Optimized time–frequency features and semi-supervised SVM to heartbeat classification

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Abstract One of the most significant indicator of heart disease is arrhythmia showing heartbeat patterns. Thus, early and accurate detection of arrythmia types by categorization of heartbeats is important.

In this paper, we introduce an ECG beat classifier system integrating two main parts: Feature extraction and classification. For the first part, we consider the features observed in the time frequency (t, f) plane where the ECG is projected using a variant of Stockwell transform. For the second part, the framework of semi supervised SVM with asymmetric costs (AS3VM) has been applied for assessment of the obtained feature sets performance. Notice that four heartbeat types have been considered: normal beats (N), left and right bundle branch blocks (L and R) and premature ventricular contractions (V).

The proposed method has been evaluated on PhysionNet's MIT-BIT arrythmia database. The obtained results show that the suggested approach achieves significant separability of the classes and thus, able to make prediction accuracies of 99.35%, 98.73%, 98.57% and 99.44% for respectively N, L, R and V beats.

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1 Introduction

The Electrocardiogram (ECG) is the electrical manifestation of the contractile activity of the heart. The ECG waveform is characterized by the waves: P, QRS and T. The analysis of the electrocardiogram is one of the most important problems in modern biomedical signal analysis. By segmenting the ECG signal, it is possible to derive a number of informative measurements from the characteristic ECG waveforms. These can then be used to detect any potential abnormalities in the cardiac rhythm.

Long-term recordings of the ECG signal are often required and usually obtained using the popular ambulatory Holter recorders. The analysis is performed offline by cardiologists. Due to the high number of beats to evaluate, this task is very expensive and time consuming. Computer-aided classification of pathological beats is therefore of great importance. However, this is a difficult task in real situations not only because of the physiological variability of the signal, but also because of the various types of noises often present in the ECG signal.

Many algorithms dealing with heartbeat classification have been addressed previously in the literature. The accuracy of this automatic classification depends considerably on the derived heartbeat features. The most popular time beat descriptors are based on the QRS complex morphology [1] [2] [3] [4] [5]. Beat descriptors rely also on its frequency components [6] [7]

[8]. Wavelet transform at appropriate scale has been exploited for feature beats extraction in [4] [5] [9]. Recently, a time-frequency domain obtained with the S-transform has been explored to obtain a 2D morphology descriptor combined with principal component analysis (PCA) [9] [10].

All studies demonstrated that the applied classification method also influences the achieved accuracy. Thus, several discriminative techniques have been developed. Such that Artificial Neural Networks [1] [2] [11] [12] with deep learning in [11] and hybrid model based on neural networks in [12]. Other work used the K^{th} nearest-neighbor rule [2] and genetic algorithms [6]. Among all these methods, Support Vector Machines (SVMs) known to be an excellent tool for classification and regression problems has shown success in this application field e.g. [6] [8] [10].

In this paper, beat descriptors are computed in the time-frequency plane because of its ability to highlight many time varying characteristics. Such information cannot be obtained directly from signals representations in time or in frequency domains only. To project the ECG signal in a time-frequency domain, a variant of stockwell transform (ST) [13] namely, the ST with a compact support kernel (ST-CSK) introduced in [14] has been used. Without any filtering, this time-frequency representation allows us to detect R peaks, to segment the heartbeats and thus, to obtain optimized ventricular activity features. After the feature extraction step, an AS3VM classifier is used to classify heartbeats. This classifier tasks has the advantage of using few labeled data. It also integrates distinct weights for the classes depending on their priors.

The rest of this paper is organized as follows. Section 2 presents the methodology namely, the projection of the signal on the time-frequency domain, feature extraction in such domain and then classification. Experimental results are shown in section 3. Section 4 concludes the paper.

2 Methods

2.1 Database

It is necessary to use a standard database to evaluate and compare the obtained result with other published studies. Thus data for the evaluation of our proposal are obtained from standard datasets [15] in Physionet. This database contains 48 records. Each record is of length 30 min with 360 Hz sampling frequency. They were recorded in two channels (modified limb lead II and modified limb lead VI) of surface ECG from Holter recorders. They present a variety of waveforms, artifacts, complex ventricular, junctional and supraventricular arrhythmias and conduction abnormalities. Each record is accompanied by an annotation file in which each ECG beat has been identified by expert cardiologists. These labels referred to as 'truth' annotation and are used to develop the classifier and to evaluate its performance in the testing phase.

