with, but the channel state information (CSI) was needed. However, these works implement resource allocation in a centralized fashion. In such schemes, in order to optimize the system performance, complete and precise CSI is required to be available, so they are neither scalable nor robust to channel estimation errors.

For distributed resource allocation[9], CSI is not needed. And game theory is an effective method to study the behavior of users in distributed schemes[10]. Moreover, pricing mechanism was introduced to improve the efficiency of nash equilibrium[11, 12]. Power control based on pricing was used to optimize the PU′s utility[13]. Gao et al.[14] studied the spectrum trading with PU and SUs and modeled the trading process as a monopoly market. Zhang et al.[15] optimized the utilities of PU and SUs based on stackelberg game, and provided a payment mechanism to divide the access time among multiple SUs. However, the SUs′ payments were not the actual material compensation and their cooperative powers were set to be fixed. In [16, 17], the PU′s transmission rate was maximized based on game theory. With a signal-to-noise rate (SNR) constraint, a distributed power control algorithm was proposed to allow all SUs to access the channel at the same time. In [18], a game-theoretical framework was proposed to investigate a power-adaptive cooperation mechanism in cognitive radio networks. However, the SUs′ performance requirement and power consumption for transmitting its own information

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were not taken into consideration.

In our model, we focus on distributed resource allocation in the property-rights model, and propose a pricing-based spectrum leasing framework, which enables the exchange between spectrum and power in cognitive radio networks. Specifically, by helping the PU′s data transmission as cooperative relays, the selected SUs gain access time to the spectrum owned by the PU. Moreover, we design a payment mechanism to divide the access time among multiple SUs fairly and effectively. The main contributions of this paper are as follows:

1) We propose a novel pricing model for spectrum leasing in secondary spectrum market, and the pricing model can be considered as a mechanism enabling the exchange between spectrum and power. The PU attempts to maximize its utility by setting the price of spectrum. Then, the selected SUs have the right to decide their power levels to help PU′s transmission, aiming to obtain corresponding access time to the spectrum.

2) We employ a stackelberg game to optimize the benefits of both PU and SUs. Specifically, the game is divided into two levels. The PU plays the seller-level game, and by setting an optimal price for spectrum, it attracts SUs to employ higher power levels and maintains enough transmission time for itself at the same time. SUs play the buyer-level game, each SU purchases a proper portion of spectrum access time to maximize its utility.

3) We prove that there is a unique equilibrium in the proposed stackelberg game, and we derive the corresponding optimal strategies of PU and SUs at stackelberg equilibrium (SE). Besides, numerical results show that our novel pricing model for spectrum leasing is effective, and the performance of both PU and SUs is improved, compared to the traditional mechanism without cooperation.

The rest of the paper is organized as follows. Section 2 describes the system model and gives the utility function for PU and SUs, respectively. The detail analyses for the game model are presented in Section 3. Section 4 describes an implementation protocol for the proposed game model. Simulation results are shown in Section 5. Finally, conclusions are drawn in Section 6.

2 System model and utility function

2.1 System model

In this section, we describe the system model for spectrum leasing and give the basic parameters in cooperative cognitive radio networks.

The system model is shown in Fig. 1, where a primary transmitter (PT) communicates with the intended primary receiver (PR). In the same bandwidth, a group of SUs, which are composed of K pairs of secondary transmitters $\{ST_i\}_{i=1}^K$ and secondary receivers $\{SR_i\}_{i=1}^K$, are seeking to exploit possible transmission opportunities. In order to improve the quality of communication, the PT leases the channel to a relay set S which is composed of k active SUs in exchange for cooperation. In our paper, no traffic requirement is imposed, thus PU and SUs do their best to transmit data as much as possible. In particular, the PU decides whether to use the entire time for direct transmission or to employ cooperation. In the latter case, the PU determines price c for its spectrum, then the selected SUs decide the amount of access time to buy. Moreover, the SU′s access time t_i is related to its cooperative power level P_i, i.e.,

$$ct_i = P_i G_{i,p} \tag{1}$$

where t_i denotes the amount of time purchased from the PU by ST_i, P_i denotes the relay power of ST_i used for cooperative transmission, which is limited by a maximum value $P_i^{\max}$, and $G_{i,p}$ denotes the channel gain between ST_i and PR.

In this paper, we study spectrum leasing in exchange for cooperation from the SUs through distributed space-time coding[19]. In our model, one time slot is divided into two parts: a fraction of the slot dedicated to PU broadcasting its data to the SUs, and PR is of duration $1-T$ $(0 \leqslant T \leqslant 1)$ (Fig. 1 (a)), while the rest T unit time of slot is used for SUs′ transmission. In the T unit time, we introduce a parameter α, dividing the fraction of slot into two parts. The fraction $0.5T + \alpha$ of the slot is used for the selected SUs relaying their received data to PT (Fig. 1 (b)), while in the last $0.5T-\alpha$ fraction of slot, the selected SUs access the channel based on time-division multiplexing access (TDMA) model, and transmit their own data (Fig. 1 (c)). We assume that the channel is modeled as independent Gaussian random variables, and the channel condition between two nodes is invariant in a slot, but generally varying between slots (i.e., Rayleigh block-fading channels). Without loss of generality, we assume that the noise power for all the links is the same, denoted by σ^2.

