Signal, Image and Video Processing

Joint Generalized Singular Value Decomposition and Tensor Decomposition for Image Super-resolution

Ying Fang, Bingo Wing-Kuen Ling, Yuxin Lin, Ziyin Huang and Yui-Lam Chan

***Abstract*—The existing methods for performing the super-resolution of the three dimensional images are mainly based on the simple learning algorithms with the low computational powers and the complex deep learning neural network based learning algorithms with the high computational powers. However, these methods are based on the prior knowledge of the images and require a large database of the pairs of the low resolution images and the corresponding high resolution images. To address this difficulty, this paper proposes a method based on the joint generalized singular value decomposition and tensor decomposition for performing the super-resolution. Here, it is not required to know the prior knowledge of the pairs of the low resolution images and the corresponding high resolution images. First, an image is represented as a tensor. Compared to the three dimensional singular spectrum analysis, the spatial structure of the local adjacent pixels of the image is retained. Second, both the generalized singular value decomposition and the Tucker decomposition are applied to the tensor to obtain two low-resolution tensors. It is worth noting that the correlation between these two low-resolution tensors is preserved. Also, these two decompositions achieve the exact perfect reconstruction. Finally, the high-resolution image is reconstructed. Compared to the de-Hankelization of the three dimensional singular spectrum analysis, the required computational complexity of the reconstruction of our proposed method is much lower. The computer numerical simulation results show that our proposed method achieves a higher peak signal to noise ratio than the existing methods.**

***Keywords*—Super-resolution, tensor, generalized singular value decomposition, Tucker decomposition.**

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1. Introduction

A high resolution image refers to an image with a high pixel density or a high definition. On the other hand, a low resolution image refers to an image with a low pixel density or a low definition. Hence, a high resolution image contains more pixels in a given area of the image compared to a low resolution image. In this case, more details of an object can be displayed. This effect is similar to increase the sampling frequency. The super-resolution is to reconstruct a high resolution image from a low resolution image. It is worth noting that the resolution of an image is further reduced due to the filtering effect imposed in the denoising and the blurring effect imposed in the image acquisition. Also, the manufacture of the high resolution sensors is very challenging even though the chip manufacturing technology is improving. However, the demands on the high resolution images are increasing particularly in the fields of the medical imaging, the satellite imaging and the computer vision. Hence, the existing imaging systems do not meet the requirements. This results to the difficulties in the medical diagnosis and the analysis of the images. On the other hand, as the super-resolution can increase the resolution of an image, the inherent limitations on the resolutions of the low resolution sensors are addressed and the relatively low cost imaging systems can be employed for displaying the super-resolution images in the high resolution devices such as the high definition liquid crystal display devices. Therefore, there is a great demand on the super-resolution and the super resolution has attracted more and more attentions in the recent years.

The existing works on the super-resolution are focused on the two dimensional images [1]. There are many existing super-resolution methods. The first type of the super-resolution methods is based on the interpolation approaches [2]-[5]. These methods mainly include the nearest neighbor interpolation methods, the bilinear interpolation methods, the bi-cubic interpolation methods and the cubic spline interpolation methods. Although the interpolation methods are with the low computational complexities, simple and easy to be implemented, only the simple interpolation functions can be used. However, since these methods are based on applying these interpolation functions to performing the smoothing operations, the natural images with the fine textures will be suffered from the artifacts. Hence, only smooth images can be generated. This limits the applications. The second type of the super-resolution methods is based on the learning approaches. These methods are mainly to learn the relationship between the low resolution images and the corresponding high resolution images from a given image database. Its advantage is that the high frequency details of the high resolution images can be obtained. Hence, these methods are conducive to the restoration of the high resolution images. However, its disadvantage is that it highly relies on the image database. Also, they require a huge number of the pairs of the low resolution images and the corresponding high resolution images for performing the training. Nevertheless, the required computational complexity for training based on the huge database is very large. Also, the hardware implementation cost is very expensive. To address this difficulty, it was recently proposed to learn the compact representations of the sampled pairs of the low resolution images and the corresponding high resolution images to obtain the prior knowledge [6]. This can reduce the required computational complexity of the training process. Besides, it was proposed to group the low resolution images and the corresponding high resolution images together in the dictionary [7] but the correlations among the color channels of the images were preserved so that the similarities among the colors of the reconstructed images were retained. However, this method does not yield the optimal size of the dictionary. Also, it is suffered from the excessive calculation problem. The third type of the super-resolution methods is based on the model reconstruction approaches. These approaches assumed or based on a prior knowledge of the model that maps the high resolution images to the low resolution images. For examples, they assumed that there are the non-local similarity prior knowledge or the sparse prior knowledge between the high resolution images and the low resolution images. Therefore, the high resolution images can be restored by solving the inverse problems of these models using the low resolution images. By using the normalization method, the effects of the ill posed problems in the inversions of the models can be suppressed and the high resolution images can be flexibly generated.

