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ORIGINAL RESEARCH

A new uncertain multi-objective programming model with chance-entropy constraint for advertising promotion

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Abstract
The COVID-19 outbreak has forced people to stay at home to prevent the spread of the virus. In this case, social media platforms have become the main communication venue for people. Online sales platforms have also become the main field for people's daily consumption. So, how to make full use of social media to carry out online advertising promotion, and then achieve better marketing, is one of the core issues that the marketing industry must pay attention to and solve. Therefore, this study takes the advertiser as the decision-maker, maximizes the number of full playing, likes, comments and forwarding, and minimizes the cost of advertising promotion as the decision-making goals, and Key Opinion Leader (KOL) selection as the decision vector. Based on this, a multi-objective uncertain programming model of advertising promotion is constructed. Among them, the chance-entropy constraint is proposed by combining the entropy constraint and the chance constraint. In addition, the multi-objective uncertain programming model is transformed into a clear single-objective model through mathematical derivation and linear weighting of the model. Finally, the practicability and effectiveness of the model are verified by numerical simulation, and decision-making suggestions for advertising promotion are put forward.

Keywords Uncertain variables · Uncertain multi-objective programming · Chance-entropy constraint · Advertising promotion · Key Opinion Leader (KOL)

1 Introduction

In the digital economy, social networks have become the largest information portals (Li et al. 2019). Users can share information with friends and fans through these social platforms such as Facebook, YouTube, Twitter, Instagram and TikTok (Chen et al. 2021). Each platform has its own unique value (Farivar et al. 2021). For example, Instagram allows users to share visual content (images or videos, etc.). In addition, during the epidemic period, people should try to avoid contact behaviors to avoid the spread of the virus. As a result, people are turning to social media platforms to buy daily necessities. This makes social media marketing the mainstream marketing method today. In the self-media era, individuals can freely access and create a variety of content on different social platforms. Among them, some influencers can influence the opinions, decisions and actions of most people, namely KOLs (Sahli 2021). At the same time, with the development of digital media and online platforms, marketers begin to explore the potential of different marketing methods in public relations and advertising promotion (Tsen and Cheng 2021). Previous studies have ignored the impact of KOLs on users (Weng and Zhang 2021). However, a large number of studies have shown that the KOL plays an extremely important role in many fields. According to the pharmaceutical marketing network, KOLs can influence doctors' medical practices, including, but not limited to, their prescribing behavior (Scher and Schett 2021). One promotion strategy adopted by the pharmaceutical industry is to hire doctors as KOLs and use these relationships to support the company's overall brand strategy and develop new products (Price et al. 2021). In the social field, Sulistyanto and Jamil (2021) aimed to determine the influence of opinion leaders on the government's epidemic prevention policy, and concluded that opinion leaders play an important thematic

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and emotional guiding role in practice. In addition, further research shows that the KOL has a strong influence, regardless of whether it is based on any social media platform. For example, Casaló et al. (2020) took fans of Instagram fashion accounts as samples, and thought that the influence of KOLs has a positive impact on fan interaction, account recommendation and advice follow. Ingenhoff et al. (2021) proposed a Twitter-based communic-ation study, that is, to identify KOLs in order to analyze their social media activities and assess their role in shaping the image of the country. Sahli (2021) pointed out that the KOL marketing on Instagram is regarded as an effective advertising channel, and its role is becoming more and more important. Thus, relying on KOLs for marketing activities has been proved to be effective in marketing, and has been preferred by many enterprises as the main marketing method of social media (Ryu and Han 2021). Nevertheless, researches on KOL marketing mostly focused on the identification of KOLs (Riquelme et al. 2019; Yin et al. 2021). In other words, the KOL is identified, which in turn enables KOLs to promote and market products. For example, Jain et al. (2020) proposed a new optimization algorithm based on social network, which uses the standard optimization function in the network to measure the reputation of users to find the top N-bit KOL. Chen (2019) proposed a leadership analysis method based on clustering to find KOLs in social networks, and experimental results show that the algorithm can effectively find KOLs in different real data sets. Sun and Bin proposed a new KOL recognition algorithm in multi-relational online social networks (Sun and Bin 2018). However, in general, companies do not choose only one KOL for marketing, but choose multiple KOLs. Then, how to plan multiple KOLs for advertising promotion will determine the final marketing effect. This is a very important subject. At present, few scholars have studied the problem of KOL selection and scheduling in social media marketing. This is a subject neglected by researchers. Therefore, this study will study the KOL selection and scheduling problems existing in the advertising promotion for social media marketing. Uncertainty exists in various fields (Ghiasi et al. 2019; Dehghani et al. 2020; Akbary et al. 2019), especially in media marketing and advertising promotion. At present, most of the researches in this area consider these uncertainties as random problems. Among them, Li and Yang (2020) carries on the optimization research from the point of view of the high uncertainty of the advertising environment, and puts forward the stochastic programming model of advertising keyword grouping. Considering the high uncertainty of the revisit date in the retail application, Shin et al. (2021) regarded it as a multi-stage random problem, and proposed a robust multi-period inventory model. However, random methods are studied on the basis of probability theory. As we all know, the prerequisite for the use of probability theory is that the probability distribution obtained must be fully close to the actual frequency, which requires a large number of sample data to obtain the probability distribution. But, it is difficult for us to get enough sample data in advertising promotion. On the one hand, the reason is that the network environment is highly uncertain. On the other hand, when we carry on the advertising promotion before the new product comes on the market, so we cant obtain the historical data, and then cant apply the probability theory to get the corresponding probability distribution. In this case, we can estimate the reliability of the effectiveness and cost of advertising promotion based on the experience and knowledge of experts. Furthermore, Kai-Ineman and Tversky (1979) shows that people often underestimate the possibility of high probability events and overestimate the possibility of low probability events, which makes the variance of reliability much greater than the frequency. At this time, if the reliability is regarded as a subjective frequency, the deduced results will be quite different from the expected results. Therefore, probability theory cant be used to solve this kind of problems. In order to deal with this kind of belief estimation based on expert experience more reasonably, Professor Liu of Tsinghua University put forward the uncertainty theory in 2007 (Liu 2007) and revised in 2010 (Liu 2010), a branch of axiomatic mathematics, and derived uncertain programming (Ahmadzade and Gao 2020), uncertain statistics (Ye and Liu 2020), uncertain network (Shi et al. 2017), uncertain process (Shi et al. 2021) and other fields. Its main applications include uncertain finance (Liu 2009a; Huang and Di 2020; Chang et al. 2020; Yu and Ning 2019; Xue et al. 2019; Li and Zhang 2021; Li et al. 2020), uncertain intensive production planning (Ning et al. 2019), uncertain vehicle routing optimization problems (Ning and Su 2017), uncertain advertising promotion (Jin et al. 2021, 2023) and so on. An effective way to deal with uncertainty is building programming models (Jin et al. 2023). For example, Khodaei et al. (2018) constructed fuzzy-based heat and power hub models for cost-emission operation of an industrial consumer using compromise programming; Saeedi et al. (2019) proposed the robust scheduling of multi-chiller system which is modeled as non-linear programming. Therefore, this study deals with the uncertainty in advertising promotion by constructing an uncertain programming model. The main contributions of this study are as follows. First, the uncertainty in advertising promotion is eliminated. Second, in uncertain environment, we discuss the influence of KOL selection diversification constraint on multi-objective advertising promotion programming. Third, we solve the multi-objective advertising promotion decision problem based on multiple social platforms. The rest of this study is organized as follows. In Sect. 2, we introduce some basic concepts of uncertainty theory. In Sect. 3, a new uncertain multi-objective programming model considering the KOL selection diversification is proposed in the multi-platform advertising promotion problem. In Sect. 4, the uncertain multi-objective programming model is transformed into a clear single-objective programming model. In Sect. 5, we use a numerical example to verify the effectiveness of our model, and provide meaningful advertising suggestions for decision-makers. Finally, the conclusion and prospect are given.

2 Preliminaries

In this section, we will mainly introduce some related definitions and theorems of Uncertainty Theory.

2.1 Uncertainty theory

Let Γ be a nonempty set (sometimes called universal set), and let L be a σ-algebra over Γ. In Uncertainty Theory, each element Λin L is called an event. A number M{Λ} will be assigned to each event Λ to indicate the belief degree with which we believe Λ will happen. There is no doubt that the assignment is not arbitrary, and the uncertain measure M must have certain mathematical properties. In order to rationally deal with belief degrees, Liu (2007) suggested the following three axioms:

Axiom 1 (Normality Axiom): M{Γ} = 1 for the universal set Γ;

Axiom 2 (Duality Axiom): M{Λ} + M{Λᶜ} = 1 for any $\Lambda \in L$;

Axiom 3 (Subadditivity Axiom): For any countable sequence of events $\{\Lambda_i\}$, there is

$$M\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \le \sum_{i=1}^{\infty} M\{\Lambda_i\} \tag{1}$$

Definition 1 (Liu 2007) The set function M is called an uncertain measure if it satis?es the normality, duality, and subadditivity axioms.

