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Output Feedback Tracking Control for a Class of Switched Nonlinear Systems with Time-varying Delay

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Abstract: This paper studies the problem of tracking control for a class of switched nonlinear systems with time-varying delay. Based on the average dwell-time and piecewise Lyapunov functional methods, a new exponential stability criterion is obtained for the switched nonlinear systems. The designed output feedback H_∞ controller can be obtained by solving a set of linear matrix inequalities (LMIs). Moreover, the proposed method does not need that a common Lyapunov function exists for the switched systems, and the switching signal just depends on time. A simulation example is provided to demonstrate the effectiveness of the proposed design scheme.

Keywords: Tracking control, switched nonlinear systems, exponential stability, time-varying delay, H_∞ controller.

1 Introduction

Switched systems are a class of hybrid dynamical systems which consist of a family of subsystems and a rule that orchestrates the switching among them. The local behavior is affected by the continuous dynamical subsystems and the discrete dynamical switching mechanisms determine the global performance. In recent years, there are lots of significant achievements both in theory development and practical applications of switched systems[1−5]. On the other hand, time-delay systems are an important class of systems, which are ubiquitous in real world, such as in chemical process, aerodynamics, and communication networks systems. Sometimes, even a small delay may affect the system performance greatly. A stable system may become unstable or chaotic behavior may appear if delay is present in the system[6−8]. Since the switched systems with time-delay have strong engineering background, they have attracted considerable attention, and some useful results have been obtained[9−18]. There are several methods used in analyzing stability of time-delay switched systems, such as the common Lyapunov function, multiple Lyapunov functions, piecewise Lyapunov function and average dwell-time. The work in [12] gives delay-dependent conditions for the exponential stability of the switched linear systems with time-varying delay by common Lyapunov function. The work in [17] gives the exponential stability criterion for a class of switched linear systems with constant time delay by combining dwell-time with the piecewise Lyapunov function.

Tracking control for time-delay switched system is widely used in robot control and guided missile control. To the best of the authors′ knowledge, the issue of tracking control has not been fully investigated for time-delay switched systems. Only a few results have been reported on tracking control for switched systems[19−26]. The stability of tracking control based on observer for time-delay switched linear systems has been investigated in [21], but the proposed method needs that a common Lyapunov function must exist for the switched systems. By using average dwell-time, a new exponential stability criterion of state feedback tracking control for switched nonlinear systems with time-varying delay has been obtained in [22]. In this paper, we aim to design an output feedback controller for a class of switched nonlinear systems with time-varying delay. Combining average dwell-time with the piecewise Lyapunov function, a new exponential stability criterion for a class of switched nonlinear systems with time-varying delay is derived. Moreover, when there exists a common Lyapunov function for the system, it will be exponentially stable under arbitrary switching law.

Notations. $\mathbf{R}^n$ denotes n-dimensional Euclidean space; $\mathbf{R}^{m\times n}$ denotes the space of $m \times n$ matrices with real entries. $L_2[0,\infty)$ is the space of square integrable functions on $[0,\infty)$, and $\mathscr{L}_1^{\rm loc}([\varrho,\infty),\mathbf{R}^n)$ is the space of locally Lebesgue integrable vector valued functions on $[\varrho,\infty)$, where ϱ is a scalar. For any given $\tau > 0$, let $C_n = C([-\tau,0],\mathbf{R}^n)$ be the Banach space of continuous mapping from $([-\tau,0],\mathbf{R}^n)$ to $\mathbf{R}^n$ with the topology of uniform convergence. I represents identity matrix with appropriate dimension. $P > 0(\geqslant,<,\leqslant 0)$ denotes a positive definite (positive semi-defined, negative definite, negative semi-definite) matrix. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimal and maximal eigenvalues of a square matrix. $\sigma_{\max}(\cdot)$ means the maximal singular value of a matrix. The superscript "T" stands for matrix transpose and the symmetric terms in a symmetric matrix are denoted by $*$. Let $x_t \in C_n$ be defined by $x_t(\theta) = x(t+\theta), \theta \in [-\tau,0]$. $\|\cdot\|$ denotes the usual 2-norm, and $\| x_t \|_{cl} = \sup_{-\tau\leqslant\theta\leqslant 0}\{\| x(t+\theta)\|, \|\dot{x}(t+\theta)\|\}$.

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2 Problem formulation and preliminaries

Consider the switched nonlinear system with time-varying delay

$$\begin{aligned}&\dot{x}(t)=f_\sigma(x(t))+A_\sigma x(t)+D_\sigma x(t-d(t))+B_\sigma u(t)+\omega(t)\\&y(t)=C_\sigma x(t)\\&x(t)=\varphi(t),\quad -\tau\leqslant t\leqslant 0\end{aligned}\tag{1}$$

where $x\in\mathbf{R}^n$ is the system state vector, $u(t)\in\mathbf{R}^p$ is the control input, $y(t)\in\mathbf{R}^q$ is the output, $\omega(t)\in\mathbf{R}^n$ is the bounded exogenous disturbance which belongs to $L_2[0,\infty)$ and $\mathscr{L}_1^{\rm loc}([\varrho,\infty),\mathbf{R}^n)$. $f_\sigma(\cdot):\mathbf{R}^n\to\mathbf{R}^n$ is a known nonlinear function; $\sigma(t):[t_0,\infty)\to M\triangleq\{1,2,\cdots,m_0\}$ is the switching signal. $d(t)$ denotes the time-varying delay satisfying $0<d(t)\leqslant\tau$; $\varphi(t)$ is a continuous vector-valued initial function. Moreover, $\sigma(t)=i$ means that the i-th subsystem is activated, A_i,B_i,C_i,D_i are known constant matrices with appropriate dimensions. Without loss of generality, we state that the following assumptions hold.

Assumption 1. $U_i\in\mathbf{R}^{n\times n},i\in M$ are known constant matrices, the function $f_i(x(t))$ is Lipschitz for all $x(t)\in\mathbf{R}^n$ and $\hat{x}(t)\in\mathbf{R}^n$, and satisfies

$$\parallel f_i(x(t))-f_i(\hat{x}(t))\parallel\leqslant\parallel U_i(x(t)-\hat{x}(t))\parallel.\tag{2}$$

Assumption 2. For $i\in M$, the subsystem (A_i,C_i,D_i) are detectable[21] and B_i are full row rank.

Definition 1[10]**.** System (1) is said to be exponentially stabilizable under control law $u(t)$ and switching law $\sigma(t)$, if the solution $x(t)$ of system (1) through $(t_0,\varphi)\in\mathbf{R}_+\times C_n$ satisfies

$$\parallel x(t)\parallel\leqslant\kappa\parallel x_{t_0}\parallel_{cl}{\rm e}^{-\lambda(t-t_0)},\quad\forall t\geqslant t_0\tag{3}$$

for some constants $\kappa\geqslant0$ and $\lambda>0$.

Suppose that the state observer is of the form

$$\begin{aligned}\dot{\hat{x}}(t)=&f_\sigma(\hat{x}(t))+A_\sigma\hat{x}(t)+D_\sigma\hat{x}(t-d(t))+\\&B_\sigma u(t)+L_\sigma(y(t)-\hat{y}(t))\\\hat{y}(t)=&C_\sigma\hat{x}(t)\end{aligned}\tag{4}$$

where $y(t)$ is the measurable output of system (1), and L_σ is the observer gain matrix to be determined later.

