A polynomial scale transformation and improved wiener process for a novel lithium-ion battery performance degradation model: Remaining useful life performance

Chao Fu, Qing Lv, Ming-Lang Tseng*, Xiancong Wu, Ming K. Lim

Abstract

This study contributes to propose a novel lithium-ion battery performance degradation model based on improved wiener process. The aging of lithium-ion batteries brings potential hazards to the power system of electric vehicles, so the health status of lithium-ion batteries needs to be evaluated. First, a polynomial scale transformation model is established to scale the cycle number to transform the nonlinear Wiener process into linear Wiener process, and model parameters are estimated by the maximum likelihood functions. Second, a performance degradation model based on the improved wiener process is constructed to estimate the remaining useful life (RUL) performance, in which the cumulative loss reaching the failure threshold is taken as the failure criterion. Finally, the proposed RUL estimation method is tested using data provided by NASA. The test results proved that the estimation errors of proposed model were controlled within 15%. The RUL estimation method proposed in this study provides a new way for the reliability evaluation of lithium-ion batteries and guarantees the safe operation of electric vehicle power system.

Keywords: polynomial scale transformation; improved wiener process; performance degradation model; remaining useful life; Lithium-ion battery

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1 Introduction

At present, governments all over the world have formulated corresponding energy conservation and emission reduction plans (Liu et al., 2021, Li et al., 2021). The electric vehicles (EVs) sales share will reach 18% in 2028 according to the prediction report of EVAdoption, and the lithium-ion battery industry has ushered in a new round of development opportunities benefited from the continuous growth of new energy vehicles (EVAdoption, 2019). The specific energy of lithium-ion batteries is the highest among the commercial rechargeable batteries, especially polymer lithium-ion batteries to realize the thinness of rechargeable batteries (Jaramillo-Cabanzo et al., 2021). The development of new energy vehicles can effectively control the consumption of fossil energy (Liu et al., 2021; Sun et al., 2021). In 2021, the number of EVs exceeded 5.5 million in the world, and it grows rapidly in the next few years (Global EV Outlook, 2019). States in the United States have formulated policies on the development of EVs and created the automobile zero emission alliance (Zhang et al., 2020). Hence, lithium-ion batteries are widely applied to EVs due to its high energy density and low pollution. To ensure the reliable operation of the power system of EVs, the health assessment of lithium-ion batteries has become a hot issue studied by experts.

The important lithium-ion battery parameters are monitored and managed by battery management system. State of charge and state of health (SOH) are key indicators to the system components and estimation has always been a hotspot and difficulty, which are employed to describe the battery degradation degree, and decreases with the increase of lithium-ion battery cycle times (Koga et al., 2021; Sun et al., 2021). Ungurean et al. (2017) pointed that the battery is considered to be invalid when SOH drops to 70% or 80% and the RUL is usually used to characterize SOH. Chen et al. (2021) argued that RUL estimation methods have focused on two types, namely model-based methods and data-driven methods. Model-based methods reflect the performance degradation process of lithium-ion batteries through establishing mathematical models, such as electrochemical model. For instance, Sadabadi et al. (2021) and Verma et al. (2020) realized the RUL estimation through the electrochemical model, but Sadabadi et al. (2021) and Verma et al. (2020) ignored to address the calculation costs, complicated structure and large unknown parameters of electrochemical model that limits its application. RUL as an important assessment indicator of SOH is a current research hotspot, which realizes accurate estimation of lithium-ion battery capacity under dynamic conditions. Qiu et al. (2020) established a mathematical model to simulate the performance degradation trajectory of lithium battery. Xue et al. (2020) pointed that the RUL estimation methods based on filtering theory establish the system state equation and measurement equation, and estimate the state quantity online. In order to improve the estimation accuracy of lithium-ion battery capacity, Lai et al. (2021) proposed an improved extended Kalman filter method. Zhang et al. (2020) improved the linear fractional Brownian motion and proposed a nonlinear drift fractional Brownian motion method to predict RUL. In this method, hidden state variables are considered and updated by particle filter. However, Lai et al. (2021) and Zhang et al. (2020) ignored that PF has the limitations of particle degradation, and particle weights cannot be updated in the estimation stage.

Especially, estimation methods on the basis of data-driven models have focused on machine learning models and time series models. Various machine learning models have been devised, such as SVM, BPNN and relevance vector machine. For instance, Ren et al. (2021) combined Long Short-Term Memory (LSTM) neural network and convolution neural network to predict the RUL of lithium-ion battery, and used automatic encoder to increase the dimension of training samples, so as to fully train the established prediction model. However, Ren et al. (2021) lacks to address the long training time and high complexity of LSTM. Machine learning models have high accuracy, but the random parameters have a greater impact on the estimation results. Common time series models include moving average (MA) model, autoregressive (AR) model and autoregressive moving average model (Fan et al., 2021). However, time series models only consider the time factor, have poor generalization ability and are susceptible to external environmental interference. In this study, a performance degradation model on the basis of improved wiener process was proposed to estimate the RUL of lithium-ion batteries. First, the single cumulative loss normal distribution was tested. Second, a third-order scale transformation model was constructed to scale the number of cycles corresponding to the cumulative capacity. Finally, the maximum likelihood estimation of drift and diffusion parameters after scale transformation was carried out to obtain the RUL probability density function to estimate RUL. The objectives of this study are as follows:

- To establish scale model to convert nonlinear wiener process into linear wiener process;
- To establish performance degradation model of the lithium-ion battery under different cumulants on the basis of the improved wiener process; and
- To estimate RUL based on lithium-ion battery performance degradation model and obtain reliability functions of RUL under different cumulants.

