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Adaptive H_∞ Control for Nonlinear Hamiltonian Systems with Time Delay and Parametric Uncertainties

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Abstract: This paper addresses the adaptive H_∞ control problem for a class of nonlinear Hamiltonian systems with time delay and parametric uncertainties. The uncertainties under consideration are some small parameter perturbations involved in the structure of the Hamiltonian system. Both delay-independent and delay-dependent criteria are established based on the dissipative structural properties of the Hamiltonian systems and the Lyapunov-Krasovskii functional approach. In order to construct the adaptive H_∞ controller, the situation that the parameter perturbation is inexistent in the system is also studied and the controller is designed. The adaptive H_∞ control problem is solved under some sufficient conditions which ensure the asymptotic stability and the L_2 gain performance of the resulted closed-loop system. Numerical example is given to illustrate the applicability of the theoretical results.

Keywords: Nonlinear Hamiltonian systems, time delay, H_∞ control, uncertainties, Lyapunov-Krasovskii function.

1 Introduction

In recent years, more and more researchers pay their attentions to time delay systems since delays are often the main causes of instability and poor performance of dynamic systems (see [1–3] and the references therein). Time delay occurs in many systems and stability analysis and synthesis for time delay systems have been one of the most challenging issues. The Lyapunov-Krasovskii (L-K) method is often used in solving the time delay problems. Stability criteria for delay systems can be classified into two categories: delay-independent and delay-dependent criteria. During the last decades, much attention has been devoted to the problems of stability analysis, robust stabilization, H_∞ controller design, etc., for time delay systems (see [4–9] and the references therein).

On the other hand, port-controlled Hamiltonian (PCH) systems have attracted increasing attentions in the field of nonlinear control theory (see [10–15] and the references therein). This class of nonlinear systems can describe not only mechanical systems but also a broad class of physical systems including passive electro-mechanical systems, power systems and their combinations. Recently, some results on Hamiltonian systems[16−18] with time delay are obtained. The stabilization problem[16], L_2-disturbance attenuation problem[17], the finite-time stability and H_∞ control design[18] of time delay PCH systems are all studied.

As is well known, the Hamiltonian function in PCH systems is considered as the sum of potential energy (excluding gravitational potential energy) and kinetic energy in physical systems, and is always a good candidate Lyapunov function for many systems[11, 13, 18, 19]. This Hamiltonian function method is simple in form, easy and effective in operation. However, when the Hamiltonian systems encounter time delay and uncertainties, the Hamiltonian function method is no longer effective directly. In this paper, we consider a class of time delay nonlinear Hamiltonian systems with parametric uncertainties and external disturbances. The adaptive H_∞ control problem of the systems under consideration is solved not only based on the dissipative structural properties of the Hamiltonian systems, but also using the L-K functional approach. In order to get the main results, we also consider the situation that the structure of the system has no parameter perturbation.

The remainder of the paper is organized as follows. Section 2 presents the problem formulation and some preliminaries. The main results are provided in Section 3. Section 4 illustrates the obtained results by a numerical example, which is followed by the conclusion in Section 5.

Notations. $\mathbf{R}^n$ denotes the n-dimensional Euclidean space and $\mathbf{R}^{n\times m}$ represents the set of all $n\times m$ matrices of real elements; $\|\cdot\|$ stands for either the Euclidean vector norm or the induced matrix 2-norm; $\|x\|_{\mathcal{C}_{n,\tau}} = \max_{t-h\leqslant\varphi\leqslant t}\|x(\varphi)\|$, where $\mathcal{C}_{n,\tau} = \mathcal{C}([-h,0],\mathbf{R}^n)$ denotes the Banach space of continuous functions mapping the interval $[-h,0]$ into $\mathbf{R}^n$; $L_2^n[0,\infty)$ denotes the set of all measurable functions $x:[0,\infty)\to\mathbf{R}^n$ that satisfy $\int_0^\infty|x(t)|^2\mathrm{d}t<\infty$; $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ stand for the maximum and the minimum of eigenvalue of a real symmetric matrix A; the notation $*$ represents the elements below the main diagonal of a symmetric matrix. In addition, for the sake of simplicity, throughout the paper, we denote $\partial H/\partial x$ by ∇H.

2 Problem statement and preliminaries

Consider the following time delay nonlinear Hamiltonian system with parametric uncertainties and disturbances

$$\begin{cases}\dot{x}=[J(x,p)-R(x,p)]\nabla H(x,p)+[J_1(x,p)-\\ \qquad R_1(x,p)]\nabla H(x_\tau,p)+g_1(x)u+g_2(x)\omega,\\ y=g_2^{\mathrm{T}}(x)\nabla H(x),\\ z=r(x)g_1^{\mathrm{T}}(x)\nabla H(x)\end{cases}\tag{1}$$

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where $x \in \mathbf{R}^n$ is the state; $u \in \mathbf{R}^m$ is the control input; $\omega \in \mathbf{R}^s$ is the disturbance input; $y \in \mathbf{R}^p$ is the output; $z \in \mathbf{R}^q$ is the penalty signal; $x_\tau = x(t-\tau) \in \mathcal{C}_{n,\tau}$ stands for the delayed state; $J(x,p)$ and $J_1(x,p) \in \mathbf{R}^{n\times n}$ are skew-symmetric matrices; $R(x,p)$ and $R_1(x,p) \in \mathbf{R}^{n\times n}$ are non-negative symmetric matrices; $\nabla H(x,p) \in \mathbf{R}^{n\times 1}$ is the gradient of the Hamiltonian function $H(x,p)$ which satisfies $H(x,p) \geqslant 0$, $H(0,0)=0$; p is an unknown bounded constant vector that denotes the parameter perturbation of the Hamiltonian structure; $g_1(x)$ and $g_2(x)$ are gain matrices of appropriate dimensions, $g_1(x)g_1^{\mathrm{T}}(x)$ is nonsingular; $r(x)$ is a weighting matrix with full column rank.

Remark 1. p in system (1) is small parameter perturbation, which makes the dissipativeness of the structure matrix unchanged. The parameter perturbations usually bring a direct impact on the states, but an indirect effect on the output of the system. Thus, the output can be chosen independent of p.

