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Multiple attribute dynamic decision making method based on some complex aggregation functions in CQROF setting

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Abstract

The q-rung Orthopair fuzzy set (QROFS) is one of the fuzzy structures which can introduce more fuzzy information than other fuzzy frames proposed by Ronald R. Yager. In this article, the dynamic multiple attribute decision making (DMADM) approach with complex q-rung Orthopair fuzzy (CQROF) information has been introduced. The ideas of CQROF variable and uncertain CQROF variables are defined and introduced new dynamic weighted averaging (DWA) operators called dynamic complex q-rung Orthopair fuzzy weighted average (DCQROFWA) operator and uncertain dynamic complex q-rung Orthopair fuzzy weighted average (UDCQROFWA) operator. For the moment, a procedure has been developed based on DCQROFWA and CQROFWA operator to solve DMADM problems where all attribute information are used in complex q-rung Orthopair fuzzy numbers (CQROFNs) collected at distinct periods, and another procedure is developed based on UDCQROFWA and CIVQROFWA operators to solve uncertain DMADM problems for interval uncertainty in which all attribute information takes in the form of complex interval-valued q-rung Orthopair fuzzy numbers (CIVQROFNs) collected at distinct periods. Finally, a comprehensive comparative analysis has been made for the proposed approach for testing its applicability and efficiency by considering a numerical example.

Keywords Dynamic multiple attribute decision-making · Complex q-rung Orthopair fuzzy numbers · DCQROFWA operator · UDCQROWA operator

Mathematics Subject Classification 06D72 · 08A72 · 03C05 · 30D05

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Abbreviations

DM	Decision makers
AOs	Aggregation operators
AHP	Analytic hierarchy process
DWA	dynamic weighted averaging
IFS	Intuitionistic fuzzy set
CIFN	Complex intuitionistic fuzzy set
PYFS	Pythagorean fuzzy set
CPYFS	Complex Pythagorean fuzzy set
QROFS	q-Rung Orthopair fuzzy set
CQROF	Complex q-Rung Orthopair fuzzy set
MADM	Multi attribute decision making
DMADM	Dynamic multi attribute decision making
DMAGDM	Dynamic multi attribute group decision making
IVIFN	Interval-valued intuitionistic fuzzy numbers
CIVQROFS	Complex interval-valued q-Rung Orthopair fuzzy set
DCQROFWA	Dynamic complex q-Rung Orthopair fuzzy weighted average
VIKOR	Multi-criteria Optimization and Compromise Solution, with pronunciation
TOPSIS	Technique for Order of Preference by Similarity to Ideal Solution
TODIM	An acronym in Portuguese for Interative Multi-criteria Decision Making
UDCQROFWA	Uncertain dynamic complex q-Rung Orthopair fuzzy weighted average
CIVQROFWA	Dynamic complex interval-valued q-Rung Orthopair fuzzy weighted average

1 Introduction

Uncertainty is one of the problems that arrived when we handled realistic situations in the science and technology environment. In that point, Zadeh (1965) invented fuzzy set (FS), which were realized in multiple-attribute decision-making (MADM) problems and multiple attribute group decision-making (MAGDM) problems. However, there are some problems that FS can not solve. To discuss this issue, Attanassov (1986) carry out the notion of an intuitionistic fuzzy set (IFS) as an extension of FS. This concept was favourably applied in different applications areas towards engineering, medical diagnosis, supply chain management, and in MADM problems (Chen 2007; Chen and Chang 2015; De et al 2000; Guo and Song 2014; Gupta et al 2018; Jana et al 2019c; Jana and Pal 2021; Jana 2021; Liu and Chen 2016; Liu et al 2019; Senapati and Chen 2021; Senapati et al 2021; Senapati and Yager 2019). However, when the sum of the membership and non-membership degrees does not lie in [0, 1], for example, $0 < 0.5 + 0.7 = 1.2 \nleq 1$, this type of situation can not be overcome by the use of IFS. To solve this kind of problem, Pythagorean fuzzy set (PYFS), as an extension of IFS, was introduced by Yager and Abbasov (2013) and Yager (2013, 2014), of which the sum of the square of membership and non-membership grade is "< 1" or "$= 1$". Since then, PYFS has received more and more attention due to its features. A large number of investigators have paid their interest to handle the decision-making problems with PYFS data. Rani et al (2020) have studied pharmacological therapy selection for type-2 diabetes in PYFS information based on new entropy and score function. Liang et al (2019) developed a decision-making method for testing quality assessment in the internet baking industry

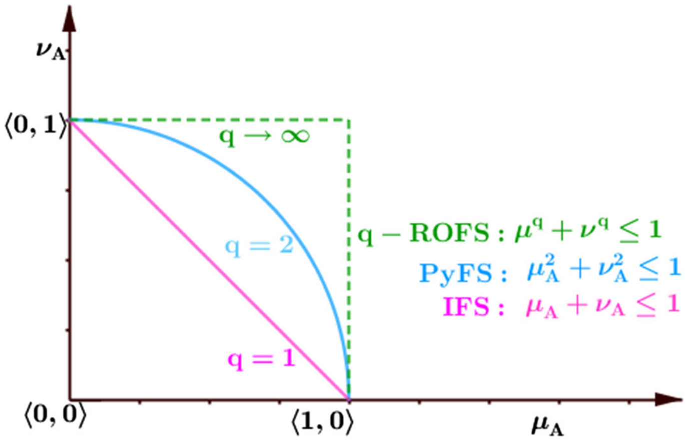

Fig. 1 Geometric representation of FS, IFS, PyFS

based on TODIM Pythagorean fuzzy VIKOR approach. Garg (2017) have proposed some generalized PYFS information aggregation using Einstein norms and then applied them to decision making problems. Ren et al (2016) have been extended PYF information based TODIM approach to multi-criteria decision making (MCDM) problems. Liang et al (2018) have utilized extended Bonferroni mean operator in the PYFS and then developed an algorithm for the application of the proposed approach. Jana et al (2019b) used solution concepts of PYFS information based on Dombi norms (Jana et al 2019; Jana et al 2019a) and applied Pythagorean Dombi operators for solving MADM problems. For more information related to PYFS, the reader may see Dick et al (2016) and Reformat and Yager (2014)) (Fig. 1).

Although IFS and PYFS can solve some uncertain situations, these sets cannot be handled all types of information entirely. As seen in this example, when a decision-maker used 0.8 as membership and 0.7 as non-membership grade, then observed that $0.8^2 + 0.7^2 = 1.13 \nleq 1$. So, PYFS cannot handle such uncertainty. For those problems, Yager (2017) have introduced Q-rung orthopair fuzzy sets (QROFS) which is more robust and usual than IFS and PYFS to lead an intricate and uncertain problems in the fuzzy frames. Furthermore, Liu and Wang (2018) have introduced QROF aggregation functions and then used them for solving a MADM problem. Liu and Liu (2018) built up a MAGDM problem based on some Bonferroni mean QROF aggregation functions. Wei et al (2018) proposed Heronian mean QROF aggregation function and used these operators to develop a MADM method. Liu et al (2018b) have been motivated to study a MAGDM problem using the power Maclaurin QROF aggregation function for accumulating QROF information. For more information regarding the application of QROF, see (Ju et al 2019; Krishankumar et al 2020; Tang et al 2020).

However, many contributors thought about what will be happened if we changed the codomain of FS by a complex number instead of [0, 1]. The essentiality of its appearance was first proposed by Ramot et al (2002), who proposed complex fuzzy sets (CFS) as a another extension of FS, where CFS is a complex-valued function, i.e., $Z_\tau(x) = M_\tau(x) \cdot e^{2i\pi N_\tau(x)}$ which follows that $0 \leq M_\tau(x), N_\tau(x) \leq 1$. Later on, researchers are more attentive to the study of CFS (Buckley 1989; Nguyen et al 2000). As in earlier, CFS has some advantages for solving complex fuzzy information. But, there are some problems or difficulties for handling complex fuzzy information by CFS. To manipulate those problems, Alkouri and Salleh (2012) proposed a complex intuitionistic fuzzy set (CIFS), where CIFS is characterized by complex-valued membership grade and complex-valued non-membership grade. The sum of the real part and imaginary part equal to or less than 1. For more results related to CIFS referred to Garg and Rani (2019), Liu and Wang (2018), Rani and Garg (2018).

It is well known that CIFS can not solve all the cases of complex-valued function, as shown in the example when the expert uses $0.8e^{2i\pi(0.7)}$ for membership grade and $0.6e^{2i\pi(0.5)}$ as a non-membership grade, we observe that $0.8 + 0.6 = 1.4 \geq 1$ and $0.7 + 0.5 = 1.2 \geq 1$. To control this type of situation, Ullah et al (2020) have proposed a complex Pythagorean fuzzy set (CPYFS), where the sum of the square of the real (imaginary) part for complex-valued membership and the sum of the square of the real (imaginary) part of complex-valued non-membership degrees are "$= 1$" or "< 1", here as in previous shows, $0.8^2 + 0.6^2 = 1$ and $0.7^2 + 0.5^2 = 0.74 \leq 1$. Thus, CPYFS received more attention from the researchers. Later on, Akram and Naz (2019) proposed a novel approach to CPYF graph. The experts have widely utilized CIFS and CPYFS. Still, due to the complexity of the decision-making problems, in some cases, where decision-makers are hesitated to give their judgement in the form of single-valued membership and non-membership degrees. The valuable pre-existing studies help the experts for depicting the uncertainties in his/her adverse decision in the complex decision-making problems. Then, it is realized that experts must have the freedom to give their preferences through intervals.

Same situations arise in the theory of CIFS and CPYFS when DMs uses these types of data because these data do not follow the condition of CIFS and the condition of CPYFS. As seen in the example, for this complex-valued membership and non-membership function $0.7e^{2i\pi(0.8)}$ and $0.8e^{2i\pi(0.9)}$, i.e., $0.7 + 0.8 = 1.6 > 1,\ 0.8 + 0.9 = 1.7 > 1$ and $0.7^2 + 0.8^2 = 1.3 > 1,\ 0.8^2 + 0.9^2 = 1.45 > 1$. To control such types of complex uncertainties, Liu et al (2019) proposed complex q-rung Orthopair fuzzy set (CQROFS). The renowned characteristic of CQROFS is that the sum of the qth power of the real (and imaginary) part for membership and the qth power of the real (imaginary) part of the non-membership degree is equal to or less than 1, i.e., $0.7^5 + 0.8^5 = 0.5 \leq 1,\ 0.8^5 + 0.9^5 = 0.92 \leq 1$. Thus, CQROFS efficiently handled complex-valued fuzzy uncertain decision-making problems. It was also noted that CQROFS is more general than CIFS and CPYFS because if the imaginary part of membership and non-membership takes zero, then CQROFS is converted into QROFS. For the parameter $q = 1$, then QROFS is reduced to IFS, and for the parameter $q = 2$, then QROFS is reduced to PYFS. Therefore, CQROFS can be considered more fuzzy information than CIFS and CPYFS. Later on, Liu et al (2020) have been utilized for the concepts of CQROF aggregation function for accumulating CQROF information. The proposed CQROF weighted average (CQROFWA) operator and CQROF weighted geometric (CQROFWG) operator, and then applied these operators to develop MADM problems. In the same environment, Garg et al (2020) have been extended the concept of CQROFS information to a complex interval-valued q-rung Orthopair set (CIVQROFS) and then used Analytic hierarchy process (AHP) and technique for order preference by similarity to ideal solution (TOPSIS) method in this environment for the study of MADM problems.