Considering SNR at PR caused by the direct transmission of PT, we have

$$\Gamma_{\rm dir} = \frac{P_p G_p}{\sigma^2} \tag{2}$$

where P_p denotes the power of PT used for its own transmission, and G_p denotes the channel gain between PT and PR.

In the case of cooperative transmission, we assume that the selected SUs and the PT are distributed via space-time coding cooperation[19]. Then the PU′s SNR with relays′ help can be denoted by[13],

$$\Gamma_{\rm coop} = \Gamma_{\rm dir} + \sum_{i\in S}\Gamma_i = \frac{P_p G_p}{\sigma^2} + \sum_{i\in S}\frac{P_i G_{i,p}}{\sigma^2}. \tag{3}$$

The transmission rate of an SU can be calculated directly by the SNR received at the corresponding SR as

$$R_i = W\log_2\left(1 + \frac{P_s G_i}{\sigma^2}\right),\quad i\in S \tag{4}$$

where W is the channel bandwidth, P_s denotes the power of SUs for their own transmission, and G_i denotes the channel gain between ST_i and SR_i. For simplicity, W will be set to be 1 in the following discussions.

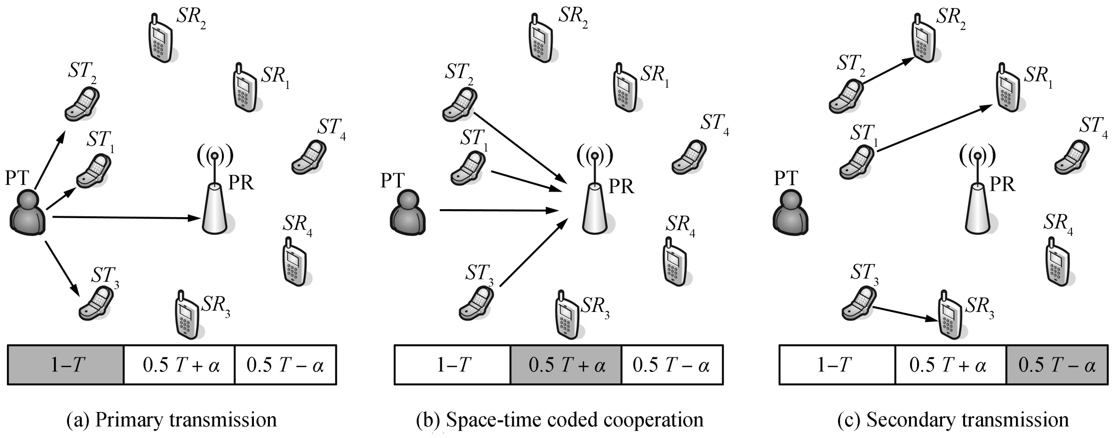


Fig. 1 System model for cooperative spectrum leasing in cognitive radio networks

2.2 Utility function

In this section, we first define the utility function for PU and SUs. Then, according to the behaviors of the PU and SUs, we formulate the pricing-based spectrum leasing problem as a stackelberg game.

Primary user/seller. The PU can be modeled as a seller, by setting an optimal price for the spectrum. It aims to not only attract SUs to employ higher power levels for its transmission, but also maintain enough transmission time for itself to obtain more benefits. Inspired by [18], the utility function of PU is defined as

$$\begin{aligned} U_p = \ & \omega_p t_p \left(\Gamma_{dir} + \sum_{i\in S} \Gamma_i \right) = \\ & \omega_p \left(1 - \frac{T}{2} + \alpha\right) \left(\frac{P_p G_p}{\sigma^2} + \sum_{i\in S} \frac{P_i G_{i,p}}{\sigma^2} \right) = \\ & \frac{\omega_p (1 - \sum\limits_{i\in S} t_i)(P_p G_p + c \sum\limits_{i\in S} t_i)}{\sigma^2} \end{aligned} \tag{5}$$

where ω_p is a predefined parameter, and describes the equivalent revenue per unit SNR contributed to the overall utility[15−18], and t_p is the time for primary transmission.

The PU′s strategy is to choose c and a set of SUs S as its relays. If c is too large, the time purchased by SUs will be small, and the SUs will make less effort for cooperation, i.e., $\Gamma_{\rm coop}$ will be relatively small. On the other hand, if c is too small, the utility of PU will also be low. Then, the seller-level game can be formulated as

$$\max_c U_p = \max_c \frac{\omega_p (1 - \sum\limits_{i\in S} t_i)(P_p G_p + c \sum\limits_{i\in S} t_i)}{\sigma^2}. \tag{6}$$

Secondary users/buyers. Each selected SU can be considered as a buyer and aims to earn the utility that not only covers its energy cost but also gains as many extra profits as possible. Therefore, the utility function of each SU is a tradeoff between its achieved data transmission rate and power cost. We define its utility function with two parts: profit and cost

$$U_i = \omega_s R_i t_i - \left[P_s t_i + P_i \left(\frac{T}{2} + \alpha \right) \right] \tag{7}$$

where ω_s is also a predefined parameter, and describes the equivalent revenue per unit data rate contributed to the overall utility.