This study presents the following contributions: (1) improving wiener process to estimate the RUL, and the estimation error was controlled within 15%; (2) establishing the third-order scale transformation model to scale the number of cycles corresponding to the cumulants; and (3) judging the health status of lithium-ion battery by RUL estimation to ensure the reliable operation of EVs.

The rest of this study is as follows. Second 2 presents the estimation methods of RUL and the application of wiener process. Section 3 introduces the principle of improved wiener process. Section 4 analyzes and discusses the simulation results. Section 5 presents the conclusions and the future research plan.

2 Method

2.1 *wiener model based on scale transformation*

The battery fails when the capacity of lithium-ion battery decreases to 70% to 80% of the rated capacity. Capacity loss in the performance degradation model is taken as the key performance parameter (Ng et al., 2014; Miao et al., 2013). The RUL of lithium-ion battery is determined through analyzing the current capacity degradation. The single capacity loss obeys the independent and identical distribution, that is, the normal distribution.

Wiener process, which is a typical stochastic process, is called Brownian motion with drift (Tsai et al., 2011). Brownian motion describes the random movement process of particles under collision, and wiener process is suitable for describing the nonstationary degradation process of product failure due to a large amount of accumulated loss. The motion is called Brownian motion if the random process $\{C(t), t\geq 0\}$ satisfies the following three constraints (Lawrynczuk, 2019):

(1) $C(t=0)=0$;

(2) $\forall t>m>0, C(t)-C(m)\sim N(0,\sigma^2(t-m))$;

(3) The random process $C(t)$ has independent and stable increments.

$\{C(t), t\geq 0\}$ is a standard Brownian motion when σ^2 is 1. The increment distribution of the wiener process depends on the time difference, so the wiener process is a homogeneous independent incremental process and obeys the normal distribution (El-Hadidy and Alfreedi, 2019). For $t(t\in[1,n])$, $C(t_k)$ is shown as follows:

$$C(t_k)=\sum_{i=1}^{k}[C(t_i)-C(t_{i-1})] \quad k\in[1,n] \tag{1}$$

The one-dimensional wiener process with drift is employed to describe the capacity degradation process of lithium-ion batteries (Lim et al. 2019).

$$U(t)=u_0+\mu\times t+\sigma\times C(t) \tag{2}$$

where u_0 is the initial value; μ indicates the degradation rate, which is used to describe the degradation trend; σ is the diffusion speed, which describes the influence of random factors on the degradation performance; $C(t)$ is the standard Brownian motion; and $U(t)$ follows the standard normal distribution $N(u_0+\mu t, \sigma^2 t)$.

The degradation amount shows an increasing trend when $\mu>0$, and the degradation amount shows a decreasing trend when $\mu<0$. The performance degradation model is established by equation (2) when the degradation rate of the product is constant. In practical engineering, the degradation rate of products is generally nonlinear. The linear wiener model cannot get accurate life estimation. However, the nonlinear degradation process can be transformed into linear degradation process by scale transformation. Therefore, the scale transformation model needs to be established to transform the time scale t.

According to the performance degradation curve of the product, the time scale conversion model is derived to convert the time t. Suppose that there is a non-negative monotonic increasing function $\beta(m)$ of t, such that $\mu t=\mu_*\beta(t)$, then the one-dimensional wiener degradation model after scale transformation is obtained:

$$U(\beta(t))=u_0+\mu_*\times\beta(t)+\sigma\times C(\beta(t)) \tag{3}$$

where μ_* is the degradation rate after scale transformation.

Let $\alpha=\beta(t)$ and $G(\alpha)=U(t)$, so that equation (3) is transformed into equation (4).

$$G(\alpha)=u_0+\mu_*\times\alpha+\sigma\times C(\alpha) \tag{4}$$

The nonlinear degradation process is transformed into linear degradation process by equation (4), and the degradation model is established according to the one-dimensional wiener process after time scale transformation. In equation (4), the degradation path is linear wiener degradation process when $\alpha=t$.

2.2 *Lithium-ion battery performance degradation modeling based on improved wiener process*

There are some cusps in the capacity degradation curve of lithium-ion battery affected by environment in the test process. The lithium-ion battery capacity degradation curve of was reconstructed by wavelet transform to reduce the influence of cusps. The reconstructed data of lithium-ion battery capacity was employed as experimental data. Then, different polynomial models were used to fit the capacity loss under different cumulants. Finally, root mean square error (*RMSE*) and coefficient of determination (*R*2) were applied to evaluate the fitting effect of different polynomial models. *RMSE* and coefficient of determination (*R*2) are expressed as follows:

$$RMSE = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(z_i - z_i^*)^2} \tag{5}$$

$$R2 = 1 - \frac{\sum_{i=1}^{m}(z_i - z_i^*)^2}{\sum_{i=1}^{m}(z_i - \overline{z})^2} \tag{6}$$

where m is the number of cycles corresponding to lithium-ion battery failure; z_i denotes the actual value of cumulative capacity loss; z_i^* represents the fitting value of cumulative capacity loss; $\overline{z}$ indicates the average value of cumulative capacity loss.