In this study, we considered four heartbeat classes which are representative for the predominant types: normal beats (N), left bundle branch block beats (L), right bundle branch block beats (R) and Ectopic beats (V). There shapes are displayed on Figure 1 (left). In agreement with the AAMI [16] recommended practice, records containing paced beats (102, 104, 107, and 217) were excluded.

2.2 Signal processing

In this paper, beat segmentation and feature extraction are done in the time-frequency plane. Thus, the first step is the time-frequency representation of the signal using ST-CSK, a variant of the ST. This latter originates from two advanced signal processing tools, the short time Fourier transform (STFT) and the continuous wavelet transform (CWT). Derived from the STFT, the standard ST of a signal $x(t)$ is given by:

$$S(\tau, f) = \int_{-\infty}^{+\infty} x(t) w(t-\tau) e^{-i2\pi f t} dt. \quad (1)$$

where the window function $w(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-t^2}{2\sigma^2}}$ is a normalized gaussian known to be compact in both time and frequency axis. The window width $\sigma = \frac{1}{|f|}$ varying inversely with frequency makes the ST performing a progressive resolution. The ST-CSK proposed in [14] overcomes practical drawbacks of the ST while preserving a large number of its useful properties.

Derived from the gaussian one, the proposed kernel $\phi(t)$ used in the ST-CSK has a polynomial form. In this case, a shifted and scaled kernel has been expressed as:

$$\phi_\lambda(t-\tau) = \begin{cases} \frac{1}{D_\gamma}(\lambda^2 - (t-\tau)^2)^\gamma, & if\ (t-\tau)^2 < \lambda^2 \\ 0, & elsewhere\ . \end{cases}, \quad (2)$$

where D_γ is a normalization parameter. The kernel width is controlled through λ and its peak is adjusted through the parameter γ allowing a tradeoff between a good autoterm resolution and a sufficient cross-term suppression. The scale parameter λ has been assumed to be a function of frequency $\lambda(f) = \frac{1}{p+f^r}$. This parameter aims to make the kernel more flexible and more adaptive to the analysed signal. Hence, the resolution in time and in frequency axis will be turned depending

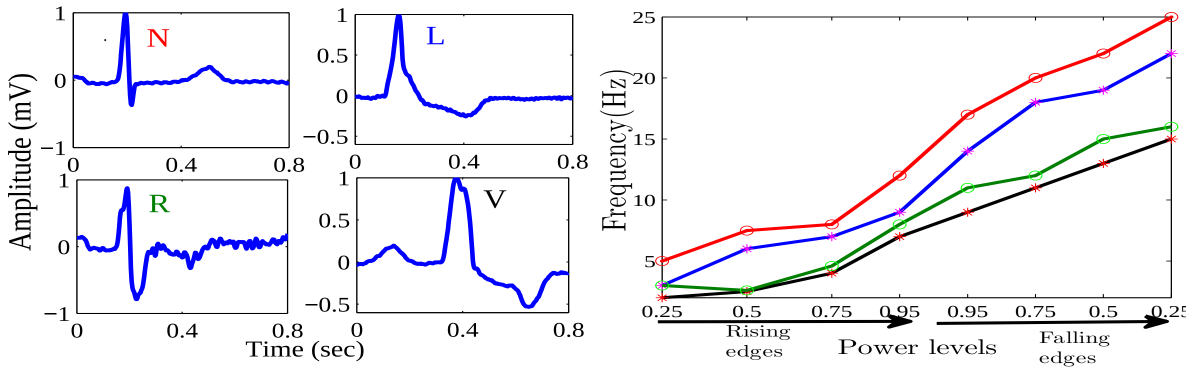

Fig. 1: *Different forms of ECG beats (Left); Relation between frequencies and power levels of different beats(Right). These power levels have been presented with the same colors as those indicating (N,L,R and V) beats in the left panel of this figure.*

on these parameters. To maximize the energy concentration, an optimization problem has been formulated under some constraints related to the width $\lambda(f)$. More details about this problem and its resolution can be found in [14]. For a time series $x(kT)$ corresponding to $x(t)$ with $k = 0, .., N-1$ and T, the time sampling interval, the output of the discrete CSK S-Transform is an N by M matrix called ST-matrix where rows represent the time and columns, the frequencies. In this paper, a frequency range $[0Hz - \frac{f_s}{2}Hz]$ is used to cover the composite ECG signal.