Axiom 4 (Product Axiom): Let (Γ_k, L_k, M_k) be uncertainty spaces for $k = 1, 2, \cdots$. The product uncertain measure M is an uncertain measure satisfying

$$M\left\{\prod_{k=1}^{\infty}\Lambda_k\right\} = \bigwedge_{k=1}^{\infty} M_k\{\Lambda_k\} \tag{2}$$

where Λ_k are arbitrarily chosen events from L_k for $k = 1, 2, \cdots$, respectively.

Definition 2 (Liu 2007) An uncertain variable is a function ξ from an uncertainty space $(\Gamma, L, M)$to the set of real numbers such that $\{\xi \in B\}$is an event for any Borel set B of real numbers. Note that the event $\{\xi \in B\}$ is a subset of the universal set Γ, i.e,

$$\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\} \tag{3}$$

We know that under the system of Probability Theory, random variables are characterized by probability distribution function or probability density function. By the same token, in the uncertain theoretical system, uncertain variables are described by uncertain distribution functions.

Definition 3 (Liu 2007) The uncertain distribution function of the uncertain variable ξ is

$$\Phi(\mathrm{x}) = M\{\xi \le x\} \tag{4}$$

where x is a real number.

Definition 4 (Liu 2007) If $\Phi(x) \in (0,1)$is continuous and monotonically increasing with respect to x and $\lim_{x\to-\infty}\Phi(x) = 0$, $\lim_{x\to+\infty}\Phi(x) = 1$, then $\Phi(x)$ is regular.

Definition 5 (Liu 2007) When $\Phi(x)$ is a regular distribution function of ξ, $\Phi^{-1}(\alpha)$ denotes the inverse uncertainty distribution of ξ, where $\alpha \in (0, 1)$.

Example 1 (Liu 2007): An uncertain variable ξ is called normal if it has a normal uncertainty distribution

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)^{-1}, \quad x \in R \tag{5}$$

denoted by $N(e, \sigma)$ where e and σ are numbers with $\sigma > 0$. The inverse distribution of normal uncertain variable $N(e, \sigma)$ is

$$\Phi^{-1}(\alpha) = e + \frac{\sqrt{3}\sigma}{\pi}\ln\frac{\alpha}{1-\alpha}, \alpha \in (0,1) \tag{6}$$

Definition 6 (Liu 2007) The optimistic value and pessimistic value to an uncertain variable ξ at a given level $0 < \alpha < 1$ are defined as

$$\xi_{\sup}(\alpha) = \sup\{\gamma | M\{\xi \ge \gamma\} \ge \alpha\} \tag{7}$$

and

$$\xi_{\inf}(\alpha) = \inf\{\gamma | M\{\xi \le \gamma\} \ge \alpha\}. \tag{8}$$

If ξ has a regular uncertainty distribution Φ, then we have

$$\xi_{\sup}(\alpha) = \Phi^{-1}(1-\alpha) \tag{9}$$

and

$$\xi_{\inf}(\alpha) = \Phi^{-1}(\alpha). \tag{10}$$

Theorem 1 (Liu 2007) *Let $\xi_i (i = 1, 2, \cdots, n)$ be uncertain variables, and let f be a real-valued measurable function. Then $f(\xi_1, \xi_2, \cdots, \xi_n)$ is an uncertain variable.*

Theorem 2 (Liu 2007) *Assume that $\xi_1, \xi_2, \cdots, \xi_n$ are independent uncertain variables with regular distribution functions $\Phi_1, \Phi_2, \cdots, \Phi_n$. If $f(x_1, x_2, \cdots, x_n)$ strongly increases with respect to $x_1, x_2, \cdots, x_m$ and strongly decreases with respect to $x_{m+1}, x_{m+2}, \cdots, x_n$, then we get*

$$\Psi^{-1}(\alpha)=f(\Phi_1^{-1}(\alpha),\cdots,\Phi_m^{-1}(\alpha),\Phi_{m+1}^{-1}(1-\alpha),\cdots,\Phi_n^{-1}(1-\alpha)), \tag{11}$$

which represents the inverse distribution of

$$\xi = f(\xi_1, \xi_2, \cdots, \xi_n). \tag{12}$$

Definition 7 (Liu 2007) Letξ be an uncertain variable. Then the expected value of ξ is defined by

$$E[\xi] = \int_0^{+\infty} M\{\xi \geq x\}\mathrm{d}x - \int_{-\infty}^{0} M\{\xi \leq x\}\mathrm{d}x \tag{13}$$

provided that the two integrals are finite.

Theorem 3 (Liu 2007) *Let ξ be an uncertain variable with uncertainty distribution Φ. Then*

$$E[\xi] = \int_0^{+\infty} (1 - \Phi(x))\mathrm{d}x - \int_{-\infty}^{0} \Phi(x)\mathrm{d}x. \tag{14}$$

Definition 8 (Liu 2007) Let ξ be an uncertain variable with finite expected value e. Then the variance of ξ is

$$V[\xi] = E[(\xi - e)^2]. \tag{15}$$

This definition tells us that the variance is just the expected value of $(\xi - e)^2$. Since $(\xi - e)^2$ is a nonnegative uncertain variable, we also have

$$V[\xi] = \int_0^{+\infty} M\left\{(\xi - e)^2 \geq x\right\}\mathrm{d}x. \tag{16}$$

2.2 Uncertain programming model

Uncertain programming model (Liu 2009b) is an optimization method involving uncertain variables in uncertain environment. When faced with unwise decisions, constraints may not be met. At this point, we need a principle to expand the constraints. Since the uncertain constraints $g_j(x, \xi) \leq 0$ do not define a deterministic feasible set, it is naturally desired that the uncertain constraints hold with confidence levels α_j. Then we have a chance-constraint as

$$M\left\{g_j(x, \xi) \leq 0\right\} \geq \alpha_j, j = 1, 2, \cdots, p. \tag{17}$$

Therefore, if the decision-maker wants to maximize the optimistic value of the objective function in uncertain environment, a chance-constrained programming model can be constructed as

$$\begin{cases} \max_x \max\limits_{\bar{f}} \bar{f} \\ \text{s.t.} \\ M\{f(x, \xi) \geq \bar{f}\} \geq \beta \\ M\left\{g_j(x, \xi) \leq 0\right\} \geq \alpha_j, j = 1, 2, \cdots, p, \end{cases} \tag{18}$$

where β and α_i are all pre-given confidence levels. And, $\max f$ is the β-optimistic value of the objective function $f(x, \xi)$. If multi-objective decision-making is needed in practical problems, a multi-objective maximized chance-constrained programming model can be constructed as

$$\begin{cases} \max\limits_x \ [\max\limits_{\bar{f}_1} \overline{f_1}, \max\limits_{\bar{f}_2} \overline{f_2}, \cdots, \max\limits_{\bar{f}_m} \overline{f_m}] \\ s.t. \\ M\left\{f_i(x, \xi) \geq \overline{f_i}\right\} \geq \beta_i, i = 1, 2, \cdots, m \\ M\left\{g_j(x, \xi) \leq 0\right\} \geq \alpha_j, j = 1, 2, \cdots, p, \end{cases} \tag{19}$$

where max $\overline{f_i}$ is the β_i-optimistic value of the objective function $f_i(x, \xi)$. Similarly, if decision-makers want to minimize the pessimistic values of some chance-constrained functions, they can build a chance-constrained programming model

$$\begin{cases} \min\limits_x \ \min\limits_{\bar{f}} \overline{f} \\ s.t. \\ M\left\{f(x, \xi) \leq \overline{f}\right\} \geq \beta \\ M\left\{g_j(x, \xi) \leq 0\right\} \geq \alpha_j, j = 1, 2, \cdots, p, \end{cases} \tag{20}$$

where β and α_j are all pre-given confidence levels. And, min $\overline{f}$ is the β-pessimistic value of the objective function $f(x, \xi)$. If multi-objective decision-making is needed in practical problems, a multi-objective minimized chance-constrained programming model can be constructed as

$$\begin{cases} \min\limits_x \ [\min\limits_{\bar{f}_1} \overline{f_1}, \min\limits_{\bar{f}_2} \overline{f_2}, \cdots, \min\limits_{\bar{f}_m} \overline{f_m}] \\ s.t. \\ M\left\{f_i(x, \xi) \leq \overline{f_i}\right\} \geq \beta_i, i = 1, 2, \cdots, m \\ M\left\{g_j(x, \xi) \leq 0\right\} \geq \alpha_j, j = 1, 2, \cdots, p \end{cases} \tag{21}$$

where min $\overline{f_i}$ is the β_i-pessimistic value of the objective function $f_i(x, \xi)$.

