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Distributed H_∞ PID Feedback for Improving Consensus Performance of Arbitrary-delayed Multi-agent System

Lin-Lin Ou[1] Jun-Jie Chen[1] Dong-Mei Zhang[1] Lin-Zhang[1] Wei-Dong Zhang[2]
[1]Department of Automation, Zhejiang University of Technology, Hangzhou 310032, China
[2]Department of Automation, Shanghai Jiaotong University, Shanghai, 200240, China

Abstract: The H_∞ proportional-integral-differential (PID) feedback for arbitrary-order delayed multi-agent system is investigated to improve the system performance. The closed-loop multi-input multi-output (MIMO) control framework with the distributed PID controller is firstly described for the multi-agent system in a unified way. Then, by using the matrix theory, the prescribed H_∞ performance criterion of the multi-agent system is shown to be equivalent to several independent H_∞ performance constraints of the single-input single-output (SISO) subsystem with respect to the eigenvalues of the Laplacian matrix. Subsequently, for each subsystem, the set of the PID controllers satisfying the required H_∞ performance constraint is analytically characterized based on the extended Hermite-Biehler theorem. Finally, the three-dimensional set of the decentralized H_∞ PID control parameters is derived by finding the intersection of the H_∞ PID regions for all the decomposed subsystems. The simulation results reveal the effectiveness of the proposed method.

Keywords: Multi-agent system, proportional-integral-differential (PID) controller, H_∞ performance index, parametric space, consensus.

1 Introduction

Rapid development in navigation, communication and computational systems has enabled greater autonomy in multi-agent systems. The main objective is to realize the cooperative control of the multi-agent system. The main advantage of the cooperative control lies in that multiple simple vehicles can replace a single complicated vehicle to finish some complex control tasks flexibly. Study of the multi-agent systems performing cooperative tasks initially began in the field of mobile robotics[1]. Their applications can also be found in car platoons[2], multi-vehicle rendezvous problems, control of unmanned air vehicles[3], cooperative data fusion of multi-sensor networks[4] and intelligent traffic control systems[5].

The centralized and distributed methods are commonly adopted to control multi-agent system. The centralized controller can be easily designed for multi-agent systems based on multi-input multi-output (MIMO) control theory. However, it is difficult to be implemented in practice due to its structural complexity and strict communication requirement for large-scaled system. A central station must be available to provide strong power to control a whole group of agents.

The distributed cooperative control of multi-agent systems has attracted more and more attention. The main advantage of this control structure is that the distributed controller imposed on each agent is implemented only based on the states and outputs of the agent and its neighbors and some inevitable physical constraints, such as limited energy and narrow bandwidths; and it has no influence on the distributed control. Thus, the distributed control method seems more promising than the centralized control method. Recent progress in the study of distributed multi-agent coordination is reviewed in [6].

The consensus generation, which can drive all the agents of the group through a local distributed protocol to reach a common value, is a main research direction in the research of distributed multi-agent coordination. The consensus problems regarding delay effects, convergence speed, complex dynamical systems and sampled-data framework were solved in [7–9]. Various modifications of the consensus protocol lead to more general system theory including input-output properties, controllability and observability[10, 11] A consensus algorithm of multi-agent second-order dynamical systems with nonsymmetrical interconnection and heterogeneous delays was studied in [12]. The leader-following consensus problem for high-order multi-agent linear dynamic systems was considered in [13]. While a lot of results about the analysis of consensus system have been obtained, the distributed control methods for the improvement of the consensus performance have not been deeply studied. Most methods focus on the design of consensus networks for increasing the convergence rate by optimizing the Fiedler eigenvalue[14]. Due to the computational complexity of the consensus network, it is desirable to design the distributed control protocol based on the dynamical output (or state) feedback.

In [15], the distributed controller design methods are proposed in a linear-fractional (LFT) framework. The networked multi-agent system was first transformed into an LFT model subject to constraints on the controller structure, and then the optimal control problem of the LFT model was cast as a convex optimization problem by defining quadratic invariance. Algebraic graph theory has been widely employed in a variety of research work dealing with such systems. Thus, another kind of distributed control methods focuses on the optimal control for the networked

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multi-agent system in terms of overall linear quadratic regulator (LQR) performance. In [16], a stabilizing distributed control design method is presented in terms of the solution of a single local LQR problem. Furthermore, the relationship between stability of the overall large-scale system, the robustness of local controllers and the spectrum of a sparsity pattern matrix has been highlighted. In [17], the coordination of a network of air vehicles is achieved by employing an LQR based on distributed receding horizon control (RHC) scheme. In [18, 19], the large-scale LQR suboptimal control problems were solved for spatially distributed systems. However, these methods become more complicated when the number of the agents increases, or the dynamical model of the agent is not simple first-order or second-order integral model but complex high-order model with time delay. Due to the advantage of the proportional-integral-differential (PID) controllers in control engineering and application, it is desirable to introduce the distributed PID feedback into the consensus system for performance improvement. The design of the distributed controllers with fixed structure (such as PID controllers) for some performance requirement in unifying theoretical framework is challenging, especially for case of directed topology graph (which may lead to complex eigenvalues).