In the above all decision-making problems, they are considering decision information where all the data take place in the same period. However, there are so many decision areas such as medical diagnosis, multi-period dynamic investment, dynamic personal selection, and dynamic military system efficient evaluation, etc.; these original data are collected at distinct periods. Xu and Yager (2008) first pointed out such type of information in their DMADM problems. They gave an application of DMADM issues based on dynamic intuitionistic fuzzy weighted average (DIFWA) operator and uncertain dynamic intuitionistic fuzzy weighted average (UDIFWA) operator to accumulate dynamic or uncertain dynamic IFS information. Further, they applied DIFWA and UDIFWA operators to solve two DMADM problems, where the argument of the attributes are used in IFNs or IVIFNs. Wei (2009) has been studied the DMADM problems where attributes values are used in IFNs or IVIFNs. He proposed some weighted geometric AOs composed from distinct periods, such as the DIFW geometric (DIFWG) operator and uncertain DIFW geometric (UDIFWG) operator to accumulate dynamic or uncertain dynamic IFS data. A procedure has been constructed based on DIFWG and IFWG operators to develop DIFMADM problems where attribute information is in IFNs. Another approach has been developed based on UDIFWG, and IVIFWG operators are derived for solving uncertain DMADM issues where all the information about attributes are given in IVIFNs composed at multiple periods. Further, he used three TOPSIS approaches to obtain the individual closeness coefficient of each alternative to positive and negative ideal option depending on the decision information provided in real number, linguistic label, and an interval number. Finally, he gave an application of the HGA and DWG operators for DMADM problems. Li et al (2015) focus on studying the dynamic multi-criteria decision making (MCDM) approach. In this approach, they applied a mathematical programming method for calculating an attribute's weight and used the BUM function to determine the time weight. Chen and Li (2011) investigated the DMADM approach based on triangular intuitionistic fuzzy numbers (TIFNs). They have been defined as the dynamic TIFNs weighted average operator in which the decision data of each attribute is in TIFNs. Furthermore, some dynamic MADM problems exist based on AOs (Chen and Li 2011; Li et al 2015; Wei 2009; Xu and Yager 2008) in an intuitionistic fuzzy environment, but they can not take into account in complex dynamic aggregation operators. On the other, present research on complex fuzzy environment (Akram and Naz 2019; Dick et al 2016; Garg et al 2020; Garg and Rani 2019; Liu et al 2020, a; Nguyen et al 2000; Rani et al 2020; Rani and Garg 2018) and till there is no research on the proposed approach in my knowledge. In this model, we try to fill this research gap to address the DCQROFMADM with CQROF information. Therefore, the proposed method is more effective and advanced than other existing methods. The aim of this paper is to propose some dynamic complex averaging operators such as DCQROFWA operator and UDCQROFWA operator to accumulate dynamic or uncertain dynamic CQROF information. Further, using operators DCQROFWA and UDCQROFWA, respectively, we shall develop two procedures for solving DMADM problems where all the attribute information are taken as CQROFNs or CIVQROFNs. The objectives of this paper are to

– a new approach is considered in connection with some complex dynamic operators
– utilize the proposed method for DCQROFMADM approach
– a case study is provided to demonstrate the method by a numerical example
– superiority of the method is verified numerically.

To do so, the remainder of this paper is organized as follows. In the next section, we review some basic concepts of complex q-rung Orthopair fuzzy set. In Sect. 3, propose a new operator called dynamic complex q-rung Orthopair fuzzy weighted average (DCQROFWA) operator. In Sect. 4, we develop DMADM problem with CQROF information where all the attribute information is used in CQROFNs take place at distinct periods. Then, an application model for DMADM has been established based on DCQROFWA and CQROFWA operators to accumulate CQROFNs collected to corresponding each period and get the most favourable one(s) alternative from the ordering of the options based on the values of score and accuracy function. In Sect. 5, another method has been developed based on UDCQROFWA and CIVQROFWA operators for solving uncertain DMADM problems under uncertain intervals in which all the attribute information are provided in CIVQROFNs collected at multiple periods. An illustrative numerical example is pointed out for the proposed model in Sect. 6. Scope of future problems aim to work, and a remark is given in Sect. 7.

2 Preliminaries

Basic concepts of Q-ROFEs are briefly reviewed in this section.

Definition 1 (Liu et al 2020) A CQROFEs is an object of the form

$$Q^{\%} = \{\langle x, \xi_c(x), \chi_c(x)\rangle | x \in \mathcal{X}\}$$

where $\xi_c = \xi_{rp}(x)e^{2i\pi\left(\xi_{ip}(x)\right)}$ and $\chi_c = \chi_{rp}(x)e^{2i\pi\left(\chi_{ip}(x)\right)}$ are the complex grade of postive and negative opinion such that $0 \leq \xi_{rp}^q + \chi_{rp}^q \leq 1$ and $0 \leq \xi_{ip}^q + \chi_{ip}^q \leq 1$, and the refusal degree of CQROFEs is denoted as $\pi_c = \left\{1 - \left(\xi_{rp}^q + \chi_{rp}^q\right)\right\}^{1/q} e^{2i\pi\left\{1-\left(\xi_{ip}^q+\chi_{ip}^q\right)\right\}^{1/q}}$. The complex Q-rung orthopair fuzzy elements (CQROFE) is simply applying as $Q^{\%}{}_1 = \left(\xi_{rp-1}e^{2i\pi(\xi_{ip-1})}, \chi_{rp-1}e^{2i\pi(\chi_{ip-1})}\right)$.

The score function value of the any CQROFE $Q^{\%}{}_{cq-1} = \left(\xi_{rp-1}e^{2i\pi(\xi_{ip-1})}, \chi_{rp-1} e^{2i\pi(\chi_{ip-1})}\right)$ is defined in the following equation

Definition 2 (Liu et al 2020)

$$CSF(Q_1^{\%}) = \frac{1}{2}\left(\xi_{rp-1} - \chi_{rp-1} + \xi_{ip-1} - \chi_{ip-1}\right) \tag{1}$$

where, $CSF(Q_1^{\%}) \in [-1, 1]$ and the accuracy function follows the equation

Definition 3 (Liu et al 2020)

$$CAF(Q_1^{\%}) = \frac{1}{2}\left(\xi_{rp-1} + \chi_{rp-1} + \xi_{ip-1} + \chi_{ip-1}\right) \tag{2}$$

where, $CAF(Q_1^{\%}) \in [0, 1]$,

then prioritized realtion between any two CQROFEs $Q^{\%}{}_1 = \left(\xi_{rp-1}e^{2i\pi(\xi_{ip-1})}, \chi_{rp-1} e^{2i\pi(\chi_{ip-1})}\right)$ and $Q^{\%}{}_2 = \left(\xi_{rp-2}e^{2i\pi(\xi_{ip-2})}, \chi_{rp-2}e^{2i\pi(\chi_{ip-2})}\right)$ is defined as follows:

(i) If $CSF(Q_1^{\%}) < CSF(Q_2^{\%})$, imply $Q_1^{\%} \prec Q_2^{\%}$
(ii) If $CSF(Q_1^{\%}) > CSF(Q_2^{\%})$, imply $Q_1^{\%} \succ Q_2^{\%}$
(iii) If $CSF(Q_1^{\%}) = CSF(Q_2^{\%})$, then

(1) If $CAF(Q_1^{\%}) < CAF(Q_2^{\%})$, imply $Q_1^{\%} \prec Q_2^{\%}$
(2) If $CAF(Q_1^{\%}) > CAF(Q_2^{\%})$, imply $Q_1^{\%} \prec Q_2^{\%}$
(3) If $CAF(Q_1^{\%}) = CAF(Q_2^{\%})$, imply $Q_1^{\%} \sim Q_2^{\%}$.

3 Dynamic complex q-rung orthopair fuzzy weighted averaging (CQROFWA) operator

To aggregate the CQROFEs, Liu et al (2020) studied complex q-rung Orthopair fuzzy weighted averaging (CQROFWA) operator. For simplicity, let CQ be the set of all ***complex q-rung orthopair fuzzy numbers.

Definition 4 Let $Q_j^{\%} = \left(\xi_{rp-j} \cdot e^{2i\pi(\xi_{ip-j})}, \chi_{rp-j} \cdot e^{2i\pi(\chi_{ip-j})}\right)$ $(j = 1, 2, \ldots, v)$ be a CQROFEs, and let $CQROFWA : \Theta^v \to \Theta$, if

$$\begin{aligned} CQROFWA(Q_1^{\%}, Q_2^{\%}, \ldots, Q_v^{\%}) &= \prod_{j=1}^{v} \left(\psi_j Q_j^{\%}\right) \\ &= \left(\left\{\left(1 - \prod_{j=1}^{v} \left(1 - \xi_{rp-j}^{q}\right)^{\psi_j}\right\}^{1/q} \right. \\ &\quad e^{2i\pi\left\{(1-\prod_{j=1}^{v}(1-\xi_{ip-j}^{q})^{\psi_j})\right\}^{1/q}}, \\ &\quad \left. \prod_{j=1}^{v} \chi_{rp-j}^{\psi_j}\, e^{2i\pi(\prod\limits_{j=1}^{v} \chi_{ip-j}^{\psi_j})}\right) \end{aligned} \tag{3}$$

where, $\psi = (\psi_1, \psi_2, \ldots, \psi_v)^T$ be such that $\psi_j > 0$, and $\sum_{j=1}^{v} \psi_j = 1$, the CQROFWA is called complex q-rung Orthopair fuzzy weighted averaging operator.

However, the CQROFWA can only be considered to accumulate complex q-rung Orthopair fuzzy information where time is not taken into account. If time is taken is considered, then CQROFEs information may be collected at different periods, then it is not suitable to lead these situations.

Definition 5 Let t be consider as time variable, then we call $\beta(t) = \left(\xi_{rp\beta(t)} \cdot e^{2i\pi(\xi_{ip\beta(t)})}, \chi_{rp\beta(t)} \cdot e^{2i\pi(\chi_{ip\beta(t)})}\right)$ be complex q-rung Orthopair fuzzy variable (CQROFV), where $\xi_{rp\beta(t)} \in [0, 1]$, $\chi_{rp\beta(t)} \in [0, 1]$, and $\xi_{ip\beta(t)} \in [0, 1]$, $\chi_{ip\beta(t)} \in [0, 1]$ such that $0 \le \xi_{rp\beta(t)} + \chi_{rp\beta(t)} \le 1$, and $0 \le \xi_{ip\beta(t)} + \chi_{ip\beta(t)} \le 1$.

For CQROFV $\beta(t) = \left(\xi_{rp\beta(t)} \cdot e^{2i\pi(\xi_{ip\beta(t)})}, \chi_{rp\beta(t)} \cdot e^{2i\pi(\chi_{ip\beta(t)})}\right)$, if $t = t_1, t_2, \ldots, t_l$, then $Q^{\%}(t_1), Q^{\%}(t_2), \ldots, Q^{\%}(t_l)$ are indicated as l distinct complex q-rung Orthopair fuzzy numbers.

Definition 6 $\beta(t_1) = \left(\xi_{rp\beta(t_1)} \cdot e^{2i\pi(\xi_{ip\beta(t_1)})}, \chi_{rp\beta(t_1)} \cdot e^{2i\pi(\chi_{ip\beta(t_1)})}\right)$ and $\beta(t_2) = \left(\xi_{rp\beta(t_2)} \cdot e^{2i\pi(\xi_{ip\beta(t_2)})}, \chi_{rp\beta(t_2)} \cdot e^{2i\pi(\chi_{ip\beta(t_2)})}\right)$ be two CQROFV, then

(1) $\beta(t_1) \bigoplus \beta(t_2)$

$$= \left\langle \left\{1 - \left(\left(1 - \xi_{rp\beta(t_1)}^{q}\right)\left(1 - \xi_{rp\beta(t_2)}^{q}\right)\right\}^{1/q}, e^{2i\pi\left\{1-\left(1-\xi_{ip\beta(t_1)}^{q}\right)\left(1-\xi_{ip\beta(t_2)}^{q}\right)\right\}^{1/q}}, \right.$$

$$\left. \chi_{rp\beta(t_1)}\chi_{rp\beta(t_2)} \cdot e^{2i\pi(\chi_{ip\beta(t_1)})} e^{2i\pi(\chi_{ip\beta(t_2)})} \right\rangle$$

(2) $\lambda\beta(t_1) = \left(1 - \left(1 - \xi_{rp\beta(t_1)}^{q}\right)^{\lambda}\right)^{1/q} \cdot e^{2i\pi\left(1-\xi_{ip\beta(t_1)}^{q}\right)^{\lambda})^{1/q}}, \chi_{rp\beta(t_1)}^{\lambda} \cdot e^{2i\pi(\chi_{ip\beta(t_1)}^{\lambda})}\Big)$,

$\lambda > 0$.

Definition 7 Let $Q^{\%}_{t_m} = \left(\xi_{rp\beta(t_m)} \cdot e^{2i\pi(\xi_{ip\beta(t_m)})}, \chi_{rp\beta(t_m)} \cdot e^{2i\pi(\chi_{ip\beta(t_m)})}\right)$, $(m = 1, 2, \ldots, l)$ be a group of CQROFEs at l distinct periods t_m $(m = 1, 2, \ldots, l)$, and let $DCQROFWA : \Theta^v \to \Theta$, where

$$DCQROFWA_{\psi(t)}\left(Q^{\%}_{t_1}, Q^{\%}_{t_m}, \ldots, Q^{\%}_{t_l}\right) = \prod_{m=1}^{l} \psi(t_m)(Q^{\%}_{t_m}) \tag{4}$$

where $\psi(t) = (\psi(t_1), \psi(t_2), \ldots, \psi(t_l))^T$ be a weight vector of the periods t_m $(m = 1, 2, \ldots, l)$, and $\psi(t_m) \in [0, 1]$, $\sum_{m=1}^{l} \psi(t_m) = 1$. Then, DCQROFWA is called dynamic complex q-rung Orthopair fuzzy weighted averaging (DCQROFWA) operator.