The reference model is given as

$$\dot{x}_r(t)=A_rx_r(t)+r(t)\tag{5}$$

where $x_r(t)\in\mathbf{R}^n$ is the reference state, A_r is a Hurwitz matrix, $r(t)$ is the bounded reference input which belongs to $L_2[0,\infty)$ and $\mathscr{L}_1^{\rm loc}([\varrho,\infty),\mathbf{R}^n)$, respectively.

Now, define tracking error $e_r(t)=x(t)-x_r(t)$, and consider the H_∞ tracking performance as[22]

$$\int_{t_0}^{\infty}{\rm e}^{-\alpha(t-t_0)}e_r^{\rm T}(t)e_r(t){\rm d}t\leqslant\gamma^2\int_{t_0}^{\infty}\bar{\omega}^{\rm T}(t)\bar{\omega}(t){\rm d}t,\ t\geqslant t_0\tag{6}$$

where $\bar{\omega}(t)=[\omega^{\rm T}(t),r^{\rm T}(t)]^{\rm T},\alpha,\gamma$ are positive constants.

Define the difference between the real state and the observer state, the observer state and the reference state as

$$e(t)=x(t)-\hat{x}(t),\quad\hat{e}_r(t)=\hat{x}(t)-x_r(t).$$

Design the output feedback controller

$$u(t)=K_\sigma\hat{x}(t)+F_\sigma x_r(t)-B_\sigma^{\rm T}(B_\sigma B_\sigma^{\rm T})^{-1}f_\sigma(\hat{x}(t))\tag{7}$$

where K_σ, F_σ are the output feedback gains. Combining (1), (4), (5) and (7), we can obtain the augmented systems

$$\begin{aligned}\dot{e}(t)=&(A_\sigma-L_\sigma C_\sigma)e(t)+D_\sigma e(t-d(t))+f_\sigma(x(t))-\\&f_\sigma(\hat{x}(t))+\omega(t)\end{aligned}\tag{8}$$

$$\begin{aligned}\dot{\hat{x}}(t)=&(A_\sigma+B_\sigma K_\sigma)\hat{x}(t)+B_\sigma F_\sigma x_r(t)+D_\sigma\hat{x}(t-d(t))+\\&L_\sigma C_\sigma e(t)\\\dot{x}_r(t)=&A_rx_r(t)+r(t).\end{aligned}\tag{9}$$

Let

$$\bar{x}(t)=\begin{bmatrix}\hat{x}(t)\\x_r(t)\end{bmatrix},\quad\bar{A}_\sigma=\begin{bmatrix}A_\sigma+B_\sigma K_\sigma&B_\sigma F_\sigma\\0&A_r\end{bmatrix},$$

$$g_\sigma(t)=\begin{bmatrix}L_\sigma C_\sigma e(t)\\r(t)\end{bmatrix},\quad\bar{D}_\sigma=\begin{bmatrix}D_\sigma&0\\0&0\end{bmatrix}.$$

Then, system (9) can be rewritten as

$$\dot{\bar{x}}(t)=\bar{A}_\sigma\bar{x}(t)+\bar{D}_\sigma\bar{x}(t-d(t))+g_\sigma(t).\tag{10}$$

Define the switching sequences of system (8) and (9)

$$\Sigma\triangleq\{(i_0,t_0),(i_1,t_1),\cdots,(i_k,t_k),\cdots\mid i_k\in M\}\tag{11}$$

which means the i_k-th subsystem is activated at time t_k.

Definition 2[20, 21]**.** For system (1), if there exist control input $u(t)$ and switching law $\sigma(t)$, such that: 1) the closed-loop (8) and (9) are exponentially stable when $\bar{\omega}(t)\equiv0$; 2) performance index (6) is satisfied when $\bar{\omega}(t)\neq0$ under zero initial conditions, that is, $x(t)=0,x_r(0)=0,\hat{x}(t)=0,t\in[-\tau,0]$. Then system (1) is said to have observer-based H_∞ model reference tracking performance.

Definition 3[1]**.** For the switched signal $\sigma(t)$ and any $t\geqslant\tau\geqslant0$, $N_\sigma(t,\tau)$ denotes the system switching times in the open interval (τ,t). If

$$N_\sigma(t,\tau)\leqslant N_0+\frac{t-\tau}{\tau_a}\tag{12}$$

holds for $\tau_a>0$ and $N_0\geqslant0$, then τ_a is called average dwell-time. Without loss of generality, as commonly used in the literature, we assume $N_0=0$.

To conclude this section, we recall the following lemmas.

Lemma 1[9]**.** Let U,V be real matrices of appropriate dimensions. Then, for any matrix $Q>0$ of appropriate dimension and scalar $\epsilon>0$, it holds that

$$UV+V^{\rm T}U^{\rm T}\leqslant\epsilon^{-1}UQ^{-1}U^{\rm T}+\epsilon V^{\rm T}QV.$$

Lemma 2[13]**.** For any constant matrix $N>0$, scalar $\tau>0$, any $t\in[0,+\infty)$, vector function $y:[t-\tau,t]\to\mathbf{R}^n$, such that the integrations in the following are well defined, then

$$\left(\int_{t-\tau}^{t}y(s){\rm d}s\right)^{\rm T}N\int_{t-\tau}^{t}y(s){\rm d}s\leqslant\tau\int_{t-\tau}^{t}y(s)^{\rm T}Ny(s){\rm d}s.$$

Consider the linear time-varying delay system (10) without switching and its homogeneous system as

$$\dot{\bar{x}}(t)=\bar{A}\bar{x}(t)+\bar{D}\bar{x}(t-d(t))+g(t)\tag{13}$$

$$\dot{\bar{x}}(t)=\bar{A}\bar{x}(t)+\bar{D}\bar{x}(t-d(t)).\tag{14}$$

It can be rewritten in operator form[21]

$$\begin{aligned}\dot{\bar{x}}(t) &= L(t, x_t) + g(t), \qquad t \geqslant \varrho\\ \dot{\bar{x}}(t) &= L(t, x_t),\ \bar{x}_\varrho = \phi, \quad t \geqslant \varrho\end{aligned}$$

where the operator $L(t,\phi)$ is linear in ϕ, and has the form $L(t,\phi) = A\phi(0) + D\phi(-d(t))$, in which $\phi(\theta) = x(t+\theta), \theta \in [-\tau, 0]$. Suppose there is an $m \in \mathscr{L}_1^{\rm loc}([\varrho,\infty), \mathbf{R}_+)$ such that

$$|\ L(t,\phi)\ | \leqslant m(t)\ |\ \phi\ | \tag{15}$$

for all $t \in (-\infty,\infty), \phi \in \mathbf{C}_n$.