In this paper, the distributed PID feedback is considered to satisfy the required H_∞ performance criterion for the consensus of multi-agent system, in which each agent is identical and has the model of arbitrary-order transfer function with time delay. The H_∞ norm requirement of the overall large-scale system is decomposed into the local performance requirement that each revised subsystem has to be satisfied. Thus, the PID controller parameters, which make each subsystem meet the local performance requirements simultaneously, must satisfy the H_∞ norm requirement of the overall system. Then, the parametric space method applicable to the system model with complex coefficients is introduced to determine the set of the PID control parameters to satisfy the local performance requirement for each subsystem. The PID parameters chosen in the resultant set can all lead to satisfactory system performance.

2 Preliminaries

Represent the communications between the agents by a directed graph $G = \{V, \varepsilon, A\}$, where $V = \{V_1, V_2, \cdots, V_n\}$ is the set of agents, and $A = [a_{ij}] \in \mathbf{R}^{n\times n}$ is the weighted adjacency matrix of G with nonnegative adjacency elements a_{ij}. An edge of G is denoted by $e_{ij} = (V_i, V_j)$. The adjacency elements associated with the edges of the graph are positive, i.e., $e_{ij} \in \varepsilon \Leftrightarrow a_{ij} > 0$. Assume that $a_{ii} = 0$. The neighborhood of agent i is defined as $N_i = \{j \in V | a_{ij} = 0\}$. The cardinal number of N_j denoted by d_j is called the degree of i. The normalized Laplacian matrix of G is defined as $L = D^{-1}(D - A)$, where $D = \mathrm{diag}(d_1, \cdots, d_n)$. Thus, we have

$$L_{ij} = \begin{cases} 1, & \text{if } i = j \\ -\dfrac{1}{d_i}, & \text{if } j \in N_i \\ 0, & \text{if } j \notin N_i. \end{cases} \tag{1}$$

The graph is connected if any two nodes i, j of the graph are connected by a path. Recall that a graph is strongly connected if and only if its Laplacian L has a single zero eigenvalue. The smallest eigenvalue of L is exactly zero and the corresponding eigenvector is given by $1 = \mathrm{Col}(1 \cdots 1)$. The Laplacian L is always ranking deficient and positive semi-definite. Moreover, the rank of L is $n-1$ if and only if G is connected[20]

$A \otimes B$ defines the Kronecker product between the matrices A and B. Let I_k be the $k \times k$ identity matrix. For a set of i matrices $\{M_1, \cdots, M_N\}$ of size $r \times s$, the direct sum is defined as the $Nr \times Ns$ block diagonal matrix $\hat{M}$ whose $r \times s$ diagonal blocks are the matrices $M_1, \cdots, M_N$, and the other entries are zero. Thus, $\hat{M}$ can be written as

$$\hat{M} = \mathrm{diag}(M_1, \cdots, M_N) = \oplus \sum_{i=1}^{N} M_i. \tag{2}$$

For a given $N \times N$ matrix Q, define an $Nk \times Nk$ matrix $Q_{(k)}$ by the equation

$$Q_{(k)} = Q \otimes I_k. \tag{3}$$

Lemma 1[21]**.** Let Q be a $N \times N$ matrix. Then M be $r \times s$ matrix with $\hat{M}$ of size $Nr \times Ns$ such that $M = I_N \otimes M = \mathrm{diag}(M, \cdots, M)$; and let $Q_{(k)} = Q \otimes I_k$, then

$$\hat{M} Q_{(s)} = Q_{(r)} \hat{M}. \tag{4}$$

3 Problem statement

We consider the problem of controlling a networked multi-agent system which consists of n identical linear time invariant (LTI) agents. Each agent has access to its own output measurements together with relative external measurements with respect to the other agents. The information exchange among these agents can be represented as the Laplacian matrix L. The dynamics of each agent is described by the following transfer function

$$G(s) = \frac{n(s)}{d(s)} \mathrm{e}^{-\theta s} \tag{5}$$

where θ is the time delay of the agent, and $N(s)$ and $D(s)$ are coprime polynomials in s, which are defined as

$$n(s) = \upsilon_b s^b + \upsilon_{b-1} s^{b-1} + \cdots + \upsilon_1 s + \upsilon_0$$

$$d(s) = s^a + \mu_{a-1} s^{a-1} + \cdots + \mu_1 s + \mu_0$$

where $\upsilon_0, \upsilon_1, \cdots, \upsilon_b$ and $\mu_0, \mu_1, \cdots, \mu_{a-1}$ are real numbers, and $a > b$. The distributed PID controller $C(s)$ imposed on each agent has the form

$$C(s) = k_p + \frac{k_i}{s} + k_d s. \tag{6}$$

where k_p, k_i and k_d are the proportional, integral and differential gains, respectively.

Now, the closed-loop multi-input and multi-output (MIMO) representation of the networked multi-agent system shown in Fig. 1 can be established. In Fig. 1, $r = [r_1, \cdots, r_N]^{\mathrm{T}}$, $y = [y_1, \cdots, y_N]^{\mathrm{T}}$, $\hat{G}(s) = \oplus \sum\limits_{i=1}^{N} G(s)$ and

$\hat{C}(s) = \oplus \sum_{i=1}^{N} C(s)$. It is clearly seen that the coupling between the agents is caused via the communication channels.

The control system design for robustness can be cased into the computation and minimization of H_∞ norm of a prescribed transfer function of the system[22]. Thus, the problem of the H_∞ optimal decentralized PID controller design is considered in this paper, and the performance requirement of the overall multi-agent system is defined with respect to a weighted complementary sensitivity function.