RMSE and *R*2 are usually chosen as evaluation indicators to evaluate the prediction results. The smaller the *RMSE* value is, the smaller the error between fitting value and actual value of polynomial model is. The closer the *R*2 ($R2 \in [0, 1]$) is to 1, the better the fitting effect of polynomial model is; the closer *R*2 is to 0, the worse the fitting effect of polynomial model is.

Suppose the cumulative capacity degradation path of the lithium-ion battery is $\beta(m)$. The capacity degradation process conforms to the linear wiener process when m is equal to $\beta(m)$. The function $\beta(m)$ is employed to scale the number of cycles of cumulative capacity loss, and the number of cumulative capacity loss after scale transformation is recorded as $m(z(m)=z(m^*))$.

$$m^* = \beta(m) \tag{7}$$

The nonlinear degradation process of cumulative capacity loss z is transformed into a linear degradation process by scaling transformation, and the nonlinear wiener process is transformed into a linear wiener process. The relationship between the number of cycles and cumulative loss is more in line with the requirements of wiener process after polynomial scale transformation. The cumulative capacity degradation model of lithium-ion battery is depicted in equation (8):

$$\begin{cases} z(m^*) = z_0 + \mu^* \times m^* + \sigma^* \times C(m^*) \\ z(m^*) \sim N(\mu^* m^*, (\sigma^*)^2 m^*) \end{cases} \tag{8}$$

where z_0 indicates the cumulative capacity loss at the initial time, and there is no capacity loss in the initial stage, so z_0 is 0; μ^* is the degradation rate of cumulative capacity loss, which reflects the lithium-ion battery performance degradation trend; m^* represents the number of cycles corresponding to the cumulative capacity loss after scale transformation; σ^* denotes the cumulative capacity diffusion speed that reflects the influence of random factors in the process of degradation; and $\beta(m^*)$obeys the standard Brownian motion.

2.3 *Principle of life estimation and reliability calculation*

The cumulative capacity loss is taken as the performance parameter to characterize the lithium-ion battery life. The number of cycles corresponding to the failure is used as the life value and the corresponding cumulative capacity loss is taken as the failure threshold. The lithium-ion battery is in failure state when the capacity of batteries decreases to 70% to 80% of the rated capacity, and the failed battery should be replaced to make the system operate safely.

Equation (9) is employed to define the RUL:

$$F = \inf\{m : z(m) \geq L \mid z(0) \leq L\} \tag{9}$$

For the improved wiener process, the key is to find the scale transformation model and transform the nonlinear degradation process into the linear degradation process. Then the probability density function $f(m^*, L)$ of lithium-ion battery life after scale transformation is obtained. For the linearized degradation process, the time distribution is inverse Gaussian distribution when wiener process reaches the failure threshold for the first time. Under the failure threshold L, the probability density function of residual life after scale transformation is as follows:

$$f(m^*; L) = \frac{L - z_0}{\sqrt{2\pi(\sigma^*)^2(m^*)^3}} e^{\frac{-(L - z_0 - \mu^* m^*)^2}{2(\sigma^*)^2 m^*}} \tag{10}$$

Generally, the expected value of RUL is taken as the life estimation value. The capacity degradation reliability function is built through equation (10):

$$R(m^*;L)=\Phi\left(\frac{L-z_0-\mu^*m^*}{\sigma^*\sqrt{m^*}}\right)-e^{\frac{-2\mu^*(L-z_0)}{(\sigma^*)^2}}\Phi\left(\frac{-L+z_0-\mu^*m^*}{\sigma^*\sqrt{m^*}}\right) \quad (11)$$

In practical engineering application, the actual value of failure threshold is difficult to determine. It is generally believed that the same batch of products have the same failure threshold, and the performance degradation path of the same batch of products is similar under the mature design condition and processing technology. The scale transformation model was established according to the historical degradation information to transform the time scale of products to estimate the RUL.

3 Results & Discussion

The battery data provided by NASA was used in the simulation experiment. Charge and discharge tests of lithium-ion batteries were carried out at room temperature. First, the battery was charged at a constant current of 1.5A, the battery was charged at constant voltage when the battery voltage reached 4.2V, and the battery was stopped charging when the charging current of the battery dropped to 0.02A. Second, the battery discharged with a constant current of 2A, the battery was stopped when the battery voltage dropped to 2.5V. The charge and discharge test of lithium-ion battery was repeated until the battery reached the failure standard, and the battery experiment was terminated. sym5 wavelet function was applied to decompose the lithium-ion battery capacity degradation data. The decomposition layer was 3 layers, and a soft threshold function was used. Then the lithium-ion battery capacity data was reconstructed and the capacity degradation curve and cumulative capacity curve after reconstruction are depicted in the Figure 1.