2.3 Beat detection

In order to obtain an automatic measurement of the ventricular activity, beat segmentation is required. Thus, R peak detection is unavoidable. This step is crucial because of different noises present in the signal and the variability of QRS shapes and their frequency content. Accordingly, a wide diversity of R peak detection algorithms have been proposed in the literature [17][18][19]. In this study, R peak detection is done in the time-frequency domain without recourse to any filtering step. The algorithm proposed in [19] has been used. However, the ST-CSK [14] has been considered since it showed better time frequency resolution and better energy concentration. After R peaks detection, a time-frequency window is defined for each beat. The Q-S segment as shown in [18] [19] is in a frequency range of [5-22.5 Hz] with at most 150 ms of width in the time axis(50 ms for the Q-R segment and 100 ms for R-S segment. Thus the beat features are computed.

2.4 Feature quantification

The QRS complex morphology and duration in ECG signal vary with origination and conduction path of the activation pulse in the heart. Thus, one can see in Figure 1 that the considered heartbeat types can be discriminated in both time domain and frequency domain. Therefore, heartbeat detection and classification methods found in literature include the use of time only features, spectral only features [6] [7] or combination of time only with frequency only features [8] [26]. ECG segments containing respectively N, L, R and V beats are projected in the time-frequency plane as shown in Figure 2. One can see that we can compute beat discriminative features in this plane. These features will successfully represent joint TF structure of the signal. Investigating this approach is the aim of the first part of this paper.

Even TFDs are rich in information, all the (t,f) considered window or matrix cannot be used as features for the classification. This would significantly increase the problem dimensionality. To avoid this, a representative set of features describing the relevant information of the ventricular activity are to be extracted from the CSK-ST matrix. Some frequency features such as the spectral flux, spectral entropy and spectral flatness are often employed for detecting abnormalities in biomedical signals [22]. Here, these spectral features extended to the joint time-frequency domain [23][24] are considered.

1. **Time-frequency flatness**: spectral flatness also known as Wiener entropy measures the width and uniformity of the power spectrum. In [24], this feature has been extended to the (t, f) domain. It is

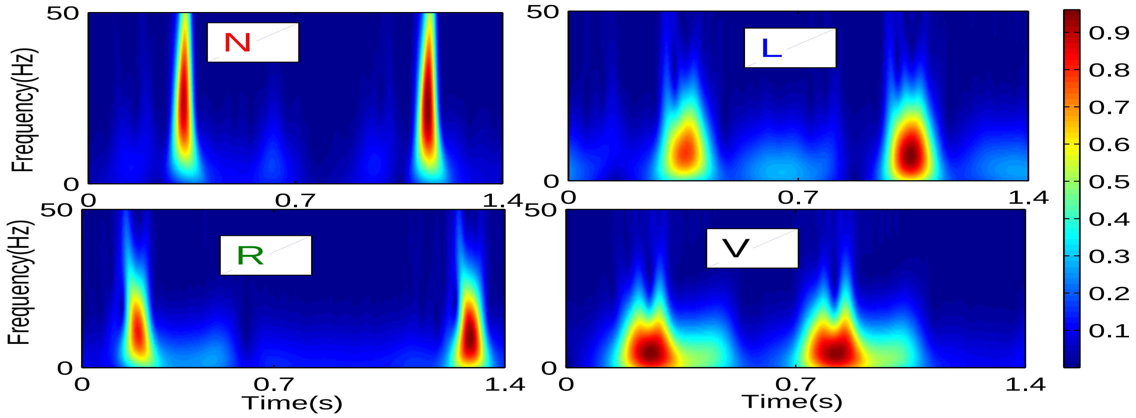

Fig. 2: *ECG segments projection in the time-frequency plane using ST-CSK.*

defined as the geometric mean of the $S(n,k)$ normalized by its arithmetic mean.

$$SF_{(t,f)} = MN \frac{\prod_{n=1}^{N} \prod_{k=1}^{M} S(n,k)}{\sum_{n=1}^{N} \sum_{k=1}^{M} S(n,k)} \tag{3}$$

where M and N are respectively the time and frequency dimensions of the considered ventricular activity window. To avoid zero products in (3), all zeros of the ST are replaced by very small values (i.e. epsilon in MATLAB).

