Multi-Criteria Decision Making in Linguistic Values of Neutrosophic Trapezoidal Fuzzy Multi- numbers

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Research Article

Keywords: Neutrosophic fuzzy set, Neutrosophic Trapezoidalfuzzy number, Neutrosophic fuzzy multiset. NeutrosophicTrapezoidal fuzzy multi-numbers

Posted Date: May 18th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1596658/v1

Multi-Criteria Decision Making in Linguistic Values of Neutrosophic Trapezoidal Fuzzy Multi- numbers

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Abstract: Neutrosophic trapezoidal fuzzy multi-numbers (NTFMN) are a specific neutrosophic fuzzy multiset on a real number set that decision makers can use to represent their neutrosophic fuzzy multi-preference information. The occurrences in the NTFMN are several, with the potential of the same or memberships of truth, indeterminacy, and falsehood. Neutrosophic trapezoidal fuzzy multi-numbers(NTFMN) are a type of neutrosophic fuzzy numbers on a real number set that are used to anticipate the solution to multi-criteria decision-making situations. In this study, we begin by defining NTFMN and then develop a set of multiple-criteria decision-making issues with NTFMN alternatives. The expected value method is used to rank neutrosophic trapezoidal fuzzy multi-numbers and choose the best option. Finally, a numerical example is provided to demonstrate the method's practicality and efficiency.

Keywords: *Neutrosophic fuzzy set, Neutrosophic Trapezoidalfuzzy number, Neutrosophic fuzzy multiset. NeutrosophicTrapezoidal fuzzy multi-numbers.*

1 Introduction

Since the beginning of optimization techniques, the study of LPP has piqued people's curiosity. Optimization strategies are still one of the most widely used operations research methodologies. These techniques are progressing and developing, both in terms of methodology and applications. Innovative analytic treatments are being pursued in the direction of theoretical progress, and additional fields of application are emerging. The evaluation of membership values is not attainable to one's satisfaction in many real-world circumstances due to a lack of data accessible. Furthermore, evaluating non-membership values is not always viable, and hesitancy persists in an indeterministic stage of the process. In the representation of an uncertain quantity, a fuzzy number is essential. Fuzzy numbers are a unique type of fuzzy sets which are of importance in solving Fuzzy Linear Programming Problems (FLPPs).

In the study of fuzzy set theory, ordering fuzzy numbers is an important topic. To rank fuzzy numbers, one fuzzy number must be compared to the others, but it can be difficult to tell which of them is smaller or larger. To rank fuzzy numbers, a variety of approaches have been proposed in the literature. An IFN appears to be a better fit for describing uncertainty as a generalisation of fuzzy numbers. Defining and analysing IFNs has sparked a slew of research projects. Several intriguing features of different types of IFNs are also examined. IFN research has recently gotten a lot of interest because it's better for fixing IFLPPs. Some researchers have developed a variety of ranking techniques for ordering IFNs.There aren't just complicated challenges requiring multiple criteria; some criteria may have an impact on a specific difficulty. However, in order to achieve the best result, all of the options must share similar criteria that clearly lead to more informed and better judgments. Traditional decision-making approaches assume that all criteria and their corresponding weights are given in discrete values, and that rating and ranking alternatives is simple.

The application of the classical decision-making approach in a real-world decision situation may confront substantial practical limits due to criteria that may contain imprecision in the information. In many cases, the performance of the criteria can only be expressed qualitatively or by using linguistic terms, which certainly demands a more appropriate method. In many circumstances, the criteria's performance can only be stated qualitatively or in language words, necessitating the use of a more appropriate method. The best circumstance for a decision-making dilemma is when all of the criteria's ratings and degrees of importance are known accurately, allowing them to be arranged in a clear order. Many real-world decision-making situations, on the other hand, occur in a context in which the goals, limitations, and consequences of various actions are unknown. As a result, when the decision scenario includes both fuzzy and crisp facts, the optimal condition for a typical decision-making problem may not be achieved. Classical decision-making procedures are unsuccessful in dealing with such issues since they are designed for problems where all performance criteria are represented by clear figures. The application of fuzzy set theory in the field of decision making is justified when the desired goals or their attainment cannot be described or appraised crisply, but only as fuzzy sets. Fuzzy multi-criteria approaches have been used in a variety of domains, including banking, urban distribution centres, water shed allocation, safety assessment, and the development of corporate organization’s performance. Fuzzy logic is used to analyse quantitative and qualitative data in a variety of applications. FMCDM's various approaches aid in the completion of numerous subtasks involving evaluation and ranking using various ways.Each strategy is distinct in its own way. There are numerous approaches for dealing with fuzzy multi-criteria decision-making challenges. Depending on changes in the business sector, Multiple Attribute Decision Making (MADM) has been one of the fastest expanding topics in recent decades.

Literature Review: The notion of Fuzzy Sets provided by Zadeh [29] and its extension presented by Attanassov as Intuitionistic Fuzzy Sets [2] have solved various problems involving vagueness and uncertainty during the last two decades. Later Cuong et.al [7] proposed the Picture Fuzzy Set by adding a neutral membership function to Intuitionistic Fuzzy Sets. There are fundamental challenges in uncertainty today that FS, IFS, and PFS cannot solve. Liu et al. presented the interaction operators, weighted interaction operators in terms of intuitionistic fuzzy numbers. Liu et al. also used the Dombi geometric Bonferroni mean operator to solve multi-attribute decision-making problems.Chen et al. presented a ranking of intuitionistic fuzzy numbers based on the comparison of likelihood relations. Chen et al. also addressed issues with intuitionistic fuzzy multi-attribute decision making. Liu et al. constructed triangular intuitionistic fuzzy numbers by combining triangular fuzzy numbers and intuitionistic fuzzy sets.Wang et al. presented fuzzy numbers, intuitionistic fuzzy weighted operator and fuzzy number intuitionistic fuzzy weighted averaging operator ranking approaches, as well as hybrid operators on this division, to cope with multi-attribute situations.A precise answer to the scenario of the statement Movie X would be a hit cannot be given by the human brain. Neutrosophic fuzzy sets clearly handle such scenarios. Wang et al. [23] presented single-valued Neutrosophic sets in 2012 and expanded the work to include interval-valued neutrosophic sets in 2013. In the topic of neutrosophic sets and neutrosophic numbers, a lot of research has been done. Ye [28] provided clearly simplified neutrosophic sets and their aggregation operations. Wang and Li [24] and Yang and Pang [26] suggested multivalued neutrosophic sets and their operations. Deli et al. [8] investigated bipolar Neutrosophic sets. Few researchers have solved decision-making difficulties in a Neutrosophic fuzzy environment. [13]; Dey et al., [10]; Pramanik et al.,[14]; Deli et al.,[9]; Ali et al.,[1]; Tian et al.,[21]. Biswas et al. [3] pioneered trapezoidal Neutrosophic fuzzy sets. Ye 2015 [27]; Tan et al., [20]; Pramanik et al., [13]; Biswas et al., [4 - 6]; Jana et al., [12] developed and solved decision-making models in a Trapezoidal Neutrosophic Numbers setting.