Decompose all functions related to the uncertain parameters p as

$$\nabla H(x,p) = \Delta_H(x,p) + \nabla H(x)$$

$$\nabla H(x_\tau,p) = \Delta_H(x,p) + \nabla H(x_\tau)$$

$$J(x,p) = \Delta_J(x,p) + J(x),\ \ J_1(x,p) = \Delta_{J_1}(x,p) + J_1(x)$$

$$R(x,p) = \Delta_R(x,p) + R(x),\ \ R_1(x,p) = \Delta_{R_1}(x,p) + R_1(x)$$

where $\Delta_i(x,0)=0$ $(i=H,J,J_1,R,R_1)$. We denote the corresponding nominal functions as $H(x)=H(x,0)$, $H(x_\tau)=H(x_\tau,0)$, $J(x)=J(x,0)$, $J_1(x)=J_1(x,0)$, $R(x)=R(x,0)$, and $R_1(x)=R_1(x,0)$, while the system (1) becomes

$$\left\{\begin{aligned} \dot{x} &= [J(x)-R(x)]\nabla H(x) + [J_1(x)-R_1(x)]\cdot \\ &\quad\ \nabla H(x_\tau) + g_1(x)u + g_2(x)\omega \\ y &= g_2^{\mathrm{T}}(x)\nabla H(x) \\ z &= r(x)g_1^{\mathrm{T}}(x)\nabla H(x). \end{aligned}\right. \tag{2}$$

The adaptive H_∞ control problem of the system (1) can be described as follows: Given a disturbance attenuation level $\gamma > 0$, find an adaptive control law

$$\left\{\begin{aligned} u &= \alpha(x,\hat{\theta}) \\ \dot{\hat{\theta}} &= \beta(x,\hat{\theta}). \end{aligned}\right. \tag{3}$$

So that the L_2 gain (from ω to z) of the closed-loop system is less than or equal to γ, i.e.,

$$\int_0^T \|z(t)\|^2 \mathrm{d}t \leqslant \gamma^2 \int_0^T \|\omega(t)\|^2 \mathrm{d}t, \quad \forall\ \omega \in L_2[0,T] \tag{4}$$

is satisfied for the closed-loop system, and meanwhile the closed-loop system under the control law (3) can be asymptotically stable when $\omega = 0$.

We seek to investigate the H_∞ control problem of system (1) for the two cases of time delay

Case 1. τ is an unknown constant.

Case 2. $\tau = d(t)$ is a time-varying continuous function which satisfies the following conditions

$$0 \leqslant d(t) \leqslant h \tag{5}$$

and

$$\dot{d}(t) \leqslant \mu < 1 \tag{6}$$

where h and μ are known positive scalars.

The following assumptions are supposed to be satisfied.

Assumption 1. The Hamiltonian function $H(x)$ and its gradient $\nabla H(x)$ satisfy

1) $H(x) \in \mathbf{C}^2$;
2) $\epsilon_1(\|x\|) \leqslant H(x) \leqslant \epsilon_2(\|x\|)$;
3) $\iota_1(\|x\|) \leqslant \nabla^{\mathrm{T}} H(x)\cdot\nabla H(x) \leqslant \iota_2(\|x\|)$

where ϵ_1, ϵ_2, ι_1, ι_2 all belong to class $\mathcal{K}$ functions.

Remark 2. Assumption 1 not only guarantees the existence of $\nabla H(x)$ and $Hess(H(x))$, but also guarantees that both $H(x)$ and $\nabla H(x)$ are bounded in terms of x. We shall note that the assumption is not very conservative to Hamiltonian functions and the majority of Hamiltonian functions in Hamiltonian systems can easily satisfy these conditions.

Assumption 2. $R(x,p) \geqslant A$, $A \geqslant 0$ is a constant matrix.

Assumption 3. There exists an appropriate dimensioned matrix $\Phi(x)$ such that

$$\begin{aligned} \{[J(x,p)-R(x,p)] + [J_1(x,p)-R_1(x,p)]\} \times \\ \Delta_H(x,p) = g_1(x)\Phi^{\mathrm{T}}(x)\theta \end{aligned} \tag{7}$$

where θ is a constant parameter vector subjecting to p.

Remark 3. Assumption 3 is the matched condition. In most cases, we can find $\Phi(x)$ and θ such that (7) holds. Similar assumption can be found in [20].

Assumption 4. $\Delta_{J_1}(x,p)$, $\Delta_{R_1}(x,p)$ satisfy

$$\Delta_{J_1}(x,p) - \Delta_{R_1}(x,p) = 2E(x)\Sigma(x,p) \tag{8}$$

where $E(x)$ is a known functional matrix with appropriate dimensions and $\Sigma(x,p)$ satisfies $\Sigma^{\mathrm{T}}(x,p)\Sigma(x,p) \leqslant I$.

To obtain the main results of this paper, the following lemma will be needed.

Lemma 1.[21] For given matrices $Z=Z^{\mathrm{T}}$, R and S with appropriate dimensions,

$$Z + RD(t)S + S^{\mathrm{T}}D^{\mathrm{T}}(t)R^{\mathrm{T}} < 0 \tag{9}$$

holds for all $D(t)$ satisfying $D^{\mathrm{T}}(t)D(t) \leqslant I$ if and only if there exists a scalar $\varepsilon < 0$ such that

$$Z + \varepsilon^{-1}RR^{\mathrm{T}} + \varepsilon S^{\mathrm{T}}S < 0. \tag{10}$$

3 Main results

In this section, we will put forward the adaptive H_∞ controller design approach for the time delay Hamiltonian systems (1) and both delay-independent and delay-dependent criteria will be given considering different cases of time delay. For the sake of clarifying the main idea, in every subsection, we will study system (2) in which the parameter perturbation dose not exist, that is, $p=0$ in (1) firstly.

3.1 Delay-independent result

In this subsection, we consider the time delay of Case 1 and develop delay-independent analysis. Firstly, the H_∞ control problem of system (2) is considered and a result is given below.

Theorem 1. Consider system (2). Suppose Assumption 1 holds. If there exist matrices $P_1 = P_1^{\mathrm{T}} > 0$,

$M_1 = M_1^{\mathrm{T}} > 0$ such that

$$\Lambda_1 = \begin{bmatrix} \Lambda_{1,1} & \Lambda_{1,2} & \frac{1}{2}g_2(x) \\ * & -P_1 & 0 \\ * & * & -\frac{1}{2}\gamma^2 I \end{bmatrix} < 0 \tag{11}$$

where

$$\Lambda_{1,1} = -R(x) - g_1(x)g_1^{\mathrm{T}}(x)(P_1 + M_1) + P_1 + \frac{1}{2}g_1(x)r^{\mathrm{T}}(x)r(x)g_1^{\mathrm{T}}(x)$$
$$\Lambda_{1,2} = \frac{1}{2}[J_1(x) - R_1(x)]$$

then the H_∞ control problem of system (2) can be solved by the following control law

$$u = -g_1^{\mathrm{T}}(x)(M_1 + P_1)\nabla H(x). \tag{12}$$

Proof. Substituting (12) into (2) yields

$$\begin{cases} \dot{x} = [J(x) - R(x)]\nabla H(x) + [J_1(x) - R_1(x)]\nabla H(x_\tau) - \\ \quad g_1(x)g_1^{\mathrm{T}}(x)(P_1 + M_1)\nabla H(x) + g_2(x)\omega \\ y = g_2^{\mathrm{T}}(x)\nabla H(x) \\ z = r(x)g_1^{\mathrm{T}}(x)\nabla H(x). \end{cases} \tag{13}$$