Lemma 3 (Variation-of-constants). Let $\bar{x}(\varrho,\phi,g)(t)$ denote the solution of system (13), and $\bar{x}(\varrho,\phi,0)(t)$ denote the solution of the corresponding homogeneous system (14). Denote $\bar{x}(\varrho,\phi,0)(t+\theta)$ by $\bar{x}_t(\varrho,\phi,0)(\theta), -\tau \leqslant \theta \leqslant 0$, $X_0(\theta) = \begin{cases} 0, & -\tau \leqslant \theta < 0\\ I, & \theta = 0\end{cases}$. If $\bar{x}_t(\varrho,\phi,0) \triangleq T(t,\varrho)\phi$, then $T(t,\varrho)$ is a continuous linear operator. And if (15) is satisfied and $g(t) \in \mathscr{L}_1^{\rm loc}([\varrho,\infty), \mathbf{R}^{2n})$, then

$$\bar{x}_t(\varrho,\phi,g) = T(t,\varrho)\phi + \int_\varrho^t T(t,s)X_0 g(s)\mathrm{d}s, \quad t \geqslant \varrho. \tag{16}$$

Proof. The proof follows the same lines as in [21, 27]. □

3 Main results

3.1 Observer gain matrix design

Consider the simplified system of (8) when $\omega(t) = 0$

$$\begin{aligned}\dot{e}(t) = \ & (A_\sigma - L_\sigma C_\sigma)e(t) + D_\sigma e(t-d(t)) +\\ & f_\sigma(x(t)) - f_\sigma(\hat{x}(t)).\end{aligned} \tag{17}$$

Theorem 1. For system (17), if there exist scalar $\alpha > 0, \tau > 0$, matrices $H_i > 0, G_i > 0, \bar{L}_i, \forall i \in M$ and any invertible matrix Y_i with appropriate dimensions, such that the following LMIs hold

$$\Theta_i = \begin{bmatrix} \varphi_{11i} & \varphi_{12i} & Y_i & -\tau Y_i D_i\\ * & \varphi_{22i} & Y_i & -\tau Y_i D_i\\ * & * & -I & 0\\ * & * & * & -\tau \mathrm{e}^{-\alpha\tau} G_i \end{bmatrix} < 0 \tag{18}$$

and the average dwell-time satisfies

$$\tau_{a_1} > \frac{\ln \mu_1}{\alpha} \tag{19}$$

where $\varphi_{11i} = Y_i(A_i+D_i) + (A_i+D_i)^{\rm T} Y_i^{\rm T} - \bar{L}_i C_i - C_i^{\rm T}\bar{L}_i^{\rm T} + \alpha H_i + U_i^{\rm T} U_i, \varphi_{12i} = H_i - Y_i + (A_i+D_i)^{\rm T} Y_i^{\rm T} - C_i^{\rm T}\bar{L}_i^{\rm T}, \varphi_{22i} = \tau G_i - Y_i - Y_i^{\rm T}$ and $\mu_1 \geqslant 1$ satisfies

$$H_i \leqslant \mu_1 H_j, \quad G_i \leqslant \mu_1 G_j, \quad \forall i,j \in M, i \neq j. \tag{20}$$

Then, system (17) is exponentially stable and the observer gain matrix is given by $L_i = Y_i^{-1}\bar{L}_i$.

Proof. Choose the piecewise Lyapunov functional candidate as

$$\begin{aligned} V(e(t)) &= V_\sigma(e(t)) = V_{1\sigma}(e(t)) + V_{2\sigma}(e(t))\\ V_{1\sigma}(e(t)) &= e^{\rm T}(t) H_\sigma e(t)\\ V_{2\sigma}(e(t)) &= \int_{-\tau}^0 \int_{t+\theta}^t \mathrm{e}^{-\alpha(t-s)} \dot{e}^{\rm T}(s) G_\sigma \dot{e}(s)\mathrm{d}s\mathrm{d}\theta. \end{aligned} \tag{21}$$

During any interval $[t_k, t_{k+1})$, we let $\sigma(t) = i_k = i$, then $V(e(t)) = V_i(e(t))$. Along the trajectories of system (17), the time derivative of $V_i(e(t))$ is given as

$$\begin{aligned}\dot{V}_i(e(t)) = \ & 2e^{\rm T}(t) H_i \dot{e}(t) - \alpha V_{2i}(e(t)) + \tau \dot{e}^{\rm T}(t) G_i \dot{e}(t) -\\ & \int_{t-\tau}^t \mathrm{e}^{-\alpha(t-s)} \dot{e}^{\rm T}(s) G_i \dot{e}(s)\mathrm{d}s.\end{aligned} \tag{22}$$

From Assumption 1, we can get

$$\begin{aligned}&[f_i(x(t)) - f_i(\hat{x}(t))]^{\rm T}[f_i(x(t)) - f_i(\hat{x}(t))] \leqslant\\ &\quad e^{\rm T}(t) U_i^{\rm T} U_i e(t).\end{aligned} \tag{23}$$

Substituting (23) into (22), one has

$$\begin{aligned}&\dot{V}_i(e(t)) + \alpha V_i(e(t)) \leqslant e^{\rm T}(t)(\alpha H_i + U_i^{\rm T} U_i)e(t) +\\ &\quad 2e^{\rm T}(t) H_i \dot{e}(t) + \tau \dot{e}^{\rm T}(t) G_i \dot{e}(t) - \mathrm{e}^{-\alpha\tau} \int_{t-d(t)}^t \dot{e}^{\rm T}(s) G_i \dot{e}(s)\mathrm{d}s -\\ &\quad [f_i(x(t)) - f_i(\hat{x}(t))]^{\rm T}[f_i(x(t)) - f_i(\hat{x}(t))].\end{aligned} \tag{24}$$

For the free weighting matrix Y_i and $\int_{t-d(t)}^t \dot{e}(s)\mathrm{d}s = e(t) - e(t-d(t))$, it holds that

$$\begin{aligned}&2[e^{\rm T}(t), \dot{e}^{\rm T}(t)] \begin{bmatrix} Y_i\\ Y_i\end{bmatrix} [(A_i + D_i - L_i C_i)e(t) - \dot{e}(t) -\\ &\quad D_i \int_{t-d(t)}^t \dot{e}(s)\mathrm{d}s + f_i(x(t)) - f_i(\hat{x}(t))] = 0.\end{aligned} \tag{25}$$

Let $\xi(t,s) = [e^{\rm T}(t), \dot{e}^{\rm T}(t), f_i^{\rm T}(x(t)) - f_i^{\rm T}(\hat{x}(t)), \dot{e}^{\rm T}(s)]^{\rm T}$. Substituting (25) into (24), it yields

$$\dot{V}_i(e(t)) + \alpha V_i(e(t)) \leqslant \frac{1}{d(t)} \int_{t-d(t)}^t \xi^{\rm T}(t,s) \bar{\Theta}_i \xi(t,s)\mathrm{d}s \tag{26}$$

where $\bar{\Theta}_i = \begin{bmatrix} \varphi_{11i} & \varphi_{12i} & Y_i & -d(t) Y_i D_i\\ * & \varphi_{22i} & Y_i & -d(t) Y_i D_i\\ * & * & -I & 0\\ * & * & * & -d(t)\mathrm{e}^{-\alpha\tau} G_i\end{bmatrix} < 0$. By proper transformation to (18), we can get $\bar{\Theta}_i < 0$, then

$$\dot{V}_i(e(t)) + \alpha V_i(e(t)) \leqslant 0. \tag{27}$$

Let t_k^- be the left limit of t_k, which is before the switching at t_k. Using (20) and (21), one has

$$V_i(e(t_k)) \leqslant \mu_1 V_j(e(t_k^-)), \quad \forall i,j \in M, \quad i \neq j. \tag{28}$$

