



# Game-theoretic analysis of competition between access service providers utilizing a Nash genetic algorithm

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## Abstract

Fiber-to-the-home (FTTH) technology is a promising solution for providing advanced service delivery to end-users, but its implementation requires substantial capital expenditures. To minimize investment risks and aid decision-making for access service providers, we propose a game theoretic framework based on a modified Nash genetic algorithm. We illustrate how this framework can be applied to analyze the competition between access providers offering a flat-rate FTTH service on multiple geographical areas. Each provider determines its price for all areas simultaneously and decides whether to invest on a particular area depending on the anticipated revenues. Two distinct demand models are adopted to describe different types of consumer behavior. A solution engine, based on a modified mixed-variable Nash genetic algorithm is implemented under an open-source license. The significance and practical implications of the equilibrium points obtained for both single and multiple area games are discussed. The proposed framework and the solution engine developed, aid both providers and regulatory bodies to analyze competitive environments. They can also be used to implement decision support tools for similar problems as well. The paper concludes by pointing out further research directions in this context.

**Keywords** Game theory · Nash genetic algorithm · Access providers · Communications · Competition · Broadband services

## 1 Introduction

The communications industry plays a fundamental role in the global economy and has been driving the development of the internet and the world wide web for decades. The market for communication services is continuously growing and was valued at 1.7 trillion USD in 2019 with an annual growth rate of 5% until 2027

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(Research 2020). The COVID-19 pandemic accelerated the adoption of e-learning and teleworking, highlighting the need for upgrades in information and communication (ICT) infrastructures (Cooper 2020). The advent of 5G (Campbell et al. 2017) ushered in new business opportunities in domains such as massive machine-type, enhanced mobile broadband and ultra-reliable low latency communications (Jiang and Liu 2017). Access networks providing connectivity to user premises are crucial to meeting the performance requirements of 5G.

Various actors are involved in the development of the communications industry, including users, device manufacturers, network, content, service, cloud providers and regulatory bodies (Battistella et al. 2013). Given the high stakes, investment decisions need to be based on rigorous frameworks. The motivation behind this work, lies in the establishment of a strategic analysis tool for access provider competition operating at a national or regional scale. We focus on Fiber-to-the-Home (FTTH) which is considered the next generation evolution of access networks. Due to their inherent low propagation loss, optical cables provide unprecedented bandwidth  $\times$  distance products (Ramaswami et al. 2009), ensuring reliable gigabit-per-second access rates at large distances, unlike wireless technologies where issues such as line-of-sight, terrain, etc may undermine service delivery. FTTH is nevertheless associated with high deployment costs involving roadworks, cable installation, termination, testing and investments in active equipment. This renders its deployment an economic rather than technical issue, determined by factors such as customer density, average income per capita, existing copper infrastructure, subsidization policies etc. Providers tend to deploy their FTTH network, starting from densely populated areas and gradually expanding to sparsely populated ones, taking competition into account. On the other hand, regulatory bodies need to analyze such strategies to regulate the market, taking the user's best interest at heart (Doove et al. 2001). In our work, we address this problem in the context of *game theory*, which is an established mathematical framework for analyzing the interaction among rational competing agents (Osborne et al. 2004).

Game theory offers valuable insights in scenarios featuring multiple rational decision-makers, each striving to make their own locally optimal decisions. Nash equilibria serve as solutions that establish stable points where all parties simultaneously achieve their optimization goals. These equilibria represent situations in which rational players lack any motivation to alter their decisions. Given that access providers generally act as rational players seeking to maximize their revenues in a competitive landscape, game theory emerges as an apt framework for analysis in these contexts (Maillé and Tuffin 2014). Finding Nash equilibria presents a considerable challenge, especially when dealing with complex utility functions and large decision spaces. Analytical solutions are often only attainable under simplified assumptions. In cases involving the intricate interplay of strategic decision-making, numerical methods become a necessity. One such method is the application of a Nash genetic algorithm (Sefrioui and Perlaux 2000). This approach combines the fundamental principles of genetic algorithms and game theory. Players are allocated chromosome populations reflecting different potential decisions. During each iteration of the algorithm, players in turn strive to locally optimize their utility functions. Over successive iterations, the algorithm steadily converges towards a solution that

corresponds to a Nash equilibrium. This iterative and evolutionary process enables the algorithm to navigate a landscape of strategic interactions and identify equilibrium points efficiently.

Using this framework, the paper addresses a critical issue involving the competition among multiple FTTH providers vying for market dominance in various geographic areas. These regions may exhibit diversity in terms of population, service demand, and deployment costs. Each provider must make strategic decisions regarding the expansion of services into specific areas and the adjustment of a uniform service fee, all with the goal of optimizing individual profits. The primary objective is to pinpoint Nash equilibrium points where providers lack any incentive to modify their tariff policies and expansion decisions and market has reached a stable state. Adapting this framework to the situations at hand, providers can evaluate the best course of action while regulatory bodies can shape their policy decisions in order to achieve the desired outcome in terms of broadband service penetration, etc.

The rest of the paper is organized as follows: In Sect. 2, we present existing literature in game theory approaches and explain our envisaged contribution compared to it. In Sect. 3, we further describe the two alternative demand models. In Sect. 4, we highlight the details of the genetic algorithm used in order to calculate the Nash equilibrium. Section 5, provides some indicative results under different settings interpreted from a strategic point-of-view. Section 6 summarizes our work and provides some directions for future work.

## 2 Related work an contribution

Game theoretic approaches are frequently used in operations research (Taleizadeh and Noori-daryan 2016; Sun et al. 2017; Migdalas 2002; Adler et al. 2021; Shubik 2002) and have been applied in various aspects of ICT, not necessarily focusing on competition. A detailed literature review (Trestian et al. 2012; Altman et al. 2006; Maillé and Tuffin 2014) is outside the scope of this paper and we only mention some pertinent works. In Maillé and Tuffin (2008), a slotted resource allocation game with several providers is analyzed. Unlike our case, this work considers wireless service providers having fixed capacity during each time slot and user demand can be split among providers. In El Azouzi et al. (2003), both the effect of pricing and quality-of-service (QoS) decisions of service providers is considered using game theory. The authors assume a single area game, with a linear demand model and no option is given to the providers not to expand. In Angelou and Economides (2009), real-options and game theory are applied to evaluate ICT investments and oligopoly under multi-criteria perspectives. The paper considers two firms that can decide whether to invest and how much to produce under a utility function inspired by the analytic hierarchy process (AHP). The framework is applied in the case of a construction company planning of laying fiber cables along a newly constructed motorway and offering dark-fiber service to other telecom providers. This situation is quite different than our scenario considered, where the service providers compete over multiple areas under various user demand models. In Wen and Fu (2014) game theory is used to study the collusion and competition strategies between

service providers. The authors adopt evolutionary and Hotelling approaches to analyze methods of eliminating operator collusion at a government level. In Valaskova et al. (2019), a general game-theoretic treatment of the oligopoly among Slovakian service providers is given. In Wang (2020), the impact of bundling in network provider strategy is analyzed, using evolutionary game theory. The study focuses on the problem of the increasing adoption of over-the-top services and the threat it poses to telecommunication providers.

Compared to existing literature, we adopt the game-theoretic approach to describe the area-by-area competition among providers in FTTH deployment. In our scenario, each provider  $j$  can decide whether or not to expand in a specific area  $i$  and can also set the price  $p_j$  it charges (considered flat in all regions). In practice, regions may correspond to provinces, municipalities or communities, each with its own deployment-related costs (Rokkas et al. 2010). In this context, Nash equilibria determine the optimal expansion and pricing strategy in the sense that no competitor has incentive to deviate from it. A number of other frameworks have been considered in ICT such as auction mechanisms especially for cloud resource management and pricing (Sharghivand et al. 2021) or network bandwidth allocation (Dramitinos et al. 2007; Afraz and Ruffini 2019). The present work focuses on provider competition described by alternative demand models discussed below. We do not focus on Stackelberg games (Riahi and Riahi 2020; Maillé and Tuffin 2014) where one or more providers commit to expanding and the rest adapt to this action. In FTTH, new cables must be installed at the customer premises anyway, unlike previous generation copper-based access technologies where decisions of the incumbent operator could largely determine the market, due to its existing infrastructure. Given the demographics of each area  $i$  and the prices  $\{p_k\}$  charged by the providers, the number of area subscribers  $n_{ij}$  attracted from access provider  $j$  is estimated by the demand model. Examples assume  $n_{ij}$  being a linear superposition of  $p_k$  (Farahat and Perakis 2009), attraction models (Allon and Federgruen 2008), finite-state continuous-time Markov chains (Maillé et al. 2009), approaches based on the standard microeconomic framework, Chouhan et al. (2021), simulation (Audestad et al. 2001) and empirical approximations based on curve fitting (Christodoulos et al. 2010). Except but special cases, the demand model intricacies may render analytical solution for Nash equilibria intractable and one must resort to numerical techniques.

## 2.1 Contribution

In our work, we modify a Nash genetic algorithm (Nash GA) based on the work of Sefrioui and Perlaux (2000) to provide a unified approach for mapping both real and binary decision variables using a scheme proposed for conventional GAs (Haupt 2007). Compared to previous works, our contribution can be summarized as follows:

- We focus on the case of service providers competing over the provision of FTTH service in a region. We consider both single and multi-area games where providers have the option to expand or not in certain areas depending on the anticipated

revenues. This scenario has not been previously analyzed in the literature with this level of detail.

- We highlight the details of a Nash GA suited for access provider competition, including initialization, convergence and unified treatment of decision variables. We also elaborate on the practical implications of the obtained Nash equilibria. Although Nash genetic algorithms have been used recently for analyzing general competitive games (Konak et al. 2022), here we formulate a game which is particularly suited for the communications sector and the scenario at hand.
- We consider two types of consumer behavior. The *limited interaction* case is based on an attraction model (Bernstein and Federgruen 2004) where the relative proportion of subscribers attracted by any two providers deciding to expand depends solely on the prices they charge. Under this model, the interaction between providers is therefore limited. We also explore an *enhanced interaction* model, where customers make informed decisions based on the cheapest price offered in their area. It is this price that ultimately determines the market size and competing providers attracts customers portions depending on the difference of the prices and the minimum price.
- In addition, we make our Python implementation freely available on the web (Kamalakis 2021) under an open-source license. This enables researchers to adapt our framework to their particular case studies. To our knowledge, no such open-source implementation currently exists.