The matrix decomposition approaches were recently proposed. For an example, a jointed two dimensional singular spectrum analysis and the generalized singular value decomposition based method was recently proposed for performing the super-resolution via a binary linear programming approach. Unlike the previous methods, the low resolution image blocks and the corresponding high resolution image blocks were moved one pixel vertically or horizontally. Then, the above procedures are repeated and the trajectory matrices were constructed. Here, the relationships among the neighboring pixels were exploited. By formulating the selection of the singular spectrum analysis components as a binary linear programming problem, the reconstruction error was minimized. Since the generalized singular value decomposition can preserve the correlation between two matrices where these two matrices are decomposed with the same numbers of columns [8], this can enhance the details of the low resolution images. As a result, the high resolution images can be reconstructed accurately.

However, the above methods have not been applied to the three dimensional images. This is because processing the three dimensional images or even the high dimensional data requires a very heavy computational power. However, the tensor processing has been applied in various fields recently including the processing of the high latitude data, the linear algebra based machine learning applications and the three dimensional image super-resolution based applications. For examples, a three dimensional image super-resolution method was proposed for enhancing the teeth images [9]. This method greatly reduced the total number of the hyper parameters. At the same time, a method based on the tensor super-resolution and the adversarial generation network was proposed [10] to generate the high resolution three dimensional images by exploring the tensor structure. Compared with the traditional two dimensional image super-resolution based learning methods, the tensor super-resolution based methods can exploit the relationships among the image sequences at different time instants via the learning process. Unlike many existing three dimensional image super-resolution methods that the three dimensional images are represented as the two dimensional matrices in the preprocessing, this method processed the three dimensional tensors directly [9].

Based on the advantages of both the tensor decomposition and the generalized singular value decomposition, this paper proposes a joint tensor decomposition and generalized singular value decomposition based super-resolution method to reconstruct the high resolution images from the low resolution images. The contributions of our proposed method are as follows. (i) The tensor structure is used to represent the images. This representation is more effective for representing the three dimensional images and preserves the local relationships among the neighbor pixels. (ii) By using the joint Tucker decomposition and generalized singular value decomposition to decompose the tensors, the correlation between the upsampled tensors and low resolution tensors can be well exploited. The outline of this paper is as follows. Section 2 reviews the definitions of both the generalized singular value decomposition and the tensor decomposition. Section 3 presents our proposed joint tensor decomposition and generalized singular value decomposition based method for performing the three dimensional image super-resolution. Section 4 presents the computer numerical simulation results. Finally, a conclusion is drawn in Section 5.

2. Review on the definitions of both the generalized singular value decomposition and the tensor decomposition

2.1. Definition of the generalized singular value decomposition

Let $D_i \in \mathrm{R}^{K_i \times L}$ for $i=1,2$ be two matrices. Assume that D_i are with the full column ranks. That is, $L \le K_i$ for $i=1,2$. The generalized singular value decomposition of D_i is to simultaneously factorize D_i into two singular value decomposition based equations sharing the same right unitary matrix [11]. Let $U_i \in \mathrm{R}^{K_i \times L}$ for $i=1,2$ be the left unitary matrices, $V^T \in \mathbb{R}^{L\times L}$ be the right unitary matrix and $\Sigma_i \in \mathbb{R}^{L\times L}$ for $i=1,2$ be the diagonal matrices. Let $\sigma_{i,a}$ for $a=1,\cdots,L$ be the generalized singular values of D_i. That is, $\Sigma_i = diag(\sigma_{i,a})$. Let $u_{i,a}$ and v_a for $a=1,\cdots,L$ be the a^{th} column of U_i and V, respectively. That is,

$$D_i = U_i \Sigma_i V^T = \sum_{a=1}^{L} \sigma_{i,a}(u_{i,a} \otimes v_a^T) \text{ for } i=1,2. \tag{1}$$

It is worth noting that that $\sigma_{i,a} > 0$ for $a=1,\cdots,L$ and they are sorted in the descending order. That is, $\sigma_{i,1} > \sigma_{i,2} > \ldots > \sigma_{i,L}$.