In addition, the utility function of each SU can be interpreted as: On one hand, the more access time is purchased, the more profit is obtained; On the other hand, with the increasing of the access time, the SU will definitely increase the energy cost. Therefore, each SU needs to find the optimal channel access time t_i to maximize its utility. Then, the buyer-level game is defined as

$$\begin{aligned} & \max_{t_i} U_i = \max_{t_i} \left[\omega_s R_i t_i - P_s t_i - P_i \left(\frac{T}{2} + \alpha \right) \right] \\ & \text{s.t.} \quad 0 < t_i \leqslant 1, \forall i \in S. \end{aligned} \tag{8}$$

Note that if an SU does not intend to access the channel, it can easily refuse to help the PU by setting $P_i = 0$.

According to the above analysis, (6) and (8) form a stackelberg game. The objective of this game is to find the SE point(s) from which neither the leader (PU) nor the followers (SUs) have incentives to deviate[20].

3 Game theory analysis

Based on the defined utilities and the above game formulation, we analyze the game in detail.

Generally, the SE for a stackelberg game can be obtained by finding its subgame perfect Nash equilibrium (NE). In the proposed game, it is not difficult to see that the SUs strictly compete in a non-cooperative fashion. Therefore, a non-cooperative time level selection subgame is formulated at the SUs′ side. For a non-cooperative game, NE is defined as the operating point at which no player can improve utility by changing its strategy unilaterally, assuming the other player continues to use its current strategy. At the PU′s side, since there is only one player, the best response of the PU can be readily obtained by solving (6). To achieve this, the best response functions for the followers (SUs) must be obtained first, since the leader (PU) derives its best response function based on those of the followers. For the proposed game in this paper, the SE can be obtained as follows: For a given value of c, (8) is solved first. Then, with the obtained best response functions t^* of the SUs, we solve (6) for the optimal spectrum price c^*.

At last, we prove the existence and uniqueness of the SE.

3.1 Optimal strategies of secondary users

Given the relay set S and spectrum price c decided by PU, each SU in the relay set S aims to maximize its own utility by competitively purchasing an optimal channel access time t_i. Therefore, a non-cooperative time level selection game can be formulated as $G = [S, \{T_i\}, \{U_i\}]$, where S is the player set, T_i is the strategy set of SU_i, and U_i is the utility of SU_i. Each SU_i selects its strategy within the strategy space $T_i = [t_i]_{i\in S}, 0 < t_i \leqslant 1$ to maximize its utility function $U_i(t_i, t_{-i})$.

To prove the existence of this non-cooperative game′s NE, we introduce the following proposition[21].

Proposition 1. An NE exists in game $G = [S, \{T_i\}, \{U_i\}]$, for all $i \in S$:

1) T_i is a nonempty convex, and compact subset of some Euclidean space $\mathbf{R}^k$.

2) $U_i(t)$ is continuous in t and concave in t_i.

Theorem 1. An NE exists in the non-cooperative time level selection game $G = [S, \{T_i\}, \{U_i\}]$.

Proof. Strategy space is defined to be $T_i = [t_i]_{i\in S}, 0 < t_i \leqslant 1$, and it is a nonempty, convex and compact subset of the Euclidean space $\mathbf{R}^k$.

From (7), we can see that $U_i(t)$ is continuous in t. Now we take the second order derivative with respect to t_i to prove its concavity.

$$\frac{\partial U_i}{\partial t_i} = \frac{-2ct_i - c\sum\limits_{j\in S, j\neq i} t_j - 2\alpha c}{G_{i,p}} + \omega_s R_i - P_s \tag{9}$$

$$\frac{\partial^2 U_i}{\partial {t_i}^2} = \frac{-2c}{G_{i,p}} < 0. \tag{10}$$

We can see that the second order derivative of U_i with respect to t_i is always less than 0, thus $U_i(t)$ is concave in t_i.

According to Proposition 1, an NE exists in the non-cooperative time level selection game. □

From (9), we can get SU$'_i$s best-response function by setting the first derivative of U_i with respect to $t_i = 0$.

$$\frac{\partial U_i}{\partial t_i} = \frac{-2ct_i - c\sum\limits_{j\in S, j\neq i} t_j - 2\alpha c}{G_{i,p}} + \omega_s R_i - P_s = 0. \tag{11}$$

By solving (11), we have

$$t_i^* = \frac{\theta_i - 2\alpha c - c\sum\limits_{j\in S, j\neq i} t_j}{2c},$$
$$\text{if } \frac{\theta_i - (2\alpha+1)c}{c} < \sum_{j\in S, j\neq i} t_j \leqslant \frac{\theta_i - 2\alpha c}{c}. \tag{12}$$

To facilitate the discussion, we define the selected SU$'_i$s type as $\theta_i = \omega_i R_i G_{i,p} - P_s G_{i,p} > 0$, which captures all private information of the SU. A large value of θ_i means that the SU′s own data transmission is efficient (a large transmission rate R_i), or it has good channel condition over relay link ST_i-PR (a large channel gain $G_{i,p}$).