A set is a collection of well-defined items that differ in pairs. Multisets are created by forming a set structure by permitting repetition of any element. Shinoj and John [17 - 18] offered intuitionistic fuzzy multisets, while Yager [25] proposed fuzzy multisets. Ye investigated single-valued neutrosophic fuzzy multisets in 2014[28]. Ulucay et al. [22] advocated the use of Neutrosophic Multisets to solve MCDM challenges. Many studies in the topic of Neutrosophic multi sets have recently been conducted. There appears to be no investigation on Neutrosophic trapezoidal fuzzy multi numbers in the present research. Uncertainty theory innovation plays a critical role in the definition of a true logical scientific model, basic demonstrating in designing space, and multi-standards based clinical diagnosis issue, among other things. Consider the following example of a polling result in an election: a candidate received thirty percent of the vote in favour, twenty percent against, ten percent uncertain, and forty percent undecided. There is a lack of certainty in separating the criteria give up and undecided when using IFS to handle this problem. The Neutrosophic Set (NS), which was framed by Smarandache (1999) and has three characteristic functions truth membership, indeterminacy membership, and falsity membership, can be used to overcome this unwanted condition. It is demonstrated that lone neutrosophic sets can deal with the inaccuracy, faltering, and truthiness of an uncertain number in a way that is gradually solid, legitimate, and reasonable for a decision maker. Fuzzy sets, intuitionistic fuzzy sets, interval-valued fuzzy sets, and other structures have been used to deal with ambiguous data in recent years. The introduction of neutrosophic sets has been proven to be better adapted to dealing with vagueness than the conventional set-theoretical framework. The fuzzy number can only quantify uncertainty and ambiguity, not hesitancy. It is intuitionistic and interval-valued intuitionistic fuzzy number.Only a neutrosophic number can effectively measure all three characteristics. As a result, the trapezoidal neutrosophic number garners greater attention and opens the door to new study.

1.1 Novelty and Motivation: A set is a well-defined collection of unique things; that is, the elements of a set differ in pairs. We can acquire a mathematical structure known as Multisets or Bags if we relax this limitation and allow repeated occurrences of any element.The following table throws an insight into the novelty of proposed research work. The current research available in the literature lacks to deal indeterminacy of fuzzy numbers having more number of membership and non-membership values. So the introduction of Neutrosophic fuzzy multi-numbers, plays a vital role in dealing real life problems involving truth, indeterminacy and falsity functions having more than one membership functions.

Table 1 Insight ofnovelty of the proposed method

S.No.	Type of numbers	Indeterminacy	Ambiguity	Uncertainty
1	Crisp value	Unable to handle	Unable tohandle	Unable to handle
2	Fuzzy numbers	Unable to handle	Unable to handle	Able to handle
3	Interval Fuzzy Number	Unable to handle	Unable to handle	Able to handle
4	Intuitionistic FuzzyNumber (IFN)	Inadequate to handle	Adequate to handle	Adequate to handle
5	Interval Intuitionistic Fuzzy Number	Inadequate to handle	Adequate to handle more clearly than IFN's	Adequate to handlemore clearly than IFN's
6	Neutrosophic Number	Able to handle	Able to handle	Able to handle
7	Interval Neutrosophic Number	Able to handle more accurately than	Able to handle	Able to handle

		Neutrosophic numbers		
8	Intuitionistic fuzzy multi-numbers	Unable to handle	Able to handle, if the data contains more than one memberships and non-memberships functions.	Able to handle, if the data contains more than one memberships and non-memberships functions.
9	Neutrosophic fuzzy Multi-Number	Able to handle, if the data contains more than one memberships and non-memberships functions.	Able to handle, if the data contains more than one memberships and non-memberships functions.	Able to handle, if the data contains more than one memberships and non-memberships functions.

Consider there is a panel of four possible alternatives to invest the money namely, A_1 is a car company, A_2 is a food company, A_3 is a computer company, and A_4 is a television company. The investment company takes a decision according to the following three criteria C_1 is the risk analysis, C_2 is the growth analysis, C_3 is the environmental impact analysis. For the company A_1 corresponding to C_1, we are considering Neutrosophic trapezoidal fuzzy multi-number as follows:

$$\left\langle [0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6),(0.6,0.3,0.5,0.2),(0.5,0.6,0.7,0.3) \right\rangle$$

Here the 2^{nd} element represents truth membership, 3^{rd} element represents indeterminacy and last element represents falsity membership values. In each element, there are four values, which refers to risk analysis due to various factors, but here we consider only 4 factors such as (a) evaluation given by existing stock holders (b) suggestion given by experts in that field (c) our own evaluation and (d) capability to take risk. The membership values for these 4 factors corresponding to truth, indeterminacy and falsity were expressed above. To the best our knowledge until October 2020, no work has been published on neutrosophic trapezoidal fuzzy multi-numbers. So, the present research will help the researchers in various fields to apply the proposed ranking to solve their problems, which deals multi-numbers.

1.2 Significance of present work

A generalization of neutrosophic fuzzy numbers is neutrosophic trapezoidal fuzzy multi-numbers. To the best of our knowledge, no work has been done in the proposed research field. The truth, indeterminacy, and falsity functions take single values in single valued trapezoidal fuzzy numbers, while NTFMNs take multiple values, which is an added advantage that allows one to deal with more critical real-time challenges. As a result of this research, future researchers will be able to better understand the notion of NTFMN, which will allow them to handle imprecise data more effectively.

1.3 Structure of the paper

The essential definitions of neutrosophic multiset and neutrosophic fuzzy multi - numbers are supplied in section 2. Section 3 contains a definition of the neutrosophic trapezoidal fuzzy multi number (NTFMN), some fundamental operations on NTFMN, ranking of NTFMN based on expected value, validation, and a proposal for an algorithm to solve the MCDM problem, along with a real-world problem to support the approach. The conclusion and future research work that can be expanded from the current study are presented in the final section.

2 Preliminaries of Neutrosophic Trapezoidal Fuzzy Multi Number

This section starts with basic concepts of multi-fuzzy set, intuitionistic fuzzymultiset, neutrosophic multiset and trapezoidal neutrosophic fuzzy number.

Definition 1 [7]An IFS A in X is given by

$$A = \{(x, \mu_A(x), v_A(x)), x \in X\}$$

where the functions $\mu_A(x)\colon X \to [0,1]$ and $v_A(x)\colon X \to [0,1]$define, the degree of membership and degree of non-membership of the element $x \in X$ to the set A respectively, which is a subset of X, and for every $x \in X$, $0 \le \mu_A(x) + v_A(x) \le 1$.

Obviously, every fuzzy set has the form $\{(x, \mu_A(x), \mu_{A^c}(x)), x \in X\}$.For each IFS A in X, $\Pi_A(x) = 1 - \mu(x) - v(x)$, is called the intuitionistic fuzzy index of x in A. It is obvious that $0 \le \Pi_A(x) \le 1, \forall\, x \in X$.

Definition 2 [28] An IFS $A = \{(x, \mu_A(x), v_A(x) | x \in X)\}$ is called IF-normal, if there exist at least two points $x_0, x_1 \in X$ such that $\mu_A(x_0) = 1, v_A(x_1) = 1$. It is easily seen that the given IFS A will be IF-normal if there is at least one point that surely belongs to A and atleast one point which does not belong to A.

Definition 3 [28]An IFS $A = \{(x, \mu_A(x), v_A(x) | x \in X)\}$ of the real line is called IF-convex, if $\forall x_1, x_2 \in \mathbb{R}, \forall \lambda \in [0,1]$.

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \ge \mu_A(x_1) \wedge \mu_A(x_2)$$
$$\gamma_A(\lambda x_1 + (1-\lambda)x_2) \ge \gamma_A(x_1) \wedge \gamma_A(x_2)$$

Definition4 [28]An IFS $A = \{(x, \mu_A(x), v_A(x) | x \in X)\}$of the real line is called an IFN if

(i) A is IF-normal,
(ii) A is IF-convex,
(iii) μ_A is upper semicontinuous and γ_A is lower semicontinuous and
(iv) $A = \{(x \in X | v_A(x) < 1\}$is bounded.