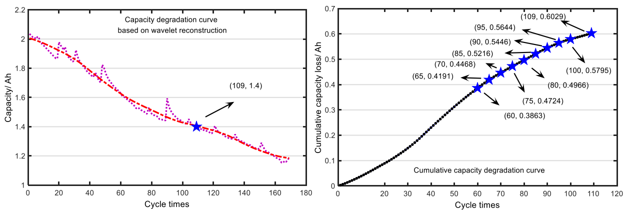


Figure 1. Wavelet reconstruction curve and cumulative capacity curve

The reconstructed lithium-ion battery capacity degradation data was smoother and reflected the degradation characteristics of the original data. In this study, it is considered that the battery fails when the battery decays to 70% of the rated capacity (1.4Ah). The number of cycles corresponding to the failure is 109, and the corresponding failure threshold L is 0.6029. Figure 1 revealed that the cumulative capacity was non-linear, so the polynomial scale transformation was employed to linearize the cumulative capacity loss.

3.1 *Polynomial scale transformation model*

The first 60, 70, 80 and 90 cumulants were fitted to obtain the polynomial scale transformation model. Then the polynomial scale transformation model was applied to scale the cycle number corresponding to the lithium-ion battery capacity accumulation. The first-order, second-order, third-order and fourth-order models were established to fit the capacity loss accumulation curves. The *RMSE* and coefficient of determination ($R2$) were employed to evaluate the fitting effect of each polynomial model. The expressions of each order model are revealed in Table 1.

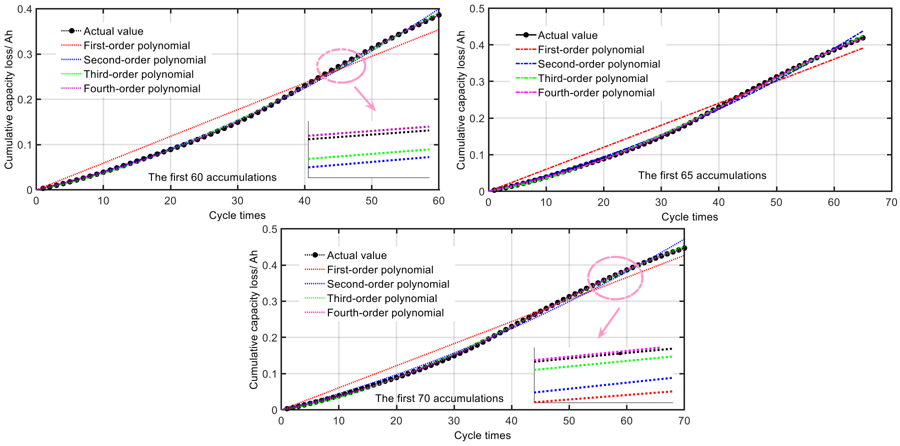
Table 1. Polynomial fitting models

Model	Equation
First-order model	$y_1(x)=p_1\times x$
Second-order model	$y_2(x)=p_1\times x^2+p_2\times x$
Third-order model	$y_3(x)=p_1\times x^3+p_2\times x^2+p_3\times x$
Fourth-order model	$y_4(x)=p_1\times x^4+p_2\times x^3+p_3\times x^2+p_4\times x$

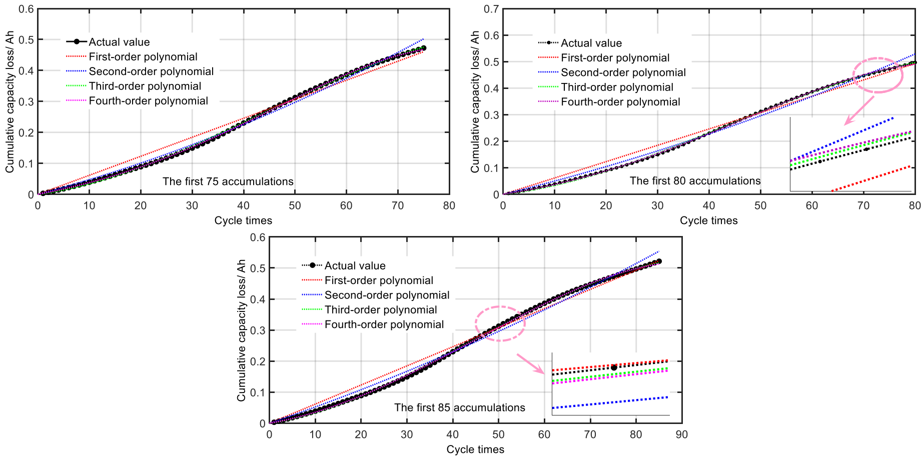
Note: p_i is the polynomial fitting coefficient.

First-order, second-order, third-order and fourth-order polynomial models were established to fit the cumulative capacity values of lithium-ion batteries at 60, 65, 70, 75, 80, 85, 90, 95 and 100 times. The fitting effects of each order model are depicted in Figure 2.