Choose a Lyapunov function as

$$V_1(x, x_t) = H(x) + \int_{t-\tau}^{t} \nabla^{\mathrm{T}} H(x(s))P_1\nabla H(x(s))\mathrm{d}s. \tag{14}$$

Calculating the derivative of $V_1(x, x_\tau)$ along the trajectories of the closed-loop system (13), we have

$$\begin{aligned} \dot{V}_1(x, x_\tau) = & -\nabla^{\mathrm{T}} H(x)[J(x) - R(x) + g_1(x)g_1^{\mathrm{T}}(x)(P_1 + \\ & M_1)]\nabla H(x) + \nabla^{\mathrm{T}} H(x)g_2(x)\omega + \\ & \nabla^{\mathrm{T}} H(x)[J_1(x) - R_1(x)]\nabla H(x_\tau) + \\ & \nabla^{\mathrm{T}} H(x)P_1\nabla H(x) - \\ & \nabla^{\mathrm{T}} H(x_\tau)P_1\nabla H(x_\tau). \end{aligned} \tag{15}$$

Using inequality (11), we further have

$$\begin{aligned} & \dot{V}_1(x, x_\tau) - \frac{1}{2}(\gamma^2\|\omega\|^2 - \|z\|^2) = \\ & -\nabla^{\mathrm{T}} H(x)[R(x) + g_1(x)g_1^{\mathrm{T}}(x)(P_1 + M_1) - P_1]\nabla H(x) + \\ & \nabla^{\mathrm{T}} H(x)g_2(x)\omega + \nabla^{\mathrm{T}} H(x)[J_1(x) - R_1(x)]\nabla H(x_\tau) - \\ & \nabla^{\mathrm{T}} H(x_\tau)P_1\nabla H(x_\tau) - \frac{1}{2}\gamma^2\omega^{\mathrm{T}}(t)\omega(t) + \\ & \frac{1}{2}\nabla^{\mathrm{T}} H(x)g_1(x)r^{\mathrm{T}}(x)r(x)g_1^{\mathrm{T}}(x)\nabla H(x) = \\ & \eta_1^{\mathrm{T}}(t)\Lambda_1\eta_1(t) \leqslant 0 \end{aligned} \tag{16}$$

where

$$\eta_1(t) = \begin{bmatrix} \nabla^{\mathrm{T}} H(x) & \nabla^{\mathrm{T}} H(x_\tau) & \omega^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}. \tag{17}$$

Consequently, the inequality (4) holds. That means the L_2 gain of the closed-loop system is less than or equal to γ.

In the following, we consider the stability of the closed-loop system when $\omega = 0$.

Since $H(x) \in \mathbf{C}^2$, $\nabla^{\mathrm{T}} H(x(s))P_1\nabla H(x(s))$ is continuous. We conclude by the condition $P_1 > 0$ that

$$\int_{t-\tau}^{t} \nabla^{\mathrm{T}} H(x(s))P_1\nabla H(x(s))\mathrm{d}s \geqslant 0. \tag{18}$$

Furthermore, according to 3) in Assumption 1, we have

$$\begin{aligned} & \int_{t-\tau}^{t} \nabla^{\mathrm{T}} H(x(s))P_1\nabla H(x(s))\mathrm{d}s \leqslant \\ & \lambda_{p_1}\int_{t-\tau}^{t} \iota_2(\|x(s)\|)\mathrm{d}s \leqslant \\ & \tau\lambda_{p_1}\iota_2(\max_{t-\tau\leqslant s\leqslant t}\|x(s)\|) = \\ & \tau\lambda_{p_1}\iota_2(\|x\|_{\mathcal{C}_{n,\tau}}) \end{aligned} \tag{19}$$

where $\lambda_{p_1} = \lambda_{\max}(P_1) > 0$.

Using 2) in Assumption 1, we get

$$V_1(x, x_\tau) \leqslant \epsilon_2(\|x\|) + \tau\lambda_{p_1}\iota_2(\|x\|_{\mathcal{C}_{n,\tau}}). \tag{20}$$

Let $\varpi_1(\|x\|_{\mathcal{C}_{n,\tau}}) = \epsilon_2(\|x\|) + \tau\lambda_{p_1}\iota_2(\|x\|_{\mathcal{C}_{n,\tau}})$. Obviously, it is a class $\mathcal{K}$ function. So

$$\epsilon_1(\|x\|) \leqslant V_1(x, x_\tau) \leqslant \varpi_1(\|x\|_{\mathcal{C}_{n,\tau}}). \tag{21}$$

By evaluating the time-derivative of $V_1(x, x_\tau)$ along the trajectories of (13) with $\omega = 0$, we obtain

$$\begin{aligned} & \dot{V}_1(x, x_\tau) = \\ & -\nabla^{\mathrm{T}} H(x)[R(x) - g_1(x)g_1^{\mathrm{T}}(x)(P_1 + M_1) - P_1]\nabla H(x) + \\ & \nabla^{\mathrm{T}} H(x)[J_1(x) - R_1(x)]\nabla H(x_\tau) - \\ & \nabla^{\mathrm{T}} H(x_\tau)P_1\nabla H(x_\tau) = \\ & \begin{bmatrix} \nabla H(x) \\ \nabla H(x_\tau) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \Gamma_1 & \Gamma_2 \\ * & -P_1 \end{bmatrix} \begin{bmatrix} \nabla H(x) \\ \nabla H(x_\tau) \end{bmatrix} \leqslant 0 \end{aligned} \tag{22}$$

where

$$\Gamma_1 = -R(x) - g_1(x)g_1^{\mathrm{T}}(x)(P_1 + M_1) + P_1$$
$$\Gamma_2 = \frac{1}{2}[J_1(x) - R_1(x)].$$

Thus, there exists υ_1 which belongs to class $\mathcal{K}$ functions such that

$$\dot{V}_1(x, x_\tau) \leqslant -\upsilon_1(\|x\|). \tag{23}$$

According to L-K theorem, we can obtain that the system (2) is asymptotically stable independent of delay. □

Next, as for system (1), an adaptive H_∞ control result is described as follows.