Theorem 2. The NE of the non-cooperative time level selection game is unique.

Proof. By Theorem 1, it is shown that an NE exists in the non-cooperative game. Therefore, the NE satisfies $t = I(t) = (I_1(t), I_2(t), \cdots, I_k(t))$, where $I_i(t)$ is the best-response function of SU$_i$, given the other SUs′ selected strategies t_{-i}. The key aspect of the uniqueness is to prove that the best-response correspondence $I_i(t)$ is a standard function. A function is said to be standard if it satisfies the following properties:

1) Positivity: Given the constraint $\sum_{j\in S, j\neq i} t_j \leqslant \frac{\theta_i - 2\alpha c}{c}$, the best-response function is always positive.

2) Monotonicity: Suppose t and t' are different time vectors, and the vector inequality $t \geqslant t'$ means that $t_i \geqslant t'_i$, $\forall i \in \{1, 2, \cdots, k\}$. If $\forall i \neq j$, $i, j \in \{1, 2, \cdots, k\}$, there are $I_j([t_1, \cdots, t_i, \cdots, t_j, \cdots, t_k]) \leqslant I_j([t_1, \cdots, t'_i, \cdots, t_j, \cdots, t_k])$ and $I_i([t_1, \cdots, t_i, \cdots, t_j, \cdots, t_k]) \leqslant I_i([t_1, \cdots, t'_i, \cdots, t_j, \cdots, t_k])$, then the monotonicity can be shown to hold. Therefore, the problem reduces to prove $\frac{\partial I_j(t)}{\partial t_i} \leqslant 0$ and $\frac{\partial I_i(t)}{\partial t_i} \leqslant 0$. According to (16), we have $\frac{\partial I_i(t)}{\partial t_i} = 0$ and $\frac{\partial I_j(t)}{\partial t_i} = -\frac{1}{2}$, thus monotonicity holds for the best-response function.

3) Scalability: for all $\beta > 1$, we have $\beta I_i(t) - I_i(\beta t) = \frac{(\theta_i - 2\alpha c)\beta - c\beta\sum_{j\in S, j\neq i} t_j}{2c} - \frac{\theta_i - 2\alpha c - c\beta\sum_{j\in S, j\neq i} t_j}{2c} = \frac{(\theta_i - 2\alpha c)(\beta - 1)}{2c} > 0$.

In conclusion, the best-response function $I_i(t)$, which is positive, monotonic and scalable, is a standard function. Therefore, there exists a unique NE point for non-cooperative game $G = [S, \{T_i\}, \{U_i\}]$.

Then, we can obtain the unique NE of the non-cooperative game by solving (11) for SU_i, $i \in S$:

$$t_i^* = \frac{(1+k)\theta_i - X - 2\alpha c}{(1+k)c} \tag{13}$$

where k is the number of relays, i.e., $k = |S|$, and $X = \sum_{i\in S} \theta_i$. Since the channel access time has corresponding restriction, i.e., $0 < t_i^* \leqslant 1$, we have the following constraint from (13):

$$\frac{2\alpha c + X}{(1+k)} < \theta_i \leqslant \frac{(1+k+2\alpha)c + X}{(1+k)}. \tag{14}$$

This constraint will be used to select cooperative relay set S.

3.2 Optimal strategies of the primary user

Based on the analytical results of the SUs′ channel access time selection game, the leader of the stackelberg game, PU, chooses its optimal price to maximize its utility according to (6). By substituting (13) into (5), we can obtain

$$U_p = \frac{\omega_p(1 - \sum_{i\in S} t_i^*)(P_p G_p + c\sum_{i\in S} t_i^*)}{\sigma^2}. \tag{15}$$

Corollary 1. The minimum and maximum price is $c^{\min} = \frac{P_p G_p(1+k) + X}{1+k+2\alpha k}$ and $c^{\max} = \sum_{i\in S}(P_i^{\max} G_{i,p})$, respectively.

Proof. In reality, we can see that the necessary condition for PU to lease spectrum for cooperation is $U_{\rm dir} \leqslant U_{\rm coop}$, where $U_{\rm dir} = \omega_p \Gamma_{\rm dir}$ denotes the utility of PU in the case of direct transmission (i.e., traditional mechanism without cooperation), and $U_{\rm coop} = \omega_p t_p \Gamma_{\rm coop}$ denotes the utility of PU in the case of cooperative transmission. According to

(2) and (3), we get

$$c \geqslant \frac{P_p G_p (1+k) + X}{(1+k+2\alpha k)}. \tag{16}$$

Besides, the PU cannot increase the spectrum price without constraint, due to that the SUs′ cooperation ability is limited. Then we obtain

$$c \leqslant \sum_{i \in S} (P_i^{\max} G_{i,p}). \tag{17}$$

□

As is mentioned above, the optimization problem for PU is to maximize its utility through the selection of spectrum price c^*. Therefore, by the first derivative of U_p with respect to c, we have

$$\frac{\partial U_p}{\partial c} = \frac{\omega_p}{\sigma^2} \left[\frac{Z - Yc^2}{(1+k)^2 c^2} \right] \tag{18}$$

where $Y = 4\alpha^2 k^2 + 2\alpha k^2 + 2\alpha k$, and $Z = X^2 + P_p G_p X(1+k)$. In the following, we will analyze the optimal spectrum price c^* in detail.