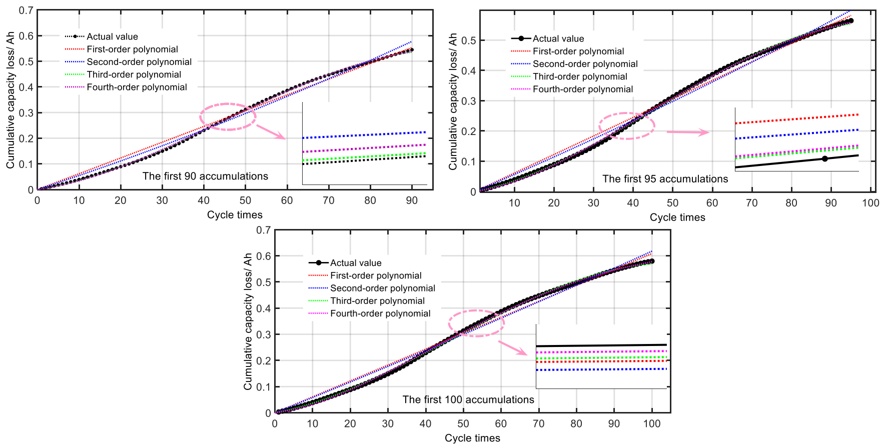
Figure 2 indicated that the fitting accuracy of the first-order and second-order models were poor, and the fitting curves had a large deviation from the true value curves. The fitting curves of the third-order model and the fourth-order model could reflect the degradation trend of battery cumulative capacity loss. To reduce the calculation cost, this study used the third-order fitting model to change the time scale of the accumulation capacity loss. The estimated parameters of the third-order scale transformation model are in Appendix.



(a) Fitting results of polynomial models for the first 60 to 70 cumulative capacity losses



(b) Fitting results of polynomial models for the first 75 to 85 cumulative capacity losses



(c) Fitting results of polynomial models for the first 90 to 100 cumulative capacity losses

Figure 2. Fitting results under different accumulations

3.2 *Life estimation and reliability evaluation*

The probability density function curves of lithium-ion battery life under different cumulants were drawn by the obtained probability density functions, as revealed in Figure 3.

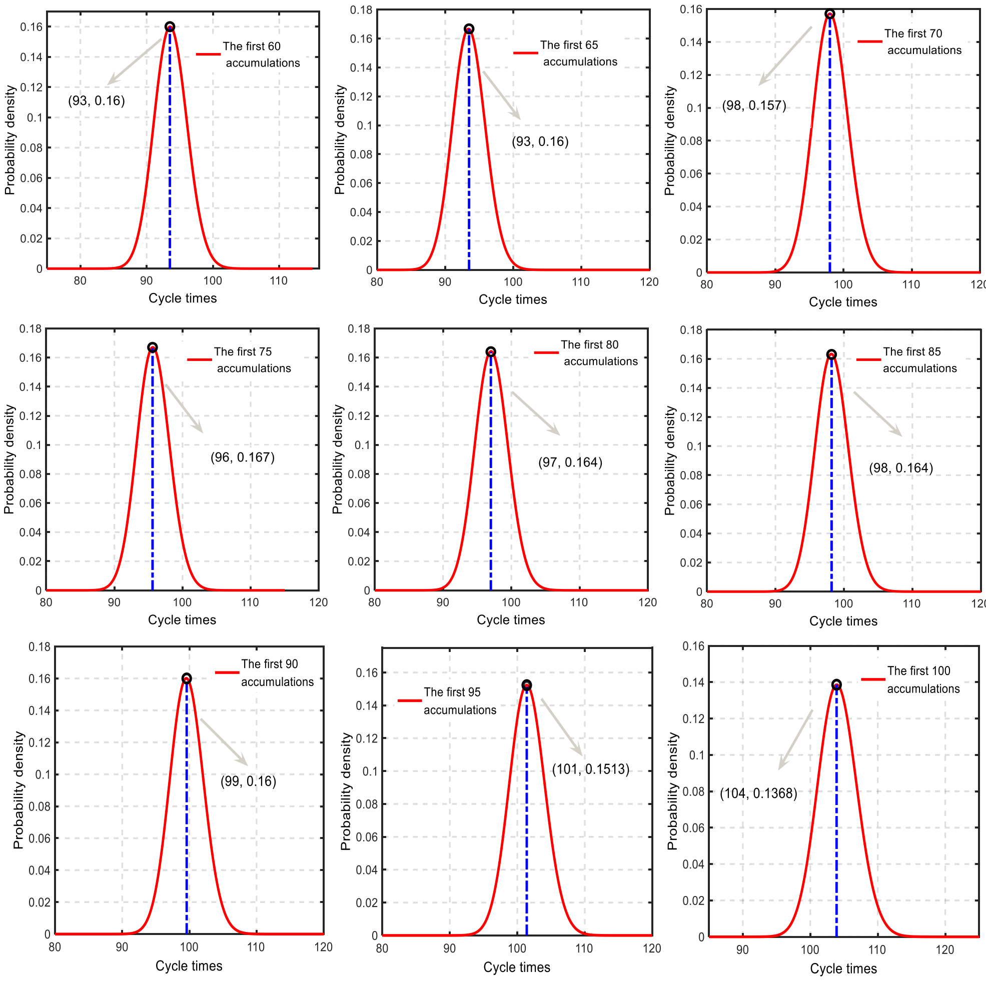


Figure 3.Probability density curves under different cumulants

The expected value of the probability density function is taken as the battery life estimation value. The life estimation values under different cumulants were obtained from Figure 3 were not the same. The lithium-ion battery life estimated value had a large deviation from the actual value when the cumulative capacity loss was small. This is due to the small amount of cumulants that cannot reflect the capacity degradation trend. With the increase of accumulations, the estimated RUL value was more and more close to the actual value. The historical degradation information was more complete when the cumulants increased, and the parameter estimation in scale transformation model was more accurate. Therefore, the RUL estimation value with more cumulants was more accurate. The battery life estimation belongs to long-term estimation when the cumulative capacity loss is small, and the life estimation belongs to short-term estimation with the increase of cumulants. The short-term estimation accuracy is higher than the long-term estimation as the battery capacity degradation information becomes more complete.