2. **Time-frequency flux:** this parameter measures the variation of the signal energy both along time and along frequency axes.

$$FL_{(t,f)} = \sum_{n=1}^{N-1} \sum_{k=1}^{M-1} |S(n+1,k+1) - S(n,k)| \tag{4}$$

3. **Energy concentration**: Many results of the literature review [21] indicate that using energy concentration CM as a feature, is a very powerful tool in a classification process. This parameter also used in [14] is defined as:

$$CM = \frac{1}{\sum_1^N \sum_1^M |\overline{S(\tau,f)}|} \tag{5}$$

Where the module of the used version of the S-transform is normalized as:

$$\overline{S(\tau,f)} = \frac{S(\tau,f)}{\sqrt{\sum_1^N \sum_1^M |S(\tau,f)|^2 d\tau df}}. \tag{6}$$

2.5 Classification

To classify the patterns, the framework of semi supervised learning approach addresses this problem by taking advantage of using few limited labeled data in order to train precise classifier while requiring less human effort and time.

For this purpose, SVM classifier has been considered. This classifier with a good generalization capability is derived from the structural risk minimization principle [25]. The supervised SVM needs a large set of labelled data to build a good model deducting the label of each new point. However, the labelled data are costly and time consuming. The aim of semi-supervised learning is to use the information contained in the unlabelled data as a distribution of the data to improve the classifier performance. The problem of semi-supervised SVM consists to find the labels $y_1^*, y_2^*, ...y_m^*$ of the unlabelled data by looking for an hyperplane maximizing the margin and minimizing the cost error for both labelled data $(x_1, y_1), (x_2, y_2), ...(x_n, y_n)$ and unlabelled data $x_1^*, ...x_m^*$. Notice that the training error of each class are penalized differently. This is often recommended in biomedical applications. For an optimal generalization performance, the unlabelled data must be less penalized than the labelled ones. The optimization problem of the AS3VM is set as:

$$\left\{ \begin{array}{l} \min_{w,b,\xi_i,\xi_j^*,y_j^*} \frac{1}{2}\|w\|^2 + C^+ \sum_{i/y_i=+1} \xi_i + C^- \sum_{i/y_i=-1} \xi_i + \\ \qquad C_+^* \sum_{j/y_j^*=+1} \xi_j^* + C_-^* \sum_{j/y_j^*=-1} \xi_j^* \ , \\ y_i(w, \phi(x_i) + b) \geq 1 - \xi_i \quad i = 1, \ldots, n \ , \\ y_j^*(w, \phi(x_j^*) + b) \geq 1 - \xi_j^* \quad j = 1, \ldots, m \ , \\ \xi_i \geq 0 \ , \quad \xi_j^* \geq 0, \quad y_j^* \in \{1, -1\} \ . \end{array} \right. \tag{7}$$

Where ξ_i and ξ_j^* are the slack variables that relax the constraints and define a loss function penalizing the training and testing errors respectively. C and C^* are trade-off constants for training and testing examples respectively. ϕ denotes a nonlinear transformation that maps data to a higher dimensional space. The dual problem to maximize can be expressed as:

$$\begin{cases} \max_{\alpha,\alpha^*} \sum_{i=1}^{n} \alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{r=1}^{n} \alpha_i\alpha_r y_i y_r K(x_i,x_r) + \\ \sum_{j=1}^{m} \alpha_j^* - \frac{1}{2}\sum_{j=1}^{m}\sum_{l=1}^{m} \alpha_j^*\alpha_l^* y_j^* y_l^* K(x_j,x_l) - \\ \sum_{i=1}^{n}\sum_{j=1}^{m} \alpha_i\alpha_j^* y_i y_j^* K(x_i,x_j^*) \\ \sum_{i=1}^{n} \alpha_i y_i + \sum_{j=1}^{m} \alpha_j^* y_j^* = 0 \ , \\ C^+ \geq \alpha_{i/y_i=+1} \geq 0 \ \ , C^- \geq \alpha_{i/y_i=-1} \geq 0 \\ C_+^* \geq \alpha^*_{j/y_j^*=+1} \geq 0 \ \ , C_-^* \geq \alpha^*_{j/y_j^*=-1} \geq 0 \end{cases} \tag{8}$$