Our proposed framework can be extended to incorporate alternative demand modelling. It is particularly useful when the analytic solution for Nash equilibria can not be found (e.g. due to demand-related model complexities). In all cases assumed here, the algorithm converges well and numerically locates an equilibrium point even in multiple area games, where the dimension of the search space increases.

### 3 Provider interaction modeling

In this section, we consider the main components needed in order to describe the provider interaction from a game-theoretic standpoint.

#### 3.1 Game-theoretic framework

The *action profile*  $s_j$  of each provider  $j$  consists of the variables that it can vary to obtain the optimal profitability. These are:

- the price  $p_j$  charged for the service, assumed flat for all areas  $i$ .
- the variables  $b_{ij}$  reflecting whether or not he invests on  $i$  ( $b_{ij} = 1$  and  $b_{ij} = 0$  respectively).

We therefore can write:

$$\mathbf{s}_j = \{p_j, b_{1j}, \dots, b_{Nj}\} \quad (1)$$

where  $N$  is the number of areas considered. Note that  $p_j$  are continuous variables while  $b_{ij}$  are binary. The decisions of provider  $j$  are prioritized under a *utility function*,  $u_j$ , related to the actions of other players as well. The utility function  $u_j = u_j(\mathbf{s})$  maps the action profile  $\mathbf{s}$  which comprises of the ensemble of combined player actions,

$$\mathbf{s} = (\mathbf{s}_1, \dots, \mathbf{s}_j, \dots, \mathbf{s}_M) \quad (2)$$

to real numbers. In (2),  $M$  is the number of providers. Let  $\mathbf{s}_{-j}$  be the action profile obtained without considering the action of player  $j$ ,

$$\mathbf{s}_{-j} = (\mathbf{s}_1, \dots, \mathbf{s}_{j-1}, \mathbf{s}_{j+1}, \dots, \mathbf{s}_M) \quad (3)$$

and  $(\mathbf{s}_j^*, \mathbf{s}_{-j})$  be the action profile obtained from  $\mathbf{s}$  when the action of player  $j$  is replaced by  $\mathbf{s}_j^*$ ,

$$(\mathbf{s}_j^*, \mathbf{s}_{-j}) = (\mathbf{s}_1, \dots, \mathbf{s}_{j-1}, \mathbf{s}_j^*, \mathbf{s}_{j+1}, \dots, \mathbf{s}_M) \quad (4)$$

*Nash's equilibrium* corresponds to an action profile  $\mathbf{s}^* = (\mathbf{s}_1^*, \dots, \mathbf{s}_M^*)$  where no player can improve its utility function by unilaterally changing his action, i.e.:

$$u_j(\mathbf{s}_j^*, \mathbf{s}_{-j}^*) \geq u_j(\mathbf{s}_j, \mathbf{s}_{-j}^*) \quad (5)$$

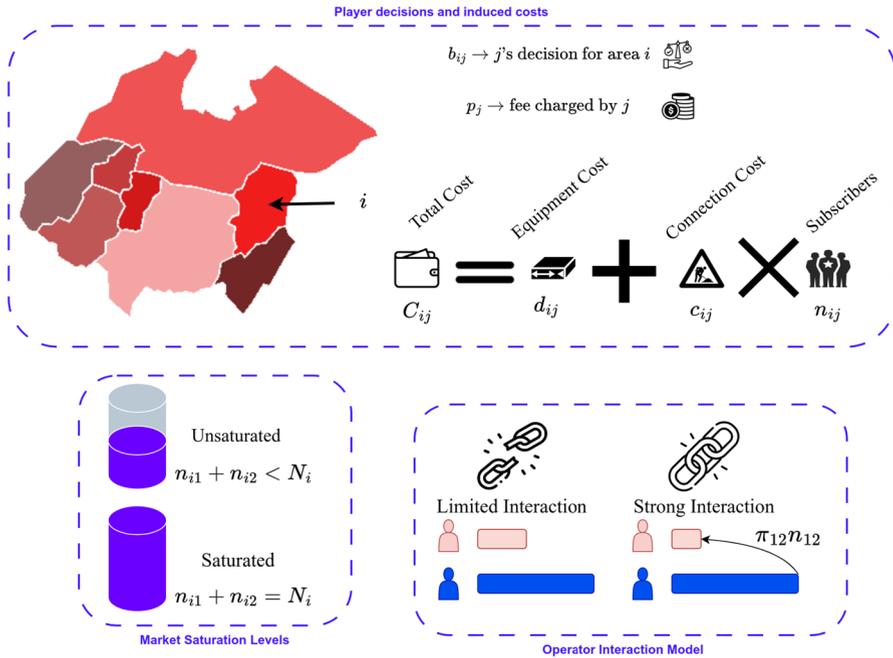
for every player  $j$  and every possible action  $\mathbf{s}_j$  of player  $j$ . This situation corresponds to a *steady state*  $\mathbf{s}^*$  that when reached, the players do not have any reason to choose different actions (Osborne et al. 2004). We assume that players know each others' preferences, and behave rationally in a predictable way, which is typical for network providers seeking to maximize their profits. The utility function of each provider is related to its profit as discussed in the next section.

### 3.2 Utility functions

The utility function reflects the total profit of each provider from all areas in the game. Figure 1 shows the main parameters involved. These are:

- the number  $n_{ij}$  of customers attracted by each provider  $j$  in each area  $i$ .
- a fixed cost component  $d_{ij}$  related to deploying the service by provider  $j$  in area  $i$ , including investments in the provider's points-of-presence (PoPs), etc.
- the average cost  $c_{ij}$  of connecting each subscriber including roadworks, fiber cable installation and optical equipment.
- the variables  $b_{ij}$  which denote whether provider  $j$  decides to expand in area  $i$ .
- the total cost  $C_{ij}$  incurred if provider  $j$  decides to move in area  $i$ . As explained in Fig. 1, this is given by:

$$C_{ij} = c_{ij}n_{ij} + d_{ij} \quad (6)$$



**Fig. 1** Illustration of the various parameters involved in the utility function calculation, the market saturation levels and the provider interaction models

- the fee  $p_j$  charged by each provider  $j$  in all areas he decided to expand.

Both  $d_{ij}$  and  $c_{ij}$  may depend on network architecture. In passive optical network (PON) deployments, fiber is shared from the PoP of the provider up to an optical splitter located in a central point of a neighborhood. Customers are then connected to the splitter by dedicated fibers. This approach leads to reduced fiber roll-out and is generally preferred (Ramaswami et al. 2009). In any case, the profit  $P_{ij}$  obtained for provider  $j$  in area  $i$  is  $P_{ij} = p_j n_{ij} - C_{ij}$  if  $b_{ij} = 1$  or zero otherwise. Equivalently:

$$P_{ij} = (p_j n_{ij} - C_{ij}) b_{ij} \tag{7}$$

The utility function  $u_j(\mathbf{s})$  is obtained by summing up profits over all areas  $i$ :

$$u_j(\mathbf{s}) = \sum_{i=1}^N P_{ij} = \sum_{i=1}^N (p_j n_{ij} - C_{ij}) b_{ij} \tag{8}$$

We relate  $n_{ij}$  in (8) to  $\mathbf{s}$  given by (2) through models discussed in the next subsection. The total number of subscribers attracted by each provider is simply:

$$n_j = \sum_i n_{ij} \tag{9}$$

### 3.3 Demand modeling

Figure 1 illustrates two market regimes: the unsaturated regime, where there are some customers that have not yet acquired the service and the saturated regime where all customers have subscribed to one of the providers. We also illustrate two different interaction models: the limited interaction model where there is little interaction between providers and an enhanced interaction model. The details will be explained in the next subsections.

#### 3.3.1 Limited interaction model

The *limited interaction model* is based on existing attraction models widely used in the literature (Bernstein and Federgruen 2004) and more specifically multi-nomial logit (MNL) models (Li et al. 2019; Besbes and Sauré 2016). The number of customers attracted varies exponentially with price,

$$n_{ij} \propto b_{ij}e^{-\alpha p_j} \tag{10}$$

In (10), the constant  $\alpha$  captures the ease with customers are attracted to providers. If  $j$  chooses not to expand in region  $i$  ( $b_{ij} = 0$ ) then no customers are attracted.

Figure 1 illustrates the situation when  $M = 2$  providers are competing in one area  $i$ . Each attracts a number of subscribers  $n_{ij}$  from the total potential customers  $N_i$ . Naturally, the provider with the lowest price attracts more customers. If the prices are high, there can be many customers that decide not to adopt, i.e.  $n_{i1} + n_{i2} < N_i$ , still relying on previous generation copper-based last mile technologies. In this *unsaturated* market regime, we have:

$$n_{ij} = b_{ij}N_i e^{-\alpha p_j} = N_i e_{ij} \tag{11}$$

where  $e_{ij}$  is given by:

$$e_{ij} = b_{ij}e^{-\alpha p_j} \tag{12}$$

and corresponds to the market share of each provider. Note that the ratio of customers attracted by any two providers  $j = q$  and  $j = r$  deciding to invest ( $b_{iq} = b_{ir} = 1$ ), is:

$$\frac{n_{iq}}{n_{ir}} = \frac{e_{iq}}{e_{ir}} = e^{-\alpha(p_q - p_r)} \tag{13}$$

The above equation implies that the ratio of attracted customers between any two providers depends on their price difference. Using (11), it is easy to show that if a provider invests ( $b_{ij} = 1$ ), then the normalized rate of change  $r_j$  of attracted customers with price, equals:

$$r_j \stackrel{\text{def}}{=} \frac{1}{n_{ij}} \frac{\partial n_{ij}}{\partial p_j} = \alpha, \text{ assuming that } b_{ij} = 1 \tag{14}$$

Equation (14) suggests that the relative rate of change of attracted customers with respect to price of an investing provider is equal to  $\alpha$  assuming unsaturated market. Let  $E_i$  be defined as:

$$E_i = \sum_j e_{ij} \tag{15}$$

In the unsaturated regime,  $E_i < 1$  and  $(2\alpha)^{-1}$  is the price decrease required to increase the provider market share by a factor of  $e^{1/2} \cong 1.65$  (i.e 65%).