Definition 5 Let X be a universe of discourse, then a neutrosophic set $\tilde{N}$ in X is given by

$$\tilde{N} = \{\langle x, T_{\tilde{N}}(x), I_{\tilde{N}}(x), F_{\tilde{N}}(x) \rangle | x \in X\}, \quad (2)$$

where $T_{\tilde{N}}(x), I_{\tilde{N}}(x)$ and $\mathrm{F}_{\tilde{N}}(x)$are called truth-membershipfunction, indeterminacy-membership function and falsity membership function, respectively. They are respectively defined by $T_{\tilde{N}}(x) \subset \left]^-0,1^+\right[$, $I_{\tilde{N}}(x) \subset \left]^-0,1^+\right[$, and $\mathrm{F}_{\tilde{N}}(x) \subset \left]^-0,1^+\right[$such that $0^- \le T_{\tilde{N}}(x) + I_{\tilde{N}}(x) + \mathrm{F}_{\tilde{N}}(x) \le 3^+$.

Definition6 Aneutrosophic trapezoidal fuzzy number is denoted by $\tilde{n} = \langle (t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4) \rangle$ in a universe of discourse X. The parameters satisfy the relation $t_1 \le t_2 \le t_3 \le t_4$, $i_1 \le i_2 \le i_3 \le i_4$ and $f_1 \le f_2 \le f_3 \le f_4$. Its truth-membership function, indeterminacy-membership function and falsity membership function are defined as follows

$$T_{\tilde{N}}(x) = \begin{cases} \dfrac{x - t_1}{t_2 - t_1}\ ; t_1 \le x \le t_2 \\ 1\ ; t_2 \le x \le t_3 \\ \dfrac{t_4 - x}{t_4 - t_3}\ ;\ t_3 \le x \le t_4 \\ 0; otherwise \end{cases} \tag{3}$$

$$I_{\tilde{N}}(x) = \begin{cases} \dfrac{i_2 - x}{i_2 - i_1}\ ; i_1 \le x \le i_2 \\ 0\ ; i_2 \le x \le i_3 \\ \dfrac{x - i_3}{i_4 - i_3}\ ;\ i_3 \le x \le i_4 \\ 1; otherwise \end{cases} \tag{4}$$

and

$$F_{\tilde{N}}(x) = \begin{cases} \dfrac{f_2 - x}{f_2 - f_1}\ ; f_1 \le x \le f_2 \\ 0\ ; f_2 \le x \le f_3 \\ \dfrac{x - f_3}{f_4 - f_3}\ ;\ f_3 \le x \le f_4 \\ 1; otherwise \end{cases} \tag{5}$$

As a special case, in the above definition when $t_2 = t_3, i_2 = i_3$ and $f_2 = f_3$ then we will get a neutrosophic triangular fuzzy number.

Definition 7[16]Let X be a non-empty set. A multi-fuzzy set A on X is given by

$$A = \{\langle x, \mu_A^1(x), \mu_A^2(x), ..., \mu_A^p(x), x \in E \rangle\}, \tag{1}$$

where the functions $\mu_A^i(x) : X \to [0,1]$ for all $i \in \{1,2,...,p\}$ such that $\mu_A^1(x) \ge \mu_A^2(x) \ge ... \ge \mu_A^p(x)$, for all $x \in E$.

Definition 8[17-18]Let X be a non-empty set. A intuitionistic fuzzy multiset A on X is given by

$$A = \{\langle x, (\mu_A^1(x), \mu_A^2(x), ..., \mu_A^p(x)), (\nu_A^1(x), \nu_A^2(x), ..., \nu_A^p(x)) : x \in X \rangle\}, \tag{2}$$

where the functions $\mu_A^i(x) : X \to [0,1]$ and $\nu_A^i(x) : X \to [0,1]$ such that $0 \le \mu_A^i(x) + \nu_A^i(x) \le 1$ for all $i \in \{1,2,...,p\}$. Also the membership sequence $(\mu_A^1(x), \mu_A^2(x), ..., \mu_A^p(x))$ is decreasingly ordered sequence of elements, $\mu_A^1(x) \geq \mu_A^2(x) \geq ... \geq \mu_A^p(x)$, for all $x \in X$ and the corresponding non-membership sequence $(\nu_A^1(x), \nu_A^2(x), ..., \nu_A^p(x))$ is neither decreasing nor increasing function.

Definition 9[19]Let X be a universe of discourse, then a neutrosophic set $\tilde{N}$ in X is given by

$$\tilde{N} = \{\langle x, T_{\tilde{N}}(x), I_{\tilde{N}}(x), F_{\tilde{N}}(x)\rangle | x \in X\}, \quad (3)$$

where $T_{\tilde{N}}(x), I_{\tilde{N}}(x)$ and $\mathrm{F}_{\tilde{N}}(x)$ are called truth-membershipfunction, indeterminacy-membership function and falsity membership function, respectively. They are respectively defined by $T_{\tilde{N}}(x) \subset]^-0,1^+[$, $I_{\tilde{N}}(x) \subset]^-0,1^+[$, and $\mathrm{F}_{\tilde{N}}(x) \subset]^-0,1^+[$ such that $0^- \leq T_{\tilde{N}}(x) + I_{\tilde{N}}(x) + \mathrm{F}_{\tilde{N}}(x) \leq 3^+$.

Definition 10[27]Let X be a nonempty set with generic elements in X denoted by x. Thesingle valued neutrosophic multiset (SVNM) A drawn from X is characterized by the three functions: count truthmembershipof CT_A, count indeterminacy-membership of CI_A, and count falsity-membership of CF_A such that $CT_A(x): X \to R, CI_A(x): X \to R, CF_A(x): X \to R$ where R is the set of all real number multisets in the real unit interval [0, 1]. Then, a SVNM A is denoted by

$$A = \{\langle x, (T_A^1(x), T_A^2(x), ..., T_A^p(x)), (I_A^1(x), I_A^2(x), ..., I_A^p(x)), (F_A^1(x), F_A^2(x), ..., F_A^p(x))\rangle : x \in X\} \quad (4)$$

where the truth-membership sequence $(T_A^1(x), T_A^2(x), ..., T_A^p(x))$, the indeterminacy-membership sequence $(I_A^1(x), I_A^2(x), ..., I_A^p(x))$, and the falsity membership sequence $(F_A^1(x), F_A^2(x), ..., F_A^p(x))$ may be in decreasing or increasing order, and the sum of $T_A^i(x), I_A^i(x), F_A^i(x) \in [0,1]$ satisfies the condition $0 \leq \sup T_A^i(x) + \sup I_A^i(x) + \sup F_A^i(x) \leq 3$, for $x \in X$ and $i = 1,2,...,q$.

In simplified form a SVNM set A is denoted by $A = \{\langle x, T_A^i(x), I_A^i(x), F_A^i(x)\rangle : x \in X, i = 1,2,...,q\}$

Definition 11[3] Aneutrosophic trapezoidal fuzzy number is denoted by $\tilde{n} = \langle (t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4)\rangle$ in a universe of discourse X. The parameters satisfy the relation $t_1 \leq t_2 \leq t_3 \leq t_4$, $i_1 \leq i_2 \leq i_3 \leq i_4$ and $\mathrm{f}_1 \leq f_2 \leq f_3 \leq f_4$. Its truth-membership function, indeterminacy-membership function and falsity membership function are defined as follows

$$T_{\tilde{N}}(x) = \begin{cases} \dfrac{x - t_1}{t_2 - t_1}\ ;\ t_1 \leq x \leq t_2 \\ 1\ ;\ t_2 \leq x \leq t_3 \\ \dfrac{t_4 - x}{t_4 - t_3}\ ;\ t_3 \leq x \leq t_4 \\ 0; otherwise \end{cases}$$

$$I_{\tilde{N}}(x)=\begin{cases}\frac{i_2-x}{i_2-i_1}\ ;\ i_1\le x\le i_2\\ 0\ \ ;i_2\le x\le i_3\\ \frac{x-i_3}{i_4-i_3}\ ;\ i_3\le x\le i_4\\ 1;otherwise\end{cases}$$

and

$$F_{\tilde{N}}(x)=\begin{cases}\frac{f_2-x}{f_2-f_1}\ ;\ f_1\le x\le f_2\\ 0\ \ ;f_2\le x\le f_3\\ \frac{x-f_3}{f_4-f_3}\ ;\ f_3\le x\le f_4\\ 1;otherwise\end{cases}$$

As a special case , in the above definition when $t_2=t_3, i_2=i_3$ and $f_2=f_3$ then we will get a neutrosophic triangular fuzzy number .