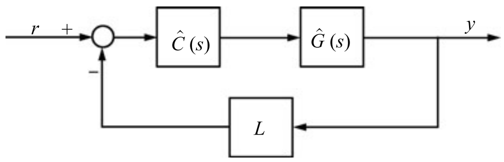

Fig. 1 The closed-loop representation of the multi-agent system

The transfer function from r to y is given by

$$T(s,k_p,k_i,k_d) = [I + L\hat{G}(s)\hat{C}(s)]^{-1}\hat{G}(s)\hat{C}(s) \tag{7}$$

Define $W(s)$ as a stable weighting function to specify the performance requirements. Then, for the known agent dynamics and communication topology, the design objective is to determine the values of the distributed PID control parameters such that the following performance requirement is satisfied.

$$||W(s)T(s,k_p,k_i,k_d)||_\infty < \gamma \tag{8}$$

where γ is the system performance that wanted.

4 Design method of distributed H_∞ PID feedback controller

In [17, 23], the analysis and controller synthesis methods are presented for the networked multi-agent system by the modal decomposition technique. They allow checking the system stability, designing a distributed stabilizing controller; and their computational complexity does not increase with the number of the agents. However, such results cannot be extended to the H_∞ distributed controllers. In [24], the distributed controller design procedures are performed under H_2 or H_∞ criteria for the distributed static state feedback controller based on the decomposition technique and LMI toolbox. Such a decomposition-based method leads to a conservative design since the performance of the multi-agent system cannot be guaranteed. Furthermore, the order of the resultant H_∞ controller is always larger than or equal to the order of the agent. Similar to the result in [25], the design method of the H_∞ distributed PID controllers is presented in this section.

Assume that L is a normal matrix. Since L is diagonalizable, there exists a nonsingular matrix R such that $L = R^{-1}\Lambda R$, where Λ is a diagonal matrix in which $\lambda_1, \lambda_2, \cdots, \lambda_N$ are the non-singular values of L. Define the weighting function $W(s) = \frac{W_n(s)}{W_d(s)}$, where $W_n(s)$ and $W_d(s)$ are coprime polynomials and $W_d(s)$ is stable. Then, define $\delta_i(s,k_p,k_i,k_d)$ and $\upsilon_i(s,k_p,k_i,k_d,\varphi)$ as

$$\delta_i(s,k_p,k_i,k_d) = sd(s)\mathrm{e}^{\theta s} + \lambda_i(k_d s^2 + k_p s + k_i)n(s) \tag{9}$$

$$\upsilon_i(s,k_p,k_i,k_d,\varphi) = sW_d(s)d(s)\mathrm{e}^{\theta s} + (k_d s^2 + k_p s + k_i)n(s)\left[\lambda_i W_d(s) + \mathrm{e}^{j\varphi}\frac{W_n(s)}{\gamma(s)}\right]. \tag{10}$$

Here, φ is the independent variable ranging from 0 to 2π .

Theorem 1. Take

$$p_i(s,k_p,k_i,k_d) = \frac{G(s)C(s)}{1+\lambda_i G(s)C(s)}. \tag{11}$$

The necessary and sufficient conditions that the PID gains satisfy $||W(s)T(s,k_p,k_i,k_d)||_\infty < \gamma$ are that, for each λ_i, the following three conditions are all satisfied:

1) All the zeros of each $\delta_i(s,k_p,k_i,k_d)$ belong to the open left-half complex plane.

2) Each $\upsilon_i(s,k_p,k_i,k_d,\varphi)$ is stable for all φ in $[0,2\pi)$.

3) $|p_i(\infty,k_p,k_i,k_d)| < \gamma$.

Proof. we first prove condition (1) can guarantee the stability of the multi-agent system. The characteristic function of the whole system is

$$\delta(s) = \det[I + L\hat{G}(s)\hat{C}(s)]. \tag{12}$$

Since $L = R^{-1}\Lambda R$, in terms of Lemma 1, we have

$$\det\left[I + L\hat{C}(s)\hat{G}(s)\right] = \det\left[R^{-1}R + R^{-1}\Lambda\hat{C}(s)\hat{G}(s)R\right] = \prod_{i=1}^{N'}\det\left[I + \lambda_i C(s)G(s)\right] = \frac{\mathrm{e}^{-N'\theta s}}{[sd(s)]^{N'}}\prod_{i=1}^{N'}\det\left[sd(s)\mathrm{e}^{\theta s} + \lambda_i(k_d s^2 + k_p s + k_i)n(s)\right] \tag{13}$$

where N' is the number of the non-zero singular values of L. From (12), the stability of the networked multi-agent system is equivalent to the PID controller stabilizing each plant with the form (11). Thus, condition (1) ensures that the multi-agent system is stable.