Theorem 4 (Liu 2007) *Let $g(x, \xi_1, \xi_2, \cdots, \xi_n)$ be a constrain function, in which $\xi_1, \xi_2, \cdots, \xi_n$ are independent, with uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$ separately. If $g(x, \xi)$*

strongly increases with respect to $\xi_1, \xi_2, \cdots, \xi_s$ *and strongly decreases with respect to* $\xi_{s+1}, \xi_{s+2}, \cdots, \xi_n$, *then*

$$M\left\{g(x, \xi_1, \xi_2, \cdots, \xi_n) \le 0\right\} \ge \alpha \tag{22}$$

is equivalent to

$$\begin{aligned} &g(x, \Phi_1^{-1}(\alpha), \cdots, \Phi_s^{-1}(\alpha), \Phi_{s+1}^{-1}(1-\alpha),\\ &\cdots, \Phi_n^{-1}(1-\alpha)) \le 0. \end{aligned} \tag{23}$$

3 Model building

3.1 Background introduction

With the development of Internet, social media has become an indispensable part of human life. The rise of social media has not only promoted human social convenience, but also improved people's consumption patterns. The shopping or selling acts through social media are called social media marketing. The emergence of the epidemic has accelerated the development of social media marketing and made it become the mainstream marketing mode. The foreign marketing platforms mainly include Youtube, Facebook, Twitter, while the domestic ones mainly include TikTok, Kuaishou and so on.

When a new product comes to market, companies usually advertise the new product. As we all know, there are two main ways of advertising promotion. One is a traditional way, including television advertising promotion and so on. The other is social media advertising promotion, which mainly includes information flow promotion and KOL promotion. Among them, the information flow promotion of social media is similar to the advertising promotion of traditional media. The promotion form is direct and clear, which is easy to arouse the disgust of the audience. Therefore, combined with the life way of modern human, most companies now adopt KOL promotion based on social media. Based on this, we take the advertising promotion of the new product as the background, take the advertiser as the decision-maker, adopt the KOL advertising promotion method based on social media, and discuss how to make the promotion decision in order to optimize the advertising promotion.

Taking Tik Tok as an example, Giant Star Map is a professional service module for advertising promotion. The specific steps are as the Fig. 1.

Step 1: Account recharge. Advertisers are required to pay a deposit in advance.
Step 2: Talent screening. Advertisers can choose KOLs for advertising promotion through platform recommendations or other channels.
Step 3: Fill in the brief. Advertisers can fill in specific advertising promotion needs, and then reach two-way cooperation.
Step 4: Order cooperation. The advertiser pays the money to reach a partnership.
Step 5: Post-cast data. After KOL's advertising promotion tasks are completed, the platform will announce promotion data to advertisers, including pageviews, completion rate, and interaction rate (likes, comments, and forwarding), etc.

In the above procedure, step 2 is the core issue for advertisers to make advertising promotion decisions. So, how to choose KOL talents reasonably so as to make the best advertising promotion decision? Firstly, according to the number of fans of the KOL, the type of audience, the matching degree of the required promotion content with the KOL style, and the marketing ability of the KOL, the KOL is classified into grades. Secondly, determine the number of KOLs to choose each level according to the actual situation.

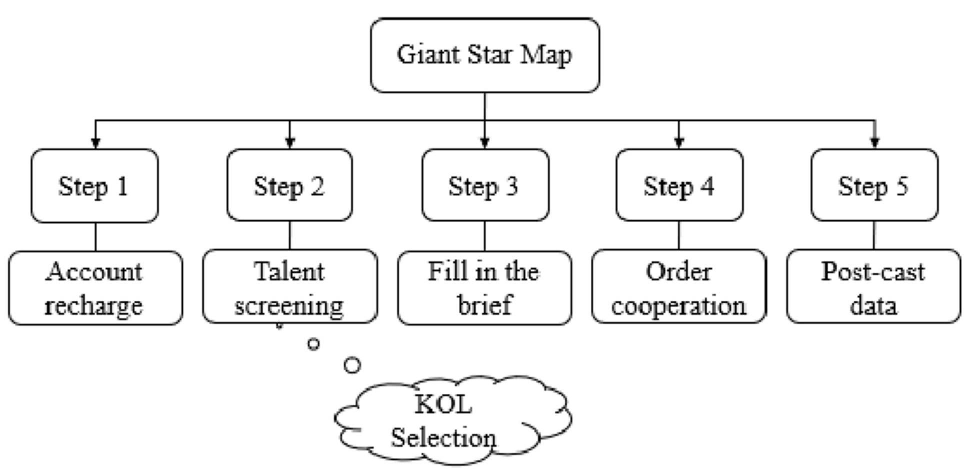

Fig. 1 TikTok-based advertising promotion process

For example, as a platform that sells clothing and cosmetics as its main products, Vipshop serves mainly the women. Therefore, decision makers should firstly classify KOLs based on the number of KOL female fans, the degree of matching between KOL styles and promoted products, and the previous promotion data of products of the same type; then, design the optimal promotion decision.

3.2 Parameter introduction

Before introducing the model, let's introduce the relevant parameters of the model.

j indicates a KOL individual, $j = 1, 2, \cdots, J$.

i indicates the level which KOL belongs to, $i = 1, 2, \cdots, I$.

k indicates the social media platform for advertising promotion, $k = 1, 2, \cdots, K$.

$x_{i,k}$ indicates the percentage of the KOLs total number at *i-th* level in the *k-th* platform, $0 < x_{i,k} < 1$.

$\xi_{i,k}$ indicates the full number of advertising videos posted by the KOL at *i-th* level in the *k-th* platform, $\xi_{i,k} > 0$.

$\tau_{i,k}$ indicates the number of likes received by the advertising video posted by the KOL at *i-th* level in the *k-th* platform, $\tau_{i,k} > 0$.

$\eta_{i,k}$ indicates the number of comments obtained from advertising videos posted by the KOL at *i-th* level in the *k-th* platform during advertising promotion, $\eta_{i,k} > 0$. Among them, the number of comments that hold a positive attitude towards video content is expressed as $\eta^+_{i,k}$; otherwise, it is expressed as $\eta^-_{i,k}$.

$\pi_{i,k}$ indicates the number of forwardings received by the advertising video posted by the KOL at *i-th* level in the *k-th* platform, $\pi_{i,k} > 0$.

$c_{i,k}$ indicates the advertising expenses paid by the decision maker to the KOL at *i-th* level in the *k-th* platform, $c_{i,k} > 0$.

$\tilde{\alpha}$ indicates the threshold of promotion risk.

$\tilde{\gamma}_k$ indicates the uncertain threshold of the KOL selection diversification.

3.3 Objective functions

For the advertising promotion on traditional media, the cost of advertising promotion in different TV stations and different time is different. This is because the audience is different on different platforms and in different time. Therefore, the advertising promotion effect in traditional media can be approximately measured by the number of viewers. However, for advertising promotion on social media platforms, we can observe more promotion data, such as the number of views, completion rate, likes, forwardings, comments, collections and so on. As consequently, we take the complete playing times, likes, comments and forwardings of the advertising video as the standard to measure the effectiveness of advertising promotion. Simultaneously, we take the advertiser as the decision-maker, so minimizing the promotion cost should also be the goal of advertising promotion programming.