Case 1. $Y \leqslant \frac{Z}{(c^{\max})^2}$, i.e., $\alpha \leqslant \frac{1}{2}\sqrt{\frac{(1+k)^2 (c^{\max})^2 + 4Z}{4k^2 (c^{\max})^2}} - \frac{k+1}{4k}$.

In this case, no matter what value c takes, U_p monotonously increases with c. Therefore, the PU will choose the maximum price $c^{\max}$ so as to increase its utility as much as possible. The reason that leads the PU to such an extreme decision is that the utility improved by increasing transmission time is not comparable with that by increasing SUs′ cooperative transmission power levels. Then, according to (17), the optimal price c^* can be set as

$$c_1^* = c^{\max} = \sum_{i \in S} (P_i^{\max} G_{i,p}). \tag{19}$$

Case 2. $\frac{Z}{(c^{\max})^2} < Y < \frac{Z}{(c^{\min})^2}$, i.e., $\frac{1}{2}\sqrt{\frac{(1+k)^2 (c^{\max})^2 + 4Z}{4k^2 (c^{\max})^2}} - \frac{k+1}{4k} < \alpha < \frac{1}{2}\sqrt{\frac{(1+k)^2 (c^{\min})^2 + 4Z}{4k^2 (c^{\min})^2}} - \frac{k+1}{4k}$.

In this case, the time used by SUs to relay data for PU (Fig. 1 (b)) is more than that for their own data transmission (Fig. 2 (c)). Therefore, the PU′s data transmission time is relatively more. With the increase of c, U_p is an increasing function of c at first, then, it is a decreasing function. The PU will choose a proper price c to attract SUs to employ higher power levels and maintain enough transmission time for itself at the same time, and the optimal price c^* is given by

$$c_2^* = \sqrt{\frac{X^2 + P_p G_p X(1+k)}{Y}}. \tag{20}$$

Case 3. $Y \geqslant \frac{Z}{(c^{\min})^2}$, i.e., $\alpha \geqslant \frac{1}{2}\sqrt{\frac{(1+k)^2 (c^{\min})^2 + 4Z}{4k^2 (c^{\min})^2}} - \frac{k+1}{4k}$.

In this case, no matter what value c takes, U_p is monotonously decreases with c. Therefore, the PU will select minimum price $c^{\min}$ so as to increase its utility as much as possible. The reason that leads the PU to such an extreme decision is that the PU′s utility does not significantly improve through cooperative transmission. To be honest, the PU would prefer direct transmission. Then, the optimal price c^* can be set according to (16) as

$$c_3^* = c^{\min} = \frac{P_p G_p (1+k) + X}{1+k+2\alpha k}. \tag{21}$$

3.3 Analyses of the stackelberg equilibrium

In this section, we will prove that the solution (13) and (20) can be considered to be an SE[18].

Property 1. For $\forall i \in S$, the optimal channel access time t_i^* decreases with c.

Proof. From (13), we can take the first order derivative of t_i^* with respect to c as

$$\frac{\partial t_i^*}{\partial c} = -\frac{(1+k)\theta_i - X}{(1+k)c^2} < 0. \tag{22}$$

If PU wants to get more cooperative power by setting higher price for the spectrum, i.e., with an increasing c, the selected SUs will have less incentive to take part in the cooperative process, and they will buy less channel access time t_i. □

Property 2. The PU′s utility U_p is concave in c.

Proof. The secondary derivative of U_p with respect to c can be obtained as

$$\frac{\partial^2 U_p}{\partial c^2} = -\frac{2\omega_p Z}{\sigma^2 c^3 (1+k)^2} < 0. \tag{23}$$

So the PU′s utility function U_p is concave in c. □

Theorem 3. t_i, $\forall i \in S$, and c given by (13) and (20) can be considered to be a unique SE for the model used in this paper.

Proof. When PU broadcasts the spectrum price c, the SUs will choose an optimal channel access time t_i as shown in (13). From Theorems 1 and 2, we prove that $(t_1^*, t_2^*, \cdots, t_i^*, \cdots, t_k^*)$ is the unique NE of the non-cooperative game. Therefore, for $\forall t_i' > 0$, $U_i(t_1^*, t_2^*, \cdots, t_i', \cdots, t_k^*) \leqslant U_i(t_1^*, t_2^*, \cdots, t_i^*, \cdots, t_k^*)$, i.e., $(t_1^*, t_2^*, \cdots, t_i^*, \cdots, t_k^*)$ are the optimal strategies for the selected SUs. Also, from Property 2, PU′s utility function U_p is concave in c, then PU can always find the optimal price c^*, i.e., for $\forall c' \in [c^{\min}, c^{\max}]$, $U_p(c') \leqslant U_p(c^*)$. Therefore, t_i, $\forall i \in S$, and c given by (13) and (20) constitute the unique SE for the model used in this paper. □

4 Implementation protocol

In this section, based on the analytical results of the stackelberg game, we propose a cooperation protocol to dynamically select the parameters in the cooperative cognitive radio networks.