where α_i and α_j^* are the Lagrange multipliers respectively for learning and testing set.
$K(x_i,x_j) =< \phi(x_i),\phi(x_j) >$ is the kernel. An algorithm for resolving this problem with a combinatorial optimization approach is given in [27]. The idea is to fix $y_1^*, y_2^*, ...y_m^*$ with the supervised SVM and to increment gradually C_+^* and C_-^* starting with small variations to target C^* by respecting the balancing constraint defined as :

$$R = \frac{1}{2}[1 + \frac{1}{m}\sum_{j=1}^{m} y_j^*] \tag{9}$$

Where R represents the ratio estimated from a priori knowledge or from the set of labeled data. The constraints (9) is added to avoid the degeneration of the solution by attributing all the unlabeled data to the same class when the size of unlabeled data is very large compared to the labeled one. After resolving the dual problem (8) and thus, computing the lagrangian multiplier, the classifier output is given by equation (10)

$$f(x) = \sum_{i=1}^{n} \alpha_i y_i K(x_i,x) + \sum_{j=1}^{m} \alpha_j^* y_j^* K(x_j^*,x) + b \tag{10}$$

and the decision is the sign of $f(x)$. Thus, $y(x) = sign(f(x))$. Notice that the multi-class classification task using one against all method consist to consider one class as positive and the rest of classes as negative.

3 Experimental results and discussion

In a classification task, extracting characteristics is a very decisive step. In this work, we proceeded to heartbeat feature extraction in the time-frequency plane. This method allowed us to quantify each beat by only a three-element vector. In Figure 3, the patterns are mapped in a two dimensional space. Thus, it can be seen that the extracted features separate successfully all the classes.

For heartbeat recognition, a two-step approach based on the SVM (one-vs-all approach) has been used for the multi-class classification. The input of the classifier is a set of three element vectors x_i.

To fine tune a global classifier, a general learning set is randomly selected in the records. For semi supervised classification, small labeled learning sets have been used to perform many tests. These sets contain about 5 to 50 samples from each class taken from the first 5 minutes of the corresponding record. Notice that a balance in the number of beats in each category is taken into consideration. from each class taken from the first 5 minutes of the corresponding record. Notice that a balance in the number of beats in each category is taken into consideration..

In order to illustrate the effectiveness of the classification methodology, we simply apply standard statistical data visualization technique by its projection onto a 2D space as shown in Figure 2. In this Figure, a comparison can also done between standard SVM (left) and the AS3VM (right). One can see how the hyperplane and the SVM margin have been readjusted by the AS3VM to improve the classification results. This, despite the fact that in some cases, ambiguous regions are observed particularly between N and R and between R and L. Moreover, it should be noted that these classes (N, L and R) are often considered in the same class (Normal) in agreement with the AAMI recommended practice.

In addition to illustrative results, correct classification and misclassification are quantified with four metrics such that True Positive (TP), True Negative (TN), False Positive (FP) and False Negative (FN). Thus, statistical parameters are evaluated using these metrics to compare detection algorithms. The sensitivity S_e and the accuracy A_c of the classifier are respectively defined as:

$$S_e = \frac{TP}{TP+FN} \ and \ A_c = \frac{TP+TN}{TP+FP+TN+FN}$$

Table 1 summarizes the classification results achieved by the AS3VM classifier compared to those obtained with a standard SVM classifier. The classification performance on each heartbeat class was measured.

Many studies dealing with heartbeat classification using the same database are found in the literature. The S-transform has already been used to heartbeat description in [9][10]. It should be noted that in both studies, the use of filtering and R peak detection stages are optional since the S-Transform allows us to access to the frequency content of the QRS complexes without recourse to any preprocessing stage. Thus, this detection can be done using local spectra as proposed in [19].