Subsequently, we consider the performance requirement $||W(s)T(s,k_p,k_i,k_d)||_\infty < \gamma$. From (7), we have

$$T(s,k_p,k_i,k_d) = [I + R^{-1}\Lambda R\hat{G}(s)\hat{C}(s)]^{-1}\hat{G}(s)\hat{C}(s) = R^{-1}H(s)R \tag{14}$$

where

$$H(s,k_p,k_i,k_d) = (I + \hat{C}(s)\hat{G}(s)\Lambda)^{-1}\hat{C}(s)\hat{G}(s). \tag{15}$$

From (15), it is known that $H(s,k_p,k_i,k_d)$ is a diagonal matrix. In terms of (14), $||T(j\omega,k_p,k_i,k_d)||^2$ can be written as

$$||T(j\omega,k_p,k_i,k_d)||^2 = H(j\omega,k_p,k_i,k_d)RR^{-1}H^*(j\omega,k_p,k_i,k_d). \tag{16}$$

Here ω is frequency. Since L is a normal matrix, it is seen that R is unitary, i.e., $RR^* = I$. Thus, we have

$$||T(j\omega,k_p,k_i,k_d)|| = ||H(j\omega,k_p,k_i,k_d)||. \tag{17}$$

Thus, we have

$$||T(j\omega,k_p,k_i,k_d)||_\infty < \gamma \Leftrightarrow ||H(j\omega,k_p,k_i,k_d)||_\infty < \gamma. \tag{18}$$

In terms of the definition of H_∞ norm, we have

$$||W(s)T(s,k_p,k_i,k_d)|| = \sup_{\omega} \sigma_{\max}(W(j\omega)T(j\omega,k_p,k_i,k_d)). \quad (19)$$

It follows that

$$||W(s)T(s,k_p,k_i,k_d)||_\infty < \gamma \Leftrightarrow ||W(s)\frac{C(s)G(s)}{1+\lambda_i C(s)G(s)}|| < \gamma \quad (20)$$

for $\forall i = 1,\cdots,N'$. It can be computed that

$$W(s)\frac{C(s)G(s)}{1+\lambda_i C(s)G(s)} = \frac{W_n(s)(k_d s^2 + k_p s + k_i)n(s)}{sW_d(s)d(s)\mathrm{e}^{\theta s} + \lambda_i W_d(s)(k_d s^2 + k_p s + k_i)n(s)}. \quad (21)$$

According to the results in [24], if the right inequality in (20) holds, then the following quasi polynomial is stable for all φ in $[0, 2\pi)$.

$$\upsilon_i(s,k_p,k_i,k_d,\varphi) = sW_d(s)d(s)\mathrm{e}^{\theta s} + (k_d s^2 + k_p s + k_i)[\lambda_i W_d(s) + \mathrm{e}^{j\varphi} W_n(s)]. \quad (22)$$

Furthermore, when $\omega \to \infty$, $|p_i(j\omega,k_p,k_i,k_d,\varphi)| < \gamma$. As a result, if conditions (2) and (3) are satisfied, the inequality $||W(s)T(s,k_p,k_i,k_d)||_\infty < \gamma$ must hold. This completes the proof of Theorem 1. □

From Theorem 1, it is seen that the synthesis of the distributed H_∞ PID controllers for the networked multi-agent system is cast into simultaneous quasi polynomial stabilization problem. If we can find the regions of the PID control parameters ensuring the stability of $\delta(s,k_p,k_i,k_d,)$ and $\upsilon(s,k_p,k_i,k_d)$, then the values of the PID control parameters satisfying $||W(s)T(s,k_p,k_i,k_d)||_\infty < \gamma$ can be derived by determining the intersection of such PID control regions. Two approaches of presenting the PID control region for the stability of $\upsilon(s,k_p,k_i,k_d)$ will be given. These two approaches can also be directly applied to $\delta(s,k_p,k_i,k_d,)$ if taking $W_n(s) = 0$ and $W_d(s) = 1$.

Based on the results in [26], the (k_d,k_i) region for a fixed k_p value can be obtained to guarantee the stability of $\upsilon(s,k_p,k_i,k_d)$ in (10). The linear programming characterization of the PID controllers that can ensure the stability of the complex quasi polynomials $\upsilon(s,k_p,k_i,k_d)$ are developed on the basis of the extended Hermite-Biehler Theorem.

Let

$$L(s) = sW_d(s)d(s) \quad (23)$$

and

$$M(s) = n(s)\left[\lambda_i W_d(s) + \mathrm{e}^{j\varphi}\frac{W_n(s)}{\gamma}\right] \quad (24)$$

The quasi polynomial (10) is transformed into

$$\upsilon_i(s,k_p,k_i,k_d) = L(s)\mathrm{e}^{\theta s} + (k_d s^2 + k_p s + k_i)M(s). \quad (25)$$

It is seen that $L(s)$ is a real polynomial and $M(s)$ is a complex polynomial. Then $L(s)$ and $M(s)$ can be written as

$$L(s) = s^e + c_{e-1}s^{e-1} + \cdots + c_1 s + c_0$$

$$M(s) = (a_f + jb_f)s^f + (a_{f-1} + jb_{f-1})s^{f-1} + \cdots + (a_0 + jb_0)$$

where $a_0, a_1, \cdots, a_f$, $b_0, b_1, \cdots, b_f$ and $c_0, c_1, \cdots, c_{e-1}$ are all real, $(a_f + jb_f) \neq 0$ and $e > f + 2$. Let $s = \frac{jz}{\theta}$. We have

$$L\left(\frac{\mathrm{j}z}{\theta}\right) = L_r(z) + \mathrm{j}L_i(z) \quad (26)$$

$$M\left(\frac{\mathrm{j}z}{\theta}\right) = M_r(z) + \mathrm{j}M_i(z). \quad (27)$$

