Some Bounds on Binary LCD Codes

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Abstract

A linear code with a complementary dual (or LCD code) is defined to be a linear code C whose dual code C^{\perp} satisfies $C \cap C^{\perp} = \{\mathbf{0}\}$. Let LCD[n,k] denote the maximum of possible values of d among [n,k,d] binary LCD codes. We give exact values of LCD[n,k] for $1 \leq k \leq n \leq 12$. We also show that LCD[n,n-i]=2 for any $i \geq 2$ and $n \geq 2^i$. Furthermore, we show that $LCD[n,k] \leq LCD[n,k-1]$ for k odd and $LCD[n,k] \leq LCD[n,k-2]$ for k even.

Keywords: binary LCD codes boundslinear code

1 Introduction

A linear code with complementary dual (or LCD code) was first introduced by Massey [7] as a reversible code in 1964. Afterwards, LCD codes were extensively studied in literature and widely applied in data storage, communications systems, consumer electronics, and cryptography.

In [8] Massey showed that there exist asymptotically good LCD codes. Esmaeili and Yari [6] identified a few classes of LCD quasi-cyclic codes. For bounds of LCD codes, Tzeng and Hartmann [12] proved that the minimum distance of a class of reversible codes is greater than that given by the BCH bound. Sendrier [11] showed that LCD codes meet the asymptotic Gilbert-Varshamov bound using the hull dimension spectra of linear codes. Recently, Dougherty et al. [5] gave a linear programming bound on the largest size of an LCD[n,d]. Constructions of LCD codes were studied by Mutto and Lal [10]. Yang and Massey [15] gave a necessary and sufficient condition for a cyclic code to have a complementary dual. It is also shown by Kandasamy et al. [13] that maximum rank distance codes generated by the trace-orthogonal-generator matrices are LCD codes. In 2014, Calet and Guilley [3] introduced several constructions of LCD codes and investigated an application of LCD codes against side-channel attacks(SCA). Shortly after, Mesnager et al. [9] provided a construction of algebraic geometry LCD codes which could be good candidates to be resistant against SCA. Recently Ding et al. [4] constructed several families of reversible cyclic codes over finite fields.

The purpose of this paper is to study exact values of LCD[n, k] (see [5]) which is the maximum of possible values of d among [n, k, d] binary LCD codes. We give exact values of LCD[n, 2] in Section

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2. In Section 3, we investigate LCD[n,k] and show that LCD[n,n-i]=2 for any $i\geq 2$ and $n\geq 2^i$. We prove that $LCD[n,k]\leq LCD[n,k-1]$ for k odd and that $LCD[n,k]\leq LCD[n,k-2]$ for k even using the notion of principal submatrices. In Section 4, we give exact values for LCK[n,d]. We have included tables for LCD[n,k] for $1\leq k\leq n\leq 12$ and LCK[n,d] for $1\leq d\leq n\leq 12$.

$2 \quad LCD[n, 2]$

Let GF(q) be the finite field with q elements. An [n,k] code C over GF(q) is a k-dimensional subspace of $GF(q)^n$. If C is a linear code, we let

$$C^{\perp} = \{ \mathbf{u} \in V \mid \mathbf{u} \cdot \mathbf{w} = 0 \text{ for all } \mathbf{w} \in C \}.$$

We call C^{\perp} the dual or orthogonal code of C.

Definition 2.1. A linear code with complementary dual (LCD code) is a linear code C satisfying $C \cap C^{\perp} = \{0\}$.

We note that if C is an LCD code, then so is C^{\perp} because $(C^{\perp})^{\perp} = C$. The following proposition will be frequently used in the later sections.

Proposition 2.2. ([8])

Let G be a generator matrix for a code over GF(q). Then $det(GG^T) \neq 0$ if and only if G generates an LCD code.

Throughout the rest of the paper, we consider only binary codes. Dougherty et al. [5] introduced LCD[n, k] which denotes the maximum of possible values of d among [n, k, d] binary LCD codes. Formally we can define it as follows.

Definition 2.3. $LCD[n, k] := \max\{d \mid there \ exists \ a \ binary[n, k, d] \ LCD \ code\}.$

Dougherty et al. [5] gave a few bounds on LCD[n, k] and exact values of LCD[n, k] for k = 1 only.

Now we obtain exact values of LCD[n, k] for k = 2 for any n.

Lemma 2.4. $LCD[n,2] \leq \lfloor \frac{2n}{3} \rfloor$ for $n \geq 2$.