2.2. Definition of the tensor decomposition

A tensor is a multidimensional data and the dimension of the tensor is called the order of the tensor. It can be regarded as the generalization of the one dimensional vectors and the two dimensional matrices in the multidimensional space. Let $\mathcal{A} \in \mathbb{R}^{I_1\times I_2\times I_3}$ be a three mode tensor of size $I_1 \times I_2 \times I_3$. By setting all the indices except one index in $\mathcal{A}$ unchanged, a fiber is defined as the vector of the data in $\mathcal{A}$ at this index. Let $A_{ij:}$ be this fiber. The fiber unfolding refers to the process of reordering the elements in a tensor into a matrix. Figure 1 shows an example of the fiber unfolding of a three mode tensor.

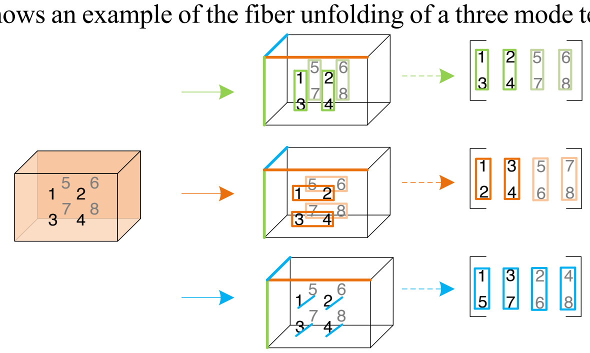


Fig. 1 Fiber unfolding of a three mode tensor.

The common tensor decompositions include the CP decomposition and the Tucker decomposition. In this session, only the Tucker decomposition is reviewed [12]. Let $\mathcal{A} \in \mathbb{R}^{I_1\times I_2\times\cdots\times I_k}$ and $\mathcal{G} \in \mathbb{R}^{R_1\times R_2\times\cdots\times R_k}$ be an original tensor and a small core tensor, respectively. Here, I_j and R_j are the orders of $\mathcal{A}$ and $\mathcal{G}$ at the j^{th} dimension, respectively. Let $X_i \in \mathbb{R}^{I_i\times R_i}$ for $i=1,2,\cdots,k$ be a set of factor matrices in the i^{th} mode. In general, their rows are orthonormal one another. The Tucker decomposition approximates $\mathcal{A}$ as the multiplication of $\mathcal{G}$ and X_i. That is,

$$\mathcal{A} = \mathcal{G} \times_1 X_1 \times_2 X_2 \times \cdots \times_k X_k, \tag{2}$$

In general, there are two main implementation methods for performing the Tucker decomposition. They are the high order singular value decomposition algorithm and the high order orthogonal iteration algorithm. In this session, the high order singular value decomposition algorithm is reviewed. The flow chart of this algorithm is shown below:

Algorithm 1. The high order singular value decomposition algorithm for performing the Tucker decomposition

Input: $\mathcal{A}$.

Output: X_1, X_2, $\cdots$, and X_k as well as $\mathcal{G}$.

Step 1: Let A_n be the mode Kolda horizontal fiber unfolding of $\mathcal{A}$.

Step 2: Perform the singular value decomposition on A_n. Let the left unitary matrix be U_n.

Step 3: Let X_n be the first R_n columns of U_n, where $R_n \le I_n$.

Step 4: Let $\mathcal{G} = \mathcal{A} \times_1 (X_1)^T \times_2 (X_2)^T \times \cdots \times_k (X_k)^T$.