It is observed from (25) that both the real and imaginary parts of $\upsilon_i(s,k_p,k_i,k_d,\varphi)$ depend on all the three gains: k_p, k_i and k_d. To overcome this problem, we construct a new quasi polynomial in which the imaginary part depends only on k_p and the real part depends only on k_i and k_d. Multiplying two sides of (25) by $M(-s)$, we have

$$\upsilon_i\left(\frac{\mathrm{j}z}{\theta},k_p,k_i,k_d,\varphi\right)M(-\frac{\mathrm{j}z}{\theta}) = p(z,k_i,k_d) + \mathrm{j}q(z,k_p) \quad (28)$$

where

$$p(z,k_i,k_d) = p_1(z) + \left(k_i - \frac{k_d z^2}{\theta^2}\right)[M_r^2(z) + M_i^2(z)] \quad (29)$$

$$q(z,k_p) = q_1(z) + \frac{zk_p[M_r^2(z) + M_i^2(z)]}{\theta}. \quad (30)$$

Here

$$p_1(z) = [L_r(z)M_r(z) + L_i(z)M_i(z)]\cos(z) - [L_i(z)M_r(z) - L_r(z)M_i(z)]\sin(z) \quad (31)$$

$$q_1(z) = [L_i(z)M_r(z) - L_r(z)M_i(z)]\cos(z) + [L_r(z)M_r(z) + L_i(z)M_i(z)]\sin(z). \quad (32)$$

In order to derive the (k_d,k_i) region for which $\upsilon_i(s,k_p,k_i,k_d,\varphi)$ is stable, some definitions are first given:

Definition 1. Let $\underline{Z} = -2l\pi - \zeta$ and $\bar{Z} = 2l\pi - \zeta$, where

$$\xi = \begin{cases} -\dfrac{\pi}{2}, & \text{if } e \text{ is even and } a_f b_f = 0 \\ 0, & \text{if } e \text{ is odd and } a_f b_f = 0 \\ \arctan(\dfrac{-b_f}{a_f}) \text{ or } \pi + \arctan(\dfrac{-b_f}{a_f}), & \text{if } e{+}f \text{ is even} \\ \arctan(\dfrac{a_f}{b_f}) \text{ or } \pi + \arctan(\dfrac{a_f}{b_f}), & \text{if } e{+}f \text{ is odd.} \end{cases}$$

For a given value of k_p, let $z_1 < z_2 < \cdots < z_{c-1}$be the real distinct zeros of $q(z,k_p)$ in (28) in the interval $(\underline{Z},\bar{Z})$, and assume $z_0 = \underline{Z}$ and $z_c = \bar{Z}$. Denote $\varsigma_f = a_f + jb_f$ as the leading coefficient of $M(s)$ and define i_t as

$$i_t = \mathrm{sgn}[p(z_t,k_i,k_d)] = \begin{cases} 0, & \text{if } M(\dfrac{-\mathrm{j}z_t}{\theta}) = 0 \\ -1 \text{ or } 1, & \text{if } M(\dfrac{-\mathrm{j}z_t}{\theta}) \neq 0 \end{cases}$$

where $t = 0, 1, 2, \cdots, c$.

Definition 2. Let $F = \{i_0, i_1, \cdots, i_c\}$ or $\{i_0, i_1, \cdots, i_{c-1}\}$. Then, the signature $\sigma(F)$ is denoted by

$$\sigma(F) = \begin{cases} \dfrac{1}{2}\left[[i_0 + 2\sum\limits_{t=1}^{c-1} i_t \times (-1)^t + (-1)^c i_c]\right] \times \\ (-1)^{c-1}\mathrm{sgn}\left[q(z_{c-1}^+)\right], \\ \quad \text{if } f \text{ is odd and } \varsigma_f \text{ is not purely imaginary,} \\ \quad \text{or } f \text{ is even and } \varsigma_f \text{ is not purely real} \\ \dfrac{1}{2}\left[2\sum\limits_{t=1}^{c-1} i_t \times (-1)^t\right] \times (-1)^{c-1}\mathrm{sgn}\left[q(z_{c-1}^+)\right], \\ \quad \text{if } f \text{ is odd and } \varsigma_f \text{ is purely imaginary,} \\ \quad \text{or } f \text{ is even and } \varsigma_f \text{ is purely real.} \end{cases}$$

Remark 1. According to the Theorem 6 in [27], the region of k_p which makes $\delta(s, k_p, k_i, k_d)$ stable can be known, and denoted by S_1. For a fixed φ^*, similar to [27], it can be inferred the region of k_p that makes $\upsilon(\frac{jz}{\theta}, k_p, k_i, k_d, \varphi^*)$ stable. Then for all $\varphi^* \in [0, 2\pi)$, the intersection region of all k_p's regions can be signed by S_2. The intersection of S_1 and S_2 is the region of k_p satisfying $||W(s)T(s, k_p, k_i, k_d)||_\infty < \gamma$.

Theorem 2. Let $l(M)$ and $r(M)$ denote the numbers of left half-plane and right half-plane zeros of $M(s)$, respectively. For a fixed k_p, if there exists one string I satisfying

$$\sigma(I) = (4l^* + e) - [l(M) - r(M)] \tag{33}$$

the set of (k_d, k_i) ensuring the stability of $\upsilon(s, k_p, k_i, k_d)$ is the intersection of the following inequalities:

$$[k_i - A(z_t)k_d + B(z_t)]i_t > 0, \quad \text{for } \forall i_t \in F \text{ and } i_t \neq 0. \tag{34}$$

Here, $A(z_t) = \frac{z^2}{\theta^2}$ and $B(z_t) = \frac{p_1(z_t)}{[M_r^2(z)+M_i^2(z)]}$. If the strings $F_1, F_2, \cdots, F_k$ all satisfy (33), then the set of (k_d, k_i) is the union of the regions of i satisfying (34) for $F_1, F_2, \cdots, F_k$.