The life estimated values were obtained through the probability functions and lithium-ion battery life reliability functions. Table 2 lists the estimated results.

Table 2. Estimated results of improved wiener model

Cycle time	Actual life	Calculated life value	Absolute error
60		93	-16
65		93	-16
70		98	-11
75		96	-13
80	109	97	-12
85		98	-11
90		99	-10
95		101	-8
100		104	-5

Table 2 depicted that the errors between the estimated value and the real value of the improved wiener process were different under different cumulants. The biggest estimation errors under 60 and 65 cumulants were 16 due to the capacity degradation information was not complete in the early stage of prediction, and less cumulants could not accurately reflect the overall degradation trend. Hence, the scale transformation model obtained by fitting cannot accurately reflect the degradation trend. The estimated errors of the improved wiener process decreased with the increase of cumulants. This is because the degradation information is more complete with the increase of accumulations, and the scale transformation model can more accurately reflect the capacity degradation trend, so the estimated result of improved wiener process is closer to the real value. The estimated battery life value of the improved wiener process is smaller than the actual life. Because the estimated value is smaller than the actual value, engineers can replace aging batteries in advance to avoid causing safety accidents due to excessive battery aging in practical engineering application. The life value and reliability are estimated by the improved wiener process to realize on-line monitoring and early warning of loss to determine the maintenance plan and improve the system reliability through fault pretreatment.

In terms of Lithium-ion battery RUL, many studies applied data-driven models to this field and this study adopted the classic Support Vector Machine (SVM) model and Back Propagation Neural Network (BPNN) model as comparison models to predict RUL. The data reconstructed by wavelet was used as the training sample and the test sample of the SVM and BPNN models. The first 60 to 100 capacity data were taken as the training samples of the SVM and BPNN models to analyze the prediction results. The predictive results of SVM model are presented in Table 3.

SVM model did not predict the lithium-ion battery life value when the number of training samples was 60 and 65. The SVM model has high requirements for the number of training samples. Table 3 revealed that SVM model accurately predicted the RUL with the increase of sample number. When the number of training samples reached 70 or more, SVM model could predict the Lithium-ion battery RUL. We can find that SVM is sensitive to the number of training samples. SVM model cannot accurately predict the life value when the number of training samples is small; SVM model accurately predicts the life value when there are enough training samples.

The first 60 to 100 capacity data were used as the training samples of the BPNN model, and then the BPNN model was employed to predict the lithium-ion battery life value under different training samples. The prediction results of the BPNN model are depicted in Table 4.

Table 3. Calculation results of SVM model

Number of training samples	Actual life	Calculated life value	Absolute error
60		—	—
65		—	—
70		113	6
75		111	2
80	109	111	2
85		112	3
90		111	2
95		111	2
100		111	2

Table 4. Calculation results of BPNN model

Number of training samples	Actual life	Calculated life value	Absolute error
60		—	—
65		—	—
70		104	-5
75		104	-5
80	109	113	4
85		112	3
90		113	4
95		110	1
100		110	1

The prediction results of BPNN model were similar to those of the SVM model. The BPNN model did not predict the RUL when the numbers of training samples were 60 or 65. Similarly, the prediction results of BPNN model are sensitive to the number of training samples. Although the BPNN model has a higher predictive accuracy, the predictive results of the model are basically bigger than the actual life value of the battery, which results in the failed battery cannot be detected in time and continue to aging. Figure 4 depicts the relative errors between the predicted value and the actual value of three predictive models.

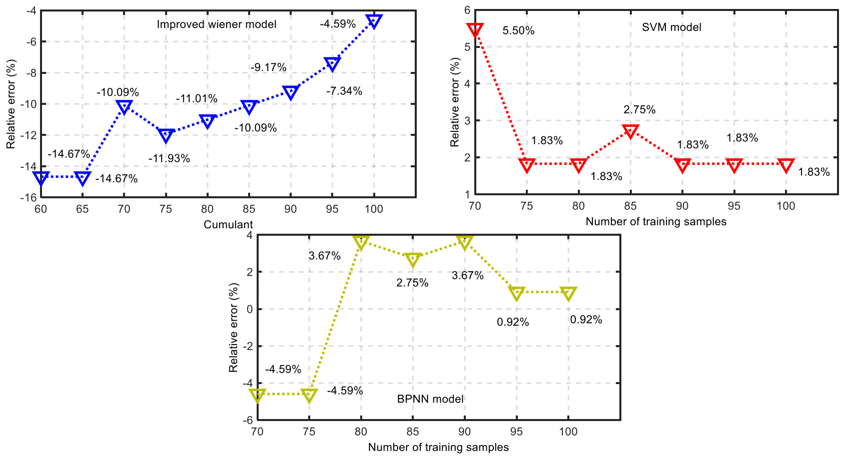


Figure 4. Relative errors

The estimated errors of improved wiener process were controlled within 16% to meet the needs of practical engineering. The estimated life values of the improved wiener process are all smaller than the actual battery life value, which makes engineers replace the failed battery more efficiently. In addition, the predicted values of the data-driven models are greater than the failure threshold of the battery, causing the battery to continue to run in the failure mode. Therefore, the proposed improved wiener process has certain application advantages on the premise of satisfying the prediction accuracy.