Definition12[11]. Let $\tilde{n}=\langle(t_1,t_2,t_3,t_4),(i_1,i_2,i_3,i_4)\rangle$ be an intuitionistic fuzzy number in the set of real numbers R. Then the expected value of $\tilde{n}$ is given by

$$EV(\tilde{n})=\frac{E_*(A)+E^*(A)}{2}.$$

3 The proposed methodology

In this section,we introduce the definition of neutrosophic trapezoidal fuzzy multi-number and some of basic operations related to it. Also expected value for the neutrosophic trapezoidal fuzzy multi - number was givenand ranking of neutrosophic trapezoidal fuzzy multi numbers based on expected value was proposed.

3.1Neutrosophic Trapezoidal Fuzzy Multi Number and ranking by expected values

Definition 7. The neutrosophic trapezoidal fuzzy multi number (NTFMN) is given by

$\tilde{n}=\langle[a,b,c,d];(\alpha^1,\alpha^2,...,\alpha^p),(\beta^1,\beta^2,...,\beta^p),(\gamma^1,\gamma^2,...,\gamma^p)\rangle$ where $\alpha^i,\beta^i,\gamma^i\in[0,1], i=1,2,...,p$ and $a,b,c,d\in\Re$

Its truth-membership function is defined by

$$T_{\tilde{n}}(x)=\begin{cases}\dfrac{(x-a)}{(b-a)}\alpha^{i}\ ;\ a\le x\le b\\ \alpha^{i}\quad ;b\le x\le c\\ \dfrac{(d-x)}{(d-c)}\alpha^{i};\ c\le x\le d\\ 0;otherwise\end{cases}$$

The indeterminacy membership function is given by

$$I_{\tilde{n}}(x)=\begin{cases}\dfrac{(b-a)+\beta^{i}(x-a_1)}{(b-a_1)};\ a_1\le x\le b\\ \beta^{i}\quad ;b\le x\le c\\ \dfrac{(x-c)+\beta^{i}(d_1-x)}{(d_1-c)};\ c\le x\le d_1\\ 1;otherwise\end{cases}$$

and the falsity membership function is given by

$$F_{\tilde{n}}(x)=\begin{cases}\dfrac{(b-a)+\gamma^{i}(x-a_1)}{(b-a_1)};\ a_1\le x\le b\\ \gamma^{i}\quad ;b\le x\le c\\ \dfrac{(x-c)+\gamma^{i}(d_1-x)}{(d_1-c)};\ c\le x\le d_1\\ 1;otherwise\end{cases}$$

Definition 8 Let $\tilde{n}_1=\left\langle[a_1,b_1,c_1,d_1];\left(\alpha^1_{n_1},\alpha^2_{n_1},\ldots,\alpha^p_{n_1}\right),\left(\beta^1_{n_1},\beta^2_{n_1},\ldots,\beta^p_{n_1}\right),\left(\gamma^1_{n_1},\gamma^2_{n_1},\ldots,\gamma^p_{n_1}\right)\right\rangle$ and $\tilde{n}_2=\left\langle[a_2,b_2,c_2,d_2];\left(\alpha^1_{n_2},\alpha^2_{n_2},\ldots,\alpha^p_{n_2}\right),\left(\beta^1_{n_2},\beta^2_{n_2},\ldots,\beta^p_{n_2}\right),\left(\gamma^1_{n_2},\gamma^2_{n_2},\ldots,\gamma^p_{n_2}\right)\right\rangle$ be any two NTFMNs and $\lambda\neq 0$ be any real number. Then 1.

$$\tilde{n}_1\oplus\tilde{n}_2=\left\langle\begin{array}{l}[a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2];\left(\alpha^1_{n_1}\wedge\alpha^1_{n_2},\alpha^2_{n_1}\wedge\alpha^2_{n_2},\ldots,\alpha^p_{n_1}\wedge\alpha^p_{n_2}\right),\\ \left(\beta^1_{n_1}\vee\beta^1_{n_2},\beta^2_{n_1}\vee\beta^2_{n_2},\ldots,\beta^p_{n_1}\vee\beta^p_{n_2}\right),\left(\gamma^1_{n_1}\vee\gamma^1_{n_2},\gamma^2_{n_1}\vee\gamma^2_{n_2},\ldots,\gamma^p_{n_1}\vee\gamma^p_{n_2}\right)\end{array}\right\rangle$$

2.

$$\tilde{n}_1\otimes\tilde{n}_2=\begin{cases}\left\langle\begin{array}{l}[a_1a_2,b_1b_2,c_1c_2,d_1d_2];\left(\alpha^1_{n_1}\wedge\alpha^1_{n_2},\alpha^2_{n_1}\wedge\alpha^2_{n_2},\ldots,\alpha^p_{n_1}\wedge\alpha^p_{n_2}\right),\\ \left(\beta^1_{n_1}\vee\beta^1_{n_2},\beta^2_{n_1}\vee\beta^2_{n_2},\ldots,\beta^p_{n_1}\vee\beta^p_{n_2}\right),\left(\gamma^1_{n_1}\vee\gamma^1_{n_2},\gamma^2_{n_1}\vee\gamma^2_{n_2},\ldots,\gamma^p_{n_1}\vee\gamma^p_{n_2}\right)\end{array}\right\rangle,d_1>0,d_2>0\\ \left\langle\begin{array}{l}[a_1d_2,b_1c_2,c_1b_2,d_1a_2];\left(\alpha^1_{n_1}\wedge\alpha^1_{n_2},\alpha^2_{n_1}\wedge\alpha^2_{n_2},\ldots,\alpha^p_{n_1}\wedge\alpha^p_{n_2}\right),\\ \left(\beta^1_{n_1}\vee\beta^1_{n_2},\beta^2_{n_1}\vee\beta^2_{n_2},\ldots,\beta^p_{n_1}\vee\beta^p_{n_2}\right),\left(\gamma^1_{n_1}\vee\gamma^1_{n_2},\gamma^2_{n_1}\vee\gamma^2_{n_2},\ldots,\gamma^p_{n_1}\vee\gamma^p_{n_2}\right)\end{array}\right\rangle,d_1<0,d_2>0,\\ \left\langle\begin{array}{l}[d_1d_2,c_1c_2,b_1b_2,a_1a_2];\left(\alpha^1_{n_1}\wedge\alpha^1_{n_2},\alpha^2_{n_1}\wedge\alpha^2_{n_2},\ldots,\alpha^p_{n_1}\wedge\alpha^p_{n_2}\right),\\ \left(\beta^1_{n_1}\vee\beta^1_{n_2},\beta^2_{n_1}\vee\beta^2_{n_2},\ldots,\beta^p_{n_1}\vee\beta^p_{n_2}\right),\left(\gamma^1_{n_1}\vee\gamma^1_{n_2},\gamma^2_{n_1}\vee\gamma^2_{n_2},\ldots,\gamma^p_{n_1}\vee\gamma^p_{n_2}\right)\end{array}\right\rangle,d_1<0,d_2<0\end{cases}$$

