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Commentary on "D-Intuitionistic Hesitant Fuzzy Sets and Their Application in Multiple Attribute Decision Making"

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Abstract

Background Li and Chen (Cognit Comput. 2018; 10:496–505) proposed the concept of the D-intuitionistic hesitant fuzzy set as well as proposed a method for comparing two D-intuitionistic fuzzy sets.
Method Li and Chen have proposed the concept of the D-intuitionistic hesitant fuzzy set by introducing the degree of belief of the decision maker regarding the opinion of an expert in the existing definition of an intuitionistic hesitant fuzzy set.
Results In future, other researchers may use Li and Chen's comparing method in their research work. However, after a deep study, it is observed that Li and Chen's comparing method fails to differentiate two distinct D-intuitionistic fuzzy sets.
Conclusion It is inappropriate to use Li and Chen's comparing method.

Keywords Intuitionistic fuzzy set · Hesitant fuzzy set · D-intuitionistic hesitant fuzzy set

Introduction

Li and Chen [1] pointed out the limitations of the hesitant fuzzy set and generalized hesitant fuzzy sets. Also, to overcome the limitations, Li and Chen [1] proposed the concept of the D-intuitionistic hesitant fuzzy set.

A D-intuitionistic hesitant fuzzy set α is represented as $\alpha = \langle (d_1, \{(\mu_1, \nu_1)\}), (d_2, \{(\mu_2, \nu_2)\}), \dots, (d_p, \{(\mu_p, \nu_p)\}) \rangle$, where,

(i) The intuitionistic fuzzy number (μ_i, ν_i) represents the views of the i^{th} expert.
(ii) d_i represents the degree of belief of the decision maker regarding the views of the i^{th} expert such that $0 \le \sum_{i=1}^{p} d_i \le 1$.
(iii) p represents the number of decision makers.
(iv) $\mu_i + \nu_i \le 1 \forall i$.
(v) $0 \le \mu_i \le 1 \forall i$.
(vi) $0 \le \nu_i \le 1 \forall i$.

Li and Chen [1] also proposed a method for comparing two D-intuitionistic hesitant fuzzy sets. In future, other researchers may use Li and Chen's comparing method [1] in their research work. In this paper, it is shown that Li and Chen's comparing method [1] fails to differentiate two distinct D-intuitionistic fuzzy sets. Hence, it is inappropriate to use Li and Chen's comparing method [1].

Li and Chen's Comparing Method

Li and Chen [1] proposed the following method for comparing two D-intuitionistic hesitant fuzzy sets $\alpha_1 = \langle (d_{11}, \{(\mu_{11}, \nu_{11})\}), (d_{21}, \{(\mu_{21}, \nu_{21})\}), \dots, (d_{p1}, \{(\mu_{p1}, \nu_{p1})\}) \rangle$ and $\alpha_2 = \langle (d_{12}, \{(\mu_{12}, \nu_{12})\}), (d_{12}, \{(\mu_{12}, \nu_{12})\}), \dots, (d_{q2}, \{(\mu_{q2}, \nu_{q2})\}) \rangle$.

Check that $S(\alpha_1) < S(\alpha_2)$ or $S(\alpha_1) > S(\alpha_2)$ or $S(\alpha_1) = S(\alpha_2)$ where, $(\alpha_1) = \sum_{i=1}^{p} \left(\frac{d_{i1}}{2} (\mu_{i1} + 1 - \nu_{i1}) \right) + (1 - \sum_{i=1}^{p} d_{i1})\theta$, $S(\alpha_2) = \left(\sum_{i=1}^{q} \frac{d_{i2}}{2} (\mu_{i2} + 1 - \nu_{i2}) \right) + (1 - \sum_{i=1}^{q} d_{i2})\theta\cdot$

Case (i): If $S(\alpha_1) < S(\alpha_2)$, then $\alpha_1 \prec \alpha_2$.
Case (ii): If $S(\alpha_1) > S(\alpha_2)$, then $\alpha_1 \succ \alpha_2$.
Case (iii): If $S(\alpha_1) = S(\alpha_2)$, then $\alpha_1 = \alpha_2$.

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Inappropriateness of Li and Chen's Comparing Method

In this section, some numerical examples are considered to show the inappropriateness of Li and Chen's comparing method [1].