2.3. Definition of the generalized singular value decomposition of the tensor

Performing the generalized singular value decomposition on the tensors is presented below. Let $\mathcal{A}_i \in \mathbb{R}^{K_i\times L\times M}$ for $i=1,2$ be two third order tensors. Assume that the pairs of matrices obtained by performing the fiber unfolding on $\mathcal{A}_i$ are the pairs of the full column rank matrices. That is, $LM \le K_i$. Hence, $\mathcal{A}_i$ can be unfolded into three pairs of the full column rank matrices. Let A_i, A_{ix} and A_{iy} for $i=1,2$ be these three pairs of the unfolded matrices. Besides, $\mathcal{A}_i$ can be factorized using various matrices with their columns being orthonormal one another [11]. Let $U_i \in \mathbb{R}^{K_i\times LM}$, $V_x^T \in \mathbb{R}^{L\times L}$ and $V_y^T \in \mathbb{R}^{M\times M}$ be these matrices. Here, these matrices are obtained by performing the generalized singular value decomposition on the unfolded pairs of the matrices. That is,

$$U_i \Sigma_i V^T = A_i = [\cdots, \mathcal{A}_{i,:lm}, \cdots] \in \mathbb{R}^{K_i\times LM}, \tag{3}$$

$$V_x \Sigma_{ix} U_{ix}^T = A_{ix}^T = [\cdots, \mathcal{A}_{i,k:m}, \cdots] \in \mathbb{R}^{L\times MK_i}, \tag{4}$$

and

$$V_y \Sigma_{iy} U_{iy}^T = A_{iy}^T = [\cdots, \mathcal{A}_{i,kl:}, \cdots] \in \mathbb{R}^{M\times K_i L}, \tag{5}$$

in which $\mathcal{A}_{i,:lm}$ is the matrix containing the K_i rows as well as from the l^{th} column to the m^{th} column of $\mathcal{A}_i$. Then, we have

$$\mathcal{A}_i = \mathcal{G}_i \times_a U_i \times_b V_x \times_c V_y = \sum_{a=1}^{LM}\sum_{b=1}^{L}\sum_{c=1}^{M} g_{i,abc}(u_{i,a} \otimes v_{x,b}^T \otimes v_{y,c}^T), \tag{6}$$

where a, b and c are the first order subscript variable, the second order subscript variable and the third order subscript variable of $\mathcal{A}_i$, respectively. Here, $\otimes$ denotes the outer product. Moreover, by applying the generalized singular value decomposition on the pair of A_i for $i=1,2$, we have $U_i^T A_i = \Sigma_i V^T$. Then, G_i can be obtained by

$$G_i = U_i^T A_i (V_y^{-T} \otimes V_x^{-T}) = \Sigma_i V^T (V_y^{-T} \otimes V_x^{-T}) \text{ for } i=1,2. \tag{7}$$

Here, $\otimes$ denotes the Kronecker operator. Hence, $\mathcal{G}_i$ can be synthesized from G_i.

3. Our Proposed Method

This section proposes a three dimensional image super resolution method via performing the generalized singular value decomposition on the tensors. Our proposed algorithm consists of two stages. They are the decomposition stage and the reconstruction stage. The block diagram of our proposed method is shown in Figure 2. Also, the procedures for performing the decomposition stage and the reconstruction stage are summarized in Algorithm 2 and Algorithm 3, respectively. For the decomposition stage, a three dimensional image is represented as the tensor. The sizes of both the second mode and the third mode of the tensor are first reduced by 2 via the bicubic interpolation. In this case, the low resolution tensor is obtained. It is worth noting that this process is an irreversible. Let $\mathcal{A}_1$ be the obtained tensor by performing the upsampling in the second mode of the low resolution tensor. Let $\mathcal{A}_2$ be the obtained tensor by performing the upsampling in the first mode of $\mathcal{A}_1$ .Then, by performing the generalized singular value decomposition on $\mathcal{A}_1$ and $\mathcal{A}_2$, $\mathcal{A}_1$ and $\mathcal{A}_2$ are simultaneously factorized using various matrices with their columns being orthonormal one another. For the reconstruction stage, the noise vectors in U_1 and U_2 are removed according to the magnitudes of the elements in $\sum_1$ and $\sum_2$, respectively. Then, by $U_i^T A_i = \sum_i V^T$, $\mathcal{G}_1$ and $\mathcal{G}_2$ are reconstructed. Finally, the high resolution image is obtained.