Proof. By the Griesmer Bound [14], any binary linear [n, k, d] code satisfies

$$n \ge \sum_{i=0}^{k-1} \left\lceil \frac{d}{2^i} \right\rceil.$$

Letting k=2, we have $n \geq d + \frac{d}{2}$. Hence

$$d \le \left\lfloor \frac{2n}{3} \right\rfloor.$$

Therefore any LCD [n, k, d] code must satisfy this inequality.

Proposition 2.5. Let $n \ge 2$. Then $LCD[n,2] = \lfloor \frac{2n}{3} \rfloor$ for $n \equiv 1, \pm 2, \text{ or } 3 \pmod{6}$.

Proof. We only need to show the existence of LCD codes with minimum distance achieving the bound $d = \left\lfloor \frac{2n}{3} \right\rfloor$.

(i) Let $n \equiv 1 \pmod{6}$, i.e. n = 6m + 1 for some positive integer m. Consider the code with generator matrix

$$G = \left[\begin{array}{c|c} 1 \dots 1 \\ 0 \dots 0 \\ 2m+1 \end{array} \middle| \begin{array}{c} 1 \dots 1 \\ 1 \dots 1 \\ 2m-1 \end{array} \middle| \begin{array}{c} 0 \dots 0 \\ 1 \dots 1 \\ 2m+1 \end{array} \right].$$

This code has minimum weight $4m = \left\lfloor \frac{2(6m+1)}{3} \right\rfloor$ and $GG^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, i.e., $\det(GG^T) = 1 \neq 0$. Therefore this code is an LCD code.

(ii) Let $n \equiv \pm 2 \pmod{6}$, i.e., n = 6m + 2 for some non negative integer m, or n = 6m - 2 for some positive integer m. Consider the code with generator matrix

$$G = \left[\begin{array}{c|c} 1 \dots 1 \\ 0 \dots 0 \\ 2m+k \end{array} \middle| \begin{array}{c} 1 \dots 1 \\ 1 \dots 1 \\ 2m \end{array} \middle| \begin{array}{c} 0 \dots 0 \\ 1 \dots 1 \\ 2m+k \end{array} \right]$$

for k = 1, -1.

If k = 1, this code has minimum weight $4m + 1 = \left\lfloor \frac{2(6m+2)}{3} \right\rfloor$ and $GG^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, i.e., $det(GG^T) = 1 \neq 0$. Therefore this code is an LCD code.

If k=-1, this code has minimum weight $4m-2=\left|\frac{2(6m-2)}{3}\right|$ and $GG^T=\left|\begin{array}{cc}1&0\\0&1\end{array}\right|$, i.e., $det(GG^T) = 1 \neq 0$. Therefore this code is an LCD code.

(iii) Let $n \equiv 3 \pmod{6}$, i.e., n = 3i for some positive odd integer i. Consider the code with generator matrix

$$G = \left[\begin{array}{c|c} 1 \dots 1 \\ 0 \dots 0 \\ \end{array} \middle| \begin{array}{c} 1 \dots 1 \\ 1 \dots 1 \\ \end{array} \middle| \begin{array}{c} 0 \dots 0 \\ 1 \dots 1 \\ \end{array} \right].$$

 $G = \left[\begin{array}{c|c} 1 \dots 1 & 0 \dots 0 \\ 0 \dots 0 & 1 \dots 1 \\ 1 \dots 1 & 1 \dots 1 \\ 1 \dots 1 & 1 \dots 1 \end{array}\right].$ This code has minimum weight $2i = \left[\begin{array}{c} 2(3i) \\ 3 \end{array}\right]$ and $GG^T = \left[\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right]$, i.e., $\det(GG^T) = 1 \neq 0$. Therefore this code is an LCD code.

Proposition 2.6. Let $n \ge 2$. Then $LCD[n, 2] = \left| \frac{2n}{3} \right| - 1$ for $n \equiv 0, -1 \pmod{6}$.

Proof. (i) Let $n \equiv 0 \pmod{6}$. Consider the generator matrix G in (iii) of the proof of Proposition 2.5, this time taking i to be an even integer. If the weight of any row of G is increased by one, the weight of the sum of the two rows is decreased by one. Hence, G is the only generator matrix for a binary code that achieves the upper bound, up to equivalence. Clearly, $det(GG^T) = 0$ and so the code is not LCD. It follows that there are no LCD code with minimum distance $\left|\frac{2n}{3}\right|$ for $n \equiv 0 \pmod{6}$.