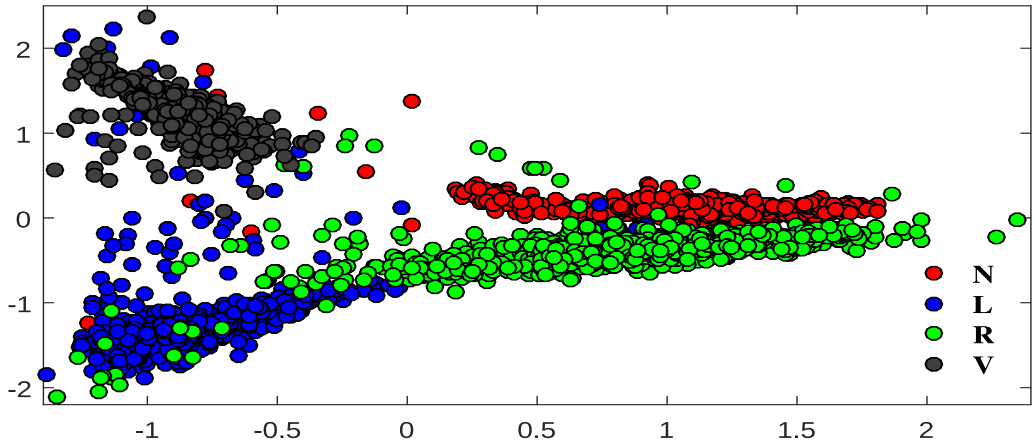

Fig. 3: *Scatter plots showing the separability of the classes when characterized in the time-frequency plane.*

Table 1: *Performances comparison between SVM and AS3VM classifiers.*

Classifier	*Se*(%)				*Sp*(%)				Ac (%)			
	N	L	R	V	N	L	R	V	N	L	R	V
SVM	91.94	79.67	93.48	90.07	98.45	98.19	94.12	99.37	97.15	93.96	94.02	97.40
AS3VM	98.42	85.97	98.65	96.18	99.61	99.20	95.57	99.88	99.38	96.28	96.06	99.13

Table 2: *Performances comparison between the proposed approach and other published methods.*

Method	Features	Train ratio (%)				S_e(%)				A_c(%)			
		N	L	R	V	N	L	R	V	N	L	R	V
[4]	14	13	40	40	40	99.85	100	99.79	99.29	99.77	99.99	99.98	99.91
[5]	30	50	50	50	50	99.72	99.43	99.12	96.03	98.75	99.92	99.92	99.46
[8]	15	30				98.2			98.2	98.8			98.8
[10]	−−	30				−−				89.10			
[11]	−−	90	90	90	90	98.17	95.78	94.37	90.43	97.27	98.36	99.29	99.07
[12]	140	90	90	90	90	95.09	87.15	97.00	98.82	97.89	98.1	99.4	98.3
Proposed	3	4.7	6.7	6.3	7.9	98.42	96.84	97.72	97.53	99.35	98.73	98.57	99.44

Table 2 provides a quantitative comparison between the proposed method and earlier published approaches. In terms of training set size, our method displays the best percentage, namely an average of 6.4% of the entire database. However the other studies used between 30% and 90% of the considered data. This, thanks to the time-frequency features leading to a good separability between classes. In addition, the feature size is minimal compared to other methods. this is important when considering algorithmic complexity.

In the same table, we can observe that the proposed method yielded to butter or comparable performances in terms of average sensitivity and accuracy respectively of 97.63% and 99.02%. In [4], the authors used morphological and dynamic features and obtained an overall accuracy performance of 99.3% but with 2.4% of rejection. In [8], a set of 15 temporal only and frequency only features have been used. With no rejection, the achieved accuracy was 97.2%. It should be noted that in that work, N, R, L beats have been considered in the same class. In the most recent reference [11], deep learning approach has been used. Its benefits are to minimize the number of preprocessing techniques but the training phase is computationally intensive and slow since it uses 90% of the data.

4 Conclusion

This paper investigates a new approach for heartbeat classification based on time-frequency features and semi supervised learning.

The ECG signal is projected in the time-frequency domain using the S-Transform with a compact support kernel. Notice that the ST matrix allows us access to any frequency range. Therefore, the preprocessing phase is unnecessary. In this plane, each beat is detected and segmented as a 2D window where a feature vector is extracted.