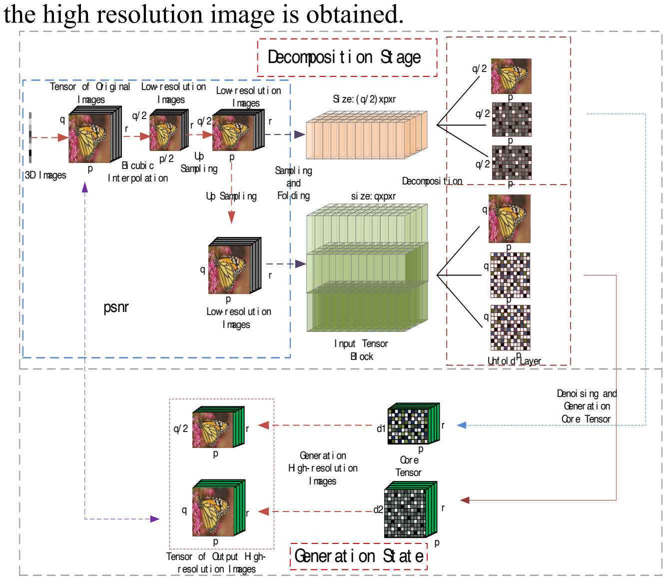


Fig. 2 The block diagram of our proposed method

Algorithm 2. Decomposition stage

Input: Let $\mathcal{A} \in \mathrm{R}^{p\times q\times r}$ be a three dimensional image. Here, r=3.

Step 1: Represent the three dimensional image as the tensor.

Step 2: Employ the bicubic interpolation to reduce the size of the tenor. Let $\overline{\mathcal{A}} \in \mathrm{R}^{\frac{p}{2}\times\frac{q}{2}\times r}$ be the tensor with the reduced size.

Step 3: Let $\mathcal{A}_1 \in \mathrm{R}^{\frac{p}{2}\times q\times r}$ be the tensor obtained by performing the upsampling on $\overline{\mathcal{A}}$. Likewise, let $\mathcal{A}_2 \in \mathrm{R}^{p\times q\times r}$ be the tensor obtained by performing the upsampling on $\mathcal{A}_1$.

Step 4: Decompose $\mathcal{A}_1$ into A_1 , A_{1x} and A_{1y} as well as $\mathcal{A}_2$ as A_2 , A_{2x} and A_{2y} .

Step 5: Compute the generalized singular value decomposition on the pairs of A_i , A_{ix}^T and A_{iy}^T .

Algorithm 3. Reconstruction stage

Step 1: Let $\sum_1$ and $\sum_2$ be the diagonal matrices obtained by performing the generalized singular value decomposition on the pair of A_i .

Step 2: Choose the column vectors in U_1 and U_2 .

Step 3: Generate $\mathcal{G}_1 \in \mathrm{R}^{d_1\times q\times r}$ and $\mathcal{G}_2 \in \mathrm{R}^{d_2\times q\times r}$.

Step 4: Generate the high resolution images.

4. Computer Numerical Simulation Results

In this section, the effectiveness of our proposed method is illustrated via a set of color images with various types of contents. In particular, there are ten images in the image set including the butterfly image, the baby image, the bird image, the coastguard image, the face image, the flowers image, the foremen image, the woman image, the zebra image and the comic image. The sizes of these images are shown in Table 1. Figure 3 shows these low resolution images.

Table 1 The sizes of the images in the image set.

Images	Sizes	Images	Sizes
butterfly	256x256x3	flowers	362x500x3
baby	256x256x3	foreman	288x352x3
bird	288x288x3	woman	288x352x3
coastguard	288x352x3	zebra	391x586x3
face	279x276x3	comic	361x250x3

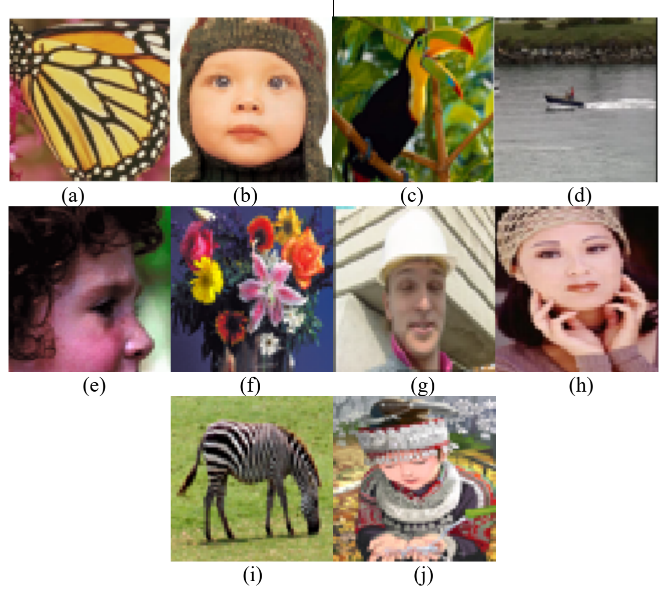


Fig. 3 The low resolution images. (a) The butterfly image. (b) The baby image. (c) The bird image. (d) The coastguard image. (e) The face image. (f) The flowers image. (g) The foremen image. (h) The woman image. (i) The zebra image. (j) The comic image.