Theorem 2. Consider system (1). Suppose Assumptions 1–4 hold. If there exist matrices $P_2 = P_2^{\mathrm{T}} > 0$, $Q_1 = Q_1^{\mathrm{T}} > 0$, $M_2 = M_2^{\mathrm{T}} > 0$ and a scalar $\varepsilon_1 > 0$ such that

$$\Lambda_2 = \begin{bmatrix} \bar{\Lambda}_{1,1} & \bar{\Lambda}_{1,2} & \frac{1}{2}g_2(x) \\ * & \bar{\Lambda}_{2,2} & 0 \\ * & * & -\frac{1}{2}\gamma^2 I \end{bmatrix} < 0 \tag{24}$$

where

$$\bar{\Lambda}_{1,1} = -A - g_1(x)g_1^{\mathrm{T}}(x)(M_2+P_2) + \varepsilon_1^{-1}E(x)E^{\mathrm{T}}(x) + P_2 + \frac{1}{2}g_1(x)r^{\mathrm{T}}(x)r(x)g_1^{\mathrm{T}}(x)$$
$$\bar{\Lambda}_{1,2} = \frac{1}{2}[J_1(x) - R_1(x)]$$
$$\bar{\Lambda}_{2,2} = -P_2 + \varepsilon_1 I$$

then the adaptive H_∞ control problem of system (1) can be solved by the following control law

$$\begin{cases} u = -g_1^{\mathrm{T}}(x)(M_2+P_2)\nabla H(x) - \Phi^{\mathrm{T}}(x)\hat{\theta} \\ \dot{\hat{\theta}} = Q_1\Phi(x)g_1^{\mathrm{T}}(x)\nabla H(x). \end{cases} \tag{25}$$

Proof. Substituting (7) and (25) into (1) yields

$$\begin{cases} \dot{x} = [J(x,p) - R(x,p)]\nabla H(x) + [J_1(x,p) - R_1(x,p)]\times \\ \quad \nabla H(x_\tau) - g_1(x)g_1^{\mathrm{T}}(x)(M_2+P_2)\nabla H(x) + \\ \quad g_2(x)\omega + g_1(x)\Phi^{\mathrm{T}}(x)(\theta - \hat{\theta}) \\ \dot{\hat{\theta}} = Q_1\Phi(x)g_1^{\mathrm{T}}(x)\nabla H(x) \\ y = g_2^{\mathrm{T}}(x)\nabla H(x) \\ z = r(x)g_1^{\mathrm{T}}(x)\nabla H(x). \end{cases} \tag{26}$$

Choose a Lyapunov function as

$$V_2(x, x_\tau, \tilde{\theta}) = H(x) + \int_{t-\tau}^{t} \nabla^{\mathrm{T}} H(x(s))P_2\nabla H(x(s))\mathrm{d}s + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}Q_1^{-1}\tilde{\theta} \tag{27}$$

where $\tilde{\theta} = \theta - \hat{\theta}$.

Calculating the derivative of $V_2(x, x_\tau, \tilde{\theta})$ along the trajectories of (26) and combining Assumption 2, we can obtain

$$\begin{aligned}\dot{V}_2(x, x_\tau, \tilde{\theta}) - \frac{1}{2}(\gamma^2\|\omega\|^2 - \|z\|^2) \leqslant \\ -\nabla^{\mathrm{T}}H(x)[A + g_1(x)g_1^{\mathrm{T}}(x)(M_2+P_2) - P_2]\nabla H(x) + \\ \nabla^{\mathrm{T}}H(x)[J_1(x,p) - R_1(x,p)]\nabla H(x_\tau) - \\ \nabla^{\mathrm{T}}H(x_\tau)P_2\nabla H(x_\tau) + \nabla^{\mathrm{T}}H(x)g_2(x)\omega - \\ \frac{1}{2}\gamma^2\omega^{\mathrm{T}}(t)\omega(t) + \frac{1}{2}\nabla^{\mathrm{T}}H(x)g_1(x)r^{\mathrm{T}}(x)\times \\ r(x)g_1^{\mathrm{T}}(x)\nabla H(x) = \\ \eta_1^{\mathrm{T}}(t)\Theta\eta_1(t)\end{aligned} \tag{28}$$

where

$$\Theta = \begin{bmatrix} \Theta_{1,1} & \Theta_{1,2} & \frac{1}{2}g_2(x) \\ * & -P_2 & 0 \\ * & * & -\frac{1}{2}\gamma^2 I \end{bmatrix}$$
$$\Theta_{1,1} = -A - g_1(x)g_1^{\mathrm{T}}(x)(M_2+P_2) + P_2 + \frac{1}{2}g_1(x)r^{\mathrm{T}}(x)r(x)g_1^{\mathrm{T}}(x)$$
$$\Theta_{1,2} = \frac{1}{2}[J_1(x,p) - R_1(x,p)].$$

From Lemma 1 and Assumption 4, we know that $\Lambda_2 < 0$ ensures $\Theta < 0$. Hence we have

$$\dot{V}_2(x, x_\tau, \tilde{\theta}) - \frac{1}{2}(\gamma^2\|\omega\|^2 - \|z\|^2) \leqslant 0 \tag{29}$$

which means that inequality (4) is satisfied.

Using the similar method as in Theorem 1, we get

$$\epsilon_1(\|\chi\|) \leqslant V_2(\chi) \leqslant \varpi_2(\|\chi\|_{\mathcal{C}_{n,\tau}}) \tag{30}$$

where $\chi = [\begin{matrix} x^{\mathrm{T}} & x_\tau^{\mathrm{T}} & \tilde{\theta}^{\mathrm{T}} \end{matrix}]^{\mathrm{T}}$, $\varpi_2(\|\chi\|_{\mathcal{C}_{n,\tau}}) = \epsilon_2(\|x\|) + \lambda_{Q_1}(\|\tilde{\theta}\|) + \tau\lambda_{P_2}\iota_2(\|x\|_{\mathcal{C}_{n,\tau}})$, $\varpi_2(\|\chi\|_{\mathcal{C}_{n,\tau}})$ belongs to class $\mathcal{K}$ functions, $\lambda_{Q_1} = \frac{1}{2}\lambda_{\max}(Q_1^{-1}) > 0$, $\lambda_{P_2} = \lambda_{\max}(P_2) > 0$.

Since (24) and (28) hold, we can easily get that when $\omega = 0$,

$$\dot{V}_2(\chi) \leqslant 0. \tag{31}$$

Thus, there exists υ_2 which is a class $\mathcal{K}$ function such that

$$\dot{V}_2(\chi) \leqslant -\upsilon_2(\|\chi\|). \tag{32}$$

According to L-K theorem, we can obtain that the system (1) is asymptotically stable independent of time delay. □

Remark 4. Theorems 1 and 2 serve for the H_∞ control design of system (1) and (2) with delay of Case 1, respectively. In fact, when the delay τ is time varying, these delay-independent results still hold. However, if τ is bounded or very small, such results often bring very conservative stability assessment (see [19]).

3.2 Delay-dependent result

In what follows, we focus on delay of Case 2 and acquire two H_∞ control results dependent on time delay for Hamiltonian system (1) and (2), respectively. Firstly, the time delay system (1) with $p = 0$ admits the following theorem.