It should be noted that the novelty of this study is the use of optimized QRS time-frequency features instead of morphological or time only and frequency only features. The illustrative results have shown that with only three features, all classes have been successfully

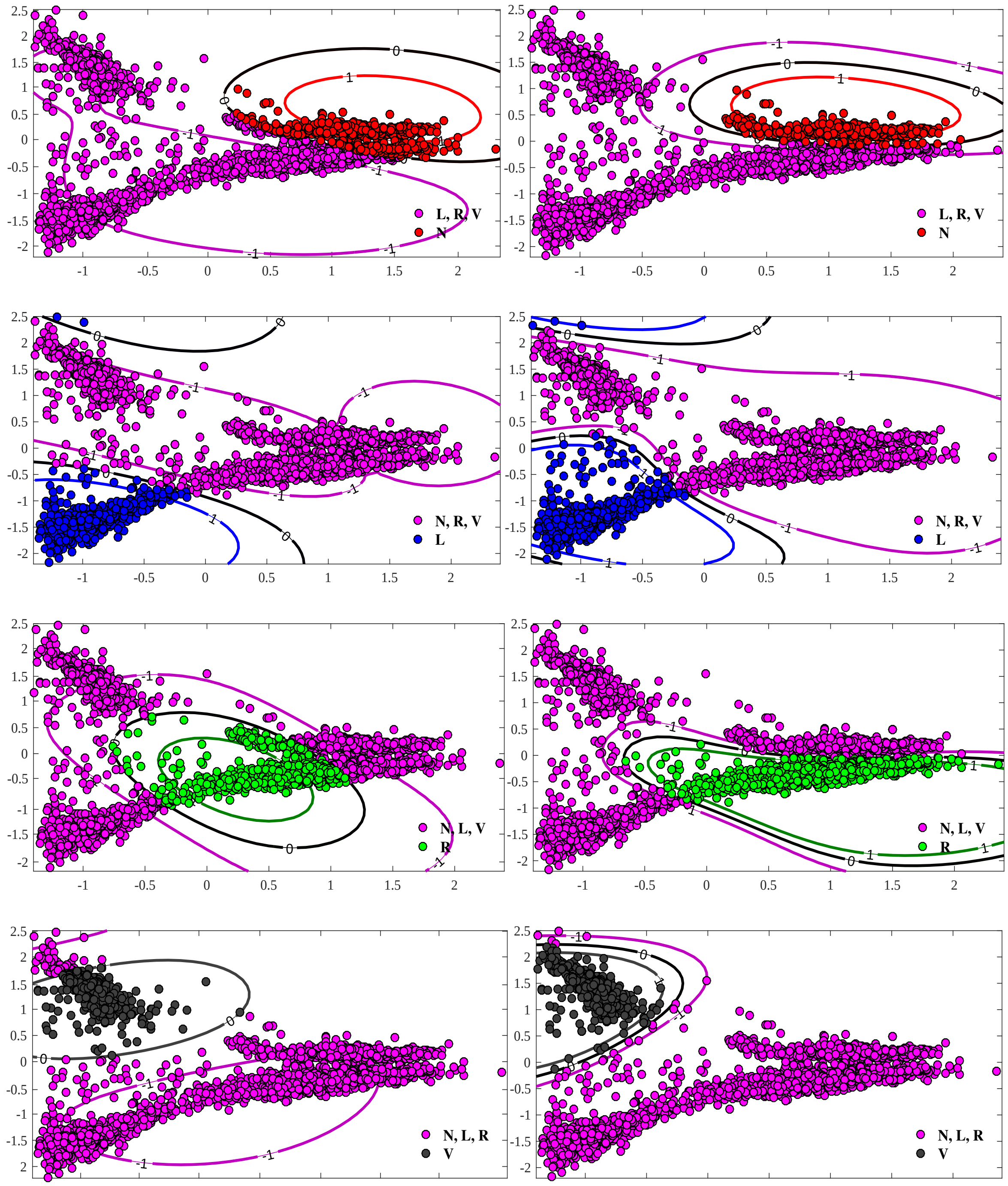

Fig. 4: Scatter plots showing the detection of different classes via one-versus-all. One can distinguish between one of the labels (corresponding color) and the rest (pink). Using SVM (left) and S3VM (right).

separated. This number of features will positively influence the complexity of our algorithm.

The AS3VM gives good performance using labelled and unlabelled data with respecting the unbalancing trade-off between classes. This classifier can be used when the labelled data size are limited. The guaranteed result (accuracy between 96.06% and 99.38%;sensitivity between 85.97% and 98.65%) can reasonably improved when including patient-specific local learning set. Furthermore, it may be possible to improve detection results by using the optimized parameters of the kernel used in the S-Transform, directly for classification instead of focusing on energy concentration.

Acknowledgments

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