Following from Definition 6, Eq. (4) can be written in the form as

$$\begin{aligned} & DCQROFWA_{\psi(t)}\left(Q^{\%}_{t_1}, Q^{\%}_{t_m}, \ldots, Q^{\%}_{t_l}\right) \\ & = \left\langle \left\{1 - \prod_{m=1}^{l}\left(1 - \xi^{q}_{rp\beta(t_1)}\right)^{\psi(t_m)}\right\}^{1/q} \cdot e^{2i\pi\left\{1-\prod_{m=1}^{l}\left(1-\xi^{q}_{ip\beta(t_m)}\right)^{\psi(t_m)}\right\}^{1/q}}, \right. \\ & \left. \prod_{m=1}^{l} \chi^{\psi(t_m)}_{rp\beta(t_m)} \cdot e^{2i\pi\left(\chi^{\psi(t_m)}_{ip\beta(t_m)}\right)} \right\rangle \end{aligned} \tag{5}$$

For DCQROFWA operator, to assign the weight vector
$\psi(t) = (\psi(t_1), \psi(t_2), \ldots, \psi(t_l))^T$ of the periods t_m $(m = 1, 2, \ldots, l)$ is a very important step. In general, $\psi(t)$ is is determine by decision maker(s) directly, or can be assigned by excited methods from Xu and Yager (2008).

4 A approach for dynamic complex q-rung Orthopair fuzzy multiple attribute decision making problem

The following notations are utilized to present the dynamic MADM with complex q-rung Orthopair fuzzy information.

(1) Here, we use discrete set of alternatives $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_u\}$ which are known.

(2) The set of attributes $\mathcal{G} = \{\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_v\}$ are known attributes, and $w = (w_1, w_2, \ldots, w_v)^T$ be the weight vector of the attributes, such that $w_j > 0$, $\sum_{j=1}^{v} w_j = 1$ for $j = 1, 2, \ldots, v$.

(3) There are l distinct periods t_m $(k = 1, 2, \ldots, l)$ whose weight is considered as $\psi(t) = (\psi(t_1), \psi(t_2), \ldots, \psi(t_l))^l$ such as $\psi(t_m) > 0$, $m = 1, 2, \ldots, l$, and $\psi(t_m) \in [0, 1]$, $\sum_{m=1}^{l} \psi(t_m) = 1$.

(4) Suppose $M(t_m) = \left(\Omega_{ij}(t_m)\right)_{u\times v} = \left(\xi_{rp(t_m)-ij} \cdot e^{2i\pi(\xi_{ip(t_m)-ij})}, \chi_{rp(t_m)-ij} \cdot e^{2i\pi(\chi_{ip(t_m)-ij})}\right)_{u\times v}$ is the complex q-rung Orthopair fuzzy decision matrix which take l distinct periods t_m $(m = 1, 2, \ldots, l)$, where $\xi_{rp(t_m)-ij} \cdot e^{2i\pi(\xi_{ip(t_m)-ij})}$ is the degree of CQROFE's grade of positive opinion for the alternative $\mathcal{A}_u$ satisfying the attribute G_v at periods t_m $(m = 1, 2, \ldots, l)$, and $\chi_{rp(t_m)-ij} \cdot e^{2i\pi(\chi_{ip(t_m)-ij})}$ is the degree of CQROFE's grade of negative opinion for the alternative $\mathcal{A}_u$ does not satisfying the attribute G_v at periods t_m $(m = 1, 2, \ldots, l)$ such as $\xi_{rp(t_m)-ij} \in [0, 1]$, $\chi_{rp(t_m)-ij} \in [0, 1]$, $\xi_{ip(t_m)-ij} \in [0, 1]$, and $\chi_{ip(t_m)-ij} \in [0, 1]$, and $0 \le \xi^{q}_{rp(t_m)-ij} + \chi^{q}_{rp(t_m)-ij} \le 1$, and $0 \le \xi^{q}_{ip(t_m)-ij} + \chi^{q}_{ip(t_m)-ij} \le 1$.

Based on the above-supplied information, we develop an Algorithm to order and select the favourable one(s). The Algorithm contains the following steps.

(Procedure I)

step 1: Applying the DCQROFWA operator:

$$\begin{aligned} & DCQROFWA_{\psi(t)}\Big(Q^{\%}(t_1), Q^{\%}(t_2), \ldots, Q^{\%}(t_l)\Big) \\ & = \Bigg\langle \Bigg\{1 - \prod_{m=1}^{l} \Big(\xi^{q}_{rp\beta(t_1)}\Big)^{\psi(t_m)}\Bigg\}^{1/q} \cdot e^{2i\pi\left\{1-\prod_{m=1}^{l}\left(1-\xi^{q}_{ip\beta(t_m)}\right)^{\psi(t_m)}\right\}^{1/q}}, \\ & \prod_{m=1}^{l} \chi^{\psi(t_m)}_{rp\beta(t_m)} \cdot e^{2i\pi\left(\chi^{\psi(t_m)}_{ip\beta(t_m)}\right)}\Bigg\rangle. \end{aligned} \tag{6}$$

To aggregate all the complex q-rung Orthopair fuzzy decision matrices $M(t_m) = \Big(\xi_{rp(t_m)-ij} \cdot e^{2i\pi(\xi_{ip(t_m)-ij})}, \chi_{rp(t_m)-ij} \cdot e^{2i\pi(\chi_{ip(t_m)-ij})}\Big)_{u\times v}$ $(m = 1, 2, \ldots, l)$ into a complex q-rung Orthopair fuzzy decision matrix, $M = ((\xi_{rp-ij} \cdot e^{2i\pi(\xi_{ip-ij})}, \chi_{rp-ij} \cdot e^{2i\pi(\chi_{ip-ij})}\Big)_{u\times v}$.

Step 2: Utilize CQROFWA operator:

$$\begin{aligned} r_i &= \Big(\xi_{rp-i} \cdot e^{2i\pi(\xi_{ip-i})}, \chi_{rp-i} \cdot e^{2i\pi(\chi_{ip-i})}\Big) = CQROFWA_w\Big(Q^{\%}_{i1}, Q^{\%}_{i2}, \ldots, Q^{\%}_{iv}\Big) \\ &= \Bigg\langle \Bigg\{1 - \prod_{i=1}^{v}\Big(\xi^{q}_{rp-i}\Big)^{w}\Bigg\}^{1/q} \cdot e^{2i\pi\left\{1-\prod_{i=1}^{v}\left(1-\xi^{q}_{ip-i}\right)^{w)}\right\}^{1/q}}, \prod_{i=1}^{v} \chi^{w}_{rp-i} \cdot e^{2i\pi\left(\chi^{w}_{ip-i}\right)}\Bigg\rangle. \end{aligned} \tag{7}$$

Step 3: Compute the score value $CSF(\Omega_i)$ $(i = 1, 2, \ldots, u)$ of the overall CQROFEs to rank all the alternatives $\mathcal{A}_i$ $(i = 1, 2, \ldots, u)$ to adopt desired choice $\mathcal{A}_i$. If the values of $CSF(\Omega_i)$ and $CSF(\Omega_j)$ are same, then we continue to evaluate accuracy degrees $CAF(\Omega_i)$ and $CAF(\Omega_j)$ based on overall CQROFEs information band rank the alternatives $\mathcal{A}_i$ depending on degree of accuracy $CAF(\Omega_i)$ and $CAF(\Omega_j)$.

Step 4: Rank all the alternative $\mathcal{A}_i$ $(i = 1, 2, \ldots, u)$ in order to choose the best one(s) in accordance with $CSF(\Omega_i)$ and $CAF(\Omega_j)$ $(i = 1, 2, \ldots, u)$.

Step 5: End.

5 Interval-valued complex q-rung Orthopair fuzzy information

Garg et al (2020) introduced complex interval-valued q-rung orthopair fuzzy set (CIVQROFES), which is a generalization of complex q-rung orthopair fuzzy set (CQROFEs), proposed by Liu et al (2020). The main characteristic of CIVQROFES is that complex grades of membership and non-membership values are interval numbers rather than exact numbers.

Definition 8 (Liu et al 2020) A CIVQROFEs is an object of the form

$$Q^{\%} = \{\langle x, \xi'_c, \chi'_c(x)\rangle | x \in \mathcal{X}\}$$

where $\xi'_c = \Big[\xi^{L}_{rp}, \xi^{R}_{rp}\Big] \cdot e^{2i\pi\left(\left[\xi^{L}_{ip}, \xi^{R}_{ip}\right]\right)}$ and $\chi'_c = \Big[\chi^{L}_{rp}, \chi^{R}_{rp}\Big] \cdot e^{2i\pi\left(\left[\chi^{L}_{ip}, \chi^{R}_{ip}\right]\right)}$ are the complex interval-valued grade of postive and negative opinion such that

$0 \le \Big(\xi^{qR}_{rp}\Big)^{q} + \Big(\chi^{qR}_{rp}\Big)^{q} \le 1$ and $0 \le \Big(\xi^{qR}_{ip}\Big)^{q} + \Big(\chi^{R}_{ip}\Big)^{q} \le 1$, and The complex interval-valued Q-rung orthopair fuzzy elements (CIVQROFE) is applying for convenience as

$$Q^{\%}{}_j = \left(\left[\xi^L_{rp-j}, \xi^R_{rp-j}\right] \cdot e^{2i\pi\left(\left[\xi^L_{ip-j}, \xi^R_{ip-j}\right]\right)}, \left[\chi^L_{rp-j}, \chi^R_{rp-j}\right] \cdot e^{2i\pi\left(\left[\chi^L_{ip-j}, \chi^R_{ip-j}\right]\right)}\right).$$

Definition 9 (Garg et al 2020; Liu et al 2020a) Let $Q^{\%}{}_{civ} = \left(\left[\xi^L_{rp}, \xi^R_{rp}\right] \cdot e^{2i\pi\left(\left[\xi^L_{ip}, \xi^R_{ip}\right]\right)}, \left[\chi^L_{rp}, \chi^R_{rp}\right] \cdot e^{2i\pi\left(\left[\chi^L_{ip}, \chi^R_{ip}\right]\right)}\right)$ be a CIVQROFEs, then score function CSF of CIVQROFN can be defined as follows:

$$CSF(Q^{\%}) = \frac{1}{2}\left(\frac{\xi^L_{rp} + \xi^R_{rp}}{2} + \frac{\xi^L_{ip} + \xi^R_{ip}}{2} - \frac{\chi^L_{rp} + \chi^R_{rp}}{2} - \frac{\chi^L_{ip} + \chi^R_{ip}}{2}\right), CSF \in [-1, 1] \quad (8)$$

Definition 10 (Garg et al 2020; Liu et al 2020a) Let $Q^{\%} = \left(\left[\xi^L_{rp}, \xi^R_{rp}\right] \cdot e^{2i\pi\left(\left[\xi^L_{ip}, \xi^R_{ip}\right]\right)}, \left[\chi^L_{rp}, \chi^R_{rp}\right] \cdot e^{2i\pi\left(\left[\chi^L_{ip}, \chi^R_{ip}\right]\right)}\right)$ be a CIVQROFEs, then accuracy function CAF of CIVQROFN can be defined as follows:

$$CAF(Q^{\%}) = \frac{1}{2}\left(\frac{\xi^L_{rp} + \xi^R_{rp}}{2} + \frac{\xi^L_{ip} + \xi^R_{ip}}{2} + \frac{\chi^L_{rp} + \chi^R_{rp}}{2} + \frac{\chi^L_{ip} + \chi^R_{ip}}{2}\right), CAF \in [0, 1]. \quad (9)$$

to evaluate the accuracy degree of CIVQROFN $Q^{\%} = \left(\left[\xi^L_{rp}, \xi^R_{rp}\right] \cdot e^{2i\pi\left(\left[\xi^L_{ip}, \xi^R_{ip}\right]\right)}, \left[\chi^L_{rp}, \chi^R_{rp}\right] \cdot e^{2i\pi\left(\left[\chi^L_{ip}, \chi^R_{ip}\right]\right)}\right)$ where $CAF \in [0, 1]$. The larger the value of CAF, the more degree of accuracy of the CIVQROFN $Q^{\%}$.

The score function CAF and the accuracy function CAF, as presented by Garg et al (2020) of the above equations. Based on the score function CSF and accuracy function CAF, in the following Garg et al (2020) set an order relation between two CIVQROFNs as follows.

Definition 11 Let $Q^{\%}{}_1 = \left(\left[\xi^L_{rp-1}, \xi^R_{rp-1}\right] \cdot e^{2i\pi\left(\left[\xi^L_{ip-1}, \xi^R_{ip-1}\right]\right)}, \left[\chi^L_{rp-1}, \chi^R_{rp-1}\right] \cdot e^{2i\pi\left(\left[\chi^L_{ip-1}, \chi^R_{ip-1}\right]\right)}\right)$

and $Q^{\%}{}_2 = \left(\left[\xi^L_{rp-2}, \xi^R_{rp-2}\right] \cdot e^{2i\pi\left(\left[\xi^L_{ip-2}, \xi^R_{ip-2}\right]\right)}, \left[\chi^L_{rp-2}, \chi^R_{rp-2}\right] \cdot e^{2i\pi\left(\left[\chi^L_{ip-2}, \chi^R_{ip-2}\right]\right)}\right)$ be two CIVQROFEs.