In this paper, both the total number of the rows and the total number of the columns of the high resolution images are chosen to be the double of those of the low resolution images so that the sizes of the high resolution images are the same as those of the original images. To perform the super-resolution, the low resolution images are first upsampled. In order to compare the effectiveness of our proposed method, a similar nonlinear adaptive method is compared. In particular, the singular value decomposition based method and the generalized singular value decomposition based method are compared. Figure 4 and Figure 5 show the super-resolution resolution images obtained via our proposed method with and without performing the denoising, respectively. On the other hand, Figure 6 and 7 show the super-resolution images obtained by the singular value decomposition based method and the generalized singular value decomposition based method, respectively. It can be seen from Figure 4 that the super-resolution images obtained by our proposed method with performing the denoising are sharp. On the other hand, it can be seen from Figure 5 to Figure 7 that the super-resolution images obtained by our proposed method without performing the denoising as well as the singular value decomposition based method and the generalized singular value decomposition based method are blurred. This demonstrates the qualitative effectiveness of our proposed method with performing the denoising. Besides, Table 1 shows the peak signal to noise ratios between the super resolution images obtained by various methods and the original images. From Table 1, it can be seen that our proposed method with performing the denoising achieves the higher peak signal to noise ratios compared with the our proposed method without performing the denoising as well as the singular value decomposition based method and the generalized singular value decomposition based method. This demonstrates the quantitative effectiveness of our proposed method with performing the denoising.

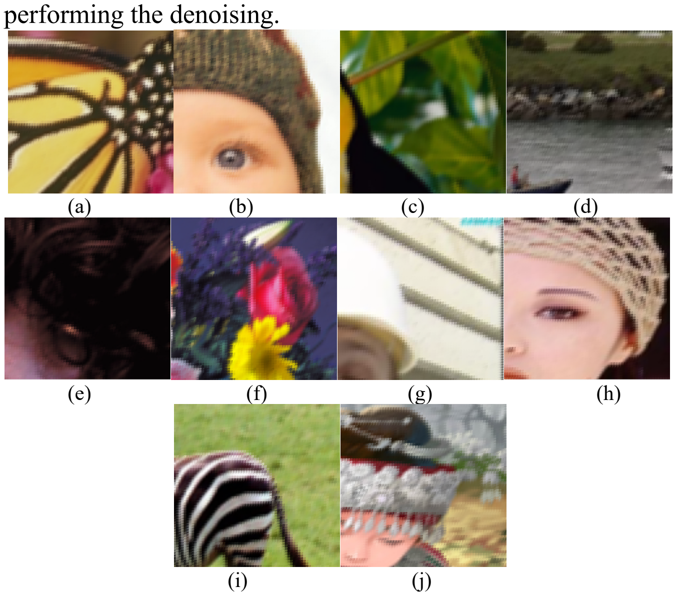


Fig. 4 The high resolution images obtained via our proposed method with performing the denoising. (a) The butterfly image. (b) The baby image. (c) The bird image. (d) The coastguard image. (e) The face image. (f) The flowers image. (g) The foremen image. (h) The woman image. (i) The zebra image. (j) The comic image.

Fig. 5 The high resolution images obtained via our proposed method without performing the denoising. (a) The butterfly image. (b) The baby image. (c) The bird image. (d) The coastguard image. (e) The face image. (f) The flowers image. (g) The foremen image. (h) The woman image. (i) The zebra image. (j) The comic image.