Let any $t \in [t_k, t_{k+1})$, then $N_\sigma(t, t_0) = k$. Combining (11), (27) and (28), we can obtain

$$\begin{aligned} V_{i_k}(e(t)) \ & \leqslant V_{i_k}(e(t_k))\mathrm{e}^{-\alpha(t-t_k)} \leqslant\\ & \mu_1 V_{i_{k-1}}(e(t_{k-1}))\mathrm{e}^{-\alpha(t-t_{k-1})} \leqslant \cdots \leqslant\\ & \mu_1^k V_{i_0}(e(t_0))\mathrm{e}^{-\alpha(t-t_0)} =\\ & \mathrm{e}^{-\alpha(t-t_0) + k\ln\mu_1} V_{i_0}(e(t_0)).\end{aligned} \tag{29}$$

From Definition 3, we can get $k\ln\mu_1 \leqslant \frac{t-t_0}{\tau_{a_1}} \ln\mu_1$. Let $2\lambda = \alpha - \frac{\ln\mu_1}{\tau_{a_1}} > 0$, then,

$$a \parallel e(t) \parallel^2 \leqslant \mathrm{e}^{-2\lambda(t-t_0)} V_{i_0}(e(t_0)) \leqslant b\, \mathrm{e}^{-2\lambda(t-t_0)} \parallel e_{t_0} \parallel_{cl}^2 \tag{30}$$

where $a = \min_{i\in M}\{\lambda_{\min}(H_i)\}, b = \max_{i\in M}\{\lambda_{\max}(H_i) + \frac{\tau^2}{2}\lambda_{\max}(G_i)\}$. From (30), we can obtain

$$\| e(t) \| \leqslant \sqrt{\frac{b}{a}} \quad \mathrm{e}^{-\lambda(t-t_0)} \| e_{t_0} \|_{cl} . \tag{31}$$

By Definition 1, we know that system (17) is exponentially stable. □

3.2 Stability and H_∞ performance

Theorem 2. For system (10), if there exist scalars $\alpha > 0, \tau > 0, \gamma > 0$, and matrices $P_i > 0, S_i > 0, \forall i \in M$, such that the following inequalities hold

$$\Phi_i = \begin{bmatrix} \phi_{11i} & P_i\bar{D}_i & P_i & \tau(\bar{A}_i + \bar{D}_i)^{\mathrm{T}} \\ * & \phi_{22i} & 0 & \tau\bar{D}_i^{\mathrm{T}} \\ * & * & -\dfrac{1}{2}\gamma^2 I & \tau I \\ * & * & * & -S_i^{-1} \end{bmatrix} < 0 \tag{32}$$

and the average dwell-time satisfies

$$\tau_a = \max\{\tau_{a_1}, \tau_{a_2}\}, \quad \tau_{a_2} > \frac{\ln \mu_2}{\alpha} \tag{33}$$

where $\phi_{11i} = P_i(\bar{A}_i + \bar{D}_i) + (\bar{A}_i + \bar{D}_i)^{\mathrm{T}}P_i + \alpha P_i + \bar{Q}$, $\phi_{22i} = -\mathrm{e}^{-\alpha\tau}S_i, \bar{Q} = \begin{bmatrix} I & -I \\ -I & I \end{bmatrix}$, and $\mu_2 \geqslant 1$ satisfies

$$P_i \leqslant \mu_2 P_j, \quad S_i \leqslant \mu_2 S_j, \quad \forall i, j \in M, i \neq j. \tag{34}$$

Then system (10) is exponentially stable, and the H_∞ model reference tracking performance in (1) is guaranteed.

Proof. From (32), using Schur complement, it yields

$$\Lambda_i = \begin{bmatrix} \phi'_{11i} & P_i\bar{D}_i & P_i \\ * & \phi_{22i} & 0 \\ * & * & -\dfrac{1}{2}\gamma^2 I \end{bmatrix} + \tau^2 \begin{bmatrix} \bar{A}_i + \bar{D}_i & \bar{D}_i & I \end{bmatrix}^{\mathrm{T}}$$
$$S_i \begin{bmatrix} \bar{A}_i + \bar{D}_i & \bar{D}_i & I \end{bmatrix} < \begin{bmatrix} -\bar{Q} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{35}$$

where $\phi'_{11i} = \phi_{11i} - \bar{Q}$.

First, consider the nominal system of system (10):

$$\dot{\bar{x}}(t) = \bar{A}_\sigma \bar{x}(t) + \bar{D}_\sigma \bar{x}(t - d(t)). \tag{36}$$

Define the piecewise Lyapunov functional candidate as

$$V(\bar{x}(t)) = V_\sigma(\bar{x}(t)) = V_{1\sigma}(\bar{x}(t)) + V_{2\sigma}(\bar{x}(t))$$
$$V_{1\sigma}(\bar{x}(t)) = \bar{x}^{\mathrm{T}}(t)P_\sigma \bar{x}(t)$$
$$V_{2\sigma}(\bar{x}(t)) = \tau \int_{-\tau}^{0}\int_{t+\theta}^{t} \mathrm{e}^{-\alpha(t-s)}\dot{\bar{x}}^{\mathrm{T}}(s)S_\sigma\dot{\bar{x}}(s)\mathrm{d}s\mathrm{d}\theta.$$

Let $-\int_{t-d(t)}^{t}\dot{\bar{x}}(s)\mathrm{d}s = z(t) = \bar{x}(t-d(t)) - \bar{x}(t)$, then $\dot{\bar{x}}(t) = (\bar{A}_i + \bar{D}_i)\bar{x}(t) + \bar{D}_i z(t)$. Then, along the trajectories of system (36), the time derivative of $V_i(\bar{x}(t))$ is given by

$$\begin{aligned}\dot{V}_i(\bar{x}(t)) \leqslant & \bar{x}^{\mathrm{T}}(t)[P_i(\bar{A}_i + \bar{D}_i) + (\bar{A}_i + \bar{D}_i)^{\mathrm{T}}P_i]\bar{x}(t) + \\ & 2\bar{x}^{\mathrm{T}}(t)P_i\bar{D}_i z(t) - \alpha V_{2i}(\bar{x}(t)) + \tau^2\dot{\bar{x}}^{\mathrm{T}}(t)S_i\dot{\bar{x}}(t) - \\ & \tau\mathrm{e}^{-\alpha\tau}\int_{t-d(t)}^{t}\dot{\bar{x}}^{\mathrm{T}}(s)S_i\dot{\bar{x}}(s)\mathrm{d}s.\end{aligned} \tag{37}$$

By Lemma 2, it can be obtained that

$$-\tau\int_{t-d(t)}^{t}\dot{\bar{x}}^{\mathrm{T}}(s)S_i\dot{\bar{x}}(s)\mathrm{d}s \leqslant -z^{\mathrm{T}}(t)S_i z(t). \tag{38}$$

Let $\eta(t) = [\bar{x}^{\mathrm{T}}(t), z^{\mathrm{T}}(t)]^{\mathrm{T}}$. Substituting (38) into (37), it yields

$$\dot{V}_i(\bar{x}(t)) + \alpha V_i(\bar{x}(t)) \leqslant \eta^{\mathrm{T}}(t)\bar{\Phi}_i\eta(t) \tag{39}$$

where

$$\begin{aligned}\bar{\Phi}_i = & \begin{bmatrix} P_i(\bar{A}_i + \bar{D}_i) + (\bar{A}_i + \bar{D}_i)^{\mathrm{T}}P_i + \alpha P_i & P_i\bar{D}_i \\ * & -\mathrm{e}^{-\alpha\tau}S_i \end{bmatrix} + \\ & \tau^2\begin{bmatrix} \bar{A}_i + \bar{D}_i & \bar{D}_i \end{bmatrix}^{\mathrm{T}} S_i \begin{bmatrix} \bar{A}_i + \bar{D}_i & \bar{D}_i \end{bmatrix}.\end{aligned} \tag{40}$$