Theorem 3. Consider system (2). Suppose Assumption 1 holds. For given scalars h and μ, if there exist matrices $K_1 = K_1^{\mathrm{T}} > 0$, $P_3 = P_3^{\mathrm{T}} > 0$, $M_3 = M_3^{\mathrm{T}} > 0$, such that

$$g_1(x)K_1g_1^{\mathrm{T}}(x) \geqslant hHess^{\mathrm{T}}(H(x))Hess(H(x)) \tag{33}$$

$$\Xi_1 = \begin{bmatrix} \Xi_{1,1} & \frac{1}{2}[J_1(x) - R_1(x)] & \frac{1}{2}g_2(x) \\ * & -(1-\mu)P_3 & 0 \\ * & * & -\frac{1}{2}\gamma^2 I \end{bmatrix} < 0 \tag{34}$$

then the H_∞ control problem of system (2) can be solved by the following control law

$$u = -[g_1^{\mathrm{T}}(x)(P_3+M_3) + \lambda_{M_3}K_1g_1^{\mathrm{T}}(x)]\nabla H(x) \tag{35}$$

where $\lambda_{M_3} = \lambda_{\max}(M_3)$,

$$\Xi_{1,1} = -R(x) - g_1(x)g_1^{\mathrm{T}}(x)(P_3+M_3) + P_3 + \frac{1}{2}g_1(x)r^{\mathrm{T}}(x)r(x)g_1^{\mathrm{T}}(x).$$

Proof. Substituting (35) into (2) yields

$$\begin{cases} \dot{x} = [J(x) - R(x)]\nabla H(x) + [J_1(x) - R_1(x)]\nabla H(x_\tau) - \\ \quad \lambda_{M_3} g_1(x) K_1 g_1^{\mathrm{T}}(x)\nabla H(x) + g_2(x)\omega - \\ \quad g_1(x) g_1^{\mathrm{T}}(x)(P_3 + M_3)\nabla H(x) \\ y = g_2^{\mathrm{T}}(x)\nabla H(x) \\ z = r(x) g_1^{\mathrm{T}}(x)\nabla H(x). \end{cases} \tag{36}$$

Let the Lyapunov function for the closed-loop system (36) be selected as

$$V_3(x, x_\tau) = H(x) + \int_{t-d(t)}^{t} \nabla^{\mathrm{T}} H(x(s)) P_3 \nabla H(x(s)) \mathrm{d}s + \bar{V}_1(x) \tag{37}$$

where

$$\bar{V}_1(x) = \int_{-h}^{0} \int_{t+\alpha}^{t} \nabla^{\mathrm{T}} H(x(s)) Hess^{\mathrm{T}}(H(x(s))) M_3 \times Hess(H(x(s))) \nabla H(x(s)) \mathrm{d}s \mathrm{d}\alpha.$$

Calculating the derivative of $V_3(x, x_\tau)$ along the trajectories of (36), and then taking advantages of (5), (6) and (33), we have

$$\begin{aligned} &\dot{V}_3(x, x_\tau) - \frac{1}{2}(\gamma^2 \|\omega\|^2 - \|z\|^2) \leqslant \\ &\quad - \nabla^{\mathrm{T}} H(x)[R(x) + g_1(x) g_1^{\mathrm{T}}(x)(P_3 + M_3) - P_3 - \\ &\quad \frac{1}{2} g_1(x) r^{\mathrm{T}}(x) r(x) g_1^{\mathrm{T}}(x)]\nabla H(x) + \nabla^{\mathrm{T}} H(x) \times \\ &\quad [J_1(x) - R_1(x)]\nabla H(x_\tau) - (1-\mu)\nabla^{\mathrm{T}} H(x_\tau) P_3 \times \\ &\quad \nabla H(x_\tau) + \nabla^{\mathrm{T}} H(x) g_2(x)\omega - \frac{1}{2}\gamma^2 \omega^{\mathrm{T}}(t)\omega(t) = \\ &\quad \eta_1^{\mathrm{T}}(t) \Xi_1 \eta_1(t). \end{aligned} \tag{38}$$

Since (34) holds, then we get

$$\dot{V}_3(x, x_\tau) - \frac{1}{2}(\gamma^2 \|\omega\|^2 - \|z\|^2) \leqslant 0 \tag{39}$$

which means that (4) is satisfied.

Since $H(x) \in \mathbf{C}^2$, $\nabla^{\mathrm{T}} H(x(s)) P_3 \nabla H(x(s))$ and $\nabla^{\mathrm{T}} H(x(s)) Hess^{\mathrm{T}}(H(x(s))) M_3 Hess(H(x(s))) \nabla H(x(s))$ are continuous, combining $P_3 > 0$, $M_3 > 0$, the inequality

$$\int_{t-d(t)}^{t} \nabla^{\mathrm{T}} H(x(s)) P_3 \nabla H(x(s)) \mathrm{d}s \geqslant 0 \tag{40}$$

and

$$\bar{V}_1(x) \geqslant 0 \tag{41}$$

hold.

Noting (5) and using 3) in Assumption 1, we have

$$\int_{t-d(t)}^{t} \nabla^{\mathrm{T}} H(x(s)) P_3 \nabla H(x(s)) \mathrm{d}s \leqslant h \lambda_{P_3} \iota_2(\|x\|_{\mathcal{C}_{n,\tau}}) \tag{42}$$

and

$$\begin{aligned} \bar{V}_1(x) \leqslant \; & \nu \lambda_{M_3} \int_{-h}^{0} \int_{t+\alpha}^{t} \iota_2(\max \|x(s)\|) \mathrm{d}s \mathrm{d}\alpha \leqslant \\ & \frac{1}{2} h^2 \nu \lambda_{M_3} \iota_2(\max_{t-h \leqslant s \leqslant t} \|x(s)\|) = \\ & \frac{1}{2} h^2 \nu \lambda_{M_3} \iota_2(\|x\|_{\mathcal{C}_{n,\tau}}) \end{aligned} \tag{43}$$

where $\lambda_{P_3} = \lambda_{\max}(P_3) > 0$, $\lambda_{M_3} = \lambda_{\max}(M_3) > 0$, $\nu = \sup_x \{\lambda_{\max}[Hess^{\mathrm{T}}(H(x)) Hess(H(x))]\}$.

Combining (42) and (43), using 2) in Assumption 1, we have

$$V_3(x, x_\tau) \leqslant \epsilon_2(\|x\|) + (h\lambda_{P_3} + \frac{1}{2} h^2 \nu \lambda_{M_3}) \iota_2(\|x\|_{\mathcal{C}_{n,\tau}}). \tag{44}$$

Let $\varpi_3(\|x\|_{\mathcal{C}_{n,\tau}}) = \epsilon_2(\|x\|) + (h\lambda_{P_3} + \frac{1}{2} h^2 \nu \lambda_{M_3}) \iota_2(\|x\|_{\mathcal{C}_{n,\tau}})$. Obviously, it is a class $\mathcal{K}$ function. Thus, $V_3(x, x_\tau)$ satisfies

$$\epsilon_1(\|x\|) \leqslant V_3(x, x_\tau) \leqslant \varpi_3(\|x\|). \tag{45}$$