3. $$\lambda\tilde{n}_1=\begin{cases}\left\langle[\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1];\left(\alpha^1_{n_1},\alpha^2_{n_1},\ldots,\alpha^p_{n_1}\right),\left(\beta^1_{n_1},\beta^2_{n_1},\ldots,\beta^p_{n_1}\right),\left(\gamma^1_{n_1},\gamma^2_{n_1},\ldots,\gamma^p_{n_1}\right)\right\rangle,\lambda>0\\ \left\langle[\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1];\left(\alpha^1_{n_1},\alpha^2_{n_1},\ldots,\alpha^p_{n_1}\right),\left(\beta^1_{n_1},\beta^2_{n_1},\ldots,\beta^p_{n_1}\right),\left(\gamma^1_{n_1},\gamma^2_{n_1},\ldots,\gamma^p_{n_1}\right)\right\rangle,\lambda<0\end{cases}$$

Example 1

Let us consider two NTFMNs

$\tilde{n}_1 = \langle [2,4,5,7]; (0.3,0.6,...,0.8), (0.7,0.4,...,0.2), (0.5.0.3....,0.1) \rangle$

$\tilde{n}_2 = \langle [1,3,5,6]; (0.7,0.2,...,0.9), (0.4,0.8,...,0.1), (0.3.0.5....,0.01) \rangle$,Then 1.

$\tilde{n}_1 \oplus \tilde{n}_2 = \langle [3,7,10,13]; (0.3,0.2,...,0.8), (0.7,0.4,...,0.2), (0.5,0.3,...,0.1) \rangle$

2. $\tilde{n}_1 \otimes \tilde{n}_2 = \langle [2,12,25,42]; (0.3,0.2,...,0.8), (0.7,0.4,...,0.2), (0.5,0.3,...,0.1) \rangle$

3. $4.\tilde{n}_1 = \langle [8,16,20,32]; (0.3,0.6,...,0.8), (0.7,0.4,...,0.2), (0.5.0.3....,0.1) \rangle$

Theorem 1 Let $\tilde{n}_1 = \left\langle [a_1, b_1, c_1, d_1]; \left(\alpha^1_{n_1}, \alpha^2_{n_1}, ..., \alpha^p_{n_1}\right), \left(\beta^1_{n_1}, \beta^2_{n_1}, ..., \beta^p_{n_1}\right), \left(\gamma^1_{n_1}, \gamma^2_{n_1}, ..., \gamma^p_{n_1}\right) \right\rangle$, $\tilde{n}_2 = \left\langle [a_2, b_2, c_2, d_2]; \left(\alpha^1_{n_2}, \alpha^2_{n_2}, ..., \alpha^p_{n_2}\right), \left(\beta^1_{n_2}, \beta^2_{n_2}, ..., \beta^p_{n_2}\right), \left(\gamma^1_{n_2}, \gamma^2_{n_2}, ..., \gamma^p_{n_2}\right) \right\rangle$ and $\tilde{n}_3 = \left\langle [a_3, b_3, c_3, d_3]; \left(\alpha^1_{n_3}, \alpha^2_{n_3}, ..., \alpha^p_{n_3}\right), \left(\beta^1_{n_3}, \beta^2_{n_3}, ..., \beta^p_{n_3}\right), \left(\gamma^1_{n_3}, \gamma^2_{n_3}, ..., \gamma^p_{n_3}\right) \right\rangle$. Then the following relations holds good.

1. $\hat{n}_1 \oplus \hat{n}_2 = \hat{n}_2 \oplus \hat{n}_1$
2. $(\hat{n}_1 \oplus \hat{n}_2) \oplus \hat{n}_3 = \hat{n}_1 \oplus (\hat{n}_2 \oplus \hat{n}_3)$
3. $\hat{n}_1 \otimes \hat{n}_2 = \hat{n}_2 \otimes \hat{n}_1$
4. $(\hat{n}_1 \otimes \hat{n}_2) \otimes \hat{n}_3 = \hat{n}_1 \otimes (\hat{n}_2 \otimes \hat{n}_3)$
5. $\lambda(\hat{n}_1 \oplus \hat{n}_2) = \lambda \hat{n}_2 \oplus \lambda \hat{n}_1; \lambda \geq 0$
6. $\lambda_1 \hat{n} \oplus \lambda_2 \hat{n} = (\lambda_1 + \lambda_2)\hat{n}; \lambda_1, \lambda_2 \geq 0.$

Proof

1.The following proof is based on the definition 3.2,

$$\tilde{n}_1 \oplus \tilde{n}_2 = \left\langle \begin{array}{l} [a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2]; \left(\alpha^1_{n_1} \wedge \alpha^1_{n_2}, \alpha^2_{n_1} \wedge \alpha^2_{n_2}, ..., \alpha^p_{n_1} \wedge \alpha^p_{n_2}\right), \\ \left(\beta^1_{n_1} \vee \beta^1_{n_2}, \beta^2_{n_1} \vee \beta^2_{n_2}, ..., \beta^p_{n_1} \vee \beta^p_{n_2}\right), \left(\gamma^1_{n_1} \vee \gamma^1_{n_2}, \gamma^2_{n_1} \vee \gamma^2_{n_2}, ..., \gamma^p_{n_1} \vee \gamma^p_{n_2}\right) \end{array} \right\rangle$$
$$= \tilde{n}_2 \oplus \tilde{n}_1$$

2.The following proof is based on the definition 3.2,

$$(\tilde{n}_1 \oplus \tilde{n}_2) \oplus \tilde{n}_3 = \left\langle \begin{array}{l} [(a_1 + a_2) + a_3, (b_1 + b_2) + b_3, (c_1 + c_2) + c_3, (d_1 + d_2) + d_3]; \\ \left(\left(\alpha^1_{n_1} \wedge \alpha^1_{n_2}\right) \wedge \alpha^1_{n_3}, \left(\alpha^2_{n_1} \wedge \alpha^2_{n_2}\right) \wedge \alpha^2_{n_3}, ..., \left(\alpha^p_{n_1} \wedge \alpha^p_{n_2}\right) \wedge \alpha^p_{n_3}\right), \\ \left(\left(\beta^1_{n_1} \vee \beta^1_{n_2}\right) \vee \beta^1_{n_3}, \left(\beta^2_{n_1} \vee \beta^2_{n_2}\right) \vee \beta^2_{n_3}, ..., \left(\beta^p_{n_1} \vee \beta^p_{n_2}\right) \vee \beta^p_{n_3}\right), \\ \left(\left(\gamma^1_{n_1} \vee \gamma^1_{n_2}\right) \vee \gamma^1_{n_3}, \left(\gamma^2_{n_1} \vee \gamma^2_{n_2}\right) \vee \gamma^2_{n_3}, ..., \left(\gamma^p_{n_1} \vee \gamma^p_{n_2}\right) \vee \gamma^p_{n_3}\right) \end{array} \right\rangle$$
$$= \tilde{n}_1 \oplus (\tilde{n}_2 \oplus \tilde{n}_3)$$

In this way the proofs of rest of the properties from 3 to 4 can be obtained.