From (35), (39) and (40), one can obtain

$$\dot{V}_i(\bar{x}(t)) + \alpha V_i(\bar{x}(t)) \leqslant 0. \tag{41}$$

By using the same means as the proof of Theorem 1, we can get

$$\| \bar{x}(t) \| \leqslant \sqrt{\frac{b_1}{a_1}} \quad \mathrm{e}^{-\lambda_1(t-t_0)} \| \bar{x}_{t_0} \|_{cl} . \tag{42}$$

where $2\lambda_1 = \alpha - \frac{\ln\mu_2}{\tau_{a_2}} > 0$, $a_1 = \min_{i\in M}\{\lambda_{\min}(P_i)\}, b_1 = \max_{i\in M}\{\lambda_{\max}(P_i) + \frac{\tau^3}{2}\lambda_{\max}(S_i)\}$. Then, system (36) is exponentially stable.

Next, the stability of system (10) will be analysed with $g_i(t) \neq 0$ when $\bar{\omega}(t) = 0$, $g_i(t) = \begin{bmatrix} L_iC_ie(t) \\ 0 \end{bmatrix}$. Since $g_\sigma(t) \in \mathscr{L}_1^{\mathrm{loc}}([\varrho, \infty), \mathbf{R}^{2n})$, for any $t \in [t_j, t_{j+1})$, by Lemma 3, we get the solution of (10) with the initial condition (t_0, ϕ_{i_0}):

$$\begin{aligned}\bar{x}_t(t_0, \phi_{i_0}, g_\sigma) = & T_{i_j}(t, t_j)\phi_{i_j} + \int_{t_j}^{t}T(t,s)X_0 g_{i_j}(s)\mathrm{d}s = \\ & T_{i_j}(t, t_j)T_{i_{j-1}}(t_j, t_{j-1})\phi_{i_{j-1}} + T_{i_j}(t, t_j) \\ & \int_{t_{j-1}}^{t_j}T(t_j,s)X_0 g_{i_{j-1}}(s)\mathrm{d}s + \int_{t_j}^{t}T(t,s)X_0 g_{i_j}(s)\mathrm{d}s = \cdots = \\ & T(t, t_0)\phi_{i_0} + \int_{t_0}^{t}T(t,s)X_0 g_\sigma(s)\mathrm{d}s\end{aligned} \tag{43}$$

where $T(t, t_0) = T_{i_j}(t, t_j)\cdot T_{i_{j-1}}(t_j, t_{j-1})\cdots T_{i_0}(t_1, t_0)$ is a continuous piecewise linear operator.

Recalling the above analysis, system (36) is exponentially stable. There exist $\eta_1 > 0, \kappa_1 > 0$ such that

$$\begin{aligned}& \| T(t, t_0) \| \leqslant \kappa_1\mathrm{e}^{-\eta_1(t-t_0)} \\ & \| T(t, s)X_0 \| \leqslant \kappa_1\mathrm{e}^{-\eta_1(t-s)}, \quad t \geqslant s \geqslant t_0.\end{aligned} \tag{44}$$

If (18), (19) and (20) hold, system (8) is exponentially stable by Theorem 1. From (31), there exists a scalar $B_0 = \sqrt{\frac{b}{a}} \| e_{t_0} \|_{cl} \max_{i\in M}\{\sigma_{\max}(L_iC_i)\}$, such that

$$\| g_\sigma(t) \| \leqslant \| L_\sigma C_\sigma \| \| e(t) \| \leqslant B_0\mathrm{e}^{-\lambda(t-t_0)}, \quad t \geqslant t_0. \tag{45}$$

Substituting (44) and (45) into (43), and choosing $\eta_1 \neq \lambda$, it yields

$$\begin{aligned}&\| \bar{x}_t(t_0,\phi_{i_0},g_\sigma) \| \leqslant \kappa_1 \mathrm{e}^{-\eta_1(t-t_0)} \| \phi_{i_0} \| + \\ &\int_{t_0}^{t} \kappa_1 \mathrm{e}^{-\eta_1(t-s)} B_0 \mathrm{e}^{-\lambda(s-t_0)} \mathrm{d}s \leqslant \\ &\kappa_1 \mathrm{e}^{-\eta_1(t-t_0)} \| \phi_{i_0} \| + \frac{\kappa_1 B_0}{\eta_1-\lambda}(\mathrm{e}^{-\lambda(t-t_0)} - \mathrm{e}^{-\eta_1(t-t_0)}) \leqslant \\ &\kappa_1(\| \phi_{i_0} \| + \frac{B_0}{|\eta_1-\lambda|})\mathrm{e}^{-\beta(t-t_0)} \end{aligned} \tag{46}$$

where $\beta=\min\{\lambda,\eta_1\}$. We know that system (10) is exponentially stable when $\bar{\omega}(t)=0$.

Second, we will prove that performance index (6) is satisfied under zero initial condition $\bar{x}(t_0)=0, e(t_0)=0$ with $\bar{\omega}(t)\neq 0$. Let $\varsigma(t)=[\bar{x}^{\mathrm{T}}(t), z^{\mathrm{T}}(t), g_i^{\mathrm{T}}(t)]^{\mathrm{T}}$. Using the same method as the above analysis, it implies

$$\begin{aligned}&\dot{V}_i(\bar{x}(t))+\alpha V_i(\bar{x}(t)) \leqslant \varsigma^{\mathrm{T}}(t)\Lambda_i\varsigma(t)+\frac{1}{2}\gamma^2 g_i^{\mathrm{T}}(t)g_i(t) \leqslant \\ &\varsigma^{\mathrm{T}}(t)\begin{bmatrix}-\bar{Q} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\varsigma(t)+\frac{1}{2}\gamma^2 g_i^{\mathrm{T}}(t)g_i(t) = \\ &-\hat{e}_r^{\mathrm{T}}(t)\hat{e}_r(t)+\frac{1}{2}\gamma^2 g_i^{\mathrm{T}}(t)g_i(t).\end{aligned} \tag{47}$$

Letting $Q=\frac{1}{2}I\in\mathbf{R}^{n\times n}$, by Lemma 1, we can get

$$\begin{aligned}-\hat{e}_r^{\mathrm{T}}(t)\hat{e}_r(t) = &-[e(t)-e_r(t)]^{\mathrm{T}}[e(t)-e_r(t)] = \\ &-e_r^{\mathrm{T}}(t)e_r(t)-e^{\mathrm{T}}(t)e(t)+2e_r^{\mathrm{T}}(t)e(t) \leqslant \\ &-e_r^{\mathrm{T}}(t)e_r(t)-e^{\mathrm{T}}(t)e(t)+e_r^{\mathrm{T}}(t)Qe_r(t)+ \\ &e^{\mathrm{T}}(t)Q^{-1}e(t) = -\frac{1}{2}e_r^{\mathrm{T}}(t)e_r(t)+\| e(t) \|^2 .\end{aligned} \tag{48}$$