Next, consider the code with generator matrix

$$G = \left[\begin{array}{c|c} 1 \dots 1 \\ 0 \dots 0 \\ \vdots + 1 \end{array} \middle| \begin{array}{c} 1 \dots 1 \\ 1 \dots 1 \\ \vdots - 1 \end{array} \middle| \begin{array}{c} 0 \dots 0 \\ 1 \dots 1 \\ \vdots \\ i \end{array} \right]$$

This code has minimum weight $2i-1 = \left\lfloor \frac{2(3i)}{3} \right\rfloor - 1$. We note that $GG^T = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, i.e., $\det(GG^T) = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ $1 \neq 0$. Therefore this code is an LCD code.

(ii) Let C be a binary code of length $n \equiv -1 \pmod{6}$, i.e., n = 3i - 1 for some positive even i. Without loss of generality, the generator matrix for C can be expressed in the following form such that the first row is the codeword whose weight is the minimum weight d.

$$G = \left[\begin{array}{c|c} 1 \dots 1 \\ 0 \dots 0 \\ \end{array} \middle| \begin{array}{c} 1 \dots 1 \\ 1 \dots 1 \\ \end{array} \middle| \begin{array}{c} 0 \dots 0 \\ 1 \dots 1 \\ \end{array} \right]$$

 $G = \left[\begin{array}{c|c} 1 \dots 1 & 0 \dots 0 \\ 0 \dots 0 & 1 \dots 1 \end{array}\right]$ Suppose $d = \left[\begin{array}{c|c} \frac{2(3i-1)}{3} & 2i-1 \text{ i.e., } i_1+i_2=2i-1 \text{ This implies that } i_3=i. \text{ Note that } i_2+i_3 \geq 2i-1 \text{ which implies } i_2 \geq i-1 \text{ and } i_1+i_3 \geq 2i-1 \text{ which implies } i_2 \geq i-1. \end{array}$ This leaves only two possible cases: $(i_1, i_2, i_3) = (i - 1, i, i), (i, i - 1, i)$, each of which gives $GG^T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, respectively. In both cases, $\det(GG^T) = 0$ and therefore they are not LCD. So there is no LCD code with minimum distance $\frac{|2n|}{|2n|}$ for n = -1 (mod 6) code with minimum distance $\lfloor \frac{2n}{3} \rfloor$ for $n \equiv -1 \pmod{6}$.

Consider the case where $(i_1, i_2, i_3) = (i - 1, i - 1, i + 1)$. Then G generates a code of minimum distance $2i - 2 = \left| \frac{2(3i-1)}{3} \right| - 1$. For this case, $\det(GG^T) = 1$ and hence the code is LCD.

3 LCD[n,k]

We begin with a rather straightforward lemma in order to prove Proposition 4.

Lemma 3.1. If $n \ge 8$, any [n, n-3, d] binary code C satisfies $d \le 2$.

Proof. Let G be a standard generator matrix of C, i.e., $G = [I_{n-3} \mid A]$ where A is an $(n-3) \times 3$ matrix. By the Singleton bound, $d \le 4$. But it is well known there is no non-trivial binary code achieving the bound. Thus, we may say that $d \leq 3$. Suppose d = 3. Then each row of A must have weight at least 2. However there are only $\binom{3}{2} + \binom{3}{3} = 4$ possible choices for the rows of A. Note that A has at least 5 rows and this contradict our assumption that d = 3. Hence $d \le 2$.

Motivated by the above lemma, we can consider more general dimension n-i rather than n-3as follows.

Proposition 3.2. Given $i \geq 2$, LCD[n, n-i] = 2 for all $n \geq 2^i$.

Proof. Let G be a standard generator matrix of an [n, n-i, d] binary code C, i.e., $G = [I_{n-i} \mid A]$ where A is an $(n-i)\times i$ matrix. By the Singleton bound, $d\leq i+1$. But there is no non-trivial binary code achieving the bound. Thus, we may say that $d \leq i$. If the minimum distance of C is at least 3, each row of A must have weight at least 2. And there are $\binom{i}{2} + \binom{i}{3} + \cdots + \binom{i}{i}$ possible choices for the rows of A. Thus, if n-i (number of rows in A) $> 2^i - i - 1$, then there exists a row of weight 1 in A. This forces the minimum distance of C to be 2, which is a contradiction. Therefore, $d \leq 2$ for all $n \geq 2^i$. Since this statement holds true for any linear code, it holds true for LCD codes as well, i.e., $LCD[n, n-i] \leq 2$ for all $n \geq 2^i$.