When $\omega = 0$, the derivative of $V_3(x, x_\tau)$ along the trajectories of (36) satisfies

$$\begin{aligned} &\dot{V}_3(x, x_\tau) \leqslant \\ &\quad - \nabla^{\mathrm{T}} H(x)[R(x) + g_1(x) g_1^{\mathrm{T}}(x)(P_3 + M_3) - P_3]\nabla H(x) + \\ &\quad \nabla^{\mathrm{T}} H(x)[J_1(x) - R_1(x)]\nabla H(x_\tau) - \\ &\quad (1-\mu)\nabla^{\mathrm{T}} H(x_\tau) P_3 \nabla H(x_\tau) = \\ &\quad \hat{\eta}_1^{\mathrm{T}}(t) \tilde{\Xi}_1 \hat{\eta}_1(t) \end{aligned} \tag{46}$$

where

$$\hat{\eta}_1(t) = \begin{bmatrix} \nabla^{\mathrm{T}} H(x) & \nabla^{\mathrm{T}} H(x_\tau) \end{bmatrix}^{\mathrm{T}}.$$

The fact that $\Xi_1 < 0$ ensures

$$\tilde{\Xi}_1 = \begin{bmatrix} \tilde{\Xi}_{1,1} & \frac{1}{2}[J_1(x) - R_1(x)] \\ * & -(1-\mu) P_3 \end{bmatrix} < 0 \tag{47}$$

holds, where $\tilde{\Xi}_{1,1} = -R(x) - g_1(x) g_1^{\mathrm{T}}(x)(P_3 + M_3) + P_3$.

Thus, we have

$$\dot{V}_3(x, x_\tau) \leqslant 0. \tag{48}$$

Therefore, there exists υ_3 which belongs to class $\mathcal{K}$ functions such that

$$\dot{V}_3(x, x_\tau) \leqslant -\upsilon_3(\|x\|). \tag{49}$$

Using L-K theorem we can conclude that the closed-loop Hamiltonian system (36) is asymptotically stable dependent of delay. □

The following theorem provides a delay-dependent adaptive H_∞ control result for system (1) with time-varying delay of Case 2.

Theorem 4. Consider system (1). Suppose Assumptions 1–4 hold. For given scalars h and μ, if there exist matrices $K_2 = K_2^{\mathrm{T}} > 0$, $P_4 = P_4^{\mathrm{T}} > 0$, $M_4 = M_4^{\mathrm{T}} > 0$, $Q_2 = Q_2^{\mathrm{T}} > 0$ and a scalar $\varepsilon_2 > 0$ such that

$$g_1(x)K_2g_1^{\mathrm{T}}(x) \geqslant hHess^{\mathrm{T}}(H(x))Hess(H(x)) \tag{50}$$

$$\Psi_1 = \begin{bmatrix} \Psi_{1,1} & \frac{1}{2}[J_1(x)-R_1(x)] & \frac{1}{2}g_2(x) \\ * & \Psi_{2,2} & 0 \\ * & * & -\frac{1}{2}\gamma^2 I \end{bmatrix} < 0 \tag{51}$$

where

$$\Psi_{1,1} = -A - g_1(x)g_1^{\mathrm{T}}(x)(P_4+M_4) + P_4 + \frac{1}{2}g_1(x)r^{\mathrm{T}}(x)r(x)g_1^{\mathrm{T}}(x) + \varepsilon_2^{-1}E(x)E^{\mathrm{T}}(x)$$

$$\Psi_{2,2} = -(1-\mu)P_4 + \varepsilon_2 I$$

then the adaptive H_∞ control problem of system (1) can be solved by the following control law

$$\begin{cases} u = -[g_1^{\mathrm{T}}(x)(P_4+M_4) + \lambda_{M_4}K_2g_1^{\mathrm{T}}(x)]\nabla H(x) - \Phi^{\mathrm{T}}(x)\hat{\theta} \\ \dot{\hat{\theta}} = Q_2\Phi(x)g_1^{\mathrm{T}}(x)\nabla H(x) \end{cases} \tag{52}$$

where $\lambda_{M_4} = \lambda_{\max}(M_4)$.

Proof. Substituting (7) and (52) into (1) yields

$$\begin{cases} \dot{x} = [J(x,p)-R(x,p)]\nabla H(x) + [J_1(x,p)-R_1(x,p)]\times \\ \quad \nabla H(x_\tau) + g_2(x)\omega - g_1(x)g_1^{\mathrm{T}}(x)(P_4+M_4)\nabla H(x) - \\ \quad \lambda_{M_4}g_1(x)K_2g_1^{\mathrm{T}}(x)\nabla H(x) + g_1(x)\Phi^{\mathrm{T}}(x)(\theta-\hat{\theta}) \\ \dot{\hat{\theta}} = Q_2\Phi(x)g_1^{\mathrm{T}}(x)\nabla H(x) \\ y = g_2^{\mathrm{T}}(x)\nabla H(x) \\ z = r(x)g_1^{\mathrm{T}}(x)\nabla H(x). \end{cases} \tag{53}$$

Choose the following Lyapunov function

$$V_4(x,x_\tau,\tilde{\theta}) = H(x) + \int_{t-d(t)}^{t} \nabla^{\mathrm{T}}H(x(s))P_4\nabla H(x(s))\mathrm{d}s + \bar{V}_2(x) + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}Q_2^{-1}\tilde{\theta}$$

where $\tilde{\theta} = \theta - \hat{\theta}$, and

$$\bar{V}_2(x) = \int_{-h}^{0}\int_{t+\alpha}^{t} \nabla^{\mathrm{T}}H(x(s))Hess^{\mathrm{T}}(H(x(s)))M_4\cdot Hess(H(x(s)))\nabla H(x(s))\mathrm{d}s\mathrm{d}\alpha.$$

Calculating the derivative of $V_4(x,x_\tau,\tilde{\theta})$ along the trajectories of the closed-loop system (53), and combining Assumption 2 and (50), it follows that

$$\begin{aligned} &\dot{V}_4(x,x_\tau,\tilde{\theta}) - \frac{1}{2}(\gamma^2\|\omega\|^2 - \|z\|^2) \leqslant \\ &\nabla^{\mathrm{T}}H(x)[-A - g_1(x)g_1^{\mathrm{T}}(x)(P_4+M_4) + P_4 + \\ &\frac{1}{2}g_1(x)r^{\mathrm{T}}(x)r(x)g_1^{\mathrm{T}}(x)]\nabla H(x) + \nabla^{\mathrm{T}}H(x)g_2(x)\omega + \\ &\nabla^{\mathrm{T}}H(x)[J_1(x,p)-R_1(x,p)]\nabla H(x_\tau) - \\ &(1-\mu)\nabla^{\mathrm{T}}H(x_\tau)P_4\nabla H(x_\tau) - \frac{1}{2}\gamma^2\omega^{\mathrm{T}}(t)\omega(t) = \\ &\eta_1^{\mathrm{T}}(t)\Omega\eta_1(t) \end{aligned} \tag{54}$$

where

$$\Omega = \begin{bmatrix} \Omega_{1,1} & \frac{1}{2}[J_1(x,p)-R_1(x,p)] & \frac{1}{2}g_2(x) \\ * & -(1-\mu)P_4 & 0 \\ * & * & -\frac{1}{2}\gamma^2 I \end{bmatrix}$$

$$\Omega_{1,1} = -A - g_1(x)g_1^{\mathrm{T}}(x)(P_4+M_4) + P_4 + \frac{1}{2}g_1(x)r^{\mathrm{T}}(x)r(x)g_1^{\mathrm{T}}(x).$$

From Lemma 1 and Assumption 4, we know that $\Psi_1 < 0$ ensures $\Omega < 0$. Since (51) holds, hence we have

$$\dot{V}_4(x,x_\tau,\tilde{\theta}) - \frac{1}{2}(\gamma^2\|\omega\|^2 - \|z\|^2) \leqslant 0 \tag{55}$$

which means that the inequality (4) is satisfied.