Similar to the reasoning in the above proof, let $e_t(t_0,\psi_{i_0},\omega)$ denote the solution of system (8) with the initial condition (t_0,ψ_{i_0}), $\psi_{i_0}=e(t_0)=0$, $T_1(t,t_0)$ denotes a continuous piecewise linear operator. As the nominal system of (8) is exponentially stable, by Lemma 3, there exist $\kappa_2>0, \eta_2>0$ such that

$$\begin{aligned}&e_t(t_0,\psi_{i_0},\omega)=T_1(t,t_0)\psi_{i_0}+\int_{t_0}^{t}T_1(t,s)X_0\omega(s)\mathrm{d}s\\ &\| T_1(t,t_0) \| \leqslant \kappa_2\mathrm{e}^{-\eta_2(t-t_0)}\\ &\| T_1(t,s)X_0 \| \leqslant \kappa_2\mathrm{e}^{-\eta_2(t-s)}, \quad t\geqslant s\geqslant t_0.\end{aligned} \tag{49}$$

According to Cauchy-Schwartz Inequality, one can get from (49):

$$\begin{aligned}&\| e(t) \|^2 \leqslant \int_{t_0}^{t}\kappa_2^2\mathrm{e}^{-\eta_2(t-s)}\mathrm{d}s\int_{t_0}^{t}\mathrm{e}^{-\eta_2(t-s)} \| \omega(s) \|^2 \mathrm{d}s \leqslant \\ &\frac{\kappa_2^2}{\eta_2}\int_{t_0}^{t}\mathrm{e}^{-\eta_2(t-s)} \| \omega(s) \|^2 \mathrm{d}s.\end{aligned} \tag{50}$$

Let $\lambda_2=\max_{i\in M}\{\sigma_{\max}(L_iC_i)\}$, it has

$$\begin{aligned}g_i^{\mathrm{T}}(t)g_i(t) = &e^{\mathrm{T}}(t)C_i^{\mathrm{T}}L_i^{\mathrm{T}}L_iC_ie(t)+r^{\mathrm{T}}(t)r(t) \leqslant \\ &\lambda_2^2 \| e(t) \|^2 + r^{\mathrm{T}}(t)r(t).\end{aligned} \tag{51}$$

Combining (47), (48), (50) and (51), it holds that

$$\begin{aligned}&\dot{V}_i(\bar{x}(t))+\alpha V_i(\bar{x}(t)) \leqslant -\frac{1}{2}e_r^{\mathrm{T}}(t)e_r(t)+(1+\frac{1}{2}\gamma^2\lambda_2^2)\\ &\quad\frac{\kappa_2^2}{\eta_2}\int_{t_0}^{t}\mathrm{e}^{-\eta_2(t-s)} \| \omega(s) \|^2 \mathrm{d}s+\frac{1}{2}\gamma^2 r^{\mathrm{T}}(t)r(t),\\ &\quad\quad t\in[t_k,t_{k+1}).\end{aligned} \tag{52}$$

Let $\Gamma(s) = -\frac{1}{2}e_r^{\mathrm{T}}(s)e_r(s) + (1 + \frac{1}{2}\gamma^2\lambda_2^2)\frac{\kappa_2^2}{\eta_2}\int_{t_0}^{s}\mathrm{e}^{-\eta_2(s-\theta)} \| \omega(\theta) \|^2 \mathrm{d}\theta+\frac{1}{2}\gamma^2 r^{\mathrm{T}}(s)r(s)$,

$$\begin{aligned}&V_{i_k}(\bar{x}(t)) = V_{i_k}(\bar{x}(t_k))+\int_{t_k}^{t}\dot{V}_{i_k}(\bar{x}(s))\mathrm{d}s \leqslant \\ &V_{i_k}(\bar{x}(t_k))-\alpha\int_{t_k}^{t}V_{i_k}(\bar{x}(s))\mathrm{d}s+\int_{t_k}^{t}\Gamma(s)\mathrm{d}s \leqslant \\ &\mu_2V_{i_{k-1}}(\bar{x}(t_{k-1}))-\mu_2\alpha\int_{t_{k-1}}^{t_k}V_{i_{k-1}}(\bar{x}(s))\mathrm{d}s+ \\ &\mu_2\int_{t_{k-1}}^{t_k}\Gamma(s)\mathrm{d}s-\alpha\int_{t_k}^{t}V_{i_k}(\bar{x}(s))\mathrm{d}s+\int_{t_k}^{t}\Gamma(s)\mathrm{d}s \leqslant \cdots \leqslant \\ &\mu_2^kV_{i_0}(\bar{x}(t_0)-\mu_2^k\alpha\int_{t_0}^{t_1}V_{i_0}(\bar{x}(s))\mathrm{d}s+ \\ &\mu_2^k\int_{t_0}^{t_1}\Gamma(s)\mathrm{d}s-\cdots-\alpha\int_{t_k}^{t}V_{i_k}(\bar{x}(s))\mathrm{d}s+\int_{t_k}^{t}\Gamma(s)\mathrm{d}s \leqslant \\ &\mu_2^{N_\sigma(t,t_0)}V_{i_0}(\bar{x}(t_0)-\alpha\int_{t_0}^{t}V_{i_k}(\bar{x}(s))\mathrm{d}s+ \\ &\int_{t_0}^{t}\mu_2^{N_\sigma(t,s)}\Gamma(s)\mathrm{d}s.\end{aligned} \tag{53}$$

Under zero initial condition, $V(\bar{x}(t_0))=0$, and $V_{i_k}(\bar{x}(t))\geqslant 0$, we can get

$$\int_{t_0}^{t}\mu_2^{N_\sigma(t,s)}\Gamma(s)\mathrm{d}s \geqslant 0. \tag{54}$$

When $t\to\infty$, pre- and post-multiplying (54) by $\mathrm{e}^{-N_\sigma(t,t_0)\ln\mu_2}$ and letting $\frac{2\kappa_2^2}{\eta_2^2-\kappa_2^2\lambda_2^2}\leqslant\gamma^2$, it can be concluded that

$$\begin{aligned}&\frac{1}{2}\int_{t_0}^{\infty}e_r^{\mathrm{T}}(s)e_r(s)\mathrm{e}^{-N_\sigma(s,t_0)\ln\mu_2}\mathrm{d}s \leqslant \\ &(1+\frac{1}{2}\gamma^2\lambda_2^2)\frac{\kappa_2^2}{\eta_2}\int_{t_0}^{\infty}\int_{t_0}^{s}\mathrm{e}^{-\eta_2(s-\theta)} \| \omega(\theta) \|^2 \mathrm{d}\theta\mathrm{d}s+ \\ &\frac{1}{2}\gamma^2\int_{t_0}^{\infty}r^{\mathrm{T}}(s)r(s)\mathrm{d}s = \\ &(1+\frac{1}{2}\gamma^2\lambda_2^2)\frac{\kappa_2^2}{\eta_2^2}\int_{t_0}^{\infty} \| \omega(s) \|^2 \mathrm{d}s+ \\ &\frac{1}{2}\gamma^2\int_{t_0}^{\infty}r^{\mathrm{T}}(s)r(s)\mathrm{d}s \leqslant \frac{1}{2}\gamma^2\int_{t_0}^{\infty}\bar{\omega}^{\mathrm{T}}(s)\bar{\omega}(s)\mathrm{d}s.\end{aligned} \tag{55}$$