(1) Maximize the full playing times of advertising videos

At present, for the promotion of social media advertising, KOL generally intersperses the advertising content inside the video. If the audience does not watch the advertisement completely, it does not necessarily pay attention to the content of the advertisement and cannot be counted as a real advertising promotion. Therefore, this study takes the full playing times of advertising video as a standard to measure the effectiveness of advertising promotion. The specific expression is

$$\sum_{k=1}^{K}\sum_{i=1}^{I}\xi_{i,k}x_{i,k} \tag{24}$$

where

$$\textit{full playing times} = \textit{full playing rate} \times \textit{views}. \tag{25}$$

In order to maximize the β_1-optimistic value of the average full playing function $\overline{f_{1,1}}$, then we can get

$$\max\left\{\overline{f_{1,1}}\middle| M\left\{\sum_{k=1}^{K}\sum_{i=1}^{I}\xi_{i,k}x_{i,k} \ge \overline{f_{1,1}}\right\} \ge \beta_1\right\}. \tag{26}$$

(2) Maximize likes of advertising videos

Giving likes to a video means that the audience is interested in the content of the video. Therefore, we also use the number of likes in the video as a measure of the effectiveness of advertising promotion. The specific expression is

$$\sum_{k=1}^{K}\sum_{i=1}^{I}\tau_{i,k}x_{i,k}. \tag{27}$$

In order to maximize the β_2-optimistic value of the average like function $\overline{f_{1,2}}$, then we can get

$$\max\left\{\overline{f_{1,2}}\middle|M\left\{\sum_{k=1}^{K}\sum_{i=1}^{I}\tau_{i,k}x_{i,k}\geq\overline{f_{1,2}}\right\}\geq\beta_2\right\}. \quad (28)$$

(3) Maximize the number of comments

In general, audiences will express their views in the comments section only when they have two extreme attitudes towards a video (very interested or not interested). If the comment is positive content, it means that the audience agrees with the video content, so it plays a role in promoting advertising. If the comment is negative, it means that the audience does not agree with the video content, so it has a restraining effect on advertising promotion. We can express it as

$$\sum_{k=1}^{K}\sum_{i=1}^{I}(\eta_{i,k}^{+}-\eta_{i,k}^{-})x_{i,k}. \quad (29)$$

In order to maximize the β_3-optimistic value of the average full playing function $\overline{f_{1,3}}$, then we can get

$$\max\left\{\overline{f_{13}}\middle|M\left\{\sum_{k=1}^{K}\sum_{i=1}^{I}(\eta_{i,k}^{+}-\eta_{ik}^{-})x_{ik}\geq\overline{f_{1,3}}\right\}\geq\beta_3\right\}. \quad (30)$$

(4) Maximize the number of forwardings

Due to the influence of the audience, the forwarding-behavior of the audience to the advertising video will promote the double promotion of the advertising video. So, We can express it as

$$\sum_{k=1}^{K}\sum_{i=1}^{I}\pi_{i,k}x_{i,k}. \quad (31)$$

In order to maximize the β_4-optimistic value of the average full playing function $\overline{f_{1,4}}$, then we can get

$$\max\left\{\overline{f_{1,4}}\middle|M\left\{\sum_{k=1}^{K}\sum_{i=1}^{I}\pi_{i,k}x_{i,k}\geq\overline{f_{1,4}}\right\}\geq\beta_4\right\}. \quad (32)$$

(5) Minimize the promotion cost

In the programming model of advertising promotion, the goal of decision-makers is to maximize their own income, so they need to strike a balance between the promotion effect and the promotion cost, that is, to maximize the promotion effect and minimize the promotion cost on the premise of satisfying the constraints. Then, the average promotion cost can be expressed as

$$\sum_{k=1}^{K}\sum_{i=1}^{I}c_{i,k}x_{i,k}. \quad (33)$$

To minimize the β'-pessimistic value of the cost function $\overline{f_2}$, we can get

$$\min\left\{\overline{f_2}\middle|M\left\{\sum_{k=1}^{K}\sum_{i=1}^{I}c_{i,k}x_{i,k}\leq\overline{f_2}\right\}\geq\beta'\right\}. \quad (34)$$

3.4 Chance-entropy constraint

The KOL at same level may have the same audience, therefore, choosing KOLs at multi-level for advertising promotion can also broaden the scope of promotion to a certain extent, and then promote the effect of advertising promotion, so as to improve the exposure of products and promote product marketing. Shannon's information entropy can be used to measure the chaos degree of information. The thermal entropy in thermodynamics indicates the confusion degree of the molecular state. In addition, entropy can also be used to measure the diversification degree of securities investment. So, this study will introduce entropy to measure the degree of diversification of KOL selection. Also, combining entropy constraint with chance constraint, we propose the chance-entropy constraint. The expression is

$$M\left\{\sum_{i=1}^{I}(-x_{i,k})\log(x_{i,k}+\varepsilon)\geq\tilde{\gamma}_k\right\}\geq\gamma \quad (35)$$

when $x_{i,k}$ is 0, formula (34) is meaningless. Therefore, we introduce a positive number ε which is small enough. At this time, the uncertainty threshold of KOL selection diversification degree on the *k-th* platform is $\tilde{\gamma}_k$, and γ is confidence value.

3.5 The uncertain multi-objective programming model

Combining the Eqs. (26), (28), (30), (32), (34) in Sect. 3.4, the Eq. (35) in Sect. 3.5, and the basic form of the uncertain programming model in Sect. 2.2, we can obtain the model

$$\left\{\begin{array}{l} \max\limits_{x} \left[\max\limits_{\overline{f_{1,1}}} \overline{f_{1,1}}, \max\limits_{\overline{f_{1,2}}} \overline{f_{1,2}}, \max\limits_{\overline{f_{1,3}}} \overline{f_{1,3}}, \max\limits_{\overline{f_{1,4}}} \overline{f_{1,4}}\right] \\ \min\limits_{x} \min\limits_{\overline{f_2}} \overline{f_2} \\ s.t. \\ \quad M\left\{\sum\limits_{k=1}^{K}\sum\limits_{i=1}^{I} \xi_{i,k} x_{i,k} \geq \overline{f_{1,1}}\right\} \geq \beta_1 \\ \quad M\left\{\sum\limits_{k=1}^{K}\sum\limits_{i=1}^{I} \tau_{i,k} x_{i,k} \geq \overline{f_{1,2}}\right\} \geq \beta_2 \\ \quad M\left\{\sum\limits_{k=1}^{K}\sum\limits_{i=1}^{I} (\eta_{i,k}^{+} - \eta_{i,k}^{-}) x_{i,k} \geq \overline{f_{1,3}}\right\} \geq \beta_3 \\ \quad M\left\{\sum\limits_{k=1}^{K}\sum\limits_{i=1}^{I} \pi_{i,k} x_{i,k} \geq \overline{f_{1,4}}\right\} \geq \beta_4 \\ \quad M\left\{\sum\limits_{k=1}^{K}\sum\limits_{i=1}^{I} c_{i,k} x_{i,k} \leq \overline{f_2}\right\} \geq \beta' \\ \quad M\left\{\sum\limits_{i=1}^{I} (-x_{i,k}) \log(x_{i,k} + \varepsilon) \geq \tilde{\gamma}_k\right\} \geq \gamma \\ \quad \sum\limits_{k=1}^{K}\sum\limits_{i=1}^{I} x_{i,k} = 1 \\ \quad x_{i,k} > 0, i = 1, 2, \cdots, I, k = 1, 2, \cdots, K. \end{array}\right. \tag{36}$$

4 Model transformation

In order to facilitate the calculation, we can transform the model (36) into an equivalent clear form through the correlation operation rules of uncertain variables.

For ease of presentation, we have made the following transformation

$$\overline{f_{1,l}}' = \max_{\overline{f_{1,l}}} \overline{f_{1,l}}, l = 1, 2, 3, 4$$

and

$$\overline{f_2}' = \min_{\overline{f_2}} \overline{f_2}.$$

Then, the specific conversion rules are shown in Theorem 5.

Theorem 5 *The multi-objective uncertain chance-constrained programming model* (36) *is equivalent to clear model*

$$\left\{\begin{array}{l} \min\limits_{x} \left[-\overline{f_{1,1}}', -\overline{f_{1,2}}', -\overline{f_{1,3}}', -\overline{f_{1,4}}'\right] \\ \min\limits_{x} \overline{f_2}' \\ s.t. \\ \quad \overline{f_{1,1}}' = \sum\limits_{k=1}^{K}\sum\limits_{i=1}^{I} \Phi_{\xi_{i,k}}^{-1}(\beta_1) x_{i,k} \\ \quad \overline{f_{1,2}}' = \sum\limits_{k=1}^{K}\sum\limits_{i=1}^{I} \Phi_{\tau_{i,k}}^{-1}(\beta_2) x_{i,k} \\ \quad \overline{f_{1,3}}' = \sum\limits_{k=1}^{K}\sum\limits_{i=1}^{I} (\Phi_{\eta_{i,k}^{+}}^{-1}(1-\beta_3) - \Phi_{\eta_{i,k}^{-}}^{-1}(\beta_3)) x_{i,k} \\ \quad \overline{f_{1,4}}' = \sum\limits_{k=1}^{K}\sum\limits_{i=1}^{I} \Phi_{\pi_{i,k}}^{-1}(\beta_4) x_{i,k} \\ \quad \overline{f_2}' = \sum\limits_{k=1}^{K}\sum\limits_{i=1}^{I} \Phi_{c_{i,k}}^{-1}(\beta') x_{i,k} \\ \quad \sum\limits_{i=1}^{I} (-x_{i,k}) \log(x_{i,k} + \varepsilon) \geq \Phi_{\tilde{\gamma}_k}^{-1}(\gamma) \\ \quad \sum\limits_{k=1}^{K}\sum\limits_{i=1}^{I} x_{i,k} = 1 \\ \quad x_{i,k} > 0, i = 1, 2, \cdots, I, k = 1, 2, \cdots, K. \end{array}\right. \tag{37}$$

Proof First, we prove that the objective function of the equivalent clear model is equivalent.