Then order relation between two CIVQROFNs $Q^{\%}{}_{civ-1}$ and $Q^{\%}{}_{civ-2}$ defined as:

(i) If $CSF(Q^{\%}{}_1) < CSF(Q^{\%}{}_2)$, imply $Q^{\%}{}_1 \prec Q^{\%}{}_2$
(ii) If $CSF(Q^{\%}{}_1) > CSF(Q^{\%}{}_2)$, imply $Q^{\%}{}_1 \succ Q^{\%}{}_2$
(iii) If $CSV(Q^{\%}{}_1) = CSF(Q^{\%}{}_2)$, then

   (1) If $CAF(Q^{\%}{}_1) < CAF(Q^{\%}{}_2)$, imply $Q^{\%}{}_1 \prec Q^{\%}{}_2$
   (2) If $CAF(Q^{\%}{}_1) > CAF(Q^{\%}{}_2)$, imply $Q^{\%}{}_1 \succ Q^{\%}{}_2$
   (3) If $CAF(Q^{\%}{}_1) = CAF(Q^{\%}{}_2)$, imply $Q^{\%}{}_1 \sim Q^{\%}{}_2$.

6 Dynamic complex interval-valued q-rung orthopair fuzzy weighted averaging (CIVQROFWA) operator

To aggregate the CIVQROFEs, Liu et al (2020) studied complex interval-valued q-rung Orthopair fuzzy weighted averaging (CIVQROFWA) operator. For simplicity, let $CIVQ$ be the set of all complex interval-valued q-rung orthopair fuzzy numbers (CIVQROFN).

Definition 12 Let $Q_j^{\%} = \left(\left[\xi_{rp-j}^L, \xi_{rp-j}^R\right] \cdot e^{2i\pi\left(\left[\xi_{ip-j}^L, \xi_{ip-j}^R\right]\right)}, \left[\chi_{rp-j}^L, \chi_{rp-j}^R\right] \cdot e^{2i\pi\left(\left[\chi_{ip-j}^L, \chi_{ip-j}^R\right]\right)}\right)$ $(j = 1, 2, \ldots, v)$ be a CIVQROFNs, and let $CIVQROFWA : \Theta^v \to \Theta$, if

$$\begin{aligned}
&CIVQROFWA(Q_1^{\%}, Q_2^{\%}, \ldots, Q_v^{\%}) \\
&= \prod_{j=1}^{v} \left(\psi_j Q_j^{\%}\right) \\
&= \left(\left[\left(1 - \prod_{j=1}^{v}\left(1 - \left(\xi_{rp-j}^L\right)^q\right)^{\psi_j}\right)^{1/q}, \right.\right. \\
&\left.\left(1 - \prod_{j=1}^{v}\left(1 - \left(\xi_{rp-j}^R\right)^q\right)^{\psi_j}\right)^{1/q}\right] \\
&\cdot e^{2i\pi\left(\left[\left(1-\prod_{j=1}^{v}(1-(\xi_{ip-j}^L)^q)^{\psi_j}\right)^{1/q}, \left(1-\prod_{j=1}^{v}\left(1-(\xi_{ip-j}^R)^q\right)^{\psi_j}\right)^{1/q}\right]\right)}, \\
&\left.\left[\prod_{j=1}^{v}\left(\chi_{rp-j}^L\right)^{\psi_j}, \prod_{j=1}^{v}\left(\chi_{rp-j}^R\right)^{psi_j}\right] \cdot e^{2i\pi\left(\left[\prod_{j=1}^{v}\left(\chi_{ip-j}^L\right)^{\psi_j}, \prod_{j=1}^{v}\left(\chi_{ip-j}^R\right)^{\psi_j}\right]\right)}\right)
\end{aligned} \tag{10}$$

where, $\psi = (\psi_1, \psi_2, \ldots, \psi_v)^T$ be such that $\psi_j > 0$, and $\sum_{j=1}^{v} \psi_j = 1$, the CIVQROFWA is called complex interval-valued q-rung Orthopair fuzzy weighted averaging operator.

However, the CIVQROFWA can only be considered to aggregate CIVQROF information where time is not taken into account. If time is taken is considered, then CIVQROF information may be collected at different periods, then it is not suitable to lead these situations.

Definition 13 Let t be consider as time variable, then we call $\beta(t) = \left(\left[\xi_{rp\beta(t)}^L, \xi_{rp\beta(t)}^R\right] \cdot e^{2i\pi([\xi_{ip\beta(t)}^L, \xi_{ip\beta(t)}^R])}, \left[\chi_{rp\beta(t)}^L, \chi_{rp\beta(t)}^R\right] \cdot e^{2i\pi(\left[\chi_{ip\beta(t)}^L, \chi_{ip\beta(t)}^R\right])}\right)$ be complex interval-valued q-rung Orthopair fuzzy variable (CIVQROFV), where $\xi_{rp\beta(t)}^L, \xi_{rp\beta(t)}^R \in [0, 1], \chi_{rp\beta(t)}^L, \chi_{rp\beta(t)}^R \in [0, 1]$, and $\xi_{ip\beta(t)}^L, \xi_{ip\beta(t)}^R \in [0, 1]$, $\chi_{ip\beta(t)}^L, \chi_{ip\beta(t)}^R \in [0, 1]$ such that $0 \leq \left(\xi_{rp\beta(t)}^R\right)^q + \left(\chi_{rp\beta(t)}^R\right)^q \leq 1$, and $0 \leq \left(\xi_{ip\beta(t)}^R\right)^q + \left(\chi_{ip\beta(t)}^R\right)^q \leq 1$.

For CIVQROFV $\beta(t)=\left(\left[\xi^L_{rp\beta(t)},\xi^R_{rp\beta(t)}\right].e^{2i\pi([\xi^L_{ip\beta(t)},\xi^R_{ip\beta(t)}])},\left[\chi^L_{rp\beta(t)},\chi^R_{rp\beta(t)}\right]\cdot e^{2i\pi(\left[\chi^L_{ip\beta(t)},\chi^R_{ip\beta(t)}\right])}\right)$, if $t=t_1,t_2,\ldots,t_l$, then $Q^{\%}(t_1),Q^{\%}(t_2),\ldots,Q^{\%}(t_l)$ are indicated as l distinct complex interval-valued q-rung Orthopair fuzzy numbers.

Definition 14 $\beta(t_1)=\left(\left[\xi^L_{rp\beta(t_1)},\xi^R_{rp\beta(t_1)}\right]\cdot e^{2i\pi([\xi^L_{ip\beta(t_1)},\xi^R_{ip\beta(t_1)}])},\left[\chi^L_{rp\beta(t_1)},\chi^R_{rp\beta(t_1)}\right]\cdot e^{2i\pi(\left[\chi^L_{ip\beta(t_1)},\chi^R_{ip\beta(t_1)}\right])}\right)$ and

$\beta(t_2)=\left(\left[\xi^L_{rp\beta(t_2)},\xi^R_{rp\beta(t_2)}\right]\cdot e^{2i\pi([\xi^L_{ip\beta(t_2)},\xi^R_{ip\beta(t_2)}])},\left[\chi^L_{rp\beta(t_2)},\chi^R_{rp\beta(t_2)}\right]\cdot e^{2i\pi(\left[\chi^L_{ip\beta(t_2)},\chi^R_{ip\beta(t_2)}\right])}\right)$ be two CQROFV, then

$$(1)\quad \beta(t_1)\bigoplus\beta(t_2)=\Bigg(\Bigg[\Big\{1-\Big(1-(\xi^L_{rp\beta(t_1)})^q\Big)\Big(1-(\xi^L_{rp\beta(t_2)})^q\Big)\Big\}^{1/q},$$
$$\Big\{1-\Big(1-(\xi^R_{rp\beta(t_1)})^q\Big)\Big(1-(\xi^R_{rp\beta(t_2)})^q\Big)\Big\}^{1/q}\Bigg]$$
$$\cdot e^2 i\pi\Bigg(\Bigg[\Big\{1-\Big(1-(\xi^L_{ip\beta(t_1)})^q\Big)\Big(1-(\xi^L_{ip\beta(t_2)})^q\Big)\Big\}^{1/q},$$
$$\Big\{1-\Big(1-(\xi^R_{ip\beta(t_1)})^q\Big)\Big(1-(\xi^R_{ip\beta(t_2)})^q\Big)^{1/q}\Bigg]\Bigg),$$
$$\Big[\chi^L_{rp\beta(t_1)}\chi^L_{rp\beta(t_2)},\chi^R_{rp\beta(t_1)}\chi^R_{rp\beta(t_2)}\Big]$$
$$\cdot e^{2i\pi([\chi^L_{ip\beta(t_1)})\chi^L_{ip\beta(t_2)},\chi^R_{ip\beta(t_1)}\chi^R_{ip\beta(t_2)})}\Big]\Bigg)$$

$$(2)\quad \lambda\beta(t_1)=\Bigg(\Bigg[\Big(1-\Big(1-(\xi^L_{rp\beta(t_1)})^q\Big)^{\lambda}\Big)^{1/q},\Big(1-\Big(1-(\xi^R_{rp\beta(t_1)})^q\Big)^{\lambda}\Big)^{1/q}\Bigg]$$
$$\cdot e^{2i\pi\left(\left[\left(1-\left(1-(\xi^L_{ip\beta(t_1)})^q\right)^{\lambda}\right)^{1/q},\left(1-\left(1-(\xi^R_{ip\beta(t_1)})^q\right)^{\lambda}\right)^{1/q}\right.}\Bigg],$$
$$\Big[\Big(\chi^L_{rp\beta(t_1)}\Big)^{\lambda},\Big(\chi^R_{rp\beta(t_1)}\Big)^{\lambda}\Big]$$
$$\cdot e^{2i\pi(\left[\left(\chi^L_{ip\beta(t_1)}\right)^{\lambda},\left(\chi^R_{ip\beta(t_1)}\right)^{\lambda}\right])}\Bigg),\ \lambda>0.$$

Definition 15 Let $Q^{\%}(t_m)=\left(\left[\xi^L_{rp\beta(t_m)},\xi^R_{rp\beta(t_m)}\right].e^{2i\pi([\xi^L_{ip\beta(t_m)},\xi^R_{ip\beta(t_m)}])},\left[\chi^L_{rp\beta(t_m)},\chi^R_{rp\beta(t_m)}\right]\cdot e^{2i\pi(\left[\chi^L_{ip\beta(t_m)},\chi^R_{ip\beta(t_m)}\right])}\right)$, $(m=1,2,\ldots,l)$ be a group of CIVQROFNs at l different periods t_m $(m=1,2,\ldots,l)$, and let $DCIVQROFWA:\Theta^v\to\Theta$, where

$$DCIVQROFWA_{\psi(t)}\Big(Q^{\%}(t_1),Q^{\%}(t_2),\ldots,Q^{\%}((t_l)\Big)=\prod_{m=1}^{l}\psi(t_m)(Q^{\%}(t_m))\quad(11)$$

where $\psi(t)=(\psi(t_1),\psi(t_2),\ldots,\psi(t_l))^T$ be a weight vector of the periods t_m $(m=1,2,\ldots,l)$, and $\psi(t_m)\in[0,1]$, $\sum_{m=1}^{l}\psi(t_m)=1$. Then, DCIVQROFWA is called uncertain dynamic complex q-rung Orthopair fuzzy weighted averaging (DCIVQROFWA) operator.

Following from Definition 14, Eq. (11) can be written in the form as

$$UDCQROFWA_{\psi(t)}\left(Q^{\%}(t_1), Q^{\%}(t_2), \ldots, Q^{\%}(t_l)\right)$$
$$\left(\left[\left(1-\prod_{m=1}^{l}\left(1-(\xi^L_{rp\beta(t_m)})^q\right)^{\psi(t_m)}\right)^{1/q}, \left(1-\prod_{m=1}^{l}\left(1-(\xi^R_{rp\beta(t_m)})^q\right)^{\psi(t_m)}\right)^{1/q}\right]\right.$$
$$\cdot e^{2i\pi\left(\left[\left(1-\prod_{m=1}^{l}\left(1-(\xi^L_{ip\beta(t_m)})^q\right)^{\psi(t_m)}\right)^{1/q}, \left(1-\prod_{m=1}^{l}\left(1-(\xi^R_{ip\beta(t_m)})^q\right)^{\psi(t_m)}\right)^{1/q}\right]\right.},$$
$$\left.\left[\left(\chi^L_{rp\beta(t_m)}\right)^{\psi(t_m)}, \left(\chi^R_{rp\beta(t_m)}\right)^{\psi(t_m)}\right]\cdot e^{2i\pi(\left[\left(\chi^L_{ip\beta(t_m)}\right)^{\psi(t_m)}, \left(\chi^R_{ip\beta(t_m)}\right)^{\psi(t_m)}\right])}\right) \tag{12}$$

For DCIVQROFWA operator, to assign the weight vector $\psi(t) = (\psi(t_1), \psi(t_2), \ldots, \psi(t_l))^T$ of the periods t_m $(m = 1, 2, \ldots, l)$ is a very important step. In general, $\psi(t)$ is is determine by decision maker(s) directly, or can be assigned by excited methods from Xu and Yager (2008).