The expected value of neutrosophic trapezoidal fuzzy multi number can be obtained as follows

Theorem 2

Let $\tilde{n} = \langle [a,b,c,d]; (\alpha^1,\alpha^2,...,\alpha^p), (\beta^1,\beta^2,...,\beta^p), (\gamma^1,\gamma^2,...,\gamma^p) \rangle$ be a NTFMN. Then

$$EX(\tilde{n}) = \frac{EX_T + EX_I + EX_F}{3} * \left(\frac{\min(\alpha^i) + \max(\beta^i, \gamma^i)}{\max(\alpha^i) + \min(\beta^i, \gamma^i)} \right)$$

$$= \frac{1}{3}\left(\frac{a+b+c+d}{4} \right) * \left(\frac{\min(\alpha^i) + \max(\beta^i, \gamma^i)}{\max(\alpha^i) + \min(\beta^i, \gamma^i)} \right) \tag{5}$$

If

$\tilde{n}_1 = \langle [a_1,b_1,c_1,d_1]; (\alpha^1_{n_1},\alpha^2_{n_1},...,\alpha^p_{n_1}), (\beta^1_{n_1},\beta^2_{n_1},...,\beta^p_{n_1}), (\gamma^1_{n_1},\gamma^2_{n_1},...,\gamma^p_{n_1}) \rangle$, then

$$EX(\hat{n}_1) = \frac{1}{12}(a_1+b_1+c_1+d_1) * \frac{\min(\alpha^1_{n_1},\alpha^2_{n_1},...,\alpha^p_{n_1}) + \max[(\beta^1_{n_1},\beta^2_{n_1},...,\beta^p_{n_1}), (\gamma^1_{n_1},\gamma^2_{n_1},...,\gamma^p_{n_1})]}{\max(\alpha^1_{n_1},\alpha^2_{n_1},...,\alpha^p_{n_1}) + \min[(\beta^1_{n_1},\beta^2_{n_1},...,\beta^p_{n_1}), (\gamma^1_{n_1},\gamma^2_{n_1},...,\gamma^p_{n_1})]} \tag{6}$$

Theorem 3(Rationality validation of proposed ranking)

The expected value defined in theorem 3.5 satisfies the following properties

Let S be the set of fuzzy quantities, and M be an ordering approach then,

A1 : For an arbitrary finite sub set of S, $\tilde{a} \in A, \tilde{a} \geq \tilde{a}$ by M on A

A2: For an arbitrary finite subset A of S, $(\tilde{a},\tilde{b}) \in A^2, \tilde{a} \geq \tilde{b}\ and \tilde{b} \geq \tilde{a}\ by$ M on A. We should have $\tilde{a} \approx \tilde{b}$ by M on A.

A3 : For an arbitrary finite subset A of S, $(\tilde{a},\tilde{b},\tilde{c}) \in A^3, \tilde{a} \geq \tilde{b}\ and \tilde{b} \geq \tilde{c}$ by M on A. We should have $\tilde{a} \approx \tilde{c}$ by M on A.

A4: For an arbitrary finite subset A of S, $(\tilde{a},\tilde{b}) \in A^2, \inf \text{supp}(\tilde{a}) \geq \sup \text{supp}(\tilde{b})$. We should have $\tilde{a} \geq \tilde{b}$ by M on A.

A5: Let S and S ' be two arbitrary finite sets of fuzzy quantities in which M can be applied and $\tilde{a}\ and \tilde{b}\ are in\ S \cap S'$, we obtain the ranking ordering $\tilde{a} > \tilde{b}\ by M\ on S'\ iff\ \tilde{a} > \tilde{b}$ by M on S.

A6 : Let $\tilde{a},\tilde{b},\tilde{a}+\tilde{c},\tilde{b}+\tilde{c}$ be the elements of S , if $\tilde{a} \geq \tilde{b}$ by M on $\{\tilde{a},\tilde{b}\}$ then $\tilde{a}+\tilde{c} \geq \tilde{b}+\tilde{c}$ by M on $(\tilde{a}+\tilde{c},\tilde{b}+\tilde{c})$.

A6' : Let $\tilde{a},\tilde{b},\tilde{a}+\tilde{c},\tilde{b}+\tilde{c}$ be the elements of S if $\tilde{a} > \tilde{b}$ by M on $\{\tilde{a},\tilde{b}\}$, then $\tilde{a}+\tilde{c} > \tilde{b}+\tilde{c}$ by M on $(\tilde{a}+\tilde{c},\tilde{b}+\tilde{c})$.

$\tilde{a}, \tilde{b}, \tilde{a}\tilde{c}, \tilde{b}\tilde{c}$ be the elements of S if $\tilde{a} \geq \tilde{b}$ by M on $\{\tilde{a}, \tilde{b}\}$, then $\tilde{a}\tilde{c} \geq \tilde{b}\tilde{c}$ by M on

A7: Let

$\left(\tilde{a}\tilde{c}, \tilde{b}\tilde{c}\right)$

For the sake of completeness, the proof of properties A4 and A6

Theorem 4

Let $\tilde{n}_1$ and $\tilde{n}_2$ be two NTFMN, if $a_{\tilde{n}_1 1} \succ a_{\tilde{n}_2 4}$ and $b_{\tilde{n}_1 1} \succ b_{\tilde{n}_2 4}$ then $\tilde{n}_1 \succ \tilde{n}_2$

Proof

It is known that

$T_{\tilde{n}_1} \succ a_{\tilde{n}_1 1}$ and $T_{\tilde{n}_1} \succ a_{\tilde{n}_1 4}; T_{\tilde{n}_2} \succ b_{\tilde{n}_2 1}$ and $T_{\tilde{n}_1} \succ b_{\tilde{n}_1 4}$

Also

$I_{\tilde{n}_1} \succ a_{\tilde{n}_1 1}$ and $\mathrm{I}_{\tilde{n}_1} \succ a_{\tilde{n}_1 4}; I \succ b_{\tilde{n}_2 1}$ and $I_{\tilde{n}_1} \succ b_{\tilde{n}_1 4}$

$F_{\tilde{n}_1} \succ a_{\tilde{n}_1 1}$ and $\mathrm{F}_{\tilde{n}_1} \succ a_{\tilde{n}_1 4}; F \succ b_{\tilde{n}_2 1}$ and $\mathrm{F}_{\tilde{n}_1} \succ b_{\tilde{n}_1 4}$

Therefore $EX(\tilde{n}_1) > EX(\tilde{n}_2) \Rightarrow \tilde{n}_1 \succ \tilde{n}_2$

Theorem 5

Let $\tilde{n}_1$ and $\tilde{n}_2$ be two NTFMN, then $EX(\tilde{n}_1 + \tilde{n}_3) > EX(\tilde{n}_2 + \tilde{n}_3) \Rightarrow \tilde{n}_1 + \tilde{n}_3 \succ \tilde{n}_2 + \tilde{n}_3$

Proof

$EX(\tilde{n}_1 + \tilde{n}_2) = EX(\tilde{n}_1) + EX(\tilde{n}_2)$

similarly

$EX(\tilde{n}_2 + \tilde{n}_3) = EX(\tilde{n}_2) + EX(\tilde{n}_3)$

$i\mathrm{f}, \tilde{n}_1 \succ \tilde{n}_2$

$EX(\tilde{n}_1 + \tilde{n}_3) > EX(\tilde{n}_2 + \tilde{n}_3) \Rightarrow \tilde{n}_1 + \tilde{n}_3 \succ \tilde{n}_2 + \tilde{n}_3$

Proposition 1 Let $\tilde{n}_1 = \left\langle [a_1, b_1, c_1, d_1]; \left(\alpha^1_{n_1}, \alpha^2_{n_1}, ..., \alpha^p_{n_1}\right), \left(\beta^1_{n_1}, \beta^2_{n_1}, ..., \beta^p_{n_1}\right), \left(\gamma^1_{n_1}, \gamma^2_{n_1}, ..., \gamma^p_{n_1}\right) \right\rangle$ and

$\tilde{n}_2 = \left\langle [a_2, b_2, c_2, d_2]; \left(\alpha^1_{n_2}, \alpha^2_{n_2}, ..., \alpha^p_{n_2}\right), \left(\beta^1_{n_2}, \beta^2_{n_2}, ..., \beta^p_{n_2}\right), \left(\gamma^1_{n_2}, \gamma^2_{n_2}, ..., \gamma^p_{n_2}\right) \right\rangle$ be any two NTFMNs.