The proof of Theorem 2 can be referred to the results in [26]. If the k_p value is fixed, the regions of (k_d, k_i) for the stability of the quasi polynomial $\upsilon_i(s, k_p, k_i, k_d)$ can be derived based on Theorem 2. For all λ_i values, the intersection of the resultant (k_d, k_i) regions must satisfy $||W(s)T(s, k_p, k_i, k_d)||_\infty < \gamma$. The results reveal that the set of the integral and derivative gains has the linear programming characterization and is a union of convex sets for a fixed proportional gain.

In terms of Theorem 1, Theorem 2 and Remark 1, an algorithm to determine the set of the PID parameters satisfying $\|W(s)T(s, k_p, k_i, k_d)\|_\infty < \gamma$ is shown as follows:

Sept 1. Observe the topology structure and determine the Laplacian matrix.

Sept 2. Compute the nonzero eigenvalues of the Laplacian matrix, and denote them as $\lambda_1, \lambda_2, \cdots$.

Sept 3. For each λ_i, determine the allowable range of k_p satisfying $||W(s)T(s, k_p, k_i, k_d)||_\infty < \gamma$ based on Remark 1.

Sept 4. Choose a k_p^* in the region of k_p.

Sept 5. Choose one of the nonzero eigenvalue and take it as λ_i.

Sept 6. In terms of Theorem 2, determine the region of (k_d, k_i) which makes $\delta_i(s, k_p, k_i, k_d)$ in (9) stable, denote it as $S_{(1,k_p^*)}$.

Sept 7. let $M(s) = N(s)[\lambda_i W_d(s) + \frac{\mathrm{e}^{j\varphi^*}W_n(s)}{\gamma})$ and $L(s) = sD(s)W_d(s)$. Then, for $\varphi \in [0, 2\pi)$, present the region of (k_d, k_i) for which $\upsilon_i(s, k_p, k_i, k_d, \varphi^*)$ in (10) is stable based on Theorem 2 and denote it as $S_{(2,k_p^*)}$.

Sept 8. Determine the region of (k_d, k_i) satisfying $|p_i(\infty, k_p, k_i, k_d)| < \gamma$, which is denoted as $S_{(3,k_p^*)}$.

Sept 9. Present the region of (k_d, k_i) (denoted as $S_{i(k_p^*)}$), which is the intersection of $S_{(1,k_p^*)}$, $S_{(2,k_p^*)}$ and $S_{(3,k_p^*)}$, i.e., $S_{i(k_p^*)} = S_{(1,k_p^*)} \cap S_{(2,k_p^*)} \cap S_{(3,k_p^*)}$.

Sept 10. Go back to Step 5, pick another nonzero eigenvalue λ_i and repeat Step 5 to Step 9.

Sept 11. Determine the intersection of the (k_d, k_i) regions (denoted as $S_{(k_p^*)}$) for all nonzero eigenvalues, i.e., $S_{(k_p^*)} = S_{1(k_p^*)} \cap \cdots \cap S_{(n-1)(k_p^*)}$.

Sept 12. By sweeping over k_p in the allowable range, repeat Step 5 to Step 11 to determine $S_{(k_p^*)}$ corresponding to different k_p^* values. Thus, the set of (k_p, k_i, k_d) which satisfies $\|W(s)T(s, k_p, k_i, k_d)\|_\infty < \gamma$ is obtained.

5 Simulation example

Consider a consensus system with four identical agents. The dynamics of each agent with the time delay is given by

$$G(s) = \frac{s+2}{s^3 + 5s^2 + 7s + 3}\mathrm{e}^{-0.5s}$$

and the topological structure is shown in Fig. 2.

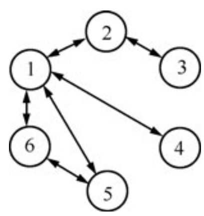

Fig. 2 Topological structure

The problem is to determine the set of the distributed PID control parameters so that $||W(s)T(s, k_p, k_i, k_d)||_\infty < 1$, where the weight $W(s)$ is chosen as $W(s) = \frac{(s+0.1)}{(s+1)}$.

From Fig. 2, the Laplacian matrix describing the interconnection of the agents is

$$L = \begin{bmatrix} 4 & -1 & 0 & -1 & -1 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 2 & -1 \\ -1 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}.$$

The non-zero singular values of L are 0.4859, 1, 2.428, 3, and 5.0861. According to Theorem 1, the quasipolynomials $\delta_i(s, k_p, k_i, k_d)$ and $\upsilon_i(s, k_p, k_i, k_d, \varphi)$ corresponding to $\lambda = 0.4859$, $\lambda = 1$, $\lambda = 2.428$, $\lambda = 3$, and $\lambda = 5.0861$ can be easily obtained from (9) and (10).