1. It is obvious that $\alpha_1 = \langle (0.6, \{(0.1,0.3)\}), (0.4, \{(0.2,0.4)\}) \rangle$ and $\alpha_2 = \langle (0.6, \{(0.3,0.5)\}), (0.4, \{(0.15,0.35)\}) \rangle$ are two distinct D-intuitionistic hesitant fuzzy sets, i.e., $\alpha_1 \neq \alpha_2$.

   While, as $S(\alpha_1) = \sum_{i=1}^{p}\left(\frac{d_{i1}}{2}\left(\mu_{i1} + 1 - v_{i1}\right)\right) + (1 - \sum_{i=1}^{p} d_{i1})\theta = \frac{0.6}{2}(0.1 + 1 - 0.3) + \frac{0.4}{2}(0.2 + 1 - 0.4) + (1 - 0.6 - 0.4)\theta = 0.24 + 0.16 = 0.40$ is equal to $S(\alpha_2) = \sum_{i=1}^{q}\left(\frac{d_{i1}}{2}\left(\mu_{i1} + 1 - v_{i1}\right)\right) + (1 - \sum_{i=1}^{q} d_{i2})\theta = \frac{0.6}{2}(0.3 + 1 - 0.5) + \frac{0.4}{2}(0.15 + 1 - 0.35) + (1 - 0.6 - 0.4)\theta = 0.24 + 0.16 = 0.40$.

   Therefore, according to Case (iii) of Li and Chen's comparing method [1], discussed in Sect. 2, $\alpha_1 \neq \alpha_2$, which is mathematically incorrect.
2. It is obvious that $\alpha_1 = \langle (0.6, \{(0.15,0.45)\}), (0.3, \{(0.2,0.3)\}) \rangle$ and $\alpha_2 = \langle (0.6, \{(0.25,0.55)\}), (0.3, \{(0.15,0.25)\}) \rangle$ are two distinct D-intuitionistic hesitant fuzzy sets i.e., $\alpha_1 \neq \alpha_2$.

   While, as $S(\alpha_1) = \sum_{i=1}^{p}\left(\frac{d_{i1}}{2}\left(\mu_{i1} + 1 - v_{i1}\right)\right) + (1 - \sum_{i=1}^{p} d_{i1})\theta = \frac{0.6}{2}(0.15 + 1 - 0.45) + \frac{0.3}{2}(0.2 + 1 - 0.3) + (1 - 0.6 - 0.3)\theta = 0.21 + 0.135 + 0.1\theta = 0.344 + 0.1\theta$ is equal to $S(\alpha_2) = \sum_{i=1}^{q}\left(\frac{d_{i1}}{2}\left(\mu_{i2} + 1 - v_{i2}\right)\right) + (1 - \sum_{i=1}^{q} d_{i2})\theta = \frac{0.6}{2}(0.25 + 1 - 0.55) + \frac{0.3}{2}(0.15 + 1 - 0.25) + (1 - 0.6 - 0.3)\theta = 0.21 + 0.135 + 0.1\theta = 0.344 + 0.1\theta\cdot$

   Therefore, according to Case (iii) of Li and Chen's comparing method [1], discussed in "Li and Chen's Comparing Method," $\alpha_1 \neq \alpha_2$, which is mathematically incorrect.

Conclusions

It can be easily concluded from "Li and Chen's Comparing Method" that Li and Chen's comparing method [1] can be used to compare two such distinct D-intuitionistic hesitant fuzzy sets α_1 and α_2 for which either the condition $S(\alpha_1) < S(\alpha_2)$ or the condition $S(\alpha_1) > S(\alpha_2)$ will be satisfied. However, Li and Chen's comparing method [1], discussed in "Li and Chen's Comparing Method", cannot be used to compare two such distinct D-intuitionistic hesitant fuzzy sets α_1 and α_2 for which the condition $S(\alpha_1) = S(\alpha_2)$ will be satisfied. To overcome this, limitation of Li and Chen's comparing method [1] may be considered as a challenging open research problem.

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Declarations

Ethical Approval This article does not contain any studies with human participants or animals performed by any of the authors.

Conflict of interest The authors declare that they have no conflict of interest.

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