



Dynamic Games and Applications: Special Issue in Memory of Chris Cannings—Introduction

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Chris Cannings was born in 1942 in Bristol and worked for almost his whole career at the University of Sheffield. He sadly died at the end of 2017. Chris was an early pioneer in the development of evolutionary games and contributed much to the early theory. He was one of the first to recognize the importance of the work of Maynard Smith and Price in 1973, and his first work on the subject was published soon after, in 1976. His initial focus was developing a strong mathematical foundation for the emerging theory, and he produced fundamental results in matrix games as well as a lot of important theory on the classic “war of attrition” game. He went on to develop new areas which grew out of this early work, including on patterns of evolutionarily stable strategies and multiplayer games. A great deal of his final work involved games on networks. Chris also developed a lot of important work in the field of mathematical genetics and produced a similarly impressive legacy in that field. This special issue, containing one review paper and eight research papers, is a tribute to his work in evolutionary games.

The special issue starts with a review paper by Bishop, Broom and Southwell, who were all PhD students of Chris from different stages of his career. This paper describes the impressive range of his work and its importance to the theory of evolutionary games. An important area of Chris’ work concerned patterns of evolutionarily stable strategies (ESSs) and the number of ESSs that matrix games could have. This corresponds to the standard quadratic optimization problem. In this issue, Bomze and Schachinger develop new perturbation methods to find ESS patterns building upon recent work by Bomze and colleagues, the most significant developments in this area for many years.

Another key area of Chris’ work was in multiplayer games, and in this issue, we have two papers further developing this area. Duong and Han develop new methods for finding the number of equilibria for multiplayer games and investigate the expected number of equilibria for certain random distributions of payoff entries, which relates to early work by Chris and by John Haigh. Bodnar, Miekisz and Vardanyan consider three-player games involving time delays, building on earlier work by Miekisz and colleagues, and show the critical effect such delays can have on the game equilibria.

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Time is also an important feature of the work of An, Wang and Zhang who consider an asymmetry game with periodic impulses, for example caused by seasonality, and show distinct results for different length periods, with longer periods leading to complexity and shorter periods promoting stability. Cressman, Koller, Garay and Garay consider mutant invasion into a multispecies scenario governed by Lotka–Volterra equations. Here, the presence of other species can have an important effect on whether mutants can replace residents. Safley, Sun and Rychtar consider a different sort of system with two types, investigating another classical game theory concept, that of signalling. Here, in a version of the Pygmalion game, where signallers signal quality, they show that dishonest signalling is prevalent.

The final two papers deal with one of the most important concepts in game theory, namely cooperative behaviour. Debarre considers a model of altruistic behaviour where individuals migrate between areas and strategy transmission from parent to offspring is not faithful, and shows that, contrary to earlier work, higher emigration can favour altruism under such circumstances. Wang, Liu and Chen consider a population with peer punishment for defection, as well as second-order punishment, where individuals can disguise themselves to avoid punishment. They show that second-order punishment is required for punishment to be effective.

As evidenced by the diverse contributions to this special issue, game-theoretic reasoning has extended well beyond its early emphasis on matrix and other classic games. This diverse approach was a feature of Chris Cannings' own work on evolutionary games. He published over 130 papers, approximately evenly split between evolutionary games and mathematical genetics, with important game theory papers appearing in journals including *Advances in Applied Probability*, the *Journal of Theoretical Biology* and *Proceedings of the Royal Society of London Series A*.

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