4 Concluding remarks

Lithium-ion batteries, which have better energy density and improve the driving mileage, are widely used in pure EVs. The safe and reliable operation of lithium-ion battery is important for EVs. RUL as a key SOH performance parameter is applied to evaluate the aging degree of lithium-ion battery. Therefore, on the basis of the improved wiener process, the performance degradation model was constructed to estimate the RUL, and was compared with data-driven models. The findings are as follows:

- A polynomial model was established to fit the lithium-ion battery cumulative loss curve. The $R2$ values of the third-order and fourth-order polynomial models reached 0.99, indicating that the fitting accuracy of two polynomial models was high.
- The third-order polynomial model was used as the scale transformation model to reduce the computational cost. The nonlinear wiener process was transformed into a linear wiener process by the third-order polynomial model.
- RUL was estimated by the performance degradation model. The estimation results revealed that the estimation error became smaller and smaller with the increase of cumulants, that is, the degradation information becomes more and more complete with the increase of cumulants, which improves the estimation accuracy of the model.
- SVM and BPNN models were more sensitive to the number of training samples. SVM and BPNN models did not predict the RUL when the number of training sample was small. Compared with SVM and BPNN models, the estimated values of RUL of the proposed model were smaller than the actual value, which could help engineers detect the failed battery more timely to ensure the safety of EV power system.

The contributions of this study are as follows: (1) a third-order time scale transformation model is established to scale the cycle numbers corresponding to the cumulative capacity loss; (2) a performance degradation model based on improved wiener process is proposed to estimate RUL; and (3) the proposed RUL estimation model has positive significance for the EV industry development.

This study proposes a novel RUL estimation model which can be applied to practical engineering; still, the limitations are existed. The mathematical statistical model is used to estimate the RUL, but the mathematical statistical model estimation accuracy is lower than the data-driven model. Future studies combine mathematical statistics model and data-driven model to establish a hybrid method under multi-time scales to improve the estimation accuracy.

Author Contributions

Chao Fu - Conceptualize, original version and finalized the final version; **Qing Lv-** Conceptualize, original version and finalized the final version; **Ming-Lang Tseng** - Conceptualize, original version and finalized the final version; Ming-**Lang Tseng-** Conceptualize, original version and finalized the final version; **Xiancong Wu** - Conceptualize, original version and finalized the final version; **Ming K. Lim** - Conceptualize, original version and finalized the final version