We first determine the allowable range of k_p satisfying $||W(s)T(s, k_p, k_i, k_d)||_\infty < \gamma$ according to Remark 1. For each different value of λ, it can be derived that the allowable ranges of k_p corresponding to $\lambda = 0.4859$, $\lambda = 1$, $\lambda = 2.428$, $\lambda = 3$, and $\lambda = 5.0861$ are $(-3.0871, 19.1474)$, $(-1.5,$

9.3037), (−0.6178, 2.8318), (−0.5, 3.1012) and (−0.2949 1.8292), respectively. Combining all these ranges, it can be seen that allowable range of k_p is $(-0.2949, 1.8292)$.

Then for a fixed k_p value, such as $k_p = 1$, determine the intersection of the stabilizing regions of (k_d, k_i) for different nonzero eigenvalues, which is shown in Fig. 3. According to the algorithm in Section 4, the (k_d, k_i) region satisfying $||W(s)T(s)||_\infty < 1$ for $k_p = 1$ can be obtained and it is shown in Fig. 4. By sweeping over $k_p \in (-0.2949, 1.8292)$, the three-dimensional set of (k_p, k_d, k_i) values satisfying $\|W(s)T(s)\|_\infty < 1$ is presented in Fig. 5.

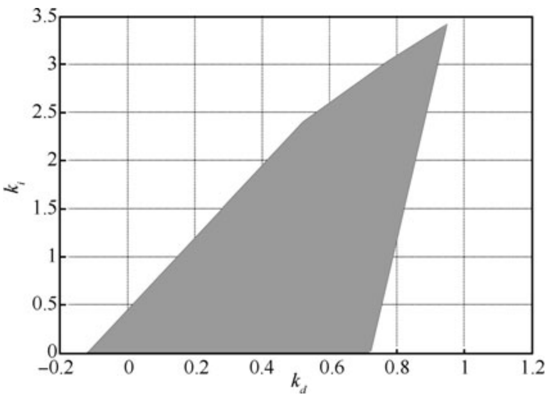

Fig. 3 Intersection of stable region of (k_d, k_i) for different λ values

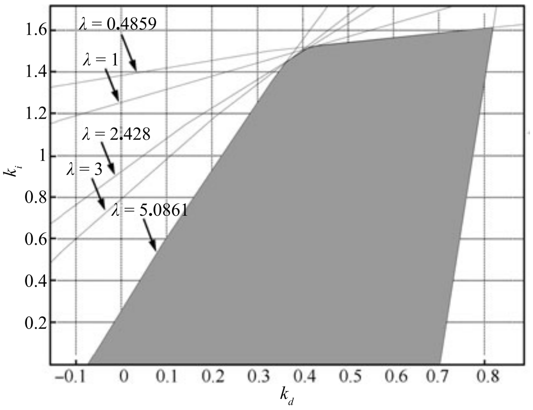

Fig. 4 The (k_d, k_i) region satisfying $\|W(s)T(s)\|_\infty < 1$ for $k_p = 1$

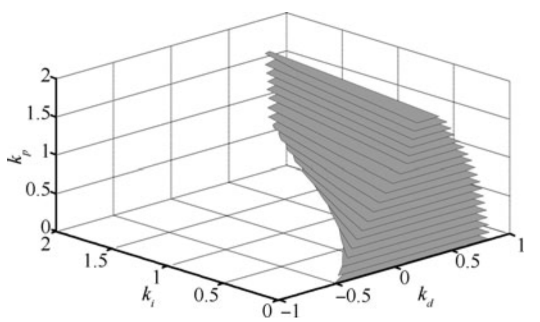

Fig. 5 The (k_p, k_d, k_i) set satisfying $\|W(s)T(s)\|_\infty < 1$

In order to check the validity of the resultant (k_p, k_d, k_i) set, three groups of (k_p, k_d, k_i) values are chosen, which are (1, 0.4, 0.5), (0.3, 0.4, 0.4) and (1, 0.5, 1.8). The points (1, 0.4, 0.5) and (0.3, 0.4, 0.4) lie inside the (k_p, k_d, k_i) set, while the point (1, 0.5, 1.8) is outside the (k_p, k_d, k_i) set. The output response curves in Figs. 6–8 shows that the points (1, 0.4, 0.5) and (0.3, 0.4, 0.4) can lead to the consensus with good performance. But for (1, 0.5, 1.8), the serious oscillation phenomenon occurs during the transient response period and the convergence rate is very slow. Fig. 9 shows the output response curves of the multi-agent system without introducing the distributed PID feedback. Comparing Fig. 9 with Fig. 7, it is easily seen that the consensus system with the distributed PID feedback has better performance.

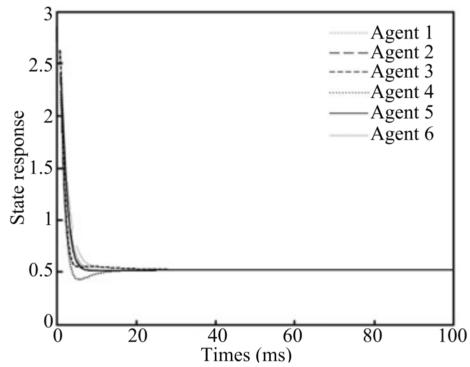

Fig. 6 The step response curves with the $(k_p, k_d, k_i) = (1, 0.4, 0.5)$ satisfying $\|W(s)T(s)\|_\infty < 1$ for $k_p = 1$