If the prices are lowered, more customers will be attracted and  $E_i$  may eventually become larger or equal to one,  $E_i \geq 1$ . This corresponds to a case where all potential customers adopt the service and the market becomes *saturated* as shown in Fig. 1. The sum of  $n_{ij}$  must equal  $N_i$ , i.e.

$$\sum_j n_{ij} = N_i \tag{16}$$

This implies that:

$$n_{ij} = \frac{e_{ij}}{\sum_k e_{ik}} N_i = \frac{e_{ij}}{E_i} N_i \tag{17}$$

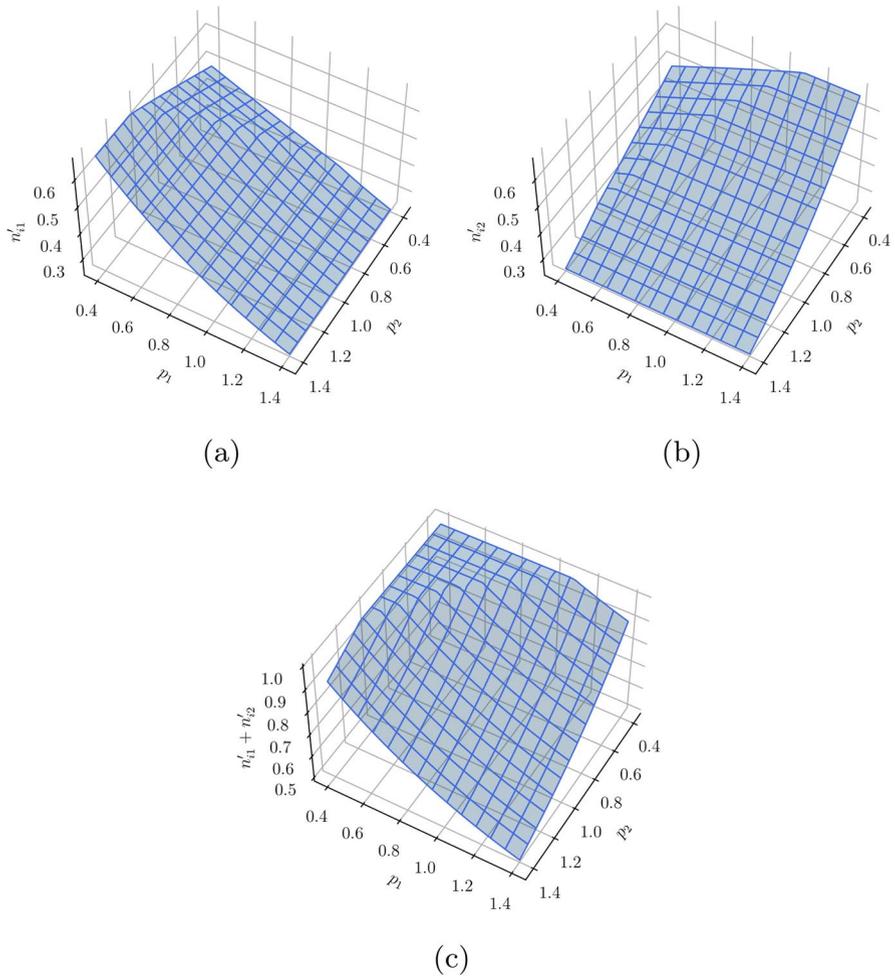
Note that the ratio of customers attracted by any two providers deciding to invest is again given by (13). To summarize:

- if  $E_i < 1$  we have an unsaturated market where (11) holds and there are potential subscribers not adopting the service,
- if  $E_i \geq 1$ , we have a saturated market where all customers have adopted the service and  $n_{ij}$  is determined by (17).

The number of attracted customers are therefore written using a two-branch function:

$$n_{ij} = \begin{cases} N_i e_{ij}, & \text{if } E_i < 1 \text{ (unsaturated)} \\ \frac{1}{E_i} N_i e_{ij}, & \text{if } E_i \geq 1 \text{ (saturated)} \end{cases} \tag{18}$$

To gain further insight in this demand model, we show the market share, i.e. the fraction of customers attracted  $n'_{ij} = n_{ij}/N_i$  from two providers  $j = 1, 2$  in Fig. 2a, b respectively, assuming that  $\alpha = 1$ . We also show the penetration  $n'_{tot} = n'_{i1} + n'_{i2}$  in Fig. 2c. In the latter figure, we observe the two market regimes: At high prices, the market is unsaturated and providers quickly attract more customers by lowering their prices. At some point, the market becomes saturated as  $n'_{tot}$  plateaus to 1. In this range, the competition between providers becomes more intense as the entire customer base is now covered and  $n'_{ij}$  depends on the fees charged by both competitors. We note that the distributions shown in Fig. 2a, b exhibit discontinuous derivatives at the branch points of (18), which combined with the discrete values  $b_{ij}$  will complicate analytical equilibrium treatments.



**Fig. 2** Numerical examples of the attraction model described in Sect. 3.3.1, in the case of two competing providers deciding to invest in an area. **a, b** Market share of each provider as a function of the service fees. **c** Total service penetration in the area

### 3.3.2 Enhanced interaction model

We now consider an *enhanced interaction model* where subscribers in an area  $i$  do a good market search to choose the provider they will adopt. The best option will be the provider  $j_{\min}^{(i)}$  with the lowest price  $p_{\min}^{(i)}$  given by:

$$p_{\min}^{(i)} = \min_j \{p_j \mid \text{provided that } b_{ij} = 1\} \tag{19}$$

Assuming an exponential law as in (11), the total number of attracted subscribers is given by:

$$n_{\text{tot}}^{(i)} = N_i e^{-\alpha p_{\text{min}}^{(i)}} \tag{20}$$

For this model it is easy to show that  $\alpha$  can be interpreted as the relative rate of change of the total number of attracted subscribers with minimum price,

$$r_{\text{tot}} \stackrel{\text{def}}{=} \frac{1}{n_{\text{tot}}^{(i)}} \frac{\partial n_{\text{tot}}^{(i)}}{\partial p_{\text{min}}^{(i)}} = \alpha \tag{21}$$

We elaborate further by assuming that there is only one provider  $j_{\text{min}}^{(i)}$  offering the cheaper rate, i.e.  $p_{\text{min}}^{(i)} < p_j$  for all  $j \neq j_{\text{min}}^{(i)}$ . We expect that some a portion  $\pi_{ij}$  of subscribers will leak away from  $j_{\text{min}}^{(i)}$  to another provider  $j$  that decides to expand in  $i$ , depending on the price difference:

$$\Delta p_{ij} = p_j - p_{\text{min}}^{(i)} \tag{22}$$

We again assume an exponential relation for the portion size:

$$\pi_{ij} \propto b_{ij} e^{-\beta \Delta p_{ij}} \tag{23}$$

where  $\beta$  is some constant describing customer leakage. A small value of  $\beta$  suggests that subscribers may leak more easily to providers charging fees higher than  $p_{\text{min}}^{(i)}$  since  $e^{-\beta \Delta p_{ij}}$  is slowly decreasing with increasing  $p_j$ . A large value of  $\beta$  implies that  $e^{-\beta \Delta p_{ij}}$  is rapidly decreasing with increasing  $p_j$ , implying smaller customer leakage. Since the portions must sum up to unity with respect to  $j$ , we can normalize  $\pi_{ij}$  as follows:

$$\pi_{ij} = \frac{b_{ij} e^{-\beta \Delta p_{ij}}}{\sum_k b_{ik} e^{-\beta \Delta p_{ik}}} \tag{24}$$

The number of subscribers is therefore:

$$n_{ij} = n_{\text{tot}}^{(i)} \pi_{ij} \tag{25}$$

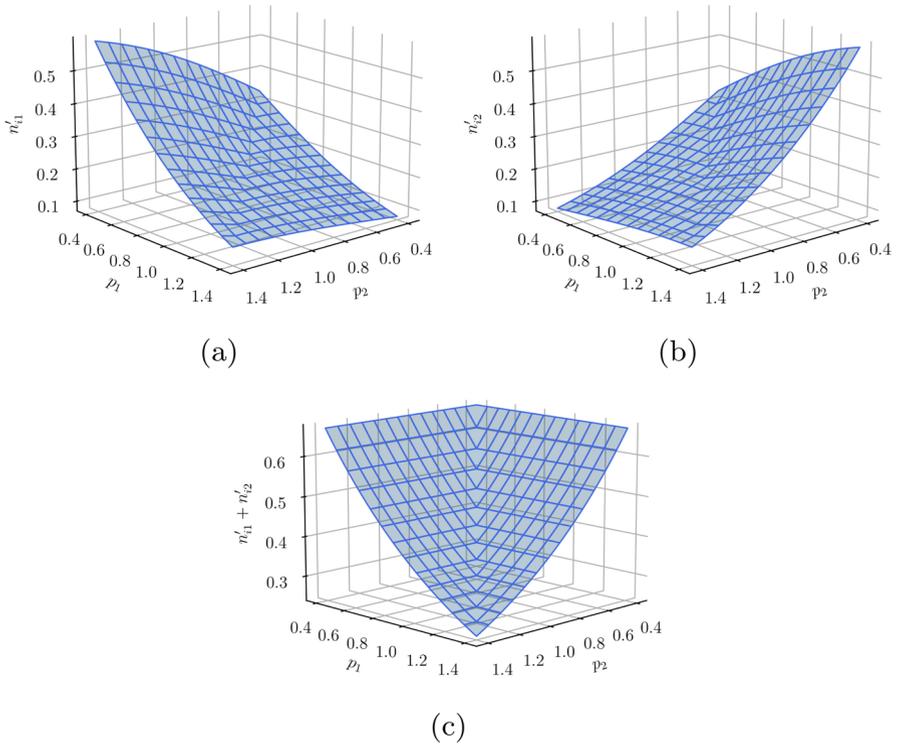
Note that (24) provides a further insight on the parameter  $\beta$ . Let us assume that only provider  $j_{\text{min}}^{(i)}$  offers the minimum price  $p_{\text{min}}^{(i)}$  while the rest offer fees  $p_j$  far greater, i.e.  $p_j \gg p_{\text{min}}^{(i)}$  for  $j \neq j_{\text{min}}^{(i)}$ . In this case  $\pi_{ij} \ll 1$  and from (24), we can show that<sup>1</sup>:

<sup>1</sup> The customer fraction  $\pi_{ij}$  is written in terms of  $f_j = A_j e^{-\beta p_j}$  with  $A_j = b_j e^{\beta p_{\text{min}}^{(i)}}$ , as follows:

$$\pi_{ij} = \frac{f_j}{\sum_k f_k}$$

and if  $\pi_{ij} \ll 1$ :

$$\frac{\partial \pi_{ij}}{\partial p_j} = \frac{-\beta f_j}{\sum_k f_k} + \frac{\beta f_j^2}{(\sum_k f_k)^2} = -\beta(\pi_{ij} - \pi_{ij}^2) \cong -\beta \pi_{ij}$$



**Fig. 3** Numerical examples of the attraction model described in Sect. 3.3.2, in the case of two competing providers deciding to invest in an area. **a, b** Market share of each provider as a function of the service fees. **c** Total service penetration in the area

$$\frac{1}{\pi_{ij}} \frac{\partial \pi_{ij}}{\partial p_j} \cong -\beta, \text{ when } j \neq j_{\min} \text{ and } p_j \gg p_{\min}^{(i)} \tag{26}$$

Therefore if one provider offers a much cheaper service, the relative decrease of the fraction  $\pi_{ij}$  of attracted customers by provider  $j$  is equal to  $\beta$ .