Next, we show that there exists an [n, n-i, 2] LCD code for $n \geq 2^i$. For i even, let $G = [I_{n-i} \mid \underbrace{\mathbf{11} \cdots \mathbf{1}}_{i}]$, and for i odd, let $G = [I_{n-i} \mid \underbrace{\mathbf{11} \cdots \mathbf{10}}_{i}]$ where $\mathbf{1}$ denotes the all one vector and $\mathbf{0}$ the all zero vector, both of which are of size $(n-i) \times 1$. In both

denotes the all one vector and $\mathbf{0}$ the all zero vector, both of which are of size $(n-i) \times 1$. In both cases, $GG^T = I_{n-i}$. Thus, G is a generator matrix for the [n, n-i] LCD code with minimum distance 2.

Hence LCD[n, n-i] = 2 for all $n \ge 2^i$.

So far we have shown the exact value of LCD[n, 2]. The relation between LCD[n, k] and LCD[n, k-1] (or LCD[n, k-2]) was unknown before. Using the idea of principal submatrices, we have Proposition 3.5 below.

Definition 3.3. Let A be a $k \times k$ matrix over a field. An $m \times m$ submatrix P of A is called a principal submatrix of A if P is obtained from A by removing all rows and columns of A indexed by the same set $\{i_1, i_2, \ldots, i_{n-m}\} \subset \{1, 2, \ldots, k\}$.

Definition 3.4. ([1]) Let A be a $k \times k$ symmetric matrix over a field. The principal rank characteristic sequence of A (simply, pr-sequence of A or pr(A)) is defined as $pr(A) = r_0]r_1r_2 \dots r_k$ where for $1 \le m \le k$

$$r_m = \begin{cases} 1 & \text{if } A \text{ has an } m \times m \text{ principal submatrix of rank } m \\ 0 & \text{otherwise} \end{cases}$$

For convenience, define $r_0 = 1$ if and only if A has a 0 in the diagonal.

Proposition 3.5. ([1]) For $k \geq 2$ over a field with characteristic 2, a principal rank characteristic sequence of a $k \times k$ symmetric matrix A is attainable if and only if it has one of the following forms:

$$(i) 0]1\overline{10} \qquad \qquad (ii) 1]\overline{010} \qquad \qquad (iii) 1]1\overline{10}$$

where $\overline{1} = 11...1$ (or empty), $\overline{0} = 00...0$ (or empty), $\overline{01} = 0101...01$ (or empty).

Proposition 3.6. We have the following:

(i) If $k \geq 3$ and k is odd,

$$LCD[n, k] < LCD[n, k - 1]$$

for any n > k.

(ii) If $k \geq 4$ and k is even,

$$LCD[n, k] \le LCD[n, k-2]$$

for any $n \geq k$.

Proof. (i) It suffices to show that any binary [n, k] LCD code C has an [n, k-1] LCD subcode for any odd $k \geq 3$. Let G be a $k \times n$ generator matrix of C. Let $A = GG^T$ which is symmetric. Then rank(A) = k. Since k is odd, case (ii) of Proposition 3.5 is not possible. Thus the only possible cases are (i) and (iii) of Proposition 3.5, which are $0 \mid 11 \dots 1$ and $1 \mid 11 \dots 1$, respectively. Hence,

n/k	1	2	3	4	5	6	7	8	9	10	11	12
1	1											
2	1	1										
3	3	2	1									
4	3	2	1	1								
5	5	2	2	2	1							
6	5	3	2	2	1	1						
7	7	4	3	2	2	2	1					
8	7	5	3	3	2	2	1	1				
9	9	6	4	4	3	2	2	2	1			
10	9	6	5	4	3	3	2	2	1	1		
11	11	6	5	4	4	4	3	2	2	2	1	
12	11	7	6	5	4	4	3	2	2	2	1	1

Table 1: LCD[n, k] for $1 \le k \le n \le 12$

there exists a principal submatrix P_1 of rank k-1 which is obtained from A by deleting some i^{th} row and column of A ($1 \le i \le k$).

Define G_1 to be a $(k-1) \times n$ matrix obtained from G by deleting the i^{th} row of G. Since $G_1G_1^T = P_1$ and $rank(P_1) = k - 1 \neq 0$, P_1 is invertible. Then the linear code C_1 with generator matrix G_1 is LCD as well.

(ii) It suffices to show that any binary [n, k] LCD code C has an [n, k-2] LCD subcode for any even $k \ge 4$. Let G be a $k \times n$ generator matrix of C and $A = GG^T$. Then rank(A) = k since C is LCD. By Proposition 3.5, we have the following three cases.

$$(i) \ 0]11...1$$
 $(ii) \ 1]0101...01$ $(iii) \ 1]11...1$

So there exists a principal submatrix P_2 of rank k-2 which is obtained from A by deleting some i^{th} , j^{th} rows and columns of A $(1 \le i \ne j \le k)$.