In the following, we consider the stability of the closed-loop system when $\omega = 0$.

Using the similar method as in Theorem 3, we get

$$\epsilon_1(\|\chi\|) \leqslant V_4(\chi) \leqslant \varpi_4(\|\chi\|_{\mathcal{C}_{n,\tau}}) \tag{56}$$

where $\chi = \begin{bmatrix} x^{\mathrm{T}} & x_\tau^{\mathrm{T}} & \tilde{\theta}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, $\|\chi\|_{\mathcal{C}_{n,\tau}} = \max_{t-h\leqslant s\leqslant t}\|\chi\|$, $\varpi_4(\|\chi\|_{\mathcal{C}_{n,\tau}}) = \epsilon_2(\|x\|) + \lambda_{Q_2}(\|\tilde{\theta}\|) + (h\lambda_{P_4} + \frac{1}{2}h^2\nu\lambda_{M_4})\iota_2(\|x\|_{\mathcal{C}_{n,\tau}})$, and $\varpi_4(\|\chi\|_{\mathcal{C}_{n,\tau}})$ belongs to class $\mathcal{K}$ functions, $\lambda_{Q_2} = \frac{1}{2}\lambda_{\max}(Q_2^{-1}) > 0$, $\lambda_{P_4} = \lambda_{\max}(P_4) > 0$, $\lambda_{M_4} = \lambda_{\max}(M_4) > 0$.

Similarly, when $\omega = 0$, we have

$$\dot{V}_4(\chi) \leqslant 0. \tag{57}$$

Thus, there exists υ_4 which belongs to class $\mathcal{K}$ functions such that

$$\dot{V}_4(\chi) \leqslant -\upsilon_4(\|\chi\|) \tag{58}$$

holds which guarantees that the time delay Hamiltonian system (1) is asymptotically stable for all time delay factors satisfying (5) and (6). □

Remark 5. Both Theorems 3 and 4 are delay-dependent results. It should be pointed out that K_1 in Theorem 3 and K_2 in Theorem 4 satisfy the same inequality. This indicates that the conditions (33) and (50) are necessary to guarantee the H_∞ control performance regardless of the parameter perturbations in the time delay Hamiltonian system under consideration.

Remark 6. The conditions obtained in Theorems 1–4 are not strict linear matrix inequalities. But with the help of some existing technique related to functions and inequalities, one can always solve these inequalities constraints. As the theorems show, most matrices need not to be solved but require the existence of themselves. In particular, when the state and gain matrices in system (1) only depend on parameter p, the conditions become linear matrix inequalities (LMIs) and can be easily resolved by the LMI toolbox of Matlab.

Remark 7. When the state matrices of system (1) $J(x,p)$, $R(x,p)$, $J_1(x,p)$ and $R_1(x,p)$ have time delay, they can be rewritten as $J(x,x_\tau,p)$, $R(x,x_\tau,p)$, $J_1(x,x_\tau,p)$,

$R_1(x, x_\tau, p)$, we may decompose them as

$$\begin{aligned} J(x, x_\tau, p) &= \Delta_J(x, x_\tau, p) + J(x, x_\tau, 0) \\ J_1(x, x_\tau, p) &= \Delta_{J_1}(x, x_\tau, p) + J_1(x, x_\tau, 0) \\ R(x, x_\tau, p) &= \Delta_R(x, x_\tau, p) + R(x, x_\tau, 0) \\ R_1(x, x_\tau, p) &= \Delta_{R_1}(x, x_\tau, p) + R_1(x, x_\tau, 0). \end{aligned}$$

It can be concluded that Theorem 4 still works provided that Assumptions 2–4 be replaced by the following assumptions, respectively.

Assumption 2′. $R(x, x_\tau, p)$ satisfies $R(x, x_\tau, p) \geqslant A^*$, A^* is a constant matrix.

Assumption 3′. There exists an appropriate dimensioned matrix $\Phi(x, x_\tau)$ such that

$$\begin{aligned} \{[J(x, x_\tau, p) - R(x, x_\tau, p)] + [J_1(x, x_\tau, p)-& \\ R_1(x, x_\tau, p)]\}\Delta_H(x, p) = g_1(x)\Phi^{\mathrm{T}}(x, x_\tau)\theta& \end{aligned} \tag{59}$$

where θ is a constant parameter vector subjecting to p.

Assumption 4′. $J_1(x, x_\tau, p)$, $R_1(x, x_\tau, p)$ satisfy

$$J_1(x, x_\tau, p) - R_1(x, x_\tau, p) \leqslant Y + 2E(x)\Sigma(x_\tau, p)) \tag{60}$$

where $\Delta_{J_1}(x, x_\tau, p) - \Delta_{R_1}(x, x_\tau, p) = 2E(x)\Sigma(x_\tau, p)$, Y is a constant matrix, $E(x)$ is a known matrix with appropriate dimensions and $\Sigma(x_\tau, p)$ satisfies $\Sigma^{\mathrm{T}}(x_\tau, p)\Sigma(x_\tau, p) \leqslant I$.

In the premise of the above Assumptions, A and $J_1(x) - R_1(x)$ can be replaced by A^* and Y respectively in the matrix inequality (51) in Theorem 4.

4 Illustrative example

In this section, an example will be demonstrated to illustrate our developed theoretical results. The following example demonstrates the correctness of Theorems 2 and 4.

Consider a two-dimensional nonlinear time delay system with parameter uncertainty and disturbance of the following form

$$\begin{cases} \dot{x}_1 = -(x_2^2 + 1.5 + p)\sin x_1 - 0.5x_1^2 x_2 - 0.5px_1^2 - \\ \qquad 2\sin x_1(t-\tau) + u_1(t) + \omega_1(t) \\ \dot{x}_2 = 0.5x_1^2 \sin x_1 - x_2^3 - px_2^2 - (2-p)x_2 - 3p - \\ \qquad (1+p)x_2(t-\tau) + u_2(t) + \omega_2(t) \\ y_1 = \sin x_1 \\ y_2 = x_2 \end{cases} \tag{61}$$

where τ is time delay, p is an unknown constant and $|p| < 1$.