In case two (or more) providers  $j = j_1, j_2, \dots$ , offer the minimum price,  $p_{ij_1} = p_{ij_2} = \dots = p_{\min}^{(i)}$ , then (24) suggests that they will have the same market share at the area in question  $i$ ,

$$n_{ij_1} = n_{ij_2} = \dots \tag{27}$$

Figure 3a, b illustrate the number of subscribers attracted by two providers assuming  $\alpha = 1$  and  $\beta = 2$  while Fig. 3c shows the service penetration. The latter is consistent with the fact that the minimum of  $p_1$  and  $p_2$  determines the subscribers opting to acquire the FTTH service by either provider. As shown in Fig. 3a, b, the way in which subscribers are attracted by each provider changes around the diagonal  $p_1 = p_2$ . As long as  $p_1 < p_2$  the market favors provider 1 while provider 2 is penalized by potential clients since dropping his fee brings less customers than when

$p_2 < p_1$ . We believe this is a better way to capture the market dynamics when well-informed clientele is assumed.

### 3.3.3 Incorporation of network characteristics

In this work, we assume that demand is predominantly shaped by pricing considerations, with network attributes such as throughput and latency not explicitly incorporated into our analysis. In FTTH, stability of service performance is inherent once the fiber connection is established at the customer's premises. This stability stands in contrast to legacy copper-based access technologies, where variables like distance from the cabinet significantly influenced service quality. In any case, if we wish to integrate network-specific attributes into utility functions, a robust approach involves employing multi-criteria methodologies such as the AHP (Angelou and Economides 2009). This framework provides a systematic means of assessing the importance of individual parameters, enabling a more comprehensive analysis of network characteristics within the broader context of demand and pricing dynamics. By incorporating AHP, we can assess other wireless/wire line technologies, acknowledging the interplay between network attributes and consumer demand.

### 3.4 Parameter extraction

Before closing this section, we should note that the utility functions in (8), involve both deployment cost-related parameters ( $c_{ij}$  and  $d_{ij}$ ) and demand-related parameters ( $\alpha$  and  $\beta$ ). The former are roughly estimated by the provider's engineers given a detailed map of the area and the house-hold geographical distribution, resulting in a fiber cable roll-out plan. A detailed calculation of deployment-related costs (trenching, road restoration, fiber termination, etc) as in Rokkas et al. (2010), can be used to obtain an estimate of the values of  $c_{ij}$  and  $d_{ij}$ . Estimation of the demand-related parameters is more involved. One approach is to fit the demand parameters with actual customer data (Abdallah and Vulcano 2021)). The detailed manner in which the demand and cost-related parameters are extracted is beyond the scope of this paper. That being said, diagrams of Figs. 2 and 3 provide some intuition on the value range of  $\alpha$  and  $\beta$ , considered in the simulations. Alternatively, we could resort to time-series forecasting adopted in Christodoulos et al. (2010) based on earlier access technology generations, nonlinear Lotka-Volterra equations (Lakka et al. 2013), multi-generational models (Michalakelis et al. 2010) or fitting demand parameters with actual data (Abdallah and Vulcano 2021). These alternatives can be incorporated in our implementation of the Nash GA described in Sect. 4.

## 4 Solution engine

The solution search space contains both the flat prices  $p_j$  of each provider which are continuous variable and the decision variables  $b_{ij}$  which are binary. This constitutes a mixed-variable problem and in light of the complexity of the utility functions

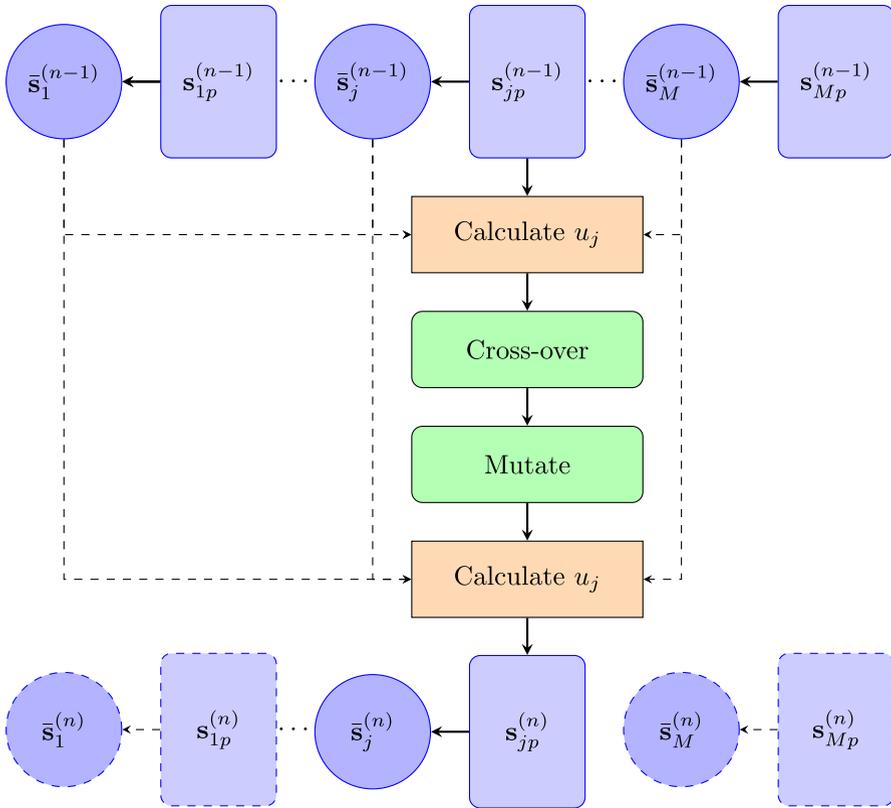


Fig. 4 Overview of the Nash GA

containing branches as in (18) or minimization in (20), analytical treatment becomes intractable. We therefore need to resort to numerical frameworks such as the Nash GA outlined in this section. An overview of the Nash GA applied in this context is depicted in Fig. 4. The details are discussed below.

### 4.1 Overview of the algorithm

In this section, we describe the details of the genetic algorithm used to solve (5) and find the optimal action profiles  $s^*$  (Sefrioui and Perlaux 2000). For each player  $j$ , we maintain a pool of  $\mathbf{P}_j^{(n)} = [s_{jp}^{(n)}]$  of  $P$  possible action plans  $s_{jp}^{(n)}$  where  $1 \leq p \leq P$ . The pool is updated in each iteration  $n$ , according to Fig. 4. Let  $\bar{s}_j^{(n-1)}$  be the best action profile of player  $j$  obtained in the previous iteration  $n - 1$  (we shall discuss how this action plan is chosen later on). At the beginning of each iteration,  $n$ , we take as a starting point the pools obtained in the previous iteration  $\mathbf{P}_j^{(n-1)}$ , i.e. we set  $\mathbf{s}_{jp}^{(n)} \leftarrow \mathbf{s}_{jp}^{(n-1)}$ .

For each player  $j$ , we first calculate the utilities  $u_{jp}$  for the action plans in his pool  $\mathbf{P}_j$  assuming that the other players  $k$  ( $k \neq j$ ) conform to the best action plan obtained at the previous iteration,  $\bar{\mathbf{s}}_k^{(n-1)}$ . Following the notation introduced in (3), this is formally written as:

$$u_{jp} = u_j\left(\bar{\mathbf{s}}_{-j}^{(n-1)}, \mathbf{s}_{jp}^{(n)}\right) \tag{28}$$

where  $(\bar{\mathbf{s}}_{-j}^{(n-1)}, \mathbf{s}_{jp}^{(n)})$  is the action profile obtain when player  $j$  chooses action  $\mathbf{s}_{jp}^{(n)}$  from its pool while the rest of the players conform to their optimal action plans obtained from the previous iteration. The next step involves the *cross-over* operation where members of the pool  $\mathbf{P}_j$  are combined to produce *offsprings*, i.e. new candidate action plans. In order to maintain population diversity, we also *mutate* the offsprings, i.e. we randomly perturb their variable values. The utility functions for these new action plans are calculated again using (28). If we obtain a higher utility function than those already found in the pool, then we replace the weakest pool members by the superior action plans. This provides the new population pool  $\mathbf{P}_j^{(n)}$  in which we identify the action  $\bar{\mathbf{s}}_j^{(n)}$  plan with the for player  $j$  with the highest utility value. This procedure is carried out for all players in order to obtain the new populations pools and the new optimal action plans to feed the next iteration  $n + 1$  of the algorithm. After a number of iterations, the algorithm will converge to a Nash equilibrium, Sefrioui and Perlaux (2000), Isakhani et al. (2020). In the following subsections we discuss several implementation aspects of the algorithm.

### 4.2 Variables

As shown in (1), the action plans  $\mathbf{s}_j$  consist of continuous variables  $p_j$  and binary variables  $b_{ij}$ , indicating a *mixed-variable* optimization problem. We modify the Nash GA proposed in Sefrioui and Perlaux (2000) to adopt the mapping of conventional GAs for mixed problems (Haupt 2007). Let  $g_{kj}$  be real variables bound inside  $[0, 1]$  for  $1 \leq k \leq N + 1$  and  $1 \leq j \leq M$ . The continuous variables  $p_j$  are mapped as follows:

$$p_j = g_{N+1,j}(P_{\max} - P_{\min}) + P_{\min} \tag{29}$$

where  $P_{\max}$  and  $P_{\min}$  are the assumed upper and lower bounds for the prices  $p_j$ ,  $P_{\min} \leq p_j \leq P_{\max}$ . We can also consider different bounds for each player if the demand-model parameters are quite different. We can choose  $P_{\min} = 0$  and set  $P_{\max}$  to a value high enough, that the number of subscribers in the absence of any competition is very low. For the binary variables  $b_{ij}$  we use the mapping,

$$b_{ij} = [g_{ij}] \tag{30}$$

where  $1 \leq i \leq N$  and  $[x]$  is the integer closest to  $x$ . Equations (29) and (30) imply that instead of using both real and binary variables, we can simply store the real

parameters  $g_{kj}^{(p)}$  bound inside  $[0, 1]$ , where  $p$  denotes the index of the pool member in the population pools and therefore treat all variables in a unified manner.