Then**t**he Expected value based ranking satisfies the following properties

1. $0 \leq EX(\tilde{n}) \leq 1$.
2. $EX(\tilde{n}_1) = EX(\tilde{n}_2)$ *iff* $\tilde{n}_1 \approx \tilde{n}_2$.
3. $EX(\tilde{n}_1) > EX(\tilde{n}_2)$ *then* $\tilde{n}_1 \succ \tilde{n}_2$
4. $EX(\tilde{n}_1) < EX(\tilde{n}_2)$ *then* $\tilde{n}_1 \prec \tilde{n}_2$

3.2 Steps of the proposed methodology to solve multicriteria decision making problem

In this section, we apply the proposed ranking of NTFMNs to solve a multi criteria decision making problem involving neutrosophic trapezoidal fuzzy multi numbers.

Let $A = (a_1, a_2, ..., a_m)$ be a set of alternatives with the criteria $C = (c_1, c_2, ..., c_n)$. The value of an alternative on a criterion $c_j, j = 1,2,...,n$ is a neutrosophic trapezoidal fuzzy multi number

$$\left[\tilde{n}_{ij}\right]=\left\langle\left[a_{ij},b_{ij},c_{ij},d_{ij}\right];\left(\alpha_{ij}^{1},\alpha_{ij}^{2},\ldots,\alpha_{ij}^{p}\right),\left(\beta_{ij}^{1},\beta_{ij}^{2},\ldots,\beta_{ij}^{p}\right),\left(\gamma_{ij}^{1},\gamma_{ij}^{2},\ldots,\gamma_{ij}^{p}\right)\right\rangle, i=1,2,\ldots,m;\ j=1,2,\ldots,n. \qquad (7)$$

Therefore we can form a decision matrix $D=\left[\tilde{n}_{ij}\right]$ in which the terms are expressed as NTFMN. The weight criterion w_i is also a NTFMN. The expected weight value is computed by the equation (5). Then the normalized expected weight value is computed by the following relation

$$N_j=\frac{EX\left(w_j\right)}{\sum_{j=1}^{n}EX\left(w_j\right)} \qquad (8)$$

Therefore, the weighted expected value for an alternative $A_i, i=1,2,\ldots,m$ is given by

$$WEX\left(A_j\right)=\sum_{j=1}^{n}N_j EX\left(\tilde{n}_{ij}\right) \qquad (9)$$

Using the above expression we can compute the weighted expected value for an alternative to rank alternatives and then to select the best in all the alternatives.

Now, we summarize the procedure as follows :

*Step 1*Construct the NTFMN multi attribute decision matrix $\left[\tilde{n}_{ij}\right]$

Step 2 Calculate the expected weight value for a criterion $c_j, j=1,2,\ldots,n$

*Step 3*Calculate the weighted expected value for an alternative $A_i, i=1,2,\ldots,m$

*Step 4*Rank the alternatives and select the best with respect to the weighted expected.

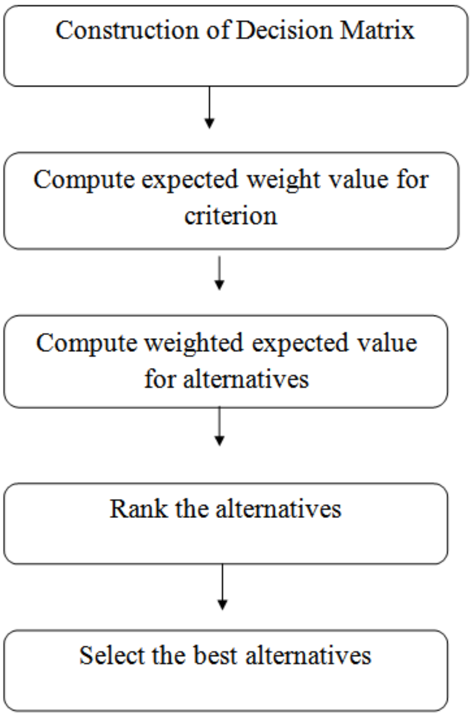

Fig 1: Flow Chart of the Proposed Procedure

4. A real case application

4.1 Problem description

In this section, we illustrate a multicriteria decision making problem under neutrosophic fuzzy multi number environment in a realistic scenario. An investor wants to select the most appropriate company to enhance profit from the money invested. Consider there are four possible alternatives to invest the money namely, A1 is a car company, A2 is a food company, A3 is a computer company,and A4 is a television company. The investment company take a decision according to the following three criteria C1 is the risk analysis, C2 is the growth analysis and C3 is the environmental impact analysis. The four possible alternatives are to be evaluated under the above three criteria by corresponding to linguistic values of neutrosophic trapezoidal fuzzy multi numbers for linguistic terms as shown in Table 2.

Table 2 Linguistic scale of neutrosophic trapezoidal fuzzy multi numbers for linguistic terms

Linguistic terms	Linguistic Values
Absolutely low	<[0.0,0.0,0.0,0.0],(0.0,0.0,0.0,0.0), (0.0,0.0,0.0,0.0), (0.0,0.0,0.0,0.0)>
Low	<[0.0,0.1,0.2,0.3],(0.6,0.3,0.5,0.7), (0.1,0.5,0.4,0.1), (0.2,0.3,0.4,0.1)>
Fairly low	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
Medium	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.8,0.1), (0.5,0.6,0.7,0.3)>
Fairly high	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>
High	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>
Absolutely high	<[1.0,1.0,1.0,1.0],(1.0,1.0,1.0,1.0), (1.0,1.0,1.0,1.0), (1.0,1.0,1.0,1.0)>

Table 3 Preference scale of alternatives and criteria weights given by five experts by linguistic scale

	k	C1	C2	C3
A1	1	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
	2	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.0,0.1,0.2,0.3],(0.6,0.3,0.5,0.7), (0.1,0.5,0.4,0.1), (0.2,0.3,0.4,0.1)>
	3	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
	4	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	5	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
A2	1	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	2	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	3	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	4	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>

	5	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
A3	1	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
	2	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
	3	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	4	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	5	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
A4	1	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
	2	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
	3	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	4	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	5	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.7,0.8,0.9,1.0],(0.6,0.8,0.4,0.5), (0.1,0.3,0.2,0.4), (0.2,0.5,0.3,0.4)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>
Weights	1	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>
	2	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6) (0.6,0.3,0.5,0.2),(0.5,0.6,0.7,0.3)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>
	3	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.1,0.2,0.3,0.4],(0.2,0.5,0.1,0.8), (0.7,0.3,0.8,0.1), (0.6,0.5,0.4,0.1)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>
	4	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>
	5	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.3,0.4,0.5,0.6],(0.3,0.2,0.4,0.6), (0.6,0.3,0.5,0.2), (0.5,0.6,0.7,0.3)>	<[0.5,0.6,0.7,0.8],(0.4,0.3,0.6,0.2), (0.1,0.6,0.3,0.7), (0.1,0.3,0.5,0.2)>