We assume that the channels are stable and the channel gains are estimated by the corresponding terminals. In addition, α, ω_p and ω_s are predefined parameters for the system. The optimal price c^* is the function of the PU′s SNR, which is changing all the time with real time channel condition. Meanwhile, c^* is also a function of selected relay set S, which complies with criteria (14).

In order to estimate the real time SNR, PU periodically collects channel conditions (G_p and $G_{i,p}$) from the PR and STs. Based on the calculated SNR, the PU enumerates all the possible cooperative relay sets S, which satisfy the relay set selection criteria (14). Then, based on each possible relay set S, the optimal price parameter $c^*(S)$ and overall utility of the PU, $U_p^*(S)$, can be calculated. From all the possible sets, the one that maximizes PU′s utility function, $S^* = \arg\max_S U_p^*(S)$ is selected to be the optimal relay set, and the optimal price c^* is set to be the corresponding price parameter with the optimal relay set S^*, i.e., $c^* = c(S^*)$.

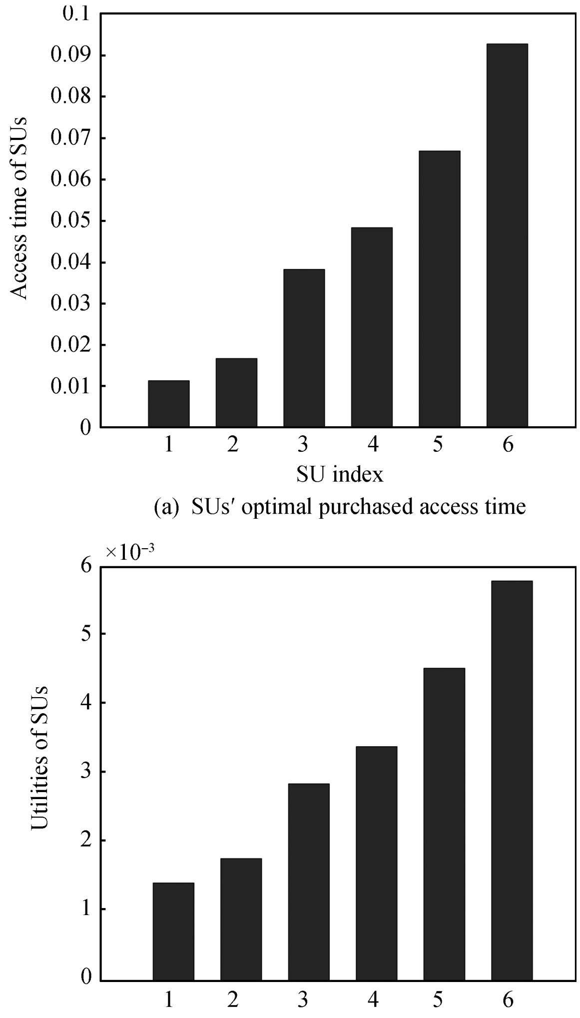
When the optimal price and relay parameters are calculated, the PT broadcasts the value of c^* and S^* to SUs. Then, the SUs can calculate their optimal channel time t_i^*, distributively according to (13). After receiving these parameters, SUs calculate t_i^* distributively according to (13). Notice that t_i depends on c, k, θ_i, and $\sum_{i\in S}\theta_i$, and only the values of c, k, and $\sum_{i\in S}\theta_i$ need to be piggybacked by the PU. After receiving c, k, and $\sum_{i\in S}\theta_i$, each selected SU can successfully calculate its optimal t_i^* distributively. In particular, t_i^* depends on the sum of the selected SUs′ types, instead of each individual type. Therefore, one SU does not need to know the others′ private information, which saves a lot of message exchanges and allows the protocol to be implemented more distributively.

5 Simulation results

In this section, we present some numerical results to show the impacts of system parameters on the optimal cooperative scheme. We consider a geometric model where the PT is located at coordinate (0, 0), the PR is located at coordinate (1, 0), and 10 STs are located randomly on a square centered at (0.5, 0) with side length $D = 1$. The SRs are located randomly on a unit square center on the corresponding STs. The propagation loss factor is set to be 2. The revenue-weighted parameters for PU and SUs are $\omega_p = 0.001$ and $\omega_s = 0.02$, and the noise level is $\sigma^2 = 10^{-4}$. Both PU and SUs transmit at a fixed power level with $P_p = 0.01$ W and $P_s = 0.005$ W, respectively, while the SU′s cooperative power is constrained by $P_i^{\max} = 0.1$ W. Specifically, in order to demonstrate the superiority of spectrum leasing scheme in cognitive radio networks, we set $\alpha = 0.1$.