### 4.3 Cross-over and mutation

In each iteration, the strongest 50% of the population members constitute the mating pool (Haupt 2007). We choose two parents through tournament selection and calculate the offspring using uniform crossover, which consists of tossing an unbiased coin and randomly selecting the value of each offspring variable from either the first or the second parent. This produces an offspring with parameters equal to  $\bar{g}_{kj}^{(p)}$  which are a mix of the parameters of the parents. Mutation is achieved by adding a random perturbation  $\Delta g_{kj}^{(p)}$  to each of the parameters determined by:

$$\Delta g_{kj}^{(p)} = m_0 r_{kj}^{(p)} g_{kj}^{(p)} \tag{31}$$

where  $0 \leq m_0 \leq 1$  is the *mutation factor* and  $r_{kj}^{(p)}$  are randomly chosen inside  $[-1, 1]$  from a uniform distribution. The values of the parameters after the mutation  $\tilde{g}_{kj}^{(p)}$ , are given by the fractional parts of the perturbed variables,

$$\tilde{g}_{kj}^{(p)} = \bar{g}_{kj}^{(p)} + \Delta g_{kj}^{(p)} - [\bar{g}_{kj}^{(p)} + \Delta g_{kj}^{(p)}] \tag{32}$$

where  $[x]$  denotes the integer part of  $x$ .

### 4.4 Initialization and termination

The initialization of the initial pools is carried out by randomly choosing the initial values of  $g_{kj}^{(p)}$ . These values yield the starting action plans  $s_{jp}^{(0)}$  for each player through the transformations discussed in Sect. 4.2. We can randomly choose one of these action plans in each pool to be the starting optimal action plan  $\bar{s}_j^{(0)}$ . The criteria for terminating the algorithm may vary depending on the requirements. In our implementation, we evaluate the degree of convergence by measuring the relative variation  $\delta u_j$  of the utility functions  $u_{jp}$  for each player pool,

$$\delta u_j = 1 - \frac{\min_p \{u_{jp}\}}{\max_p \{u_{jp}\}} \tag{33}$$

We assume that convergence is achieved if  $\delta u_j$  is smaller than a specified value  $\delta u_{\min}$  for all player pools  $\mathbf{P}_j$ .

If a complex demand model is adopted, the utility function evaluation is expected to be the predominant factor in the execution time of the algorithm. We may therefore choose to terminate the algorithm after the number of total utility function evaluations exceed a specified number  $N_{\text{evals}}$ .

**Table 1** Game parameters

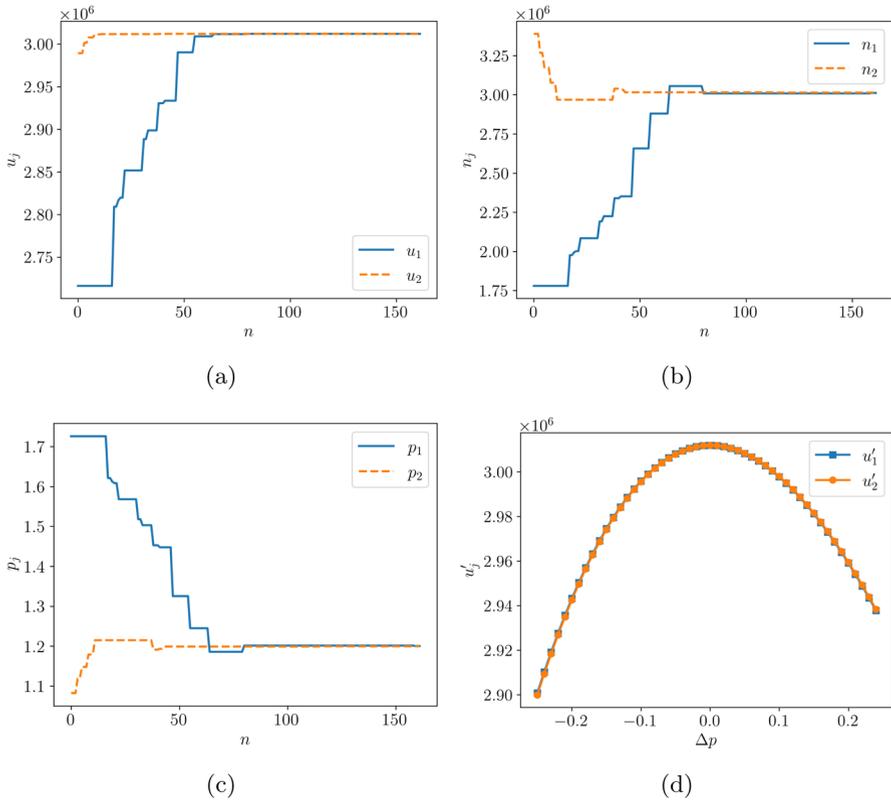
Parameter	Explanation	Value
$N_1$	Max. number of potential customers	10.000.000
$M$	Number of players	2
$N$	Number of areas	1
$c_{ij}$	Average connection cost (per subscriber)	0.2
$\alpha$	Demand coefficient	1
$\beta$	Leakage coefficient	2
$P$	Pool size per player	10
$m_0$	Mutation factor	0.1
$\delta u_{\min}$	Minimum required pool utility range	$10^{-3}$
$N_{\text{evals}}$	Maximum number of utility evaluations	$10^4$

### 4.5 Implementation

Our Python implementation is publicly available under an open-source license (Kamalakris 2021). It relies on the NumPy module which is a fundamental scientific computing package. We also make use of the Matplotlib package for visualization of the results. In our implementation, we adopted an object-oriented approach where the various data structures involved in the algorithm are represented as classes. The variables are represented by the chromosome class which provides the necessary methods for manipulating the parameters  $g_{kj}^{(p)}$  and their mapping to the actual action variables  $p_j^{(p)}$  and  $b_{ij}^{(p)}$  for each member  $p$  of a population pool for player  $j$ . The population class, deals with the pool of action plans for each player providing the necessary tools for calculating the utility of each action plan, carrying out the cross-over and mutation operations and sorting the action plans in terms of fitness. The ensemble of pools is described by the population\_group class, which handles the calculation of the utility functions at a group level (i.e. taking into account the best action plans of the other players), the calculation of the next generation of pools and keeping track of the overall convergence of the algorithm in terms of the value range of utility values for each pool and provides some rudimentary logging. The operator\_game class models the access provider competition incorporating the two demand models discussed in Sect. 3.

**Table 2** Nash equilibrium for  $N = 1$  and the limited interaction model

Parameter	Explanation	Optimal value
$u_1$	Utility (profit) for player 1	$3.01 \times 10^6$
$u_2$	Utility (profit) for player 2	$3.01 \times 10^6$
$p_1$	Price set by player 1	1.20
$p_2$	Price set by player 2	1.20
$b_{11}$	Player 1 decision variable	1 (True)
$b_{12}$	Player 2 decision variable	1 (True)
$n_1$	Total subscribers for player 1	$3.0 \times 10^6$
$n_2$	Total subscribers for player 2	$3.0 \times 10^6$



**Fig. 5** Results of the Nash GA for the case of the limited interaction model assuming all players have the same installation costs: **a** utilities (profits)  $u_j$ , **b** number of subscribers  $n_j$ , **c** prices  $p_j$  as a function of the algorithm iteration  $n$ , **d** sensitivity analysis around the equilibrium point

## 5 Results and discussion

We first discuss single area games where the provider interaction is more easily understood and then move on to multiple area games.

### 5.1 Single area games

We begin by examining a single area game,  $N = 1$  with two players,  $M = 2$ , reflecting a situation where the operators decide whether or not to offer the service at a national scale simultaneously. Table 1 summarizes the key parameters assumed. We have chosen the maximum customer number size  $N_1$  that from a practical stand-point could make sense to invest on the entire region directly and not on an area-by-area basis. The demand coefficient  $\alpha$  is equal to 1 as in Figs. 2 and 3. To negate the need to transform prices between currencies, we have adopted an arbitrary currency unit for  $p_j$  which varies in  $[0, 4]$ . Given the  $e^{-\alpha p_j}$  dependence in (10), the range  $0 \leq p_j \leq 4$

corresponds to service penetration ranges from 2% to 100%. For the enhanced interaction model, we set the leakage coefficient in (23) equal to  $\beta = 2$ . We will investigate the influence of this parameter later on in this section. We also assume an average connection cost equal to  $c_{1j} = 0.2$  per subscriber.

### 5.1.1 Limited interaction model

Table 2 summarizes the Nash equilibrium point assuming the limited interaction model. We readily see that the optimal action plans for each player are identical, due to the nature of the limited interaction model. Given that  $n_1 + n_2 < N_1$ , the market is unsaturated and each provider attracts customers independently of the other. It is in the best interest of each provider to offer the service ( $b_{1j} = 1$ ) at a price  $p_1 = p_2 \cong 1.2$  much higher than the connection cost ( $= 0.2$ ). Figure 5a shows the obtained utilities  $u_j$  corresponding to the best action plans of each player  $j$ , with respect to the iteration number  $n$  which is an indication of the convergence of the algorithm. The algorithm terminates after 162 iterations and 3584 utility function evaluations, achieving the required relative utility variation.

Figure 5b, c show the evolution of the corresponding subscriber numbers  $n_j$  and the prices  $p_j$ . We see that a similar convergence behavior is obtained, since the values of these parameters do not significantly vary after 70 iterations. Figure 5d shows a sensitivity analysis around the equilibrium values of the prices  $p_1$  and  $p_2$  of Table 2. This is carried out by calculating the utility functions

$$u'_1 = u_1(p_1 + \Delta p, b_{11}, p_2, b_{12}) \tag{34}$$

$$u'_2 = u_2(p_1, b_{11}, p_2 + \Delta p, b_{12}) \tag{35}$$

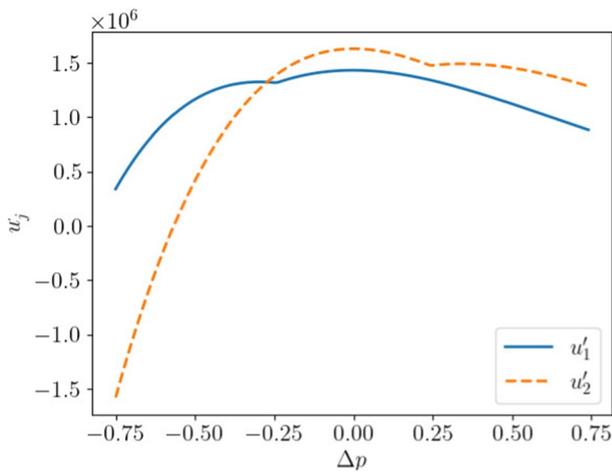
As shown in Fig. 5d,  $\Delta p = 0$  is a maximum for both  $u'_1$  and  $u'_2$ , implying that neither player can do better by changing the price. Additionally, we see that starting from the equilibrium point and choosing  $b_{11} = 0$  would imply that the utility for player 1 will drop to zero. The same holds for player 2 when setting  $b_{12} = 0$ . Thus neither provider can do better by changing their action plans and Table 2 indeed provides a Nash equilibrium.