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Consent to Participate

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Not applicable

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References

1. El-Hadidy M A, Alfreedi AA. 2019. Internal truncated distributions: applications to Wiener process range distribution when deleting a minimum stochastic volatility interval from its domain. Journal of Taibah University for Science, 13(1), 201-215.
2. EV Sales Forecasts – EVAdoption 2019, https://evadoption.com/ev-sales/ev-sales-forecasts/. Google Scholar.
3. Fan D, Sun H, Yao J, Zhang K, Yan X, Sun Z. 2021. Well production forecasting based on ARIMA-LSTM model considering manual operations. Energy, 220.
4. Global EV Outlook 2019, https://www.iea.org/publications/reports/globalevoutlook2019/. Google Scholar.
5. Jaramillo-Cabanzo DF, Ajayi BP, Meduri P, Sunkara MK. 2021. One-dimensional nanomaterials in lithium-ion batteries. Journal of Physics D-Applied Physics, 54(8), 40.
6. Koga S, Camacho-Solorio L, Krstic M. 2021. State Estimation for Lithium-Ion Batteries with Phase Transition Materials Via Boundary Observers. Journal of Dynamic Systems Measurement and Control-Transactions of the Asme, 143(4).
7. Lai X, Yi W, Cui YF, Qin C, Han XB, Sun T, . . . Zheng YJ. 2021. Capacity estimation of lithium-ion cells by combining model-based and data-driven methods based on a sequential extended Kalman filter. Energy, 216, 14.
8. Lawrynczuk M. 2019. Identification of Wiener models for dynamic and steady-state performance with application to solid oxide fuel cell. Asian Journal of Control, 21(4), 1836-1846.
9. Li LL, Liu ZF, Tseng ML, Zheng SJ, Lim MK. 2021. Improved tunicate swarm algorithm: Solving the dynamic economic emission dispatch problems. Applied Soft Computing, 108.
10. Liu ZF, Luo SF, Tseng ML, Liu HM, Li L, Mashud AHM. (2021). Short-term photovoltaic power prediction on modal reconstruction: A novel hybrid model approach. Sustainable Energy Technologies and Assessments, 45, 17.
11. Lim, H., Kim, Y. S., Bae, S. J., Sung, S. I. 2019. Partial accelerated degradation test plans for Wiener degradation processes. Quality Technology and Quantitative Management, 16(1), 67-81.
12. Liu J., Bai J. Y., Deng, Y., Chen, X. H., Liu, X. 2021. Impact of energy structure on carbon emission and economy of China in the scenario of carbon taxation. Science of the Total Environment, 762, 11.
13. Liu ZF, Li LL, Liu YW, Liu JQ, Li HY, Shen Q. 2021. Dynamic economic emission dispatch considering renewable energy generation: A novel multi-objective optimization approach. Energy, 235, 19.
14. Miao Q, Xie L, Cui HJ, Liang W, Pecht M. 2013. Remaining useful life prediction of lithium-ion battery with unscented particle filter technique. Microelectronics Reliability, 53(6), 805-810.
15. Ng SSY, Xing YJ, Tsui KL. 2014. A naive Bayes model for robust remaining useful life prediction of lithium-ion battery. Applied Energy, 118, 114-123.
16. Qiu XH, Wu WX, Wang SF. 2020. Remaining useful life prediction of lithium-ion battery based on improved cuckoo search particle filter and a novel state of charge estimation method. Journal of Power Sources, 450, 13.
17. Ren L, Dong J, Wang X, Meng Z, Zhao L, Deen MJ. 2021. A Data-Driven Auto-CNN-LSTM Prediction Model for Lithium-Ion Battery Remaining Useful Life. IEEE Transactions on Industrial Informatics, 17(5), 3478-3487.
18. Sun W., Zhang, H., Tseng, ML., Zhang W. .Li Xin. 202. Hierarchical energy optimization management of active distribution network with multi-microgrid system, Journal of Industrial and Production Engineering, DOI: 10.1080/21681015.2021.1972478
19. Sadabadi KK, Jin X, Rizzoni G. 2021. Prediction of remaining useful life for a composite electrode lithium ion battery cell using an electrochemical model to estimate the state of health. Journal of Power Sources, 481, 10.
20. Tsai CC, Tseng ST, Balakrishnan N. 2011. Mis-specification analyses of gamma and Wiener degradation processes. Journal of Statistical Planning and Inference, 141(12), 3725-3735.
21. Ungurean L, Carstoiu G, Micea MV, Groza V. 2017. Battery state of health estimation: a structured review of models, methods and commercial devices. International Journal of Energy Research, 41(2), 151-181.
22. Verma MKS, Basu S, Patil RS, Hariharan KS, Adiga SP, Kolake SM, Sung Y. (2020). On-Board State Estimation in Electrical Vehicles: Achieving Accuracy and Computational Efficiency Through an Electrochemical Model. IEEE Transactions on Vehicular Technology, 69(3), 2563-2575.
23. Xue ZW, Zhang Y, Cheng C, Ma GJ. 2020. Remaining useful life prediction of lithium-ion batteries with adaptive unscented kalman filter and optimized support vector regression. Neurocomputing, 376, 95-102.
24. Zhang C, Greenblatt JB, MacDougall P, Saxena S, Prabhakar AJ. 2020. Quantifying the benefits of electric vehicles on the future electricity grid in the midwestern United States. Applied Energy, 270, 12.

25. Zhang H, Mo ZL, Wang JY, Miao Q. 2020. Nonlinear-Drifted Fractional Brownian Motion with Multiple Hidden State Variables for Remaining Useful Life Prediction of Lithium-Ion Batteries. IEEE Transactions on Reliability, 69(2), 768-780.

Appendix

Table 1. Third-order scale transformation model

Cycle time	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$
60	-6.101×10^{-4}	1.014×10^{-1}	2.639
65	-7.996×10^{-4}	1.15×10^{-1}	2.425
70	-9.352×10^{-4}	1.255×10^{-1}	2.247
75	-1.01×10^{-3}	1.318×10^{-1}	2.134
80	-1.021×10^{-3}	1.327×10^{-1}	2.116
85	-9.766×10^{-4}	1.285×10^{-1}	2.204
90	-9.01×10^{-3}	1.208×10^{-1}	2.372
95	-8.241×10^{-4}	1.127×10^{-1}	2.562
100	-7.582×10^{-4}	1.053×10^{-1}	2.742

Note: $\hat{p}_1$, $\hat{p}_2$ and $\hat{p}_3$ are the estimated values of parameters p_1, p_2 and p_3.

Nomenclature		$U(t)$	Standard normal distribution
Acronym		μ_*	Degradation rate after scale transformation
RUL	Remaining useful life	m	The number of cycles
EVs	Electric vehicles	z_i	The actual value of cumulative capacity loss
SOH	State of health	z_i^*	The fitting value of cumulative capacity loss
MA	Moving average	$\overline{z}$	The average value of cumulative capacity loss
AR	Autoregressive	z_0	The cumulative capacity loss at the initial time
LSTM	Long Short-Term Memory	μ^*	The degradation rate of cumulative capacity loss
SVM	Support vector machine	m^*	The number of cycles
BP	Back propagation	σ^*	The cumulative capacity diffusion speed
*R*2	Coefficient of determination	L	Failure threshold
RMSE	Root mean square error	μ	Degradation rate
Notation		$C(t)$	Standard Brownian motion
u_0	Initial value	f	Probability density function