By Definition 3 and (33), $N_\sigma(s,t_0)\ln\mu_2 \leqslant \frac{s-t_0}{\tau_{a2}}\ln\mu_2 \leqslant \alpha(s-t_0)$, it follows from (55) that

$$\frac{1}{2}\int_{t_0}^{\infty}\mathrm{e}^{-\alpha(t-t_0)}e_r^{\mathrm{T}}(t)e_r(t)\mathrm{d}t \leqslant \frac{1}{2}\gamma^2\int_{t_0}^{\infty}\bar{\omega}^{\mathrm{T}}(t)\bar{\omega}(t)\mathrm{d}t.$$

□

Remark 1. The average dwell-time τ_{a1} is designed to guarantee that system (17) is exponentially stable, and τ_{a2}

can guarantee system (36) is exponentially stable. The average dwell-time τ_a of the whole system is chosen as the maximum value between τ_{a1} and τ_{a2}, which can guarantee that system (8) and (9) are exponentially stable with H_∞ performance index. In [21], a condition is required to hold that (8) has the common Lyapunov function and the designed switching law relies on the states $\bar{x}(t), \dot{\bar{x}}(t)$ and $\dot{\bar{x}}(s)$ which contains the derivative term. The proposed method just needs that (8) satisfies the average dwell-time (19) lowering the conservatism, which is one of the main contributions of this paper. The asymptotic stability of system (10) is obtained in [21], while the proposed method can guarantee that system (10) is exponentially stable with $g_i(t) \neq 0$.

3.3 H_∞ controller design

First, let

$$\Omega_{11i} = \begin{bmatrix} \psi_{11i} & B_i\bar{F}_i \\ * & \psi_{22i} \end{bmatrix}, \Omega_{12i} = \begin{bmatrix} D_iS_{1i} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Omega_{14i} = \begin{bmatrix} \psi_{17i} & 0 \\ \bar{F}_i^{\mathrm{T}}B_i^{\mathrm{T}} & X_{2i}A_r^{\mathrm{T}} \end{bmatrix}, \Omega_{15i} = \begin{bmatrix} X_{1i} \\ -X_{2i} \end{bmatrix}$$

where $\psi_{11i} = X_{1i}(A_i+D_i)^{\mathrm{T}}+(A_i+D_i)X_{1i}+\alpha X_{1i}+B_i\bar{K}_i+\bar{K}_i^{\mathrm{T}}B_i^{\mathrm{T}}, \psi_{22i} = A_rX_{2i} + X_{2i}A_r^{\mathrm{T}} + \alpha X_{2i}, \psi_{17i} = X_{1i}(A_i + D_i)^{\mathrm{T}} + \bar{K}_i^{\mathrm{T}}B_i^{\mathrm{T}}$, and $\Omega_{24i} = \Omega_{12i}^{\mathrm{T}}$.

Theorem 3. Consider the augmented system (10), and suppose that Assumptions 1 and 2 and (18)–(20) hold. For the given scalars $\tau > 0, \alpha > 0, \gamma > 0$, there exist matrices $X_{1i} > 0, X_{2i} > 0, S_{1i} > 0, S_{2i} > 0, \bar{K}_i, \bar{F}_i, \forall i \in M$, such that the following LMIs hold

$$\Omega_i = \begin{bmatrix} \Omega_{11i} & \Omega_{12i} & I & \tau\Omega_{14i} & \Omega_{15i} \\ * & -\mathrm{e}^{-\alpha\tau}\bar{S}_i & 0 & \tau\Omega_{24i} & 0 \\ * & * & -\frac{1}{2}\gamma^2 I & \tau I & 0 \\ * & * & * & -\bar{S}_i & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{56}$$

and average dwell-time satisfies (33), $\mu_2 \geqslant 1$ satisfies

$$\begin{aligned} &X_{1i} \leqslant \mu_2 X_{1j}, X_{2i} \leqslant \mu_2 X_{2j}, \\ &S_{1i} \leqslant \mu_2 S_{1j}, S_{2i} \leqslant \mu_2 S_{2j}, \quad \forall i,j \in M, i \neq j. \end{aligned} \tag{57}$$

Then, the system (10) is exponentially stable, and (1) has the H_∞ model reference tracking performance. The designed controller gain matrices are given by $K_i = \bar{K}_iX_{1i}^{-1}, F_i = \bar{F}_iX_{2i}^{-1}$.

Proof. Choose the form of positive definite matrices as $P_i = \begin{bmatrix} P_{1i} & 0 \\ * & P_{2i} \end{bmatrix}, S_i = \begin{bmatrix} \hat{S}_{1i} & 0 \\ * & \hat{S}_{2i} \end{bmatrix}$. Pre- and post-multiplying (32) by $\mathrm{diag}\{P_i^{-1}, S_i^{-1}, I, I\}$, denote $X_i = P_i^{-1} = \begin{bmatrix} X_{1i} & 0 \\ * & X_{2i} \end{bmatrix}, \bar{S}_i = S_i^{-1} = \begin{bmatrix} S_{1i} & 0 \\ * & S_{2i} \end{bmatrix}, \bar{K}_i = K_iX_{1i}, \bar{F}_i = F_iX_{2i}$, and $X_i\bar{Q}X_i = \begin{bmatrix} X_{1i} \\ -X_{2i} \end{bmatrix}\begin{bmatrix} X_{1i} & -X_{2i} \end{bmatrix}$. By using Schur complement for (32) in Theorem 2, we can easily get (56). □

Remark 2. The work in [22] proposed an exponential stability criterion for a class of switched nonlinear systems with time-varying delay. However, the designed controller relies on the system state $x(t)$ and reference state $x_r(t)$. In this paper, we design an observer to deal with the unavailable state $x(t)$ and construct an output feedback controller to track the reference signal $x_r(t)$.

4 Numerical example

Consider system (1) and reference system (5) with

$$A_1 = \begin{bmatrix} 3.5 & 1.5 \\ 1.5 & 4.5 \end{bmatrix}, B_1 = \begin{bmatrix} -0.6 & -0.2 \\ 0.2 & -0.4 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -0.2 & 0.8 \end{bmatrix}, D_1 = \begin{bmatrix} 0.25 & 0.15 \\ 0.25 & -0.15 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 4.5 & 2 \\ -1.5 & 3 \end{bmatrix}, B_2 = \begin{bmatrix} -0.5 & 0.1 \\ -0.1 & -0.25 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 1.0 & -0.7 \end{bmatrix}, D_2 = \begin{bmatrix} -0.1 & 0.1 \\ 0.2 & 0 \end{bmatrix}$$

$$A_r = \begin{bmatrix} -5 & 0.5 \\ -1.5 & -4.5 \end{bmatrix}, f_1(x(t)) = \begin{bmatrix} 0.1\cos(0.01x_1) \\ 0.1\cos(0.01x_2) \end{bmatrix}$$

$$f_2(x(t)) = \begin{bmatrix} 0.2\cos(0.01x_1) \\ 0.2\cos(0.01x_2) \end{bmatrix}.$$

We adopt the parameters below: $d(t) = 0.16+0.14\sin t, \tau = 0.3, \alpha = 1, \gamma = 0.5$, and the Lipschitz matrices are given by

$$U_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad U_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

First, let $\mu_1 = 7$, by Theorem 1, we obtain

$$H_1 = \begin{bmatrix} 0.6095 & -1.3686 \\ -1.3686 & 3.3900 \end{bmatrix}, L_1 = \begin{bmatrix} 73.9087 \\ 46.4478 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 1.0839 & -1.2598 \\ -1.2598 & 1.6531 \end{bmatrix}, L_2 = \begin{bmatrix} 23.8485 \\ 9.8203 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} 0.3475 & -0.6217 \\ -0.6217 & 1.2153 \end{bmatrix}, G_2 = \begin{bmatrix} 0.4400 & -0.6056 \\ -0.6056 & 0.9653 \end{bmatrix}$$

and the average dwell-time satisfies $\tau_{a_1} > 1.9459\,\mathrm{s}$. However, by using the proposed method in [21], we cannot obtain the feasible solutions, so we cannot get the corresponding common Lyapunov function.