(1) Objective function of full playing

Obviously, $\xi_{i,k}$ is a regular uncertain variable, whose uncertain distribution is $\Phi_{\xi_{i,k}}^{-1}(x)$. From Theorem 1, $g_{1,1} = \sum\limits_{k=1}^{K}\sum\limits_{i=1}^{I} \xi_{i,k} x_{i,k}$ is also an uncertain variable.

Due to $x_{i,k} > 0$, it is easy to know that $g_{1,1}$ strict increments to $\xi_{i,k}$. According to Theorem 2, the inverse uncertainty distribution of $g_{1,1}$ is

$$\Psi_{1,1}^{-1}(\alpha) = \overline{f_{1,1}}(\Phi_{\xi_{i,k}}^{-1}(\alpha)) = \sum_{k=1}^{K}\sum_{i=1}^{I} \Phi_{\xi_{i,k}}^{-1}(\alpha) x_{i,k}. \tag{38}$$

Therefore, the β_1-optimism value of $g_{1,1}$ is

$$\overline{f_{1,1}}' = \Psi_{1,1}^{-1}(\beta_1) = \sum_{k=1}^{K}\sum_{i=1}^{I} \Phi_{\xi_{i,k}}^{-1}(\beta_1) x_{i,k}. \tag{39}$$

(2) Objective function of liking

As same as (1), the β_2-optimistic value of $g_{1,2}$ is

$$\overline{f_{1,2}}' = \Psi_{1,2}^{-1}(\beta_2) = \sum_{k=1}^{K}\sum_{i=1}^{I} \Phi_{\tau_{i,k}}^{-1}(\beta_2) x_{i,k}. \tag{40}$$

(3) Objective function of commenting

Similarly, $\eta^+_{i,k}$ and $\eta^-_{i,k}$ are all regular uncertain variables, whose uncertain distributions are $\Phi^{-1}_{\eta^+_{i,k}}(x)and\Phi^{-1}_{\eta^-_{i,k}}(x)$ separately. From Theorem 1, $g_{1,3} = \sum_{k=1}^{K}\sum_{i=1}^{I}(\eta^+_{i,k} - \eta^-_{i,k})x_{i,k}$ is also an uncertain variable.

Due to $x_{i,k} > 0$, it is easy to know that $g_{1,3}$ strict increments to $\eta^+_{i,k}$, and strict decline to $\eta^-_{i,k}$. According to Theorem 2, the inverse uncertainty distribution of $g_{1,3}$is

$$\begin{aligned}\Psi^{-1}_{1,3}(\alpha) &= \overline{f_{1,3}}(\Phi^{-1}_{\eta^+_{i,k}}(\alpha), \Phi^{-1}_{\eta^-_{i,k}}(\alpha))\\ &= \sum_{k=1}^{K}\sum_{i=1}^{I}(\Phi^{-1}_{\eta^+_{i,k}}(1-\alpha) - \Phi^{-1}_{\eta^-_{i,k}}(\alpha))x_{i,k}.\end{aligned} \tag{41}$$

Therefore, the β_3-optimistic value of $g_{1,3}$ is

$$\begin{aligned}\overline{f_{1,3}}' &= \Psi^{-1}_{1,3}(\beta_3)\\ &= \sum_{k=1}^{K}\sum_{i=1}^{I}(\Phi^{-1}_{\eta^+_{i,k}}(1-\beta_3) - \Phi^{-1}_{\eta^-_{i,k}}(\beta_3))x_{i,k}.\end{aligned} \tag{42}$$

(4) Objective function of forwarding

In the same way as (1), the β_4-optimistic value of $g_{1,4}$ is

$$\overline{f_{1,4}}' = \Psi^{-1}_{1,4}(\beta_4) = \sum_{k=1}^{K}\sum_{i=1}^{I}\Phi^{-1}_{\pi_{i,k}}(\beta_4)x_{i,k}. \tag{43}$$

(5) Objective function of promotion cost

In the same way as (1), the β'-optimistic value of g_2 is

$$\overline{f_2}' = \Psi^{-1}_{2}(\beta') = \sum_{k=1}^{K}\sum_{i=1}^{I}\Phi^{-1}_{\pi_{i,k}}(\beta')x_{i,k}. \tag{44}$$

Then, we prove the equivalent form of the constraint function.

(6) KOL selection diversification constraint in Advertising Promotion

From Theorem 4, there is

$$\begin{aligned}&M\left\{\sum_{i=1}^{I}(-x_{i,k})\log(x_{i,k}+\varepsilon) \geq \tilde{\gamma}_k\right\} \geq \gamma\\ &\Leftrightarrow \Phi_{\tilde{\gamma}_k}(\sum_{i=1}^{I}(-x_{i,k})\log(x_{i,k}+\varepsilon)) \geq \gamma\\ &\Leftrightarrow \sum_{i=1}^{I}(-x_{i,k})\log(x_{i,k}+\varepsilon) \geq \Phi^{-1}_{\tilde{\gamma}_k}(\gamma).\end{aligned} \tag{45}$$

To sum up, the model (36) converted into model (37).

□

It is not difficult to find that both models (36) and (37) are multi-objective programming models. And there are many ways to solve the multi-objective programming model. Among them, the most commonly used solution method is the linear weighting method. Therefore, we introduce the linear weighting method to transform the multi-objective programming method into single-objective programming method. Suppose $\overline{f_1}, \overline{f_2}, \cdots, \overline{f_n}$ are objective functions, whose weight set is $\Lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)^T$, where $\lambda_i \geq 0$. So, we introduce the evaluation function

$$\overline{f} = \sum_{i=1}^{n}\lambda_i\overline{f_i}. \tag{46}$$

Therefore, the optimal solution of the model with the objective function can be regarded as the Pareto optimal solution of the model.

5 Case study

5.1 Uncertain parameters

In order to enable decision-makers to better apply the model (36) in the decision-making of social media advertising promotion, we introduce a numerical example to illustrate it. The network environment is highly uncertain, and the new products promoted by advertising do not have historical promotion data, so we often lack sample data of advertising promotion. In addition, as we have introduced in Sect. 1, there are some limitations in applying stochastic theory to solve the uncertainty of this situation. In this case, we need to introduce the expert empirical method of Uncertainty Theory to estimate the promotion effect, promotion cost and KOL selection diversification of advertising based on social media, and further use the least square method (Liu 2010) to predict the uncertain distribution. Among them, the specific uncertain parameter values are shown in Table 1.

5.2 The model

In this article, we will consider two social media platforms, and comprehensively evaluate the promotion ability of rated KOLs according to the fans number of KOLs, the type of audience, and the degree of matching between the content to be promoted and the style of KOLs. For example: if the product to be advertised is a female skin care product, the decision maker will make a comprehensive evaluation according to the number of female fans of KOLs, the degree of matching

Table 1 Main uncertain parameters

		$i=1$	$i=2$	$i=3$	$i=4$
ξ	$k=1$	N(500,50)	N(5000,100)	N(50000,500)	N(500000,1000)
	$k=2$	N(600,30)	N(4000,80)	N(55000,300)	N(620000,600)
		$i=1$	$i=2$	$i=3$	$i=4$
τ	$k=1$	N(1000,30)	N(10000,100)	N(99000,330)	N(780000,870)
	$k=2$	N(980,36)	N(8900,69)	N(100000,290)	N(1000000,788)
		$i=1$	$i=2$	$i=3$	$i=4$
η^+	$k=1$	N(87,26)	N(356,61)	N(658,56)	N(984,79)
	$k=2$	N(102,34)	N(485,57)	N(965,35)	N(1657,85)
		$i=1$	$i=2$	$i=3$	$i=4$
η^-	$k=1$	N(234,56)	N(348,64)	N(351,58)	N(321,46)
	$k=2$	N(451,58)	N(453,89)	N(468,25)	N(398,45)
		$i=1$	$i=2$	$i=3$	$i=4$
π	$k=1$	N(9,10)	N(24,16)	N(67,25)	N(99,32)
	$k=2$	N(24,12)	N(36,14)	N(78,36)	N(124,59)
		$i=1$	$i=2$	$i=3$	$i=4$
c	$k=1$	N(150,26)	N(1486,97)	N(9875,156)	N(50000,568)
	$k=2$	N(186,34)	N(1245,56)	N(7859,132)	N(54300,368)
			$k=1$		$k=2$
$\tilde{\gamma}$			L(0.2,0.55)		L(0.15,0.6).