7 A approach for uncertain dynamic complex q-rung Orthopair fuzzy multiple attribute decision making problem

The following notations are utilized to present the dynamic MADM problems with complex interval-valued q-rung Orthopair fuzzy information.

(1) Here, we use discrete set of alternatives $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_u\}$ which are known.
(2) The set of attributes $\mathcal{G} = \{\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_v\}$ are known attributes, and $w = (w_1, w_2, \ldots, w_v)^T$ be the weight vector of the attributes, such that $w_j > 0$, $\sum_{j=1}^{v} w_j = 1$ for $j = 1, 2, \ldots, v$.
(3) There are l distinct periods t_m $(k = 1, 2, \ldots, l)$ whose weight is considered as $\psi(t) = (\psi(t_1), \psi(t_2), \ldots, \psi(t_l))^l$ such as $\psi(t_m) > 0$, $m = 1, 2, \ldots, l$, and $\psi(t_m) \in [0, 1]$, $\sum_{m=1}^{l} \psi(t_m) = 1$.
(4) Suppose $M(t_m) = \left(\Omega_{ij}(t_m)\right)_{u\times v} = \left(\left(\left[\zeta^L_{rp(t_m)-ij}, \zeta^R_{rp(t_m)-ij}\right]\cdot e^{2i\pi\left(\left[\zeta^L_{ip(t_m)-ij}, \zeta^R_{ip(t_m)-ij}\right]\right)},\right.\right.$ $\left.\left.\left[\eta^L_{rp(t_m)-ij}, \eta^R_{rp(t_m)-ij}\right]\cdot e^{2i\pi\left(\left[\eta^L_{ip(t_m)-ij}, \eta^R_{rp(t_m)-ij}\right]\right)}\right)\right)_{u\times v}$ is the complex q-rung Orthopair fuzzy decision matrix which take l distinct periods t_m $(m = 1, 2, \ldots, l)$, where $\xi_{rp(t_m)-ij}\cdot e^{2i\pi(\xi_{ip(t_m)-ij})}$ is the degree of CQROFE's grade of positive opinion for the alternative $\mathcal{A}_u$ satisfying the attribute G_v at periods t_m $(m = 1, 2, \ldots, l)$, and $\chi_{rp(t_m)-ij}\cdot e^{2i\pi(\chi_{ip(t_m)-ij})}$ is the degree of CQROFE's grade of negative opinion for the alternative $\mathcal{A}_u$ does not satisfying the attribute $\left[\xi^L_{rp(t_m)-ij}, \xi^R_{rp(t_m)-ij}\right] \subset [0, 1]$, $\left[\chi^L_{rp(t_m)-ij}, \chi^R_{rp(t_m)-ij}\right] \subset [0, 1]$, $\left[\xi^L_{ip(t_m)-ij}, \xi^R_{ip(t_m)-ij}\right] \subset [0, 1]$, and $\left[\chi^L_{ip(t_m)-ij}, \chi^R_{ip(t_m)-ij}\right] \subset [0, 1]$, and $0 \leq \left(\xi^R_{rp(t_m)-ij}\right)^q + \left(\chi^R_{rp(t_m)-ij} \leq\right)^q 1$, and $0 \leq \left(\xi^R_{ip(t_m)-ij}\right)^q + \left(\chi^R_{ip(t_m)-ij}\right)^q \leq 1$.

Based on the above-supplied information, we develop an Algorithm to order and select the favourable one(s). The Algorithm contains the following steps.

Procedure II

step 1: Applying the DCQROFWA operator:

$$\begin{aligned}
&UDCQROFWA_{\psi(t)}\Big(Q^{\%}(t_1), Q^{\%}(t_2), \ldots, Q^{\%}(t_l)\Big)\\
&= \Bigg(\Bigg[\Bigg(1-\prod_{m=1}^{l}\Big(1-(\xi^{L}_{rp\beta(t_m)})^{q}\Big)^{\psi(t_m)}\Bigg)^{1/q}, \Bigg(1-\prod_{m=1}^{l}\Big(1-(\xi^{R}_{rp\beta(t_m)})^{q}\Big)^{\psi(t_m)}\Bigg)^{1/q}\Bigg]\\
&\cdot e^{2i\pi\left(\left[\left(1-\prod_{m=1}^{l}\left(1-(\xi^{L}_{ip\beta(t_m)})^{q}\right)^{\psi(t_m)}\right)^{1/q}, \left(1-\prod_{m=1}^{l}\left(1-(\xi^{R}_{ip\beta(t_m)})^{q}\right)^{\psi(t_m)}\right)^{1/q}\right.\right.}\Bigg],\\
&\Bigg[\Big(\chi^{L}_{rp\beta(t_m)}\Big)^{\psi(t_m)}, \Big(\chi^{R}_{rp\beta(t_m)}\Big)^{\psi(t_m)}\Bigg]\cdot e^{2i\pi\left(\left[\left(\chi^{L}_{ip\beta(t_m)}\right)^{\psi(t_m)}, \left(\chi^{R}_{ip\beta(t_m)}\right)^{\psi(t_m)}\right]\right)}\Bigg)
\end{aligned} \tag{13}$$

$M(t_m) = \Big(\Big(\Big[\xi^{L}_{rp(t_m)-ij}, \xi^{R}_{rp(t_m)-ij}\Big]\cdot e^{2i\pi\left(\left[\xi^{L}_{ip(t_m)-ij}, \xi^{R}_{ip(t_m)-ij}\right]\right)}, \Big[\chi^{L}_{rp(t_m)-ij}, \chi^{R}_{rp(t_m)-ij}\Big]\cdot e^{2i\pi\left(\left[\chi^{L}_{ip(t_m)-ij}, \chi^{R}_{rp(t_m)-ij}\right]\right)}\Big)\Big)_{u\times v}$ $(m = 1, 2, \ldots, l)$ into a complex interval-valued q-rung Orthopair fuzzy decision matrix, $M = (\Big(\Big[\xi^{L}_{rp-ij}, \xi^{R}_{rp-ij}\Big]\cdot e^{2i\pi\left(\left[\xi^{L}_{ip-ij}, \xi^{R}_{rp-ij}\right]\right)}, \Big[\chi^{L}_{rp-ij}, \chi^{R}_{rp-ij}\Big]\cdot e^{2i\pi\left(\left[\chi^{L}_{ip-ij}, \chi^{R}_{ip-ij}\right]\right)}\Big)_{u\times v}$.

Step 2: Utilize CIVQROFWA operator:

$$\begin{aligned}
r_i &= \Big(\Big[\zeta^{L}_{rp-i}, \zeta^{R}_{rp-i}\Big]\cdot e^{2i\pi\left(\left[\zeta^{L}_{ip-i}, \zeta^{R}_{rp-i}\right]\right)}, \Big[\eta^{L}_{rp-i}, \eta^{R}_{rp-i}\Big]\cdot e^{2i\pi\left(\left[\eta^{L}_{ip-i}, \eta^{R}_{ip-i}\right]\right)}\Big)\\
&= CIVQROFWA_{w}\Big(Q^{\%}_{cq-i1}, Q^{\%}_{cq-i2}, \ldots, Q^{\%}_{cq-iv}\Big)\\
&\Bigg\langle\Bigg[\Bigg\{1-\prod_{i=1}^{v}\Big(1-\Big(\xi^{L}_{rp-i}\Big)^{q}\Big)^{w_i}\Bigg\}^{1/q}, \Bigg\{1-\prod_{i=1}^{v}\Big(1-\Big(\xi^{R}_{rp-i}\Big)^{q}\Big)^{w_i}\Bigg\}^{1/q}\Bigg]\\
&\cdot e^{2i\pi\left(\left[\left\{1-\prod_{i=1}^{v}\left(1-\left(\xi^{L}_{ip-i}\right)^{q}\right)^{w_i}\right\}^{1/q}, \left\{1-\prod_{i=1}^{v}\left(1-\left(\xi^{L}_{ip-i}\right)^{q}\right)^{w_i}\right\}^{1/q}\right]\right)},\\
&\Bigg[\prod_{i=1}^{v}\Big(\chi^{L}_{rp-i}\Big)^{w_i}, \prod_{i=1}^{v}\Big(\chi^{R}_{rp-i}\Big)^{w_i}\Bigg]\cdot e^{2i\pi\left(\left[\prod_{i=1}^{v}\left(\chi^{L}_{ip-i}\right)^{w_i}, \prod_{i=1}^{v}\left(\chi^{L}_{ip-i}\right)^{w_i}\right]\right)}\Bigg\rangle.
\end{aligned} \tag{14}$$

Step 3: Compute the score value $CSF(\Omega_i)$ $(i = 1, 2, \ldots, u)$ of the overall CQROFEs to rank all the alternatives $\mathcal{A}_i$ $(i = 1, 2, \ldots, u)$ to adopt desired choice $\mathcal{A}_i$. If the values of $CSF(\Omega_i)$ and $CSF(\Omega_j)$ are same, then we continue to evaluate accuracy degrees $CAF(\Omega_i)$ and $CAF(\Omega_j)$ based on overall CQROFEs information band rank the alternatives $\mathcal{A}_i$ depending on degree of accuracy $CAF(\Omega_i)$ and $CAV(r_j)$.

Step 4: Rank all the alternative $\mathcal{A}_i$ $(i = 1, 2, \ldots, u)$ in order to choose the best one(s) in accordance with $CSF(\Omega_i)$ and $CAF(\Omega_j)$ $(i = 1, 2, \ldots, u)$.

Step 5: End.

Table 1 Decision matrix for expert[1]

	$\mathcal{G}_1$	$\mathcal{G}_2$
$\mathcal{A}_1$	$(0.5e^{2i\pi(0.2)}, 0.4e^{2i\pi(0.5)})$	$(0.5e^{2i\pi(0.6)}, 0.3e^{2i\pi(0.2)})$
$\mathcal{A}_2$	$(0.6e^{2i\pi(0.3)}, 0.3e^{2i\pi(0.4)})$	$(0.4e^{2i\pi(0.3)}, 0.2e^{2i\pi(0.6)})$
$\mathcal{A}_3$	$(0.4e^{2i\pi(0.3)}, 0.4e^{2i\pi(0.3)})$	$(0.3e^{2i\pi(0.5)}, 0.4e^{2i\pi(0.3)})$
$\mathcal{A}_4$	$(0.4e^{2i\pi(0.4)}, 0.3e^{2i\pi(0.5)})$	$(0.4e^{2i\pi(0.3)}, 0.4e^{2i\pi(0.2)})$
$\mathcal{A}_5$	$(0.7e^{2i\pi(0.6)}, 0.2e^{2i\pi(0.3)})$	$(0.6e^{2i\pi(0.2)}, 0.2e^{2i\pi(0.4)})$
	$\mathcal{G}_3$	$\mathcal{G}_4$
	$(0.3e^{2i\pi(0.3)}, 0.5e^{2i\pi(0.5)})$	$(0.4e^{2i\pi(0.4)}, 0.3e^{2i\pi(0.4)})$
	$(0.6e^{2i\pi(0.3)}, 0.3e^{2i\pi(0.2)})$	$(0.5e^{2i\pi(0.2)}, 0.2e^{2i\pi(0.4)})$
	$(0.3e^{2i\pi(0.4)}, 0.5e^{2i\pi(0.6)})$	$(0.6e^{2i\pi(0.4)}, 0.2e^{2i\pi(0.4)})$
	$(0.6e^{2i\pi(0.5)}, 0.2e^{2i\pi(0.2)})$	$(0.5e^{2i\pi(0.2)}, 0.3e^{2i\pi(0.4)})$
	$(0.5e^{2i\pi(0.4)}, 0.3e^{2i\pi(0.3)})$	$(0.6e^{2i\pi(0.5)}, 0.1e^{2i\pi(0.2)})$

8 Numerical example

The Recent development of modern science and technology makes our society very complicated, and as a result, human decision processes have become uncertain and difficult to analyze. Contemporary society has increased day-to-day demand for information technology. The emerging software systems selection takes a vital role. The main intention of this plan is to predict the favourable software systems based on their performances, which has five options to have emerging software systems. Thus, to this problem, we introduce a numerical example to predict the potential assessment of software technology systems depicted in Ye (2014) to investigate the efficiency and applicability of the proposed method. There is a committee which selects five possible software systems $\mathcal{A}_i$ $(i = 1, 2, \ldots, 5)$. The assessment of five possible software systems are made under the following four attributes:

$\mathcal{G}_1$: Contribution about organization performance
$\mathcal{G}_2$: Effort to transform the current system
$\mathcal{G}_3$: Costs of hardware and software investment
$\mathcal{G}_4$: Outsourcing software developer reliability.