### 5.1.2 Enhanced interaction model

We next apply the algorithm to find the equilibrium for the enhanced interaction model. Table 3 shows the equilibrium point obtained by the algorithm. Unlike Table 2, player 2 now has a higher utility function  $u_2$  and achieves this by offering the service at a substantially lower price  $p_2$  which allows him to establish a leading position in the market, attracting a much higher subscriber number  $n_2$  than his competitor, player 1. On the other hand, player 1 seeks to strike a balance between lowering his rate  $p_1$  in order to attract more customers and keeping the price high enough to ensure significant revenues. Re-initializing at different

**Table 3** Nash equilibrium for  $N = 1$  and the enhanced interaction model

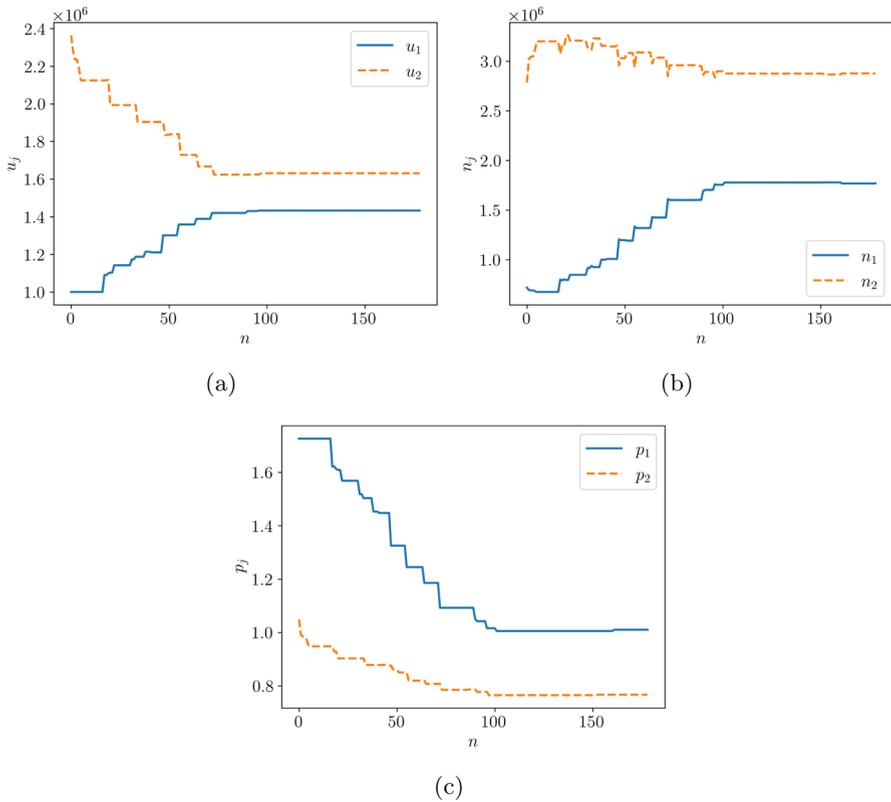
Parameter	Explanation	Optimal value
$u_1$	Utility (profit) for player 1	$1.43 \times 10^6$
$u_2$	Utility (profit) for player 2	$1.63 \times 10^6$
$p_1$	Price set by player 1	1.01
$p_2$	Price set by player 2	0.77
$b_{11}$	Player 1 decision variable	1 (True)
$b_{12}$	Player 2 decision variable	1 (True)
$n_1$	Total subscribers for player 1	$1.76 \times 10^6$
$n_2$	Total subscribers for player 2	$2.87 \times 10^6$



**Fig. 6** Price sensitivity analysis around the equilibrium point of Table 3

initial random states, we confirmed that there are two equilibrium points in which the role of player 1 and 2 are exchanged. Under this interaction model, providers may not prefer to simply lower the price to cope with the competition. A sensitivity analysis around the equilibrium point of Table 3 is carried out in Fig. 6 which shows of  $u'_1$  and  $u'_2$  in (34) and (35). Since  $\Delta p = 0$  is a maximum point for both utilities, we see that neither player alone can choose a better price strategy. We notice a bump-like behavior near  $\Delta p = -0.24$  for  $u'_1$  and  $\Delta p = +0.24$  for  $u'_2$ . At the former point, player 1 sets a price  $p_1 = 0.77 = p_2$  and hence the providers attract the same portion of subscribers from the customer pool. This turns out to be sub-optimal due to the enhanced interaction assumed. Similarly, at  $\Delta p = 0.24$ , player 2 sets the same price as 1 ( $p_2 = p_1 = 1.01$ ) and this turns out to be a sub-optimal strategy for player 2.

Figure 7 shows the convergence properties of the algorithm for the enhanced interaction demand model. Figure 7a shows the evolution of utilities  $u_j$  corresponding to the best action plans of each player. Figure 7b, c show the evolution

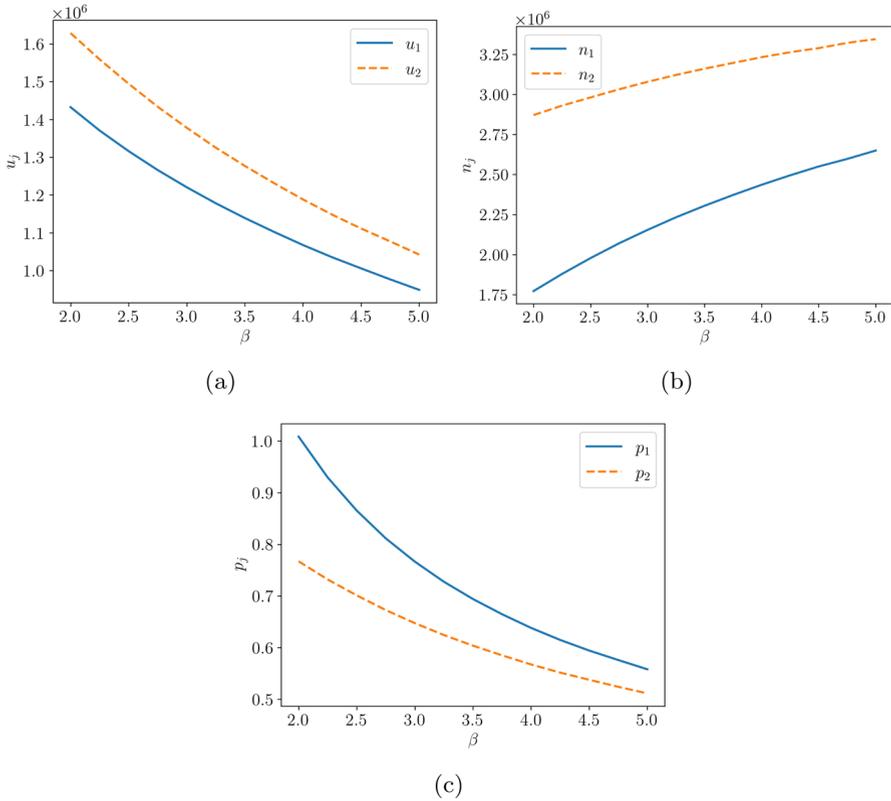


**Fig. 7** Results of the Nash GA for the case of the enhanced interaction model assuming all players have the same installation costs: **a** utilities (profits)  $u_j$ , **b** number of subscribers  $n_j$ , **c** prices  $p_j$  as a function of the algorithm iteration  $n$

of the corresponding subscriber numbers  $n_j$  and the prices  $p_j$ . In this case the algorithm terminates after 179 iterations and 3598 utility function calls, achieving the desired utility value range in both population pools.

### 5.1.3 Influence of customer awareness

The effect of  $\beta$ , describing customer leakage in the enhanced interaction case, is shown in Fig. 8. Figure 8a shows the equilibrium utility values obtained for various values of  $\beta$  in the range [2, 5] while Fig. 8b, c show the corresponding subscriber populations and optimal price settings. In Sect. 3.3.2, we discussed how  $\beta$  can be related to consumer awareness. According to (23), a high value for  $\beta$  implies that few customers will be attracted by the expensive provider. This pushes the players to lower their prices  $p_j$  (Fig. 8c), thereby attracting higher subscriber numbers  $n_j$  (Fig. 8b). The provider profits are however reduced as shown in Fig. 8a.



**Fig. 8** Nash equilibrium obtained for different  $\beta$ : **a** utilities (profits)  $u_j$ , **b** number of subscribers  $n_j$ , **c** prices  $p_j$

**Table 4** Nash equilibrium for  $N = 1$  with dissimilar providers under the limited interaction model

Parameter	Explanation	Optimal value
$u_1$	Utility (profit) for player 1	$3.01 \times 10^6$
$u_2$	Utility (profit) for player 2	$2.23 \times 10^6$
$p_1$	Price set by player 1	1.20
$p_2$	Price set by player 2	1.49
$b_{11}$	Player 1 decision variable	1 (True)
$b_{12}$	Player 2 decision variable	1 (True)
$n_1$	Total subscribers for player 1	$3.0 \times 10^6$
$n_2$	Total subscribers for player 2	$2.2 \times 10^6$

### 5.1.4 Dissimilar providers

We next take a look at a case where installation costs for players are dissimilar. Table 4 describes the equilibrium point obtained when the average connection costs

**Table 5** Nash equilibrium for  $N = 1$  with dissimilar providers under the enhanced interaction model

Parameter	Explanation	Optimal value
$u_1$	Utility (profit) for player 1	$1.88 \times 10^6$
$u_2$	Utility (profit) for player 2	$0.98 \times 10^6$
$p_1$	Price set by player 1	0.81
$p_2$	Price set by player 2	1.22
$b_{11}$	Player 1 decision variable	1 (True)
$b_{12}$	Player 2 decision variable	1 (True)
$n_1$	Total subscribers for player 1	$3.05 \times 10^6$
$n_2$	Total subscribers for player 2	$1.35 \times 10^6$

**Table 6** Multiple area game parameters

Parameter	Explanation	Value
$N_i$	Area population	1,000,000
$M$	Number of players	2
$N$	Number of areas	10
$c_{ij}$	Average connection cost (per subscriber)	$0.3(i - 1) + 0.2$
$\alpha$	Demand coefficient	1
$\beta$	Leakage coefficient	2
$P$	Pool size per player	50
$m_0$	Mutation factor	0.1
$\delta u_{\min}$	Minimum required pool utility range	$10^{-3}$
$N_{\text{evals}}$	Maximum number of utility evaluations	$10^6$

are  $c_{11} = 0.2$  (same as before) and  $c_{12} = 0.5$ . Player 2 must now compromise with a lower utility value  $u_2$  due to the reduced profit margin. Player 1’s utility remains unchanged compared to Table 2. As explained in Sect. 3.3.1, under the limited interaction model, the players attract customers independently unless the market is saturated. Table 5 outlines Nash’s equilibrium for dissimilar providers, in the case of enhanced interaction. Since player 2 has a reduced profit margin, player 1 now becomes dominant with a utility value which is higher than the utility of the dominant player in Table 3. Player 2 is forced to charge higher rates, thereby claiming smaller portion of the customers willing to pay for the service.