between the product and the style of KOL, and further grade KOLs. Here, we divide the KOL one each platform into four levels, each containing five KOLs. Therefore, combining with the parameter values in Table 1, we can get the model

$$\begin{cases} \min\limits_{x} \left[-\overline{f_{1,1}}', -\overline{f_{1,2}}', -\overline{f_{1,3}}', -\overline{f_{1,4}}' \right] \\ \min\limits_{x} \overline{f_2}' \\ s.t. \\ \overline{f_{1,1}}' = \sum\limits_{k=1}^{2}\sum\limits_{i=1}^{4} \Phi^{-1}_{\tilde{\xi}_{i,k}}(\beta_1) x_{i,k} \\ \overline{f_{1,2}}' = \sum\limits_{k=1}^{2}\sum\limits_{i=1}^{4} \Phi^{-1}_{\tau_{i,k}}(\beta_2) x_{i,k} \\ \overline{f_{1,3}}' = \sum\limits_{k=1}^{2}\sum\limits_{i=1}^{4} (\Phi^{-1}_{\eta^{+}_{i,k}}(1-\beta_3) - \Phi^{-1}_{\eta^{-}_{i,k}}(\beta_3)) x_{i,k} \\ \overline{f_{1,4}}' = \sum\limits_{k=1}^{2}\sum\limits_{i=1}^{4} \Phi^{-1}_{\pi_{i,k}}(\beta_4) x_{i,k} \\ \overline{f_2}' = \sum\limits_{k=1}^{2}\sum\limits_{i=1}^{4} \Phi^{-1}_{c_{i,k}}(\beta') x_{i,k} \\ \sum\limits_{i=1}^{4} (-x_{i,k}) \log(x_{i,k}+\varepsilon) \geq \Phi^{-1}_{\tilde{\gamma}_k}(\gamma) \\ \sum\limits_{k=1}^{2}\sum\limits_{i=1}^{4} x_{i,k} = 1 \\ x_{i,k} > 0, i = 1,2,3,4, k = 1,2. \end{cases} \tag{47}$$

Let $\Lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5) = (0.3, 0.2, 0.1, 0.1, 0.3)$ be the weight vector of the objective function in the model (47). Through the linear weighted sum method, we can get the evaluation function

$$\overline{f} = -0.3\overline{f_{1,1}}' - 0.2\overline{f_{1,2}}' - 0.1\overline{f_{1,3}}' - 0.1\overline{f_{1,4}}' + 0.3\overline{f_2}'. \tag{48}$$

5.3 Results and discussion

Let $\beta_i = \beta' = \gamma = 0.8, (i = 1, 2, 3, 4.)$, as shown as Table 2, we can get the optimal advertising promotion scheme, that is, the optimal selection of KOLs.

Let $\gamma = 0.7$, we can get the optimal selection of KOLs in Table 3. By comparing the results of Tables 2 and 3, we find that with the change of γ, the optimal objective function value and the optimal allocation ratio of KOLs also change. Among them, the complete playing times, likes, comments and forwardings of advertising videos all increase with the decrease of γ, while the cost target and overall goal of advertising promotion are increased.

Let $\gamma = 0.6$, we can get the optimal advertising promotion scheme, which is shown in Table 4.

Let $\gamma = 0.5$, we can get the optimal advertising promotion scheme in Table 5.

Let $\gamma = 0.4$, we can get the optimal advertising promotion scheme shown as Table 6.

By comparing the data from Tables 2, 3, 4, 5 and 6, we can find that with the increase of confidence γ, the value of $\overline{f_{1,1}}'$, $\overline{f_{1,2}}'$, $\overline{f_{1,3}}'$ and $\overline{f_{1,4}}'$ show decreasing trends. As shown in Figs. 2, 3, 4 and 5, we can find that these four functions are all convex functions, that is, the speed of reduction is accelerated step by step. This shows that in advertising promotion, the more diversified the KOL selection, the worse the promotion effect. Therefore, advertisers should weaken the diversity of KOL as much as possible when promoting advertisements.

In addition, as shown in Figures 6 and 7, with the increase of γ, $\overline{f_2}'$ and $\overline{f}'$ show increasing trends. In other words, the impact of the diversity of KOL selections on the cost of advertising promotion is: the more diversity of KOL selections, the higher the cost of advertising

Table 2 The change of the optimal value when $\beta_i = \beta' = \gamma = 0.8, (i = 1, 2, 3, 4)$

Optimal objective function value	Optimal allocation ratio			
$\overline{f_{1,1}}' = 5.3873e+05$	$x_{1,1} = 0.0115$ $x_{1,2} = 0.0115$	$x_{2,1} = 0.0118$ $x_{2,2} = 0.0118$	$x_{3,1} = 0.0159$ $x_{3,2} = 0.0164$	$x_{4,1} = 0.2910$ $x_{4,2} = 0.6302$
$\overline{f_{1,2}}' = 8.6150e+05$	$x_{1,1} = 0.0119$ $x_{1,2} = 0.0119$	$x_{2,1} = 0.0123$ $x_{2,2} = 0.0123$	$x_{3,1} = 0.0176$ $x_{3,2} = 0.0177$	$x_{4,1} = 0.2688$ $x_{4,2} = 0.6475$
$\overline{f_{1,3}}' = 1.0652e+03$	$x_{1,1} = 0.0091$ $x_{1,2} = 0.0050$	$x_{2,1} = 0.0154$ $x_{2,2} = 0.0155$	$x_{3,1} = 0.0379$ $x_{3,2} = 0.0689$	$x_{4,1} = 0.1196$ $x_{4,2} = 0.7287$
$\overline{f_{1,4}}' = 150.4372$	$x_{1,1} = 0.0037$ $x_{1,2} = 0.0064$	$x_{2,1} = 0.0072$ $x_{2,2} = 0.0103$	$x_{3,1} = 0.0402$ $x_{3,2} = 0.0786$	$x_{4,1} = 0.1461$ $x_{4,2} = 0.7074$
$\overline{f_2}' = 297.1734$	$x_{1,1} = 0.4737$ $x_{1,2} = 0.4370$	$x_{2,1} = 0.0333$ $x_{2,2} = 0.0560$	$x_{3,1} = 0.0000$ $x_{3,2} = 0.0000$	$x_{4,1} = 0.0000$ $x_{4,2} = 0.0000$
$\overline{f}' = -2.5773e+05$	$x_{1,1} = 0.0115$ $x_{1,2} = 0.0115$	$x_{2,1} = 0.0119$ $x_{2,2} = 0.0118$	$x_{3,1} = 0.0164$ $x_{3,2} = 0.0168$	$x_{4,1} = 0.2863$ $x_{4,2} = 0.6338$

Table 3 The change of the optimal value when $\beta_i = \beta^{'} = 0.8, \gamma = 0.7, (i = 1, 2, 3, 4)$