4.2 Problem solution

Step 1. The neutrosophic trapezoidal fuzzy multi number $\tilde{n}$ in the decision matrix and weights can be calculated by

$$\tilde{n} = \left\langle \begin{array}{l} \left[\frac{\sum_{k=1}^{5} a_{ij1}(k)}{5}, \frac{\sum_{k=1}^{5} a_{ij2}(k)}{5}, \frac{\sum_{k=1}^{5} a_{ij3}(k)}{5}, \frac{\sum_{k=1}^{5} a_{ij4}(k)}{5}, \frac{\sum_{k=1}^{5} a_{ij5}(k)}{5} \right], \left(\frac{\sum_{k=1}^{5} \alpha_{ij1}(k)}{5}, \frac{\sum_{k=1}^{5} \alpha_{ij2}(k)}{5}, \frac{\sum_{k=1}^{5} \alpha_{ij3}(k)}{5}, \frac{\sum_{k=1}^{5} \alpha_{ij4}(k)}{5}, \frac{\sum_{k=1}^{5} \alpha_{ij5}(k)}{5} \right), \\ \left(\frac{\sum_{k=1}^{5} \beta_{ij1}(k)}{5}, \frac{\sum_{k=1}^{5} \beta_{ij2}(k)}{5}, \frac{\sum_{k=1}^{5} \beta_{ij3}(k)}{5}, \frac{\sum_{k=1}^{5} \beta_{ij4}(k)}{5}, \frac{\sum_{k=1}^{5} \beta_{ij5}(k)}{5} \right), \left(\frac{\sum_{k=1}^{5} \gamma_{ij1}(k)}{5}, \frac{\sum_{k=1}^{5} \gamma_{ij2}(k)}{5}, \frac{\sum_{k=1}^{5} \gamma_{ij3}(k)}{5}, \frac{\sum_{k=1}^{5} \gamma_{ij4}(k)}{5}, \frac{\sum_{k=1}^{5} \gamma_{ij5}(k)}{5} \right) \end{array} \right\rangle$$

Therefore the decision matrix is given by

$$D = \begin{bmatrix} \left\langle \begin{matrix} [0.26,0.36,0.46,0.56],(0.28,0.34,0.32,0.6), \\ (0.54,0.36,0.58,0.26),(0.46,0.5,0.54,0.20) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.34,0.44,0.54,0.64],(0.32,0.3,0.42,0.48), \\ (0.42,0.42,0.48,0.38),(0.36,0.46,0.56,0.22) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.12,0.22,0.32,0.42],(0.3,0.4,0.24,0.74), \\ (0.56,0.34,0.66,0.12),(0.5,0.48,0.46,0.14) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} [0.5,0.6,0.7,0.8],(0.42,0.38,0.52,0.34), \\ (0.2,0.48,0.32,0.54),(0.2,0.4,0.5,0.26) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.5,0.6,0.7,0.8],(0.42,0.38,0.52,0.34), \\ (0.2,0.48,0.32,0.54),(0.2,0.4,0.5,0.26) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.34,0.44,0.54,0.64],(0.32,0.22,0.44,0.52), \\ (0.5,0.36,0.46,0.3),(0.42,0.54,0.66,0.28) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} [0.38,0.48,0.58],(0.34,0.24,0.48,0.44), \\ (0.4,0.42,0.42,0.4),(0.34,0.48,0.62,0.26) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.54,0.64,0.74,0.84],(0.44,0.4,0.56,0.26), \\ (0.1,0.54,0.28,0.64),(0.12,0.34,0.46,0.24) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.26,0.36,0.46,0.56],(0.28,0.26,0.34,0.64), \\ (0.62,0.3,0.56,0.18),(0.52,0.58,0.64,0.26) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} [0.66,0.76,0.86,0.96],(0.56,0.7,0.44,0.44), \\ (0.1,0.36,0.22,0.46),(0.18,0.46,0.34,0.36) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.62,0.72,0.82,0.92],(0.52,0.6,0.48,0.38), \\ (0.1,0.42,0.24,0.52),(0.16,0.42,0.38,0.32) \end{matrix} \right\rangle & \left\langle \begin{matrix} [0.18,0.28,0.38,0.48],(0.24,0.38,0.22,0.72), \\ (0.66,0.3,0.68,0.14),(0.56,0.54,0.52,0.18) \end{matrix} \right\rangle \end{bmatrix}$$

The weight value is given by

$$w = \begin{Bmatrix} \langle [0.38,0.48,0.58,0.68],(0.34,0.24,0.48,0.44),(0.4,0.42,0.42,0.4),(0.34,0.48,0.62,0.26) \rangle, \\ \langle [0.22,0.32,0.42,0.52],(0.26,0.32,0.28,0.68),(0.64,0.3,0.62,0.16),(0.54,0.56,0.58,0.22) \rangle \\ \langle [0.46,0.56,0.66,0.76],(0.38,0.28,0.56,0.28),(0.2,0.54,0.34,0.6),(0.18,0.36,0.54,0.22) \rangle \end{Bmatrix}$$

Step 2. Compute the expected weight value for the criterions by using the equations (5) , we get $EX(w_1) = 0.20532$, $EX(w_2) = = 0.1321$, $EX(w_3) = 0.2418$ and the normalized weight values are $N_1=0.3544$, $N_2 = 0.2281$, $N_3 = 0.4174$.

Step 3. Compute the weighted expected values of the alternatives by using the equations (9) , we have $WEX(A_1) = 0.1372$, $WEX(A_2) =0.2293$, $WEX(A_3) = 0.2069$, $WEX(A_4) = 0.2309$.

Step 4. The alternatives are ranked as $A_2 \ \phi \ A_4 \ \phi \ A_3 \ \phi \ A_1$

Thus the optimal alternative is A_2.

5 Conclusion

During the literature analysis, the ranking of IFNs was highlighted as a crucial job for solving optimization problems with uncertainty. As a result, a number of references were thoroughly discussed. Also discussed were the need for neutrosophic fuzzy sets and the extension of neutrosophic fuzzy numbers. There was a new type of neutrosophic fuzzy multiset and numbers introduced. The expected-based ranking method was used to solve the neutrosophic fuzzy multi-number problem, which is incentive-based, fast-responsive, and light-weighted, and may be used to solve any optimization problem. An strategy to solve multi-attribute decision making problems is utilised to validate the proposed method ranking with appropriate illustrations. In this framework, an investor attempts to identify the most suitable company to invest the money in from a panel of four possible alternatives with three criteria.

We defined NTFMN as a generalisation of neutrosophic fuzzy numbers in this study. Because of the scarcity of information in real-life circumstances, it is preferable to use NTFMN rather than NTFN. As a result, it is critical to take the first step toward multiplication in a neutrosophic environment. It is more difficult to rank several numbers. In this paper, we constructed the ranking using expected values and created an algorithmic strategy to dealing with multi-criteria decision-making situations. In the future, we will propose new ranking approaches to arrange neutrosophic multi-numbers and broaden our work to answer existing real-world problems in medicine, defense, investing, and other disciplines where necessity arises. In the complexity of socio-economic settings, multi-criteria decision making difficulties are critical. Because the data available in real-world problems is always ambiguous and imprecise, old solutions do not appear to be useful for dealing with these concerns. As a result, we employ neutrosophic trapezoidal fuzzy multi numbers to address such critical issues. Until recently, there haven't been many research efforts committed to ranking neutrosophic fuzzy multi numbers using a single formula. The current study is the first step in this direction. This topic has various intriguing and crucial future directions as a relatively new and exciting research area. It is possible In this paper, we offer an anticipated ranking approach for ranking neutrosophic fuzzy multi-numbers. Based on the proposed strategy, the study can be extended to evaluate novel ranking techniques. The proposed work for solving Neutrosophic fuzzy multi-number multi-criteria issues is limited to optimization problems. This method can be used to answer any mathematical issue that involves a neutrosophic fuzzy environment. To give superior ranking performance, it is required to analyse and categorise the ranking algorithms proposed thus far in a way that is appropriate for the current circumstance. In an uncertain natural world, software can be created to tackle optimization difficulties.

Declaration of Competing Interest

The authors declared that they have no conflicts of interest to this work.

We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted. Compliance with ethical standards

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