The five possible alternatives $\mathcal{A}_i$ $(i = 1, 2, \ldots, 5)$ are to be evaluated on as listed in the following matrices are given in Tables 1, 2 and 3.

Let importance of the experts be $\psi(t) = (0.2, 0.3, 0.5)^T$ of the periods t_m $(m = 1, 2, 3)$, and $w = (0.35, 0.25, 0.20, 0.30)^T$ be the weight vector of the attributes $\mathcal{G}_{cq-j}$ $(j = 1, 2, 3, 4)$. Then, we apply the proposed Procedure I to get most favourable alternative.

Step 1: Applying the DCQROFWA operator to aggregate the all the complex q-rung Orthopair fuzzy decision matrices $M(t_m)$ given in Tables 1, 2 and 3 into a complex q-rung Orthopair fuzzy decision matrix M as given in Table 4 below:

Step 2: Applying CQROFWA operator to get the overall values of r_i of the alternatives $\mathcal{A}_i$ $(i = 1, 2, 3, 4, 5)$,

$$\Omega_1 = (0.4428e^{2i\pi(0.4104)}, 0.2307e^{2i\pi(0.2647)}),\quad \Omega_2 = (0.5279e^{2i\pi(0.2695)}, 0.2311e^{2i\pi(0.3218)})$$
$$\Omega_3 = (0.4462e^{2i\pi(0.4495)}, 0.2267e^{2i\pi(0.2711)}),\quad \Omega_4 = (0.3873e^{2i\pi(0.4030)}, 0.2493e^{2i\pi(0.2713)})$$
$$\Omega_5 = (0.5324e^{2i\pi(0.3563)}, 0.2242e^{2i\pi(0.2846)}).$$

Table 2 Decision matrix for expert[2]

	$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}_3$	$\mathcal{G}_4$
$\mathcal{A}_1$	$(0.3e^{2i\pi(0.4)}, 0.4e^{2i\pi(0.2)})$	$(0.6e^{2i\pi(0.2)}, 0.2e^{2i\pi(0.4)})$	$(0.5e^{2i\pi(0.4)}, 0.3e^{2i\pi(0.3)})$	$(0.4e^{2i\pi(0.4)}, 0.3e^{2i\pi(0.2)})$
$\mathcal{A}_2$	$(0.4e^{2i\pi(0.3)}, 0.3e^{2i\pi(0.3)})$	$(0.3e^{2i\pi(0.3)}, 0.4e^{2i\pi(0.2)})$	$(0.6e^{2i\pi(0.2)}, 0.3e^{2i\pi(0.4)})$	$(0.4e^{2i\pi(0.2)}, 0.3e^{2i\pi(0.5)})$
$\mathcal{A}_3$	$(0.6e^{2i\pi(0.4)}, 0.2e^{2i\pi(0.3)})$	$(0.5e^{2i\pi(0.6)}, 0.2e^{2i\pi(0.2)})$	$(0.6e^{2i\pi(0.5)}, 0.1e^{2i\pi(0.2)})$	$(0.4e^{2i\pi(0.4)}, 0.2e^{2i\pi(0.3)})$
$\mathcal{A}_4$	$(0.3e^{2i\pi(0.4)}, 0.3e^{2i\pi(0.2)})$	$(0.3e^{2i\pi(0.5)}, 0.2e^{2i\pi(0.2)})$	$(0.4e^{2i\pi(0.3)}, 0.3e^{2i\pi(0.5)})$	$(0.3e^{2i\pi(0.5)}, 0.3e^{2i\pi(0.2)})$
$\mathcal{A}_5$	$(0.4e^{2i\pi(0.2)}, 0.2e^{2i\pi(0.3)})$	$(0.6e^{2i\pi(0.2)}, 0.3e^{2i\pi(0.4)})$	$(0.3e^{2i\pi(0.2)}, 0.4e^{2i\pi(0.3)})$	$(0.5e^{2i\pi(0.2)}, 0.2e^{2i\pi(0.3)})$

Table 3 Decision matrix for expert[3]

	$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}_3$	$\mathcal{G}_4$
$\mathcal{A}_1$	$(0.2e^{2i\pi(0.4)}, 0.3e^{2i\pi(0.4)})$	$(0.5e^{2i\pi(0.4)}, 0.2e^{2i\pi(0.2)})$	$(0.4e^{2i\pi(0.4)}, 0.3e^{2i\pi(0.3)})$	$(0.4e^{2i\pi(0.4)}, 0.2e^{2i\pi(0.3)})$
$\mathcal{A}_2$	$(0.4e^{2i\pi(0.3)}, 0.3e^{2i\pi(0.4)})$	$(0.6e^{2i\pi(0.2)}, 0.2e^{2i\pi(0.3)})$	$(0.7e^{2i\pi(0.3)}, 0.1e^{2i\pi(0.4)})$	$(0.3e^{2i\pi(0.2)}, 0.4e^{2i\pi(0.4)})$
$\mathcal{A}_3$	$(0.3e^{2i\pi(0.2)}, 0.4e^{2i\pi(0.3)})$	$(0.2e^{2i\pi(0.5)}, 0.4e^{2i\pi(0.3)})$	$(0.5e^{2i\pi(0.3)}, 0.2e^{2i\pi(0.4)})$	$(0.4e^{2i\pi(0.5)}, 0.2e^{2i\pi(0.3)})$
$\mathcal{A}_4$	$(0.4e^{2i\pi(0.4)}, 0.2e^{2i\pi(0.4)})$	$(0.3e^{2i\pi(0.2)}, 0.4e^{2i\pi(0.5)})$	$(0.4e^{2i\pi(0.3)}, 0.2e^{2i\pi(0.4)})$	$(0.2e^{2i\pi(0.4)}, 0.4e^{2i\pi(0.2)})$
$\mathcal{A}_5$	$(0.6e^{2i\pi(0.2)}, 0.2e^{2i\pi(0.6)})$	$(0.5e^{2i\pi(0.3)}, 0.3e^{2i\pi(0.1)})$	$(0.3e^{2i\pi(0.2)}, 0.3e^{2i\pi(0.4)})$	$(0.2e^{2i\pi(0.4)}, 0.5e^{2i\pi(0.4)})$

Step 3: Calculate the scores $CSF(r_{cq-i})$ $(i = 1, 2, \ldots, u)$ of complex q-rung Orthopair fuzzy preference values Ω_i $(i = 1, 2, \ldots, u)$ to rank all the alternatives $\mathcal{A}_i$ $(i = 1, 2, 3, 4, 5)$

$$
\begin{aligned}
&CSF(\Omega_1) = 0.1704, \quad CSF(\Omega_2) = 0.1223,\\
&CSF(\Omega_3) = 0.1990, \quad CSF(\Omega_4) = 0.1349,\\
&CSF(\Omega_5) = 0.1899.
\end{aligned}
$$

Step 4: The ordering of the alternatives $\mathcal{A}_i$ $(i = 1, 2, \ldots, 5)$ and selected the favourable one(s) in accordance with $CSF(\Omega_i)$ and $CSF(r_i)$ $(i = 1, 2, \ldots, 5)$ as $\mathcal{A}_3 \succ \mathcal{A}_5 \succ \mathcal{A}_1 \succ \mathcal{A}_4 \succ \mathcal{A}_2$. Thus, computed desirable alternative is $\mathcal{A}_3$.

Let $\psi(t) = (0.2, 0.3, 0.5)^T$ be the weight vector of periods t_m $(m = 1, 2, 3)$, and $w = (0.35, 0.25, 0.20, 0.30)^T$ be the weight vector of the attributes $\mathcal{G}_j$ $(j = 1, 2, 3, 4)$. If the all the alternatives $\mathcal{A}_i$ $(i = 1, 2, \ldots, 5)$ are to be evaluated by using complex q-rung Orthopair fuzzy by the decision makers based on the above attributes $\mathcal{G}_j$ $(j = 1, 2, 3, 4)$ at the periods

Table 4 Aggregate value of the alternatives

	$\mathcal{G}_1$	$\mathcal{G}_2$
$\mathcal{A}_1$	$(0.3367e^{2i\pi(0.3758)}, 0.2828e^{2i\pi(0.3397)})$	$(0.5353e^{2i\pi(0.4321)}, 0.2169e^{2i\pi(0.2462)})$
$\mathcal{A}_2$	$(0.4588e^{2i\pi(0.3000)}, 0.3000e^{2i\pi(0.3669)})$	$(0.5253e^{2i\pi(0.2395)}, 0.2462e^{2i\pi(0.2814)})$
$\mathcal{A}_3$	$(0.4564e^{2i\pi(0.3069)}, 0.3249e^{2i\pi(0.3000)})$	$(0.3643e^{2i\pi(0.5353)}, 0.3249e^{2i\pi(0.2656)})$
$\mathcal{A}_4$	$(0.3757e^{2i\pi(0.4000)}, 0.2449e^{2i\pi(0.3397)})$	$(0.3256e^{2i\pi(0.3643)}, 0.3249e^{2i\pi(0.3162)})$
$\mathcal{A}_5$	$(0.5867e^{2i\pi(0.3771)}, 0.2000e^{2i\pi(0.4243)})$	$(0.5559e^{2i\pi(0.2599)}, 0.2766e^{2i\pi(0.2000)})$
	$\mathcal{G}_3$	$\mathcal{G}_4$
	$(0.4228e^{2i\pi(0.3842)}, 0.3323e^{2i\pi(0.3323)})$	$(0.4000e^{2i\pi(0.4000)}, 0.2449e^{2i\pi(0.2814)})$
	$(0.6560e^{2i\pi(0.2774)}, 0.1732e^{2i\pi(0.3482)})$	$(0.3881e^{2i\pi(0.2000)}, 0.3194e^{2i\pi(0.4277)})$
	$(0.5132e^{2i\pi(0.4016)}, 0.1951e^{2i\pi(0.3523)})$	$(0.4588e^{2i\pi(0.4563)}, 0.2000e^{2i\pi(0.3178)})$
	$(0.4588e^{2i\pi(0.3620)}, 0.2259e^{2i\pi(0.3723)})$	$(0.3367e^{2i\pi(0.4160)}, 0.3464e^{2i\pi(0.2297)})$
	$(0.3620e^{2i\pi(0.2690)}, 0.3270e^{2i\pi(0.3464)})$	$(0.4458e^{2i\pi(0.3921)}, 0.2753e^{2i\pi(0.3194)})$

Table 5 Decision matrix for expert[1]

	$\mathcal{G}_1$	$\mathcal{G}_2$
$\mathcal{A}_1$	$([0.5, 0.6]e^{2i\pi([0.2,0.3])}, [0.3, 0.4]e^{2i\pi([0.2,0.3])})$	$([0.4, 0.5]e^{2i\pi([0.5,0.6])}, [0.2, 0.3]e^{2i\pi([0.2,0.3])})$
$\mathcal{A}_2$	$([0.5, 0.6]e^{2i\pi([0.2,0.3])}, [0.2, 0.3]e^{2i\pi([0.3,0.4])})$	$([0.4, 0.5]e^{2i\pi([0.3,0.4])}, [0.1, 0.2]e^{2i\pi([0.5,0.6])})$
$\mathcal{A}_3$	$([0.4, 0.5]e^{2i\pi([0.2,0.3])}, [0.3, 0.4]e^{2i\pi([0.2,0.3])})$	$([0.2, 0.3]e^{2i\pi([0.4,0.5])}, [0.3, 0.4]e^{2i\pi([0.2,0.3])})$
$\mathcal{A}_4$	$([0.3, 0.4]e^{2i\pi([0.3,0.4])}, [0.2, 0.3]e^{2i\pi([0.4,0.5])})$	$([0.3, 0.4]e^{2i\pi([0.2,0.3])}, [0.3, 0.4]e^{2i\pi([0.1,0.2])})$
$\mathcal{A}_5$	$([0.6, 0.7]e^{2i\pi([0.5,0.6])}, [0.1, 0.2]e^{2i\pi([0.2,0.3])})$	$([0.5, 0.6]e^{2i\pi([0.2,0.3])}, [0.2, 0.3]e^{2i\pi([0.3,0.4])})$
	$\mathcal{G}_3$	$\mathcal{G}_4$
	$([0.2, 0.3]e^{2i\pi([0.2,0.3])}, [0.4, 0.5]e^{2i\pi([0.4,0.5])})$	$([0.3, 0.4]e^{2i\pi([0.3,0.4])}, [0.2, 0.3]e^{2i\pi([0.3,0.4])})$
	$([0.5, 0.6]e^{2i\pi([0.2,0.3])}, [0.2, 0.3]e^{2i\pi([0.1,0.2])})$	$([0.4, 0.5]e^{2i\pi([0.2,0.3])}, [0.2, 0.3]e^{2i\pi([0.4,0.5])})$
	$([0.2, 0.3]e^{2i\pi([0.3,0.4])}, [0.4, 0.5]e^{2i\pi([0.5,0.6])})$	$([0.4, 0.6]e^{2i\pi([0.3,0.4])}, [0.1, 0.2]e^{2i\pi([0.3,0.4])})$
	$([0.5, 0.6]e^{2i\pi([0.4,0.5])}, [0.1, 0.2]e^{2i\pi([0.1,0.2])})$	$([0.4, 0.5]e^{2i\pi([0.2,0.3])}, [0.2, 0.3]e^{2i\pi([0.3,0.4])})$
	$([0.4, 0.5]e^{2i\pi([0.2,0.4])}, [0.2, 0.3]e^{2i\pi([0.2,0.3])})$	$([0.4, 0.6]e^{2i\pi([0.4,0.5])}, [0.1, 0.2]e^{2i\pi([0.2,0.3])})$

t_m $(m = 1, 2, 3)$, are as shown in the following Tables 5, 6, and 7. In such a case, we are applying the proposed Procedure II to get the most desirable alternative one(s).