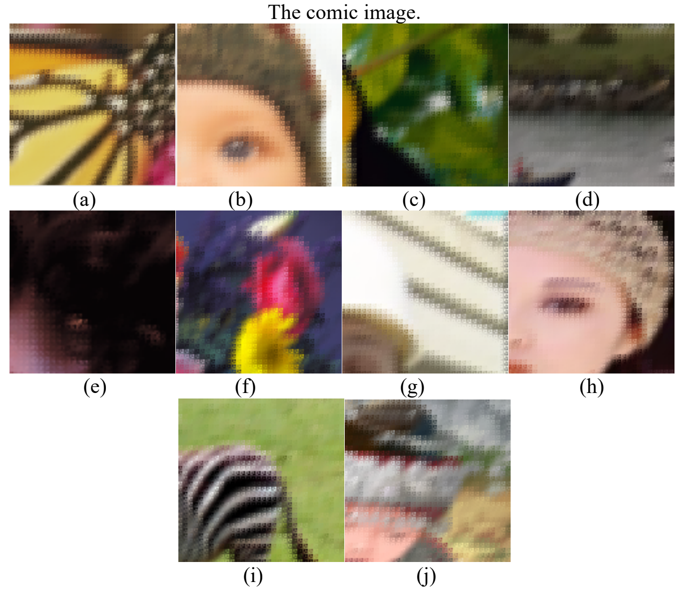
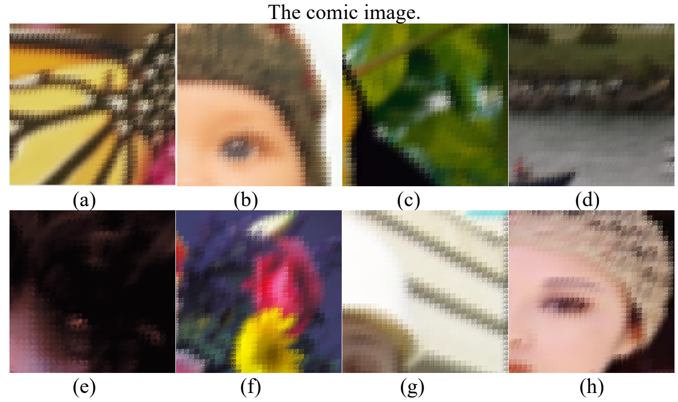


Fig. 6 The high resolution images obtained via the singular value decomposition based method. (a) The butterfly image. (b) The baby image. (c) The bird image. (d) The coastguard image. (e) The face image. (f) The flowers image. (g) The foremen image. (h) The woman image. (i) The zebra image. (j) The comic image.



(i) (j)

Fig. 7 The high resolution images obtained via the generalized singular value decomposition based method. (a) The butterfly image. (b) The baby image. (c) The bird image. (d) The coastguard image. (e) The face image. (f) The flowers image. (g) The foremen image. (h) The woman image. (i) The zebra image. (j) The comic image.

Table 2 The peak signal to noise ratio obtained by the singular value decomposition based method, the generalized singular value decomposition based method, our proposed method without performing the denoising and Our proposed method with performing the denoising.

Images	Peak signal to noise ratio			
	Singular value decomposition based method	Generalized singular value decomposition based method	Our proposed method without performing the denoising	Our proposed method with performing the denoising
butterfly	10.7156	10.7156	**11.4020**	**12.0469**
baby	9.1071	9.1071	**9.3005**	**10.0987**
bird	12.9036	12.9036	**13.0872**	**13.5177**
coastguard	11.9021	11.9021	**12.1713**	**12.4331**
face	12.2722	12.2722	**12.4027**	**12.4498**
flowers	12.8789	12.8789	**13.4029**	**13.7946**
foreman	11.0280	11.0280	**11.3171**	**12.0123**
woman	10.3008	10.3008	**10.6399**	**11.0266**
zebra	11.4232	11.4232	12.5691	12.9794
comic	11.1114	11.1114	11.8632	12.0000

5. CONCLUSION

This paper proposed a joint tensor decomposition and generalized singular value decomposition based method for performing the three dimensional image super-resolution. Compared with the existing machine learning based super-resolution methods, our proposed method requires a lower computational power. On the other hand, the conventional matrix decomposition based methods achieve the lower peak signal to noise ratios. This is because converting the three dimensional images to the matrices for processing will lose the local neighbor information of the images. On the other hand, our proposed method achieves a higher peak signal to noise ratio. This is because the tensor structures can preserve the local neighbor information of the images.

In future, our proposed method will combine with the machine learning based super-resolution methods so as to fasten the existing the machine learning based super-resolution methods but also to achieve a high peak signal to noise ratio.

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