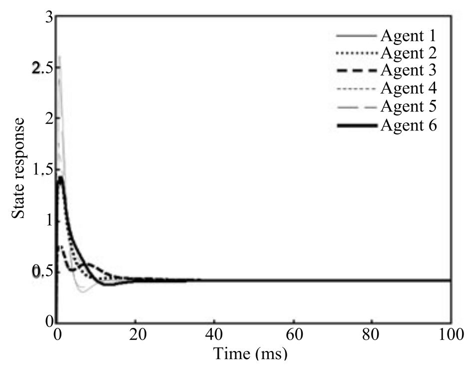

Fig. 7 The step response curves with the $(k_p, k_d, k_i) = (0.3, 0.4, 0.4)$ satisfying $\|W(s)T(s)\|_\infty < 1$ for $k_p = 1$

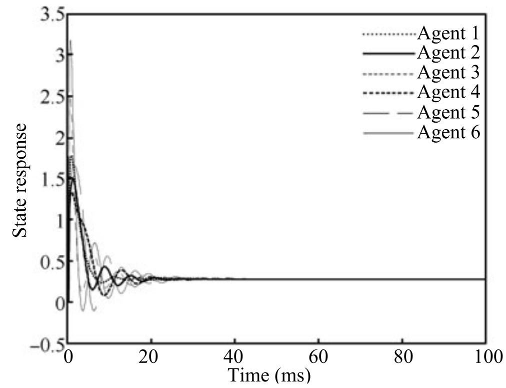

Fig. 8 The step response curves with the $(k_p, k_d, k_i) = (1, 0.5, 1.8)$ without satisfying $\|W(s)T(s)\|_\infty < 1$ for $k_p = 1$

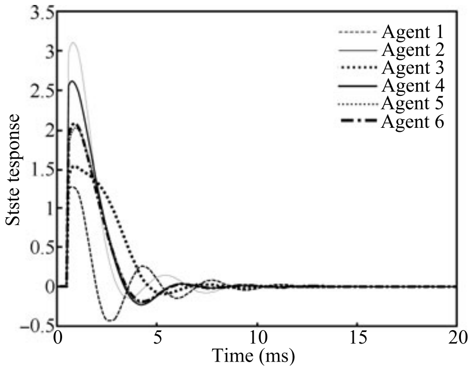

Fig. 9 The step response curves without the distributed PID feedback

6 Conclusions

In this paper, a distributed PID feedback method based on H_∞ performance criterion is proposed in a parametric manner for the consensus of arbitrary-order delayed multi-agent system. The H_∞ performance index is first decomposed based on the interconnection topology. Then the distributed PID feedback control problem for the H_∞ performance requirement is equivalently transformed into several independent H_∞ performance constraints of single-input single output (SISO) subsystem with respect to the eigenvalues of the Laplacian matrix. For each subsystem, the set of the PID controllers satisfying the required H_∞ performance constraint is further converted to simultaneous stabilization problem of a family of complex quasipolynomials and the characteristic equations. Subsequently, the sets of the H_∞ PID parameters for each subsystem are presented based on the extended Hermite-Biehler theorem. Thus, the intersection of the PID parameter sets for all the transformed quasi polynomial is the values of PID control parameters satisfying the H_∞ performance requirement. In comparison with the other distributed controller design methods, the proposed approach is easy to understand and can deal with the restriction on the structure and order of the distributed controller.

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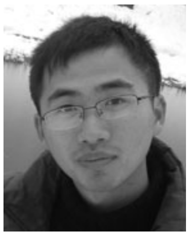
Lin-Lin Ou received her B. Sc. degree in vacuum technology and equipment and her M. Sc. degree in fluid measurement and control from Hefei University of Technology, China in 2000 and 2003, respectively, and her Ph. D. degree in control theory and engineering from Shanghai Jiao Tong University, China in 2006. She is currently an associate professor in the Department of Automation, Zhejiang University of Technology, China.

Her research interests include theoretical aspects of time-delayed systems and control, cooperative control of multi-agent system.

E-mail: linlinou@zjut.edu.cn



Jun-Jie Chen received his B. Sc. degree in automation from Zhejiang University of Technology, China in 2012. He is currently a master student.

His research interests include theoretical aspects of time-delayed systems and control.

E-mail: ilvxcjj@aliyun.com

Dong-Mei Zhang is professor at College of Science, Zhejiang University of Technology China.

Her research interests include robust control, distributed control and polynomial optimization theory.

E-mail: meidzh@zjut.edu.cn (Corresponding author).

Lin Zhang received his B. Sc. degree in electrical engineering and automation from Xuchang University, China in 2008. He received his M. Sc. degree in automation from Taiyuan University, China in 2012. He is currently a Ph. D. candidate in Zhejiang University of Technology.

His research interests include mobile robot.

E-mail: eagle_center@163.com

Wei-Dong Zhang received his B. Sc., M. Sc. and Ph. D. degrees from Zhejiang University, China in 1990, 1993, and 1996, respectively. He worked in National Key Laboratory of Industrial Control Technology as a post-doc before joining Shanghai Jiao Tong University, China in 1998 as an associate professor. Since 1999, he has been a professor at the Department of Automation, Shanghai Jiao Tong University. He has authored or coauthored over 200 journal and conference papers.

His research interests include robust control, networked control systems, and their applications in industry and aerospace.

E-mail: wdzhang@sjtu.edu.cn