Step 1: Utilize the UDCIVQROFWA operator to aggregate all complex interval-valued q-rung Orthopair fuzzy matrices $M(t_m)$ given in Tables 5,6 and 7 into a complex q-rung Orthopair fuzzy decision matrix M presented in Table 8.

Step 2: Applying the CIVQROFWA operator to get the overall values $r_i = \left(\left[\xi^L_{rp-i}, \xi^R_{rp-i}\right] \cdot e^{2i\pi\left(\left[\xi^L_{ip-i}, \xi^R_{ip-i}\right]\right)}, \left[\chi^L_{rp-i}, \chi^R_{rp-i}\right] \cdot e^{2i\pi\left(\left[\chi^L_{ip-i}, \chi^R_{rp-i}\right]\right)}\right)$ of the alternatives $\mathcal{A}_i$ $(i = 1, 2, 3, 4, 5)$ as follows:

Table 6 Decision matrix for expert^2

	$\mathcal{G}_1$	$\mathcal{G}_2$
$\mathcal{A}_1$	$([0.3, 0.4]e^{2i\pi([0.3,0.4])}, [0.3, 0.4]e^{2i\pi([0.2,0.3])})$	$([0.5, 0.6]e^{2i\pi([0.2,0.3])}, [0.1, 0.2]e^{2i\pi([0.3,0.4])})$
$\mathcal{A}_2$	$([0.3, 0.4]e^{2i\pi([0.3,0.4])}, [0.2, 0.3]e^{2i\pi([0.2,0.3])})$	$([0.3, 0.4]e^{2i\pi([0.3,0.4])}, [0.4, 0.5]e^{2i\pi([0.2,0.3])})$
$\mathcal{A}_3$	$([0.5, 0.6]e^{2i\pi([0.3,0.4])}, [0.2, 0.3]e^{2i\pi([0.2,0.3])})$	$([0.4, 0.5]e^{2i\pi([0.5,0.6])}, [0.2, 0.3]e^{2i\pi([0.1,0.2])})$
$\mathcal{A}_4$	$([0.3, 0.4]e^{2i\pi([0.3,0.4])}, [0.2, 0.3]e^{2i\pi([0.1,0.2])})$	$([0.2, 0.3]e^{2i\pi([0.4,0.5])}, [0.2, 0.3]e^{2i\pi([0.2,0.3])})$
$\mathcal{A}_5$	$([0.4, 0.5]e^{2i\pi([0.2,0.3])}, [0.1, 0.2]e^{2i\pi([0.3,0.4])})$	$([0.5, 0.6]e^{2i\pi([0.2,0.3])}, [0.3, 0.4]e^{2i\pi([0.3,0.4])})$
	$\mathcal{G}_3$	$\mathcal{G}_4$
	$([0.4, 0.5]e^{2i\pi([0.3,0.4])}, [0.2, 0.3]e^{2i\pi([0.2,0.3])})$	$([0.3, 0.4]e^{2i\pi([0.3,0.4])}, [0.2, 0.3]e^{2i\pi([0.2,0.3])})$
	$([0.5, 0.6]e^{2i\pi([0.1,0.2])}, [0.2, 0.3]e^{2i\pi([0.3,0.4])})$	$([0.3, 0.4]e^{2i\pi([0.1,0.2])}, [0.2, 0.3]e^{2i\pi([0.4,0.5])})$
	$([0.5, 0.6]e^{2i\pi([0.4,0.5])}, [0.1, 0.2]e^{2i\pi([0.2,0.3])})$	$([0.4, 0.3]e^{2i\pi([0.4,0.5])}, [0.2, 0.3]e^{2i\pi([0.2,0.3])})$
	$([0.2, 0.4]e^{2i\pi([0.2,0.3])}, [0.2, 0.3]e^{2i\pi([0.4,0.5])})$	$([0.2, 0.3]e^{2i\pi([0.4,0.5])}, [0.2, 0.3]e^{2i\pi([0.2,0.3])})$
	$([0.2, 0.3]e^{2i\pi([0.1,0.2])}, [0.4, 0.5]e^{2i\pi([0.3,0.4])})$	$([0.4, 0.5]e^{2i\pi([0.2,0.3])}, [0.2, 0.3]e^{2i\pi([0.3,0.4])})$

Table 7 Decision matrix for expert^3

	$\mathcal{G}_1$	$\mathcal{G}_2$
$\mathcal{A}_1$	$([0.2, 0.3]e^{2i\pi([0.3,0.4])}, [0.2, 0.3]e^{2i\pi([0.3,0.4])})$	$([0.4, 0.5]e^{2i\pi([0.3,0.4])}, [0.2, 0.3]e^{2i\pi([0.1,0.2])})$
$\mathcal{A}_2$	$([0.3, 0.4]e^{2i\pi([0.3,0.4])}, [0.3, 0.4]e^{2i\pi([0.4,0.5])})$	$([0.5, 0.6]e^{2i\pi([0.1,0.2])}, [0.2, 0.3]e^{2i\pi([0.2,0.3])})$
$\mathcal{A}_3$	$([0.2, 0.3]e^{2i\pi([0.2,0.3])}, [0.3, 0.4]e^{2i\pi([0.2,0.3])})$	$([0.2, 0.3]e^{2i\pi([0.4,0.5])}, [0.3, 0.4]e^{2i\pi([0.2,0.3])})$
$\mathcal{A}_4$	$([0.3, 0.4]e^{2i\pi([0.3,0.4])}, [0.2, 0.3]e^{2i\pi([0.3,0.4])})$	$([0.3, 0.4]e^{2i\pi([0.2,0.3])}, [0.4, 0.5]e^{2i\pi([0.4,0.5])})$
$\mathcal{A}_5$	$([0.5, 0.6]e^{2i\pi([0.2,0.3])}, [0.2, 0.3]e^{2i\pi([0.5,0.6])})$	$([0.4, 0.5]e^{2i\pi([0.2,0.3])}, [0.2, 0.3]e^{2i\pi([0.1,0.2])})$
	$\mathcal{G}_3$	$\mathcal{G}_4$
	$([0.4, 0.5]e^{2i\pi([0.3,0.4])}, [0.3, 0.4]e^{2i\pi([0.2,0.3])})$	$([0.3, 0.4]e^{2i\pi([0.4,0.5])}, [0.2, 0.3]e^{2i\pi([0.2,0.3])})$
	$([0.6, 0.7]e^{2i\pi([0.2,0.3])}, [0.1, 0.2]e^{2i\pi([0.3,0.4])})$	$([0.2, 0.3]e^{2i\pi([0.2,0.3])}, [0.4, 0.5]e^{2i\pi([0.3,0.4])})$
	$([0.4, 0.5]e^{2i\pi([0.2,0.3])}, [0.2, 0.3]e^{2i\pi([0.3,0.4])})$	$([0.3, 0.4]e^{2i\pi([0.4,0.5])}, [0.2, 0.3]e^{2i\pi([0.2,0.3])})$
	$([0.3, 0.4]e^{2i\pi([0.3,0.4])}, [0.2, 0.3]e^{2i\pi([0.3,0.4])})$	$([0.2, 0.3]e^{2i\pi([0.3,0.4])}, [0.4, 0.5]e^{2i\pi([0.2,0.3])})$
	$([0.2, 0.3]e^{2i\pi([0.1,0.2])}, [0.2, 0.3]e^{2i\pi([0.3,0.4])})$	$([0.2, 0.3]e^{2i\pi([0.3,0.4])}, [0.4, 0.5]e^{2i\pi([0.3,0.4])})$

$$r_1 = ([0.3753, 0.4741]e^{2i\pi([0.3331,0.4328])}, [0.1858, 0.2850]e^{2i\pi([0.1767,0.2821])})$$
$$r_2 = ([0.4326, 0.5308]e^{2i\pi([0.2440,0.3411])}, [0.1925, 0.2952]e^{2i\pi([0.2533,0.3568])})$$
$$r_3 = ([0.3777, 0.4752]e^{2i\pi([0.3649,0.4632])}, [0.1910, 0.2918]e^{2i\pi([0.1789,0.2777])})$$
$$r_4 = ([0.3072, 0.4115]e^{2i\pi([0.3169,0.4167])}, [0.2045, 0.3055]e^{2i\pi([0.2033,0.3083])})$$
$$r_5 = ([0.4373, 0.5358]e^{2i\pi([0.2828,0.3753])}, [0.1725, 0.2747]e^{2i\pi([0.2272,0.3323])}).$$

Step 3: Evaluate the score values $CSF(\Omega_i)$ of the overall values of complex interval-valued q-rung Orthopair fuzzy preference Ω_i $(i = 1, 2, 3, 4, 5)$ to ordering the alternative $\mathcal{A}_i$ $(i = 1, 2, \ldots, 5)$.

Table 8 Aggregated values of the alternatives for CIVCQROF information

	$\mathcal{G}_1$
$\mathcal{A}_1$	$([0.3367, 0.4289]e^{2i\pi([0.2853,0.3842])}, [0.2449, 0.3464]e^{2i\pi([0.2249,0.3464])})$
$\mathcal{A}_2$	$([0.3620, 0.4588]e^{2i\pi([0.2843,0.3842])}, [0.2449, 0.3464]e^{2i\pi([0.3067,0.4102])})$
$\mathcal{A}_3$	$([0.3819, 0.4750]e^{2i\pi([0.2395,0.3369])}, [0.2656, 0.3669]e^{2i\pi([0.2000,0.3000])})$
$\mathcal{A}_4$	$([0.3000, 0.4000]e^{2i\pi([0.3000,0.4000])}, [0.2000, 0.3000]e^{2i\pi([0.2285,0.3397])})$
$\mathcal{A}_5$	$([0.5020, 0.6017]e^{2i\pi([0.3194,0.4084])}, [0.1414, 0.2449]e^{2i\pi([0.3571,0.4625])})$
	$\mathcal{G}_2$
	$([0.4357, 0.5353]e^{2i\pi([0.3473,0.4414])}, [0.1625, 0.2656]e^{2i\pi([0.1597,0.2670])})$
	$([0.4387, 0.5370]e^{2i\pi([0.2415,0.3314])}, [0.2144, 0.3224]e^{2i\pi([0.2402,0.3446])})$
	$([0.2930, 0.3860]e^{2i\pi([0.4357,0.5353])}, [0.2656, 0.3669]e^{2i\pi([0.1625,0.2656])})$
	$([0.2774, 0.4000]e^{2i\pi([0.2930,0.3860])}, [0.3067, 0.4102]e^{2i\pi([0.2462,0.3571])})$
	$([0.4563, 0.5559]e^{2i\pi([0.2000,0.3000])}, [0.2259, 0.3270]e^{2i\pi([0.1732,0.2828])})$
	$\mathcal{G}_3$
	$([0.3758, 0.4736]e^{2i\pi([0.2853,0.3842])}, [0.2814, 0.3837]e^{2i\pi([0.2297,0.3323])})$
	$([0.5559, 0.6560]e^{2i\pi([0.1807,0.2774])}, [0.1414, 0.2449]e^{2i\pi([0.2408,0.3482])})$
	$([0.4160, 0.5132]e^{2i\pi([0.3069,0.4016])}, [0.1866, 0.2942]e^{2i\pi([0.2942,0.3979])})$
	$([0.3473, 0.4414]e^{2i\pi([0.3069,0.4036])}, [0.1741, 0.2766]e^{2i\pi([0.2625,0.3723])})$
	$([0.2690, 0.3620]e^{2i\pi([0.1340,0.2279])}, [0.2462, 0.3479]e^{2i\pi([0.2259,0.3270])})$
	$\mathcal{G}_4$
	$([0.3000, 0.4000]e^{2i\pi([0.3575,0.4563])}, [0.2000, 0.3000]e^{2i\pi([0.2169,0.3178])})$
	$([0.2929, 0.3881]e^{2i\pi([0.1807,0.2774])}, [0.2828, 0.3873]e^{2i\pi([0.3464,0.4472])})$
	$([0.3575, 0.4563]e^{2i\pi([0.3842,0.4836])}, [0.1741, 0.2766]e^{2i\pi([0.2169,0.3178])})$
	$([0.2690, 0.3620]e^{2i\pi([0.3256,0.4228])}, [0.2828, 0.3873]e^{2i\pi([0.2169,0.3178])})$
	$([0.3314, 0.4260]e^{2i\pi([0.3069,0.4036])}, [0.2462, 0.3571]e^{2i\pi([0.2766,0.377])})$

$$
\begin{aligned}
&CSF(\Omega_1) = 0.1714, \quad CSF(\Omega_2) = 0.1126, \\
&CSF(\Omega_3) = 0.1854, \quad CSF(\Omega_4) = 0.1076, \\
&CSF(\Omega_5) = 0.1561.
\end{aligned}
$$

Step 4: The ordering of the alternatives $\mathcal{A}_i$ $(i = 1, 2, \ldots, 5)$ and selected the favourable one(s) in accordance with $CSF(\Omega_i)$ and $CAF(\Omega_i)$ $(i = 1, 2, \ldots, 5)$ as $\mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_2 \succ \mathcal{A}_4$, and thus most favourable alternative is $\mathcal{A}_3$.