Let $x = [x_1,\ x_2]^{\mathrm{T}}$, $u = [u_1(t),\ u_2(t)]^{\mathrm{T}}$, $y = [y_1,\ y_2]^{\mathrm{T}}$ and $\omega = [\omega_1(t),\ \omega_2(t)]^{\mathrm{T}}$, the system (61) can be rewritten in the following form of nonlinear time delay Hamiltonian system

$$\begin{cases} \dot{x} = [J(x,p) - R(x,p)]\nabla H(x,p) + [J_1(x,p) - \\ \qquad R_1(x,p)]\nabla H(x_\tau, p) + g_1(x)u + g_2(x)\omega \\ y = g_2^{\mathrm{T}}(x)\nabla H(x) \end{cases} \tag{62}$$

where

$$J(x,p) = \begin{bmatrix} 0 & -0.5x_1^2 \\ 0.5x_1^2 & 0 \end{bmatrix}$$

$$J_1(x,p) = -J(x,p)$$

$$R_1(x,p) = \begin{bmatrix} 2 & 0.5x_1^2 \\ -0.5x_1^2 & 1+p \end{bmatrix}$$

$$g_1(x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R(x,p) = \begin{bmatrix} x_2^2 + 1.5 + p & 0 \\ 0 & x_2^2 + 2 - p \end{bmatrix}$$

$$g_2(x) = g_1(x)$$

and

$$H(x,p) = \sin^2(0.5x_1) + 0.5(x_2^2 + 2px_2 + p^2).$$

We give a penalty signal

$$z = r(x)g_1(x)\nabla H(x) \tag{63}$$

where $r(x) = \begin{bmatrix} 1 & 1 \\ 0.5 & 1 \end{bmatrix}$.

It is easy to verify that the Hamiltonian function $H(x, 0)$ and its gradient $\nabla H(x, 0)$ in system (61) satisfy Assumption 1. Let $\nu = 1$, $A = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$, $\theta = -p$ and $\Phi(x) = [0.5x_1^2, x_2^2 + 3]$. We can easily verify that system (61) with the above values satisfies Assumptions 2–4.

Firstly, in order to verify Theorem 2, we consider the case that the delay τ in system (61) is a constant and develop a delay-independent result by using Theorem 2. Here we set $\varepsilon_1 = 1$, $Q_1 = 1$. Using the LMI control toolbox of Matlab, the LMIs in Theorem 2 are solved to find the following matrices

$$P_2 = \begin{bmatrix} 4.3773 & 0.0000 \\ 0.0000 & 4.3773 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 4.4398 & 0.3750 \\ 0.3750 & 4.3773 \end{bmatrix}.$$

An adaptive H_∞ controller of the system (61) is obtained as

$$u = \begin{bmatrix} -8.8171\sin x_1 - 0.3750x_2 - 0.5x_1^2\hat{\theta} \\ -0.3750\sin x_1 - 8.7546x_2 - (x_2^2 + 3)\hat{\theta} \end{bmatrix}.$$

We carry the simulation results with the following choices: the initial condition $x(0) = \varphi_0 = [1, -2]^{\mathrm{T}}$, the disturbance signal $\omega = \mathrm{e}^{-t}\sin t$, the disturbance attenuation level $\gamma = 1$. The simulation results are shown in Figs. 1 and 2 with $\tau = 0.5$ and $\tau = 3$. The simulation results show that the robust H_∞ control law proposed in Theorem 2 is effective.

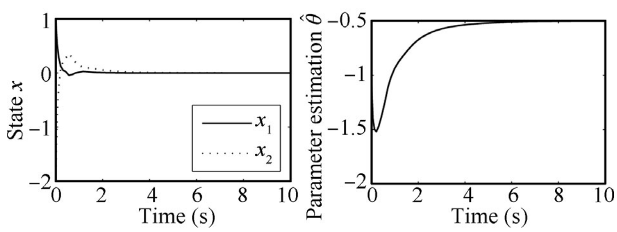

Fig. 1 Response curves with $\tau = 0.5$

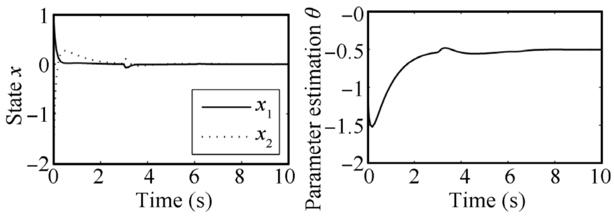

Fig. 2 Response curves with $\tau = 3$

Next we consider the case that the delay in system (61) is a time-varying continuous function and design an adaptive controller by using Theorem 4 to guarantee the asymptotical stability of the closed-loop system.

Here we set $\tau = d(t) = \frac{1}{6}(\pi + 2\arctan t)$, then we can take $\mu = 0.34$, $h = 1.05$. Through simple test works and calculations, we get that $\varepsilon_2 = 1$, $Q_2 = 1$, $K_2 = \begin{bmatrix} 1.05 & 0 \\ 0 & 1.05 \end{bmatrix}$ and the solution of (51)

$$P_4 = \begin{bmatrix} 7.3218 & 0.0000 \\ 0.0000 & 7.3218 \end{bmatrix}$$

$$M_4 = \begin{bmatrix} 6.4970 & 0.3750 \\ 0.3750 & 6.4345 \end{bmatrix}$$

satisfy all the conditions in Theorem 4. Then an adaptive H_∞ controller of the system (61) is obtained as

$$u = \begin{bmatrix} -21.0029 \sin x_1 - 0.3750 x_2 - 0.5 x_1^2 \hat{\theta} \\ -0.3750 \sin x_1 - 20.9404 x_2 - (x_2^2 + 3)\hat{\theta} \end{bmatrix}.$$

Simulation is shown in Fig. 3. From the simulation we can see that the adaptive H_∞ control law proposed in Theorem 4 is effective.

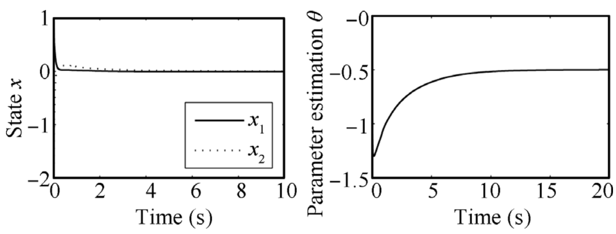

Fig. 3 Response curves with $\tau = d(t)$

5 Conclusions

In this paper, the adaptive H_∞ control problem of a class of time delay nonlinear Hamiltonian systems with parametric uncertainties and disturbances has been investigated. Based on the Lyapunov-Krasovskii functional technique, some sufficient conditions are established and adaptive controllers are designed which guarantee the asymptotic stability and L_2 gain stability of the closed-loop systems. A numerical example is provided to illustrate the theoretical developments. Especially, the results obtained in this paper have provided a new way in dealing with the H_∞ control design problem for some classes of nonlinear systems with time delay and uncertainties.

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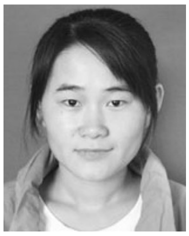

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