## 5.2 Multiple area games

### 5.2.1 Nash equilibrium

We next discuss games where each player adjusts the variables  $\beta_{ij}$  of his action plan in multiple areas ( $N > 1$ ). This scenario is more interesting from a practical point-of-view, since access providers usually take decisions on a area-by-area

**Table 7** Nash equilibrium for  $N = 10$  under the limited interaction model

Parameter	Explanation	Optimal value
$u_1$	Utility (profit) for player 1	$0.85 \times 10^6$
$u_2$	Utility (profit) for player 2	$0.85 \times 10^6$
$p_1$	Price set by player 1	2.10
$p_2$	Price set by player 2	2.10
$b_{i1}$	Player 1 decision variable	1 for $i \leq 7$ , 0 otherwise
$b_{i2}$	Player 2 decision variable	1 for $i \leq 7$ , 0 otherwise
$n_1$	Total subscribers for player 1	$0.85 \times 10^6$
$n_2$	Total subscribers for player 2	$0.85 \times 10^6$

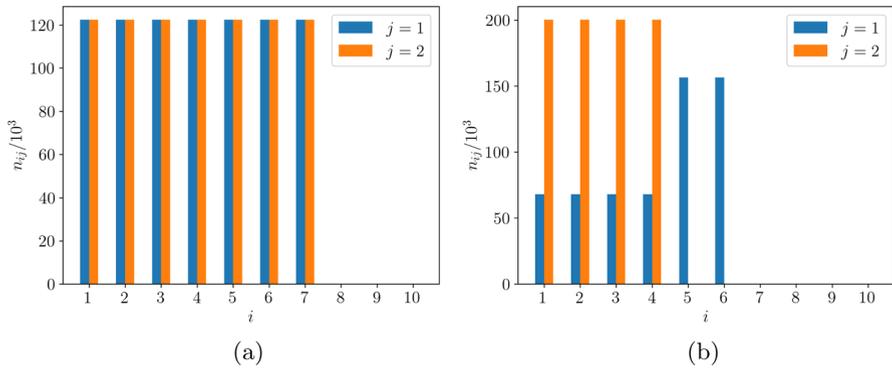
**Table 8** Nash equilibrium for  $N = 10$  under the enhanced interaction model

Parameter	Explanation	Optimal value
$u_1$	Utility (profit) for player 1	$0.42 \times 10^6$
$u_2$	Utility (profit) for player 2	$0.53 \times 10^6$
$p_1$	Price set by player 1	1.85
$p_2$	Price set by player 2	1.31
$b_{i1}$	Player 1 decision variable	1 for $i \leq 6$ , 0 otherwise
$b_{i2}$	Player 2 decision variable	1 for $i \leq 4$ , 0 otherwise
$n_1$	Total subscribers for player 1	$0.58 \times 10^6$
$n_2$	Total subscribers for player 2	$0.80 \times 10^6$

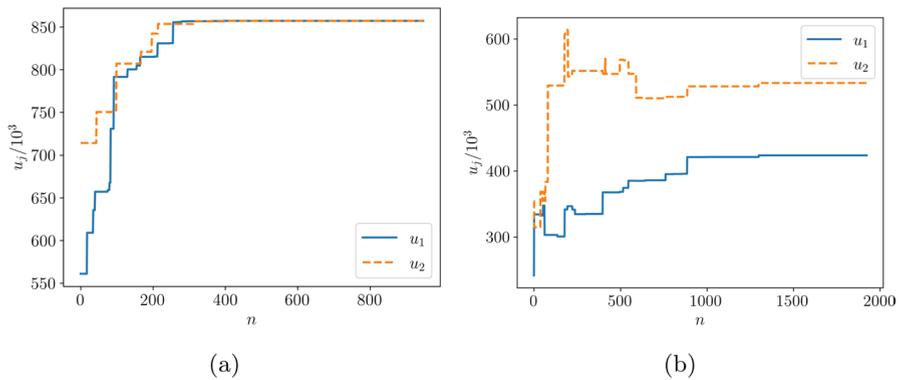
basis. Such scenarios are harder to solve because of the much higher dimension in the variable search space. We consider a ten area game ( $N = 10$ ) in which have different average customer connection cost  $c_{ij}$  which is the same for all providers ( $c_{ij} = c_i$ ). Table 6 summarizes the game parameters assumed. Seeking an analytical solution for this problem is even more involved than in single area games: one must consider many candidate sets of values for  $b_{ij}$  in addition to discontinuities in the derivatives of utility functions with respect to price.

Due to the higher dimension of the search space, we need a larger pool size per player  $P$  to ensure population diversity, and we have chosen  $P = 50$  in the simulations. The larger pool size leads to increased number of required utility function evaluations. For  $c_{ij}$ , we have assumed a simple linear dependence on  $i$ ,  $c_{ij} = 0.3(i - 1) + 0.2$ . Table 7 shows the equilibrium obtained based on the limited interaction model. The optimal action plans are the same for both players similar to the single area game (Table 2). The figure suggests that the providers prefer to invest in areas with lower installation costs  $c_{ij}$  first (smaller  $i$ ). The higher connection costs in some areas decrease the profits and providers are better off not expanding there ( $b_{i1} = b_{i2} = 0$  for  $i \geq 8$ ).

Table 8 summarizes the equilibrium point for the enhanced interaction model, where the role of the players is distinguished. Figure 9 illustrates the customer distribution  $n_{ij}$  attracted in each area  $i$  by provider  $j$ . In the limited interaction model, this



**Fig. 9** Subscribers attracted in the 10-area game for **a** the limited and **b** the enhanced interaction models

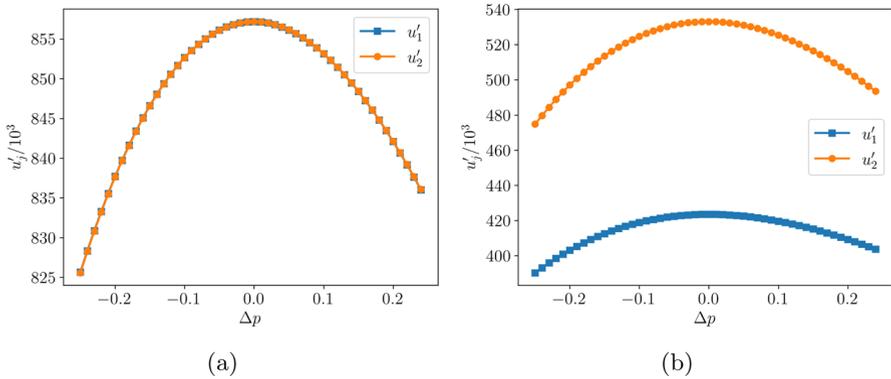


**Fig. 10** Optimal utilities for the two players as a function of the algorithm iteration  $n$  for the case of **a** the limited and **b** the enhanced interaction models respectively

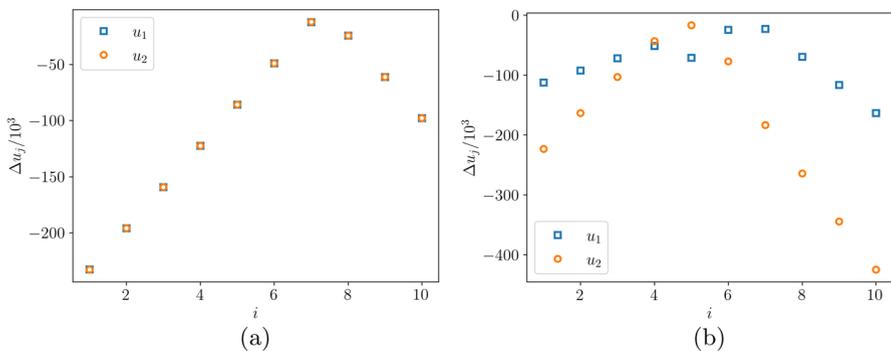
distribution is identical for both providers but in the case of enhanced interaction, the distributions differ. The dominant player (player 2) is able to achieve a higher utility value by expanding in fewer areas ( $b_{2i} = 0$ , for  $i \geq 4$ ). The alternative provider must choose to expand in less profitable areas ( $b_{1i} = 0$  for  $i \geq 6$ ) in order to make up some of the profit loss. These are examples of how our framework can assist providers in identifying their investment plans given each region characteristics.

### 5.2.2 Convergence and sensitivity analysis

We next discuss the algorithm convergence in the multiple area scenarios. In Fig. 10, we show the optimal utilities  $u_j$  for both players at each iteration of the algorithm. For brevity we omit the corresponding graphs for the subscriber number  $n_j$  and the prices  $p_j$  which more or less exhibit the same behavior. Looking at the horizontal axis scale, it is evident that convergence is slower than Figs. 5 and



**Fig. 11** Price sensitivity analysis around the equilibrium point for the ten area game for the case of **a** the limited and **b** the enhanced interaction models



**Fig. 12** Decision sensitivity analysis around the equilibrium point for the ten area game for the case of **a** the limited and **b** the enhanced interaction models

7 due to the increase in search space dimension. The search space now comprises of the two continuous variables  $p_j$  plus 20 discrete binary variables  $b_{ij}$ , instead of 2 plus 2 in the single area game. Figure 10a corresponds to limited provider interaction model where the algorithm terminates after 948 iterations (96, 796 utility function evaluations). Figure 10b shows the results of enhanced interaction where the algorithm terminates after 1, 924 iterations (196, 348 utility function evaluations). In Fig. 11 we show the sensitivity analysis with respect to  $p_j$ . We ascertain that  $\Delta p = 0$  is maximum around the estimated equilibrium point. Figure 12 shows the results of a sensitivity analysis carried out by inverting just one decision variable of the players (i.e. setting  $b_{ij} = 1$  if  $b_{ij} = 0$  and vice-versa). We readily see that for both demand models, the change  $\Delta u_j = u'_j - u_j$  is negative as expected in a Nash equilibrium. The sensitivities in the limited interaction game are identical, since the solution is completely symmetric. In the enhanced interaction model, the dominant player (player 2) has the highest sensitivity values as expected.