Optimal objective function value	Optimal allocation ratio			
$\overline{f_{1,1}}^{'} = 5.5046e + 05$	$x_{1,1} = 0.0087$	$x_{2,1} = 0.0090$	$x_{3,1} = 0.0123$	$x_{4,1} = 0.2844$
	$x_{1,2} = 0.0087$	$x_{2,2} = 0.0089$	$x_{3,2} = 0.0127$	$x_{4,2} = 0.6554$
$\overline{f_{1,2}}^{'} = 8.8080e + 05$	$x_{1,1} = 0.0092$	$x_{2,1} = 0.0095$	$x_{3,1} = 0.0139$	$x_{4,1} = 0.2615$
	$x_{1,2} = 0.0091$	$x_{2,2} = 0.0095$	$x_{3,2} = 0.0140$	$x_{4,2} = 0.6733$
$\overline{f_{1,3}}^{'} = 1.0911e + 03$	$x_{1,1} = 0.0073$	$x_{2,1} = 0.0127$	$x_{3,1} = 0.0331$	$x_{4,1} = 0.1117$
	$x_{1,2} = 0.0039$	$x_{2,2} = 0.0128$	$x_{3,2} = 0.0623$	$x_{4,2} = 0.7562$
$\overline{f_{1,4}}^{'} = 152.6793$	$x_{1,1} = 0.0027$	$x_{2,1} = 0.0055$	$x_{3,1} = 0.0346$	$x_{4,1} = 0.1370$
	$x_{1,2} = 0.0049$	$x_{2,2} = 0.0081$	$x_{3,2} = 0.0707$	$x_{4,2} = 0.7365$
$\overline{f_2}^{'} = 258.9325$	$x_{1,1} = 0.4939$	$x_{2,1} = 0.0202$	$x_{3,1} = 0.0000$	$x_{4,1} = 0.0000$
	$x_{1,2} = 0.4483$	$x_{2,2} = 0.0377$	$x_{3,2} = 0.0000$	$x_{4,2} = 0.0000$
$\bar{f}^{'} = -2.6343e + 05$	$x_{1,1} = 0.0087$	$x_{2,1} = 0.0090$	$x_{3,1} = 0.0128$	$x_{4,1} = 0.2797$
	$x_{1,2} = 0.0087$	$x_{2,2} = 0.0090$	$x_{3,2} = 0.0130$	$x_{4,2} = 0.6591$

Table 4 The change of the optimal value when $\beta_i = \beta^{'} = 0.8, \gamma = 0.6, (i = 1, 2, 3, 4)$

Optimal objective function value	Optimal allocation ratio			
$\overline{f_{1,1}}^{'} = 5.6147e + 05$	$x_{1,1} = 0.0062$	$x_{2,1} = 0.0064$	$x_{3,1} = 0.0091$	$x_{4,1} = 0.2750$
	$x_{1,2} = 0.0062$	$x_{2,2} = 0.0064$	$x_{3,2} = 0.0094$	$x_{4,2} = 0.6813$
$\overline{f_{1,2}}^{'} = 8.8971e + 05$	$x_{1,1} = 0.0067$	$x_{2,1} = 0.0070$	$x_{3,1} = 0.0106$	$x_{4,1} = 0.2518$
	$x_{1,2} = 0.0068$	$x_{2,2} = 0.0070$	$x_{3,2} = 0.0106$	$x_{4,2} = 0.6995$
$\overline{f_{1,3}}^{'} = 1.1155e + 03$	$x_{1,1} = 0.0058$	$x_{2,1} = 0.0104$	$x_{3,1} = 0.0285$	$x_{4,1} = 0.1034$
	$x_{1,2} = 0.0030$	$x_{2,2} = 0.0105$	$x_{3,2} = 0.0558$	$x_{4,2} = 0.7828$
$\overline{f_{1,4}}^{'} = 154.7805$	$x_{1,1} = 0.0019$	$x_{2,1} = 0.0041$	$x_{3,1} = 0.0292$	$x_{4,1} = 0.1270$
	$x_{1,2} = 0.0037$	$x_{2,2} = 0.0062$	$x_{3,2} = 0.0627$	$x_{4,2} = 0.7651$
$\overline{f_2}^{'} = 227.4136$	$x_{1,1} = 0.5128$	$x_{2,1} = 0.0101$	$x_{3,1} = 0.0000$	$x_{4,1} = 0.0000$
	$x_{1,2} = 0.4553$	$x_{2,2} = 0.0218$	$x_{3,2} = 0.0000$	$x_{4,2} = 0.0000$
$\bar{f}^{'} = -2.6866e + 05$	$x_{1,1} = 0.0066$	$x_{2,1} = 0.0068$	$x_{3,1} = 0.0101$	$x_{4,1} = 0.2625$
	$x_{1,2} = 0.0062$	$x_{2,2} = 0.0061$	$x_{3,2} = 0.0107$	$x_{4,2} = 0.6909$

Table 5 The change of the optimal value when $\beta_i = \beta^{'} = 0.8, \gamma = 0.5, (i = 1, 2, 3, 4)$

Optimal objective function value	Optimal allocation ratio			
$\overline{f_{1,1}}^{'} = 5.7157e + 05$	$x_{1,1} = 0.0041$	$x_{2,1} = 0.0043$	$x_{3,1} = 0.0062$	$x_{4,1} = 0.2621$
	$x_{1,2} = 0.0041$	$x_{2,2} = 0.0042$	$x_{3,2} = 0.0065$	$x_{4,2} = 0.7085$
$\overline{f_{1,2}}^{'} = 9.1524e + 05$	$x_{1,1} = 0.0047$	$x_{2,1} = 0.0049$	$x_{3,1} = 0.0077$	$x_{4,1} = 0.2390$
	$x_{1,2} = 0.0047$	$x_{2,2} = 0.0049$	$x_{3,2} = 0.0077$	$x_{4,2} = 0.7266$
$\overline{f_{1,3}}^{'} = 1.1386e + 03$	$x_{1,1} = 0.0044$	$x_{2,1} = 0.0083$	$x_{3,1} = 0.0242$	$x_{4,1} = 0.0947$
	$x_{1,2} = 0.0022$	$x_{2,2} = 0.0084$	$x_{3,2} = 0.0492$	$x_{4,2} = 0.8085$
$\overline{f_{1,4}}^{'} = 156.7492$	$x_{1,1} = 0.0013$	$x_{2,1} = 0.0030$	$x_{3,1} = 0.0243$	$x_{4,1} = 0.1165$
	$x_{1,2} = 0.0026$	$x_{2,2} = 0.0046$	$x_{3,2} = 0.0548$	$x_{4,2} = 0.7928$
$\overline{f_2}^{'} = 202.4259$	$x_{1,1} = 0.5331$	$x_{2,1} = 0.0030$	$x_{3,1} = 0.0000$	$x_{4,1} = 0.0000$
	$x_{1,2} = 0.4556$	$x_{2,2} = 0.0082$	$x_{3,2} = 0.0000$	$x_{4,2} = 0.0000$
$\bar{f}^{'} = -2.7355e + 05$	$x_{1,1} = 0.0042$	$x_{2,1} = 0.0043$	$x_{3,1} = 0.0066$	$x_{4,1} = 0.2573$
	$x_{1,2} = 0.0042$	$x_{2,2} = 0.0043$	$x_{3,2} = 0.0068$	$x_{4,2} = 0.7123$

Table 6 The change of the optimal value when $\beta_i = \beta^{'} = 0.8, \gamma = 0.4, (i = 1, 2, 3, 4)$

Optimal objective function value	Optimal allocation ratio			
$\overline{f_{1,1}}^{'} = 5.8072e + 05$	$x_{1,1} = 0.0024$ $x_{1,2} = 0.0024$	$x_{2,1} = 0.0025$ $x_{2,2} = 0.0025$	$x_{3,1} = 0.0038$ $x_{3,2} = 0.0040$	$x_{4,1} = 0.2444$ $x_{4,2} = 0.7380$
$\overline{f_{1,2}}^{'} = 9.3037e + 05$	$x_{1,1} = 0.0029$ $x_{1,2} = 0.0029$	$x_{2,1} = 0.0031$ $x_{2,2} = 0.0031$	$x_{3,1} = 0.0051$ $x_{3,2} = 0.0051$	$x_{4,1} = 0.2228$ $x_{4,2} = 0.7549$
$\overline{f_{1,3}}^{'} = 1.1604e + 03$	$x_{1,1} = 0.0033$ $x_{1,2} = 0.0016$	$x_{2,1} = 0.0065$ $x_{2,2} = 0.0065$	$x_{3,1} = 0.0202$ $x_{3,2} = 0.0428$	$x_{4,1} = 0.0857$ $x_{4,2} = 0.8334$
$\overline{f_{1,4}}^{'} = 158.5919$	$x_{1,1} = 0.0009$ $x_{1,2} = 0.0018$	$x_{2,1} = 0.0021$ $x_{2,2} = 0.0033$	$x_{3,1} = 0.0197$ $x_{3,2} = 0.0471$	$x_{4,1} = 0.1054$ $x_{4,2} = 0.8197$
$\overline{f_2}^{'} = 186.4357$	$x_{1,1} = 0.6067$ $x_{1,2} = 0.3933$	$x_{2,1} = 0.0000$ $x_{2,2} = 0.0000$	$x_{3,1} = 0.0000$ $x_{3,2} = 0.0000$	$x_{4,1} = 0.0000$ $x_{4,2} = 0.0000$
$\overline{f}^{'} = -2.7795e + 05$	$x_{1,1} = 0.0025$ $x_{1,2} = 0.0025$	$x_{2,1} = 0.0026$ $x_{2,2} = 0.0026$	$x_{3,1} = 0.0042$ $x_{3,2} = 0.0042$	$x_{4,1} = 0.2399$ $x_{4,2} = 0.7416$