Define G_2 to be a $(k-2) \times n$ matrix obtained from G by deleting the i^{th} and j^{th} rows of G. Since $G_2G_2^T = P_2$ and $rank(P_2) = k-2 \neq 0$, P_2 is invertible. Then the linear [n, k-2] code C_2 with generator matrix G_2 is LCD as well.

Since the minimum distance of a code is always less than or equal to the minimum distance of a subcode, this completes the proof of (a) and (b).

In Table 1 we give exact values of LCD[n, k] for $1 \le k \le n \le 12$. Based on Proposition 3.6 and Table 1, we conjecture the following.

Conjecture If $2 \le k \le n$, then $LCD[n, k] \le LCD[n, k-1]$. (Note: It suffices to show that this is true when k is even.)

$\mathbf{4} \quad LCK\left[n,d\right]$

We define another combinatorial function LCK[n, d].

Definition 4.1. $LCK[n,d] := \max\{k \mid there \ exists \ a \ binary \ [n,k,d] \ LCD \ code\}$

For convenience, define LCK[n, d] = 0 if and only if there is no LCD code with the given n and d.

n/d	1	2	3	4	5	6	7	8	9	10	11	12
1	1											
2	2	0										
3	3	2	1									
4	4	2	1*	0								
5	5	4	1	0	1							
6	6	4	2	2	1*	0						
7	7	6	3	2	1*	0	1					
8	8	6	4*	2	2	0	1*	0				
9	9	8	5	4	2*	2	1	0	1			
10	10	8	6	4	3	2	1	0	1*	0		
11	11	10	7	6	3	2	1	0	1	0	1	
12	12	10	7	6	4	3	2*	0	1	0	1*	0

Table 2: LCK[n, d] for $1 \le d \le n \le 12$

It is noticeable in Table 2 that more zeros appear as n gets larger. Dougherty et al. [5] showed LCK[n,d] = 0 for n even and when d = n. Now we show that this is a special case of the following general proposition.

Proposition 4.2. (i) Suppose that n is even, $k \ge 1$, and $i \ge 0$. If $n \ge 6i$, then there is no [n, k, n-2i] LCD code, i.e., LCK[n, n-2i] = 0.

(ii) Suppose that n is odd, $k \ge 1$, and $i \ge 0$. If n > 6i + 3, then there is no [n, k, n - 2i - 1] LCD code, i.e., LCK[n, n - 2i - 1] = 0.

Proof. (i) Suppose C is an LCD [n, k, n-2i] code with parameters in the hypothesis. Let G be a generator matrix of C.

If k = 1, then $GG^T = 0$ since the minimum distance n - 2i is even. Then by Proposition 2.2, there is no [n, 1, n - 2i] LCD code with n even.

Now suppose $k \geq 2$. Then there should exist an LCD [n,2,n-2i] subcode of C. By the Griesmer Bound with k=2, we obtain $n \geq n-2i+\frac{n-2i}{2}$ which implies $n \leq 6i$. Thus we can say that there is no [n,2,n-2i] code if n > 6i. When n meets the Griesmer Bound, i.e., n=6i, there is no [6i,2,4i] LCD code because by Proposition 2.6 the maximum of the possible minimum distance among any [6i,2] LCD codes is 4i-1.

(ii) A similar argument to (i) shows that there is no [n, 1, n-2i-1] LCD code with n odd because the minimum distance n-2i-1 is even.

Suppose $k \ge 2$. Then there should exist an LCD [n, 2, n-2i-1] subcode of C. By the Griesmer Bound with k=2, we have $n \ge n-2i-1+\frac{n-2i-1}{2}$ which implies $n \le 6i+3$. Thus we can say that there is no [n, 2, n-2i-1] code if n > 6i+3. That is, there is no such an LCD code.

In Table 2, the values of LCK[n,d] are given for $1 \le d \le n \le 12$. These values are obtained using Table 1, Proposition 4.2, and two tables from [5]. The values with * are the ones that are corrected here as they are incorrectly reported in Table 1 of [5].

5 Appendix

Below is an exhaustive search program written by MAGMA [2] in order to compute LCD[n, k] which run slowly for large n and k.

```
LCD:=function(n,k)
I:=IdentityMatrix(GF(2),k);

Max:=0;
for g in RMatrixSpace(GF(2),k,n-k) do
   if Determinant(I+(g*Transpose(g))) eq 1
   then if Max lt MinimumDistance
   (LinearCode(HorizontalJoin(I,g)))
   then Max:=MinimumDistance
   (LinearCode(HorizontalJoin(I,g)));
end if;
end if;
end for;
return Max;
end function;
```

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