As described in the implementation protocol in Section 4, in our proposed pricing model, the PU first needs to select the optimal relay set according to the relay selection criteria. There are two factors that affect the relay selection: location of ST and the distance between ST and SR. The location of ST affects the channel condition between SU and PU′s, i.e., the value of $G_{i,p}$. Since an SU with suitable $G_{i,p}$ will be better to help PU′s transmission, and it is also likely be selected as a relay. Meanwhile, the distance between ST and SR affects the value of G_i. A proper G_i will help SU to obtain more utility from cooperation, such SU has more incentive to take part in the cooperation. Based on the above analyses, in the following simulation, we assume that $k = 6$ SUs are selected by the PU as cooperative relays. Without loss of generality, we arrange the selected SUs according to their types: $\theta_1 < \theta_1 < \cdots < \theta_6$.

To study the behaviors of the selected SUs, we focus on the channel access time they purchase and the utilities they obtain. Figs. 2 (a) and 2 (b) show that the SU with larger type will tend to purchase more channel access time and get more revenue from the cooperation at the same time. An SU with large type means that it has good channel condition over relay link ST_i-PR, which will be better for helping PU′s data transmission. And it is also beneficial for the PU to select such SU as relay. Meanwhile, a large type indicates that the SU has good channel condition when it transmits its own data, so it tends to purchase more channel access time to transmit its own data. Therefore, with more access time and higher channel gain, the SU will obtain more utility. These results also comply with the analyses shown in (7) and (13).



(a) SUs′ optimal purchased access time

(b) SUs′ optimal utilities

Fig. 2 SUs′ optimal purchased access time and utilities

Figs. 3 and 4 present the selected SUs′ channel access time purchased and utilities obtained versus spectrum price (between the minimum to maximum), respectively. From Fig. 3, we find that the SU′s channel access time decreases with spectrum price c. This is because that the gradual increase in the price will reduce SU′s enthusiasm for cooperation. In particular, when the spectrum price is too high, the access time of some SUs will become zero (such as SU_1 and SU_2), this means that such SUs will quit from the cooperative game. Meanwhile, from Fig. 4, we can see that the SUs′ utilities also decrease with spectrum price c, and they have almost the same variation tendency as the access time. This is easy to understand from SU′s utility function in (7). Moreover, it is also shown that the SU with larger type tends to purchase more channel access time and obtain more utility with certain spectrum price, which further proves our analyses in Figs. 2 (a) and 2 (b).

Figs. 5 (a) and 5 (b) show PU′s optimal price c^* and optimal utility U_p^* versus the number of relays, respectively. We can see that both PU′s optimal price c^* and U_p^* utility increase with the number of relays. The reason is that, with the increasing of the number of relays, the competition among the SUs becomes intense, and then the PU intends to obtain more revenue by setting a larger price. Meanwhile, the more SUs participated in cooperation, the more cooperative power provided, i.e., the PU benefits from the competition among SUs. If there is only one relay in the network, the relay will have no incentive to help the PU, as there is no competition to access the channel, i.e., the utility of PU is relative smaller.