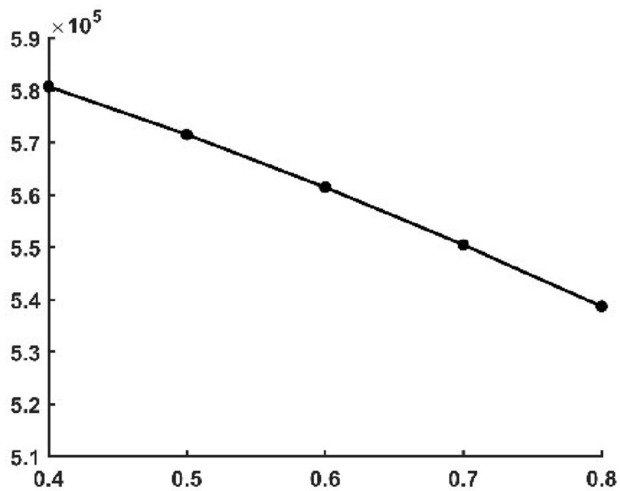

Fig. 2 The change of $\overline{f_{1,1}}^{'}$

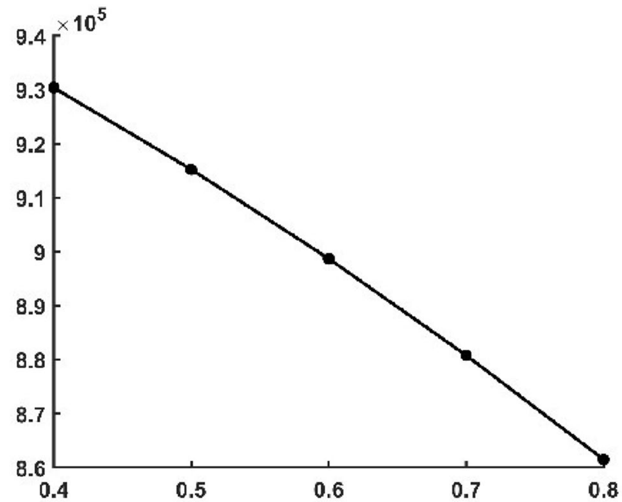

Fig. 3 The change of $\overline{f_{1,2}}^{'}$

promotion. On the other hand, it can be seen from the Fig. 6 that the image is concave, that is, the cost of advertising promotion increases with the increase of the diversification degree for KOL selection, and the speed of increase is getting faster and faster. Therefore, advertisers should try their best to reduce the selection of KOL when promoting advertising for the purpose of low cost.

6 Conclusions

In this study, we discussed the advertising promotion decision problem considering the KOL selection diversification. Among them, the full playing times of advertising videos, the number of likes, the number of positive and negative comments, the number of forwarding, and the

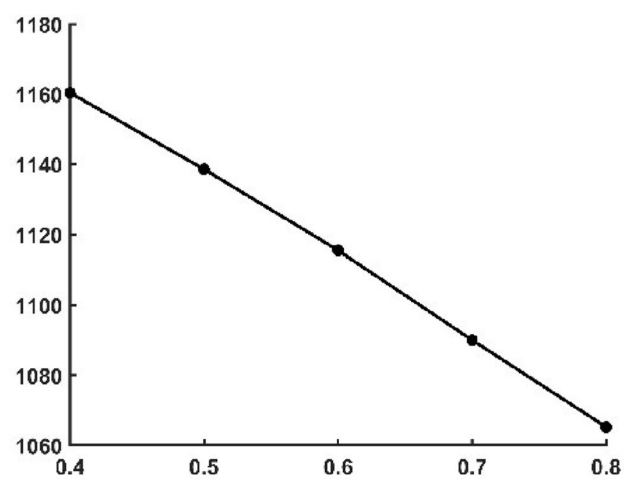

Fig. 4 The change of $\overline{f_{1,3}}^{'}$

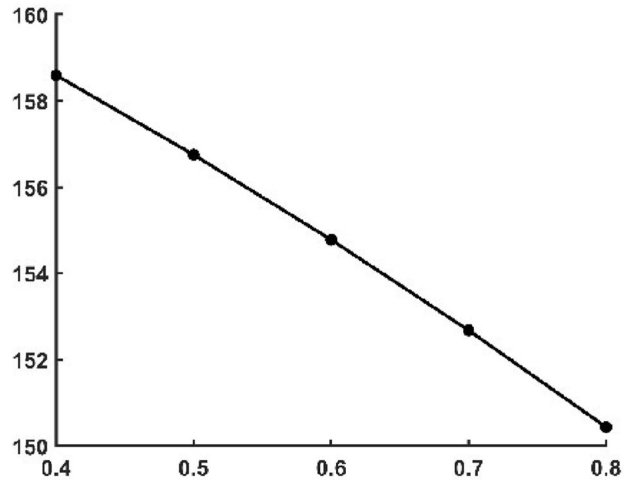

Fig. 5 The change of $\overline{f_{1,4}}'$

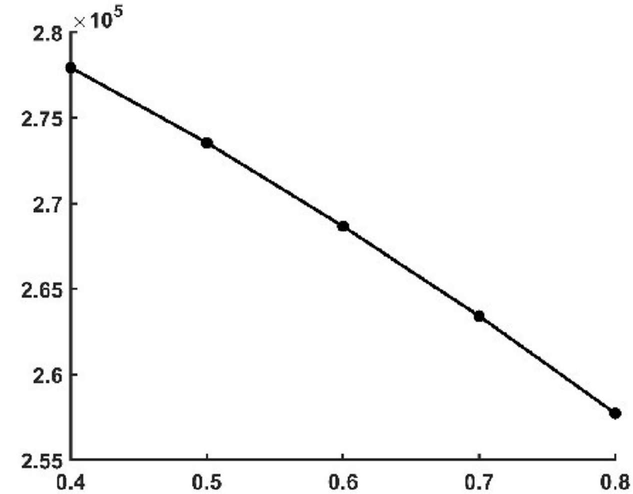

Fig. 7 The change of $-\overline{f}'$

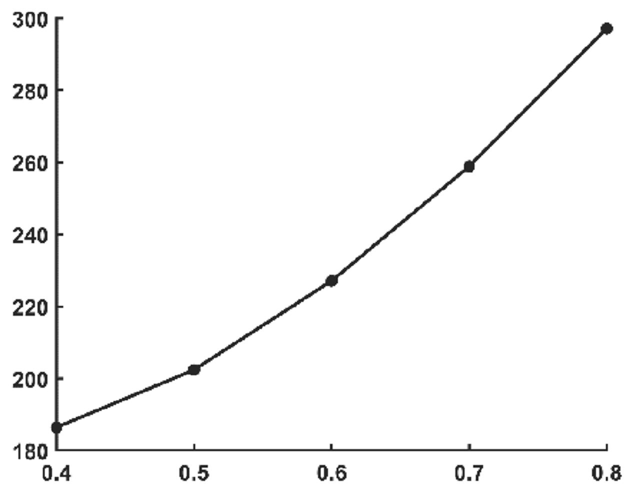

Fig. 6 The change of $\overline{f_2}'$

cost of advertising promotion are taken as uncertain variables. The KOL selection is the decision vector. First, we construct an uncertain multi-objective advertising promotion programming model with KOL selection as the decision vector for the first time, and consider the impact of KOL selection diversification constraints on the five objectives of maximizing the number of full playing, likes, comments and forwarding of advertising, and minimizing the cost of advertising promotion. Among them, we combine entropy constraint with chance constraint for the first time, and then propose the chance-entropy constraint, which is used to measure the degree of KOL selection diversification. In addition, we transform the multi-objective uncertain advertising promotion programming model into a single-objective uncertain programming model, and calculate the clear form of the model through the relevant operation rules of uncertainty theory. Finally, we conduct numerical simulation to verify the effectiveness and practicability of the model. The experimental results show that the KOL selection diversification in advertising promotion has an impact on the optimal decision-making of advertising promotion.

KOL selection in social media advertising is an interesting and novel topic. At present, our team has carried out research in this area and achieved some results. In the future research, we will conduct in-depth research and analysis on the decision-makers and executives of advertising promotion, as well as the uncertain influencing factors of advertising promotion, in order to provide more accurate and powerful decision support for advertising promotion behavior, so as to complete better advertising promotion.

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Declarations

Conflict of Interest The authors declare no conflict of interest.

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