Then, considering the system (9), and using Theorem 3, we can get the common Lyapunov function and a set of solutions

$$X_{1i} = \begin{bmatrix} 2.0301 & 0 \\ 0 & 1.9886 \end{bmatrix}, X_{2i} = \begin{bmatrix} 2.5081 & -0.0537 \\ -0.0537 & 2.7959 \end{bmatrix}$$

$$S_{1i} = \begin{bmatrix} 21.0670 & -0.0569 \\ -0.0569 & 22.7170 \end{bmatrix}, S_{2i} = \begin{bmatrix} 24.4292 & -0.0840 \\ -0.0840 & 24.3988 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} 14.3934 & -5.6477 \\ 12.1647 & 26.0282 \end{bmatrix}, K_2 = \begin{bmatrix} 20.2412 & 11.0896 \\ -13.5763 & 35.8298 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} -2.6262 & 1.3188 \\ -1.1708 & -3.7332 \end{bmatrix}, F_2 = \begin{bmatrix} -3.5098 & -1.3707 \\ 1.3485 & -6.7497 \end{bmatrix}$$

where $i = 1, 2$, thus, we can get the average dwell-time $\tau_{a_2} > 0$.

According to the switching law (33), we choose the average dwell-time $\tau_a = 2\,\mathrm{s}$. Let $w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}, r(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix}$, $\omega_i(t) = r_i(t) = \begin{cases} 2\dfrac{\sin t}{t}, & 5\,\mathrm{s} \leqslant t \leqslant 20\,\mathrm{s} \\ 0, & \text{others} \end{cases}$, $i = 1, 2$, and choose the initial conditions $\varphi(t) = 0, t \in [-0.3, 0), x(0) = [0.4, -0.3]^{\mathrm{T}}, \hat{x}(0) = [0.3, -0.2]^{\mathrm{T}}, x_r(t) = [0.6, -0.5]^{\mathrm{T}}$. Then, the simulation results are shown in Figs. 1–4.

It can be seen that the state error is exponentially stable after $2\,\mathrm{s}$ when $\bar{\omega}(t) = 0$, and the real state tracks the reference state perfectly. When the system is subjected to the bounded exogenous disturbance, we can see that it has H_∞ tracking performance under the designed controller and switching law.

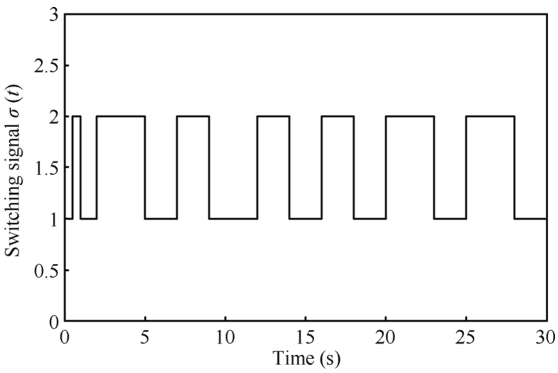

Fig. 1 The designed switching signal $\sigma(t)$

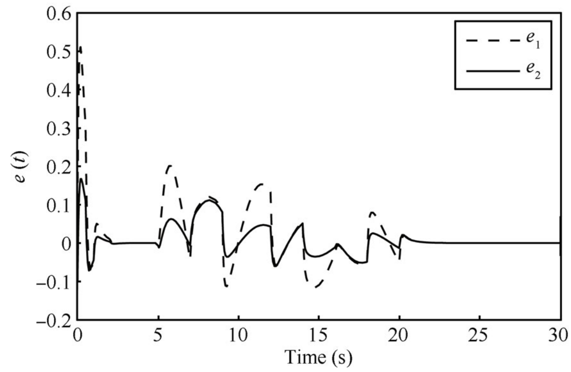

Fig. 2 State error $e(t)$

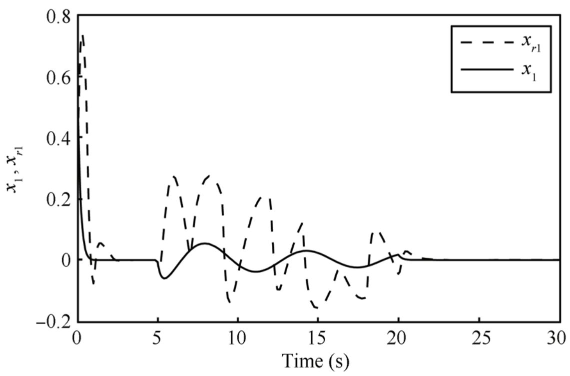

Fig. 3 The state x_1 and the reference state x_{r1}

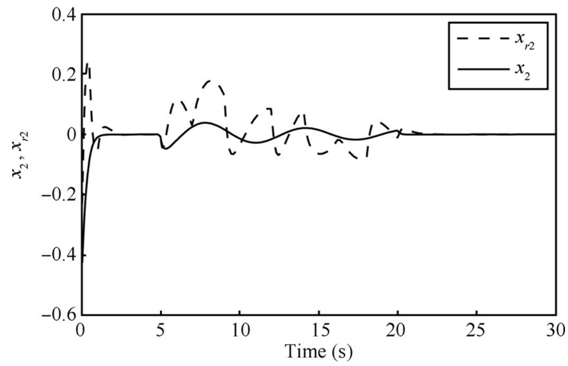

Fig. 4 The state x_2 and the reference state x_{r2}

5 Conclusions

In this paper, output feedback tracking control for switched nonlinear system with time-varying delay has been studied. A new switching law is designed and exponential stability criterion for the system is derived. Average dwell-time and piecewise Lyapunov function have been used to obtain the stability of the augmented system and H_∞ tracking performance when the system has the bounded exogenous disturbance. Besides, the designed observer and controller can be obtained by solving a set of LMIs, and the proposed method does not need that a common Lyapunov function exists for the switched nonlinear systems. A numerical example is given to show the effectiveness of the proposed method. If the nonlinear function $f_\sigma(\cdot)$ is unknown, how to deal with this problem will be our further study issue.

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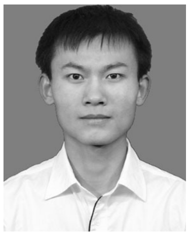

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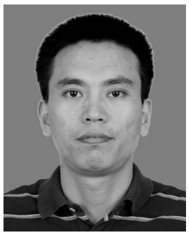

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