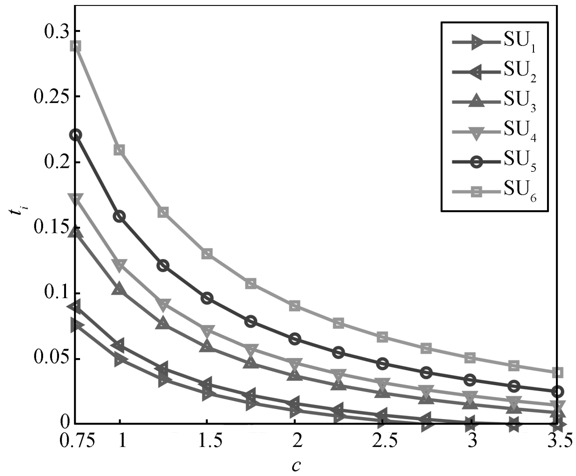


Fig. 3 SU′s purchase access time t_i versus the spectrum price c

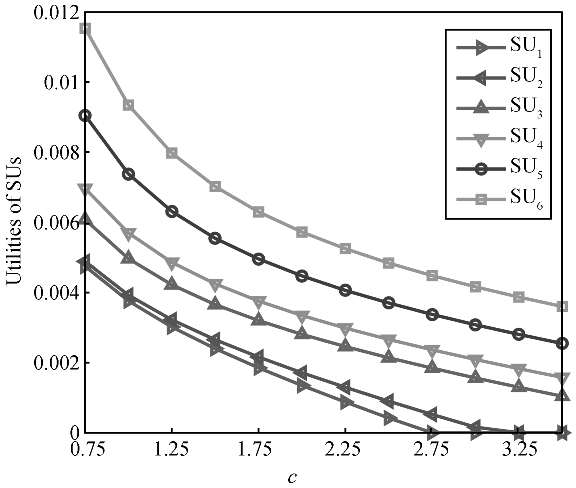
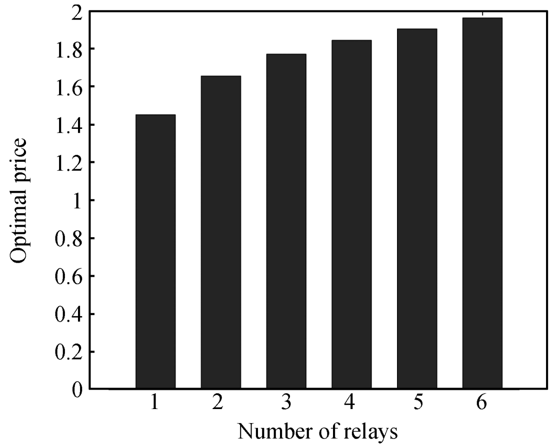
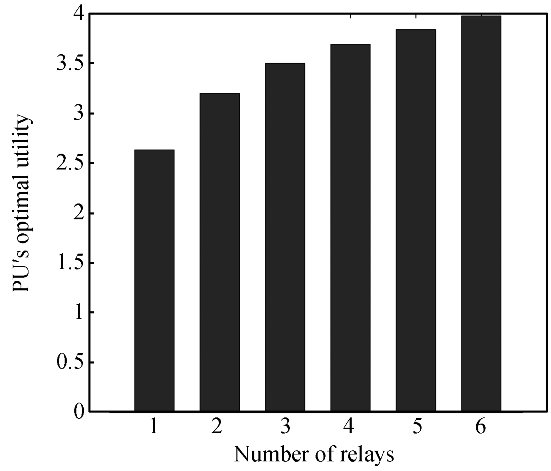


Fig. 4 SUs′ utilities versus the spectrum price c

In order to illustrate the relationship between PU′s price and utility, as well as the superiority of the spectrum leasing scheme compared to the traditional mechanism without cooperation, we present PU′s utility function U_p versus spectrum price c and the number of relays k in Fig. 6. It is shown that when the spectrum price reaches the optimal value, it inspires the best enthusiasm of SUs to participate in cooperation, and the PU′s utility is maximal at the same time. However, if the price continues to increase, the SUs′ enthusiasm of cooperation will gradually reduce for consuming too much energy, then the utility of PU tends to decrease. Moreover, we can see that, the PU′s utility has been significantly improved with the proposed spectrum leasing scheme compared to the traditional transmission without cooperation ($k = 0$). From Fig. 6, we can also see that, the more SUs participating in the cooperation transmission, the more utility PU obtains (at the optimal price), which is consistent with the analyses in Figs. 5 (a) and 5 (b).



(a) PU′s optimal price c^* versus the number of relays k



(b) PU′s optimal utility U_p^* versus the number of relays k

Fig. 5 PU′s optimal price c^* and utility U_p^* versus the number of relays k

6 Conclusions

In this paper, we propose a novel pricing model for spectrum leasing, which can be implemented by enabling the exchange between spectrum and power in cognitive radio networks. First, the PU selects SUs as cooperative relays and leases portion of time slot to SUs for their data transmission. Then, the selected SUs access the channel by cooperating with the PU competitively. Moreover, SUs′ access time is proportional to its cooperation power level, which motivates the cooperation. By formulating the problem as a stackelberg game, we are able to prove that a unique SE exists. At last, numerical results show the effectiveness of the proposed spectrum leasing model based on pricing in detail.

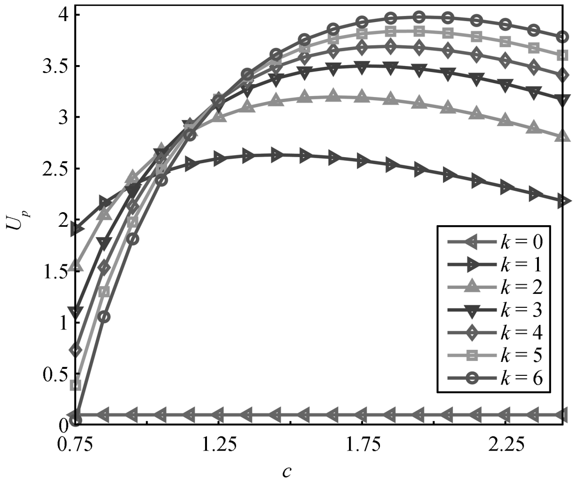


Fig. 6 PU′s utility function U_p versus the spectrum price c and the number of relays k

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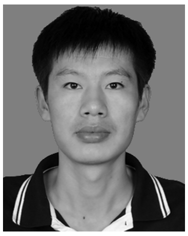

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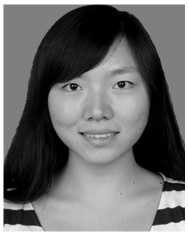

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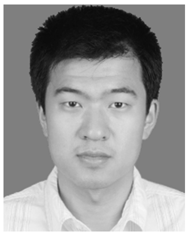

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