## 6 Conclusions and future work

We have illustrated how a properly modified Nash GA can be used in order to calculate the equilibrium point in the case of access providers which compete over the provision of a given flat-rated broadband service such as FTTH. We considered two simple demand models. In the first, the competition is limited and each operator attracts subscribers independently of the other, unless the market becomes saturated. In the second model, the total number of subscribers are determined by the provider with the lowest price and other providers draw subscribers from it depending on the difference of their charged price compared to the minimum price. We have shown that the algorithm can handle cases where there are several areas in which each provider can decide whether he wishes to expand or not, depending on the area particularities. Our work can be used to implement decision support tools which can help providers to evaluate the market potential of a broadband technology and regulatory bodies to understand the competition prospects and protect the customer interests.

Our Python-based “solution engine” is publicly available on the web. Possible extensions would be to reduce the computation time by identifying the right heuristics for initializing the algorithm in order to speed up the convergence process, instead of using random initial states. Alternative evolutionary approaches such as colonial competitive algorithms proposed in Rajabioun et al. (2008) can also be studied to ascertain whether they can speed-up convergence. Apart from the engine itself it would be interesting to consider alternative demand models in the presence of competition. These include Markov chains, curve-fitting, etc. One should also adopt a more fine-grained cost model for estimating deployment costs detailed in Skoufis et al. (2020). In FTTH, such costs depend on pre-installed fiber routes (i.e. dark fibers), the position of the provider’s point-of-presence (PoPs), population density, terrain type, etc Rokkas et al. (2010). One could also take into account various inter-relations between the territories, e.g. expanding to area B is easier when one is already present in area A. We can also consider scenarios where an incumbent player is already present in some areas therefore his decision variables may be locked. We plan to address some of these issues in future publications.

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### Declarations

**Conflict of interest** The authors have no competing interests to declare that are relevant to the content of this article.

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## References

- Abdallah T, Vulcano G (2021) Demand estimation under the multinomial logit model from sales transaction data. *Manuf Serv Oper Manag* 23(5):1196–1216
- Adler N, Brudner A, Proost S (2021) A review of transport market modeling using game-theoretic principles. *Eur J Oper Res* 291(3):808–829
- Afraz N, Ruffini M (2019) A distributed bilateral resource market mechanism for future telecommunication networks. In: 2019 IEEE Globecom Workshops (GC Wkshps). IEEE, pp 1–6
- Allon G, Federgruen A (2008) Service competition with general queueing facilities. *Oper Res* 56(4):827–849
- Altman E, Boulogne T, El-Azouzi R, Jiménez T, Wynter L (2006) A survey on networking games in tele-communications. *Comput Oper Res* 33(2):286–311
- Angelou GN, Economides AA (2009) A multi-criteria game theory and real-options model for irreversible ICT investment decisions. *Telecommun Policy* 33(10–11):686–705
- Audestad J, Gaivoronski AA, Werner A (2001) Modelling market uncertainty and competition in telecommunication environment: network providers and virtual operators. *Teletronikk* 97(4):46–64
- Battistella C, Colucci K, De Toni AF, Nonino F (2013) Methodology of business ecosystems network analysis: a case study in telecom Italia future Centre. *Technol Forecast Soc Chang* 80(6):1194–1210
- Bernstein F, Federgruen A (2004) A general equilibrium model for industries with price and service competition. *Oper Res* 52(6):868–886
- Besbes O, Sauré D (2016) Product assortment and price competition under multinomial logit demand. *Prod Oper Manag* 25(1):114–127
- Campbell K, Diffley J, Flanagan B, Morelli B, O’Neil B, Sideco F (2017) The 5g economy: how 5g technology will contribute to the global economy. *IHS Econ IHS Technol* 4:16
- Chouhan AS, Sridhar V, Rao S (2021) Service provider strategies in telecommunications markets: analytical and simulation analysis. *Sādhanā* 46(1):1–10
- Christodoulos C, Michalakelis C, Varoutas D (2010) Forecasting with limited data: combining Arima and diffusion models. *Technol Forecast Soc Chang* 77(4):558–565. <https://doi.org/10.1016/j.techfore.2010.01.009>
- Cooper PW (2020) COVID-19 and the telecommunications industry. [www.pwc.com/us/en/library/covid-19/coronavirus-telecommunication-impact.html](http://www.pwc.com/us/en/library/covid-19/coronavirus-telecommunication-impact.html). Accessed 26 Feb 2021
- Doove S, Gabbitas O, Nguyen-Hong D, Owen J (2001) Price effects of regulation: telecommunications, air passenger transport and electricity supply
- Dramitinos M, Stamoulis GD, Courcoubetis C (2007) An auction mechanism for allocating the bandwidth of networks to their users. *Comput Netw* 51(18):4979–4996
- El Azouzi R, Altman E, Wynter L (2003) Telecommunications network equilibrium with price and quality-of-service characteristics. In: *Teletraffic science and engineering*, vol 5. Elsevier, New York, pp 369–378
- Farahat A, Perakis G (2009) Profit loss in differentiated oligopolies. *Oper Res Lett* 37(1):43–46
- Haupt RL (2007) Antenna design with a mixed integer genetic algorithm. *IEEE Trans Antennas Propag* 55(3):577–582
- Isakhani H, Xiong C, Yue S, Chen W (2020) A bioinspired airfoil optimization technique using nash genetic algorithm. In: 2020 17th international conference on ubiquitous robots (UR). IEEE, pp 506–513
- Jiang D, Liu G (2017) An overview of 5g requirements. *5G Mobile Commun* 3–26
- Kamalakis T (2021) `pynashga`: a Python framework for implementing Nash genetic algorithms. GitHub
- Konak A, Kulturel-Konak S, Snyder LV (2022) Nash equilibrium sorting genetic algorithm for simultaneous competitive maximal covering location with multiple players. *Eng Optim* 54(10):1709–1723
- Lakka S, Michalakelis C, Varoutas D, Martakos D (2013) Competitive dynamics in the operating systems market: modeling and policy implications. *Technol Forecast Soc Chang* 80(1):88–105
- Li H, Webster S, Mason N, Kempf K (2019) Product-line pricing under discrete mixed multinomial logit demand. *Manuf Serv Oper Manag* 21(1):14–28
- Maillé P, Tuffin B (2008) Analysis of price competition in a slotted resource allocation game. In: IEEE INFOCOM 2008-The 27th conference on computer communications. IEEE, pp 888–896
- Maillé P, Tuffin B (2014) *Telecommunication network economics: from theory to applications*. Cambridge University Press, Cambridge

- Maillé P, Naldi M, Tuffin B (2009) Price war with migrating customers. In: 2009 IEEE international symposium on modeling, analysis & simulation of computer and telecommunication systems. IEEE, pp 1–8
- Michalakelis C, Varoutas D, Sphicopoulos T (2010) Innovation diffusion with generation substitution effects. *Technol Forecast Soc Chang* 77(4):541–557
- Migdalas A (2002) Applications of game theory in finance and managerial accounting. *Oper Res Int J* 2:209–241
- Osborne MJ et al (2004) An introduction to game theory, vol 3. Oxford University Press, New York
- Rajabioun R, Atashpaz-Gargari E, Lucas C (2008) Colonial competitive algorithm as a tool for nash equilibrium point achievement. In: International conference on computational science and its applications. Springer, Berlin, pp 680–695
- Ramaswami R, Sivarajan K, Sasaki G (2009) Optical networks: a practical perspective. Morgan Kaufmann, Burlington
- Research GV (2020) Telecom Services Market Worth 2.53 Trillion By 2027. [www.grandviewresearch.com/press-release/telecom-services-market](http://www.grandviewresearch.com/press-release/telecom-services-market). Accessed 26 Feb 2021
- Riahi S, Riahi A (2020) The non-cooperative game theory applied to telecommunication systems. *Int J Inf Sci Technol* 4(2):3–11
- Rokkas T, Katsianis D, Kamalakis T, Varoutas D (2010) Economics of time and wavelength domain multiplexed passive optical networks. *IEEE/OSA J Opt Commun Netw* 2(12):1042–1051
- Sefrioui M, Perlaux J (2000) Nash genetic algorithms: examples and applications. In: Proceedings of the 2000 congress on evolutionary computation. CEC00 (Cat. No. 00TH8512), vol 1. IEEE, pp 509–516
- Sharghivand N, Derakhshan F, Siasi N (2021) A comprehensive survey on auction mechanism design for cloud/edge resource management and pricing. *IEEE Access* 9:126502–126529
- Shubik M (2002) Game theory and operations research: some musings 50 years later. *Oper Res* 50(1):192–196
- Skoufis A, Chatzithanasis G, Dede G, Kamalakis T, Michalakelis C (2020) Technoeconomic analysis of a VDSL2/G.fast vectoring network: a case study from Greece. *NETNOMICS Econ Res Electron Netw* 21(1):83–101
- Sun J, Fu Y, Ji X, Zhong RY (2017) Allocation of emission permits using DEA-game-theoretic model. *Oper Res Int J* 17:867–884
- Taleizadeh AA, Noori-daryan M (2016) Pricing, inventory and production policies in a supply chain of pharmacological products with rework process: a game theoretic approach. *Oper Res Int J* 16:89–115
- Trestian R, Ormond O, Muntean G-M (2012) Game theory-based network selection: solutions and challenges. *IEEE Commun Surv Tutor* 14(4):1212–1231
- Valaskova K, Durica M, Kovacova M, Gregova E, Lazaroiu G (2019) Oligopolistic competition among providers in the telecommunication industry: the case of Slovakia. *Adm Sci* 9(3):49
- Wang HT (2020) Discussion of the competitive strategies of telecom operators and over-the-top service providers from the perspective of evolutionary game theory. *Decis Anal* 17(3):260–275
- Wen P-c, Fu B (2014) Analysis on evolutionary game theory of the collusion and competition strategies between telecom operators. In: 2014 international conference on management science & engineering 21th annual conference proceedings. IEEE, pp 263–270

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