From the above-using operator's output, when we used DCQROFWA and CQROFWA operators, and ordering of the alternatives is as follows $\mathcal{A}_3 \succ \mathcal{A}_5 \succ \mathcal{A}_1 \succ \mathcal{A}_4 \succ \mathcal{A}_2$, and when used UDCQROFWG and CIVQROFWA operators then ranking shows $\mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_2 \succ \mathcal{A}_4$. It is concluded that although ranking order is different for these operators but both operators provided $\mathcal{A}_3$ as the most favourable alternative.

Table 9 Influence of parameter to the ranking order by DCQROFWA operator

q	$CSV(r_1)$	$CSV(r_2)$	$CSV(r_3)$	$CSV(r_4)$	$CSV(r_5)$	Ranking
$q=1$	0.1909	0.1289	0.2184	0.1489	0.1983	$\mathcal{A}_3 \succ \mathcal{A}_5 \succ \mathcal{A}_1 \succ \mathcal{A}_4 \succ \mathcal{A}_2$
$q=3$	0.1704	0.1223	0.1990	0.1349	0.1899	$\mathcal{A}_3 \succ \mathcal{A}_5 \succ \mathcal{A}_1 \succ \mathcal{A}_4 \succ \mathcal{A}_2$
$q=5$	0.1806	0.1281	0.2126	0.1340	0.1937	$\mathcal{A}_3 \succ \mathcal{A}_5 \succ \mathcal{A}_1 \succ \mathcal{A}_4 \succ \mathcal{A}_2$
$q=10$	0.1918	0.1471	0.2252	0.1402	0.2048	$\mathcal{A}_3 \succ \mathcal{A}_5 \succ \mathcal{A}_1 \succ \mathcal{A}_2 \succ \mathcal{A}_4$

Table 10 Influence of parameter to the ranking order by UDCQROFWA operator

q	$CSV(r_1)$	$CSV(r_2)$	$CSV(r_3)$	$CSV(r_4)$	$CSV(r_5)$	Ranking
$q=1$	0.1847	0.1179	0.1949	0.1229	0.1593	$\mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_2 \succ \mathcal{A}_4$
$q=3$	0.1741	0.1126	0.1854	0.1076	0.1561	$\mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_2 \succ \mathcal{A}_4$
$q=5$	0.1721	0.1252	0.1883	0.1048	0.1630	$\mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_2 \succ \mathcal{A}_4$
$q=10$	0.1781	0.1412	0.1986	0.1049	0.1766	$\mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_2 \succ \mathcal{A}_4$

9 Flexibility and sensitivity analysis of the parameter q on decision-making results

In order to analyze the flexibility and sensitivity of the parameter q on decision making results, we set the different values of q to sort the new practical DMADM example. The ranking order of the alternatives is shown in Tables 9 and 10.

From Table 9, we see that the aggregating results are different with the increasing value of the parameter q when applying the DCQROFWA operator. For the different values of q, the ranking results are still the same as $\mathcal{A}_3 \succ \mathcal{A}_5 \succ \mathcal{A}_1 \succ \mathcal{A}_4 \succ \mathcal{A}_2$, and optimal choice is $\mathcal{A}_3$. Also, for increasing value of q, the score value of $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$, $\mathcal{A}_4$, and $\mathcal{A}_5$ are fluctuated. When the value of q is increasing, it can more reflect the attitude of the DMs is optimistic or pessimistic and optimal choice is still the same. From Table 10, we see that the aggregating results are different with the increasing value of the parameter q when applying the UDCQROFWA operator. The ranking orders are same as $\mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_2 \succ \mathcal{A}_4$ for increasing value of q and optimal choice is $\mathcal{A}_3$. Also, it noticeable that for increasing value of q, the score value of $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$, $\mathcal{A}_4$, and $\mathcal{A}_5$ are some increasing and decreasing. When the value of q is increasing, the attitude of the DMs is optimistic and pessimistic, but the optimal choice is still A_3. In general, different DM can set the different values to set the parameter q on the basis of their preferences.

10 Discussing results for comparative analysis

Some existing operators: dynamic intuitionistic fuzzy weighted geometric (DIFWG) operator and uncertain dynamic intuitionistic fuzzy weighted geometric (UDIVIFWG) operator is a particular case of the proposed operators, so, in order to compare the performance of the proposed operators under this environment, convert the considering data into intuitionistic fuzzy numbers. In that direction, set the phase term of each CQROFN and CIVQROFN (imaginary part) to be zero for $q = 1$, and apply existing operators (Xu and Yager 2008). The results are briefly discussed as follows:

Applying weight vector $\psi(t) = (0.2, 0.3, 0.5)^T$ of the periods (t_m) $(m = 1, 2, 3)$ for the expert decision matrices when $q = 1$. To study the comparative results, we used the expert's decision matrices Tables 1, 2, 3, 5, 6 and 7 without considering complex parts. We find the results as follows.

Step 1: Applying the DIFWA operator to accumulate the all IFS decision matrices $M(t_m)$ into a complex IFS decision matrix M as given in Table 11 below:

Step 2: Let $\beta_i^+ = (\beta_1^+, \beta_2^+, \ldots, \beta_m^+)$ and $\beta_i^- = (\beta_1^-, \beta_2^-, \ldots, \beta_m^-)$ define as the intuitionistic fuzzy positive ideal solution (IFPIS) and intuitionistic fuzzy negative ideal solution (IFNIS) respectively, where $\beta_i^+ = (1, 0, 0)$, as the largest v IFNs and $\beta_i^- = (0, 1, 0)$, are as the smallest v IFNs.

Step 3: Evaluate the distance between IFPIS β_i^+ and the alternative $\mathcal{A}_i$, and IFNIS β_i^- and the alternative $\mathcal{A}_i$ are as follows:

$$\begin{aligned} D(\mathcal{A}_i, \beta^+) &= \frac{1}{2}\sum_{j=1}^{v} w_j D\Big(r_{ij}, \beta_j^+\Big) = \frac{1}{2}\sum_{j=1}^{v} w_j \Big(|\xi_{ij} - 1| + |\chi_{ij} - 0| + |\pi_{ij} - 0|\Big) \\ &= \frac{1}{2}\sum_{j=1}^{v} w_j \Big(1 - \xi_{ij} + \chi_{ij} - 0 + 1 - \xi_{ij} - \chi_{ij}\Big) \\ &= \sum_{j=1}^{v} w_j \Big(1 - \xi_{ij}\Big). \end{aligned} \tag{15}$$

$$\begin{aligned} D(\mathcal{A}_i, \beta^-) &= \frac{1}{2}\sum_{j=1}^{v} w_j D\Big(r_{ij}, \beta_j^-\Big) = \frac{1}{2}\sum_{j=1}^{v} w_j \Big(|\xi_{ij}| + |\chi_{ij} - 1| + |\pi_{ij} - 0|\Big) \\ &= \frac{1}{2}\sum_{j=1}^{v} w_j \Big(1 + \xi_{ij} - \chi_{ij} + 1 - \xi_{ij} - \chi_{ij}\Big) \\ &= \sum_{j=1}^{v} w_j \Big(1 - \chi_{ij}\Big). \end{aligned} \tag{16}$$

Step 4: Calculate the closeness coefficient of each alternative by the following equation:

$$C_i(\mathcal{A}_i) = \frac{D(\mathcal{A}_i, \beta^-)}{D(\mathcal{A}_i, \beta^+) + D(\mathcal{A}_i, \beta^-)} \quad i = 1, 2, \ldots, v. \tag{17}$$

Step 5: Rank all the alternatives $\mathcal{A}_i$ $(i = 1, 2, \ldots, u)$ based on the values of the closeness coefficient, higher value of closeness coefficient $C_i(\mathcal{A}_i)$ imply the better choice alternative.

In this method, we give some main calculating results. In this direction, applying step by 1-4, at the periods t_m $(m = 1, 2, \ldots, p)$, we obtain the measures of each option from

IFPIA and IFNIA. Finally, we are finding all the closeness coefficients of each options $\mathcal{A}_i$ as follows $C(\mathcal{A}_1) = 0.5505$, $C(\mathcal{A}_2) = 0.5590$, $C(\mathcal{A}_3) = 0.5864$, $C(\mathcal{A}_4) = 0.5247$ and $C(\mathcal{A}_5) = 0.5831$. Then ordering of the alternative is $\mathcal{A}_3 \succ \mathcal{A}_5 \succ \mathcal{A}_2 \succ \mathcal{A}_1 \succ \mathcal{A}_4$, and best alternative is $\mathcal{A}_3$.

If $q = 1$, then complex interval-valued q-rung Orthopair fuzzy set (CIVQROFS) converted into complex interval-valued intuitionistic fuzzy set (CIVIFS). But, for comparing the performance of pre-existing operator UDIFWA (Xu and Yager 2008), set each phase term of the proposed operator to be zero for converting into required data for $q = 1$. The results are briefly discussed as follows:

Applying expert's weight vector $\psi(t) = (0.2, 0.3, 0.5)^T$ of the periods (t_m) $(m = 1, 2, 3)$ for the expert decision matrices.

Step 1: Applying the UDCROFWA operator to accumulate the all IVIFN decision matrices $M(t_m)$ into a complex IVIFN decision matrix M as given in Table 12 below:

Step 2: Let $\beta_i^+ = (\beta_1^+, \beta_2^+, \ldots, \beta_m^+)$ and $\beta_i^- = (\beta_1^-, \beta_2^-, \ldots, \beta_m^-)$ define as the uncertain intuitionistic fuzzy positive ideal solution (UIFPIS) and uncertain intuitionistic fuzzy negative ideal solution (UIFNIS) respectively, where $\beta_i^+ = ([1, 1], [0, 0], [0, 0])$, as the largest v UIFNs and $\beta_i^- = ([0, 0], [1, 1], [0, 0])$, are as the smallest v UIFNs.

Step 3: Evaluate the distance between UIFPIS β_i^+ and the alternative $\mathcal{A}_i$, and UIFNIS β_i^- and the alternative $\mathcal{A}_i$ are as follows:

$$D(\mathcal{A}_i, \beta^+) \tag{18}$$

$$= \frac{1}{4}\sum_{j=1}^{v} w_j D\Big(r_{ij}, \beta_j^+\Big) \tag{19}$$

$$= \frac{1}{4}\sum_{j=1}^{v} w_j \Big(|\xi_{ij}^L - 1| + |\xi_{ij}^R - 1| + |\chi_{ij}^L - 0| + |\chi_{ij}^R - 0| + |\pi_{ij}^L - 0| + |\pi_{ij}^L - 0|\Big)$$

$$= \frac{1}{4}\sum_{j=1}^{v} w_j \Big(2 - (\xi_{ij}^L + \xi_{ij}^R) + \chi_{ij}^L + \chi_{ij}^R + 1 - \xi_{ij}^L - \xi_{ij}^R + 1 - \chi_{ij}^L - \chi_{ij}^R\Big)$$

$$= \frac{1}{2}\sum_{j=1}^{v} w_j \Big(2 - (\xi_{ij}^L + \xi_{ij}^R)\Big). \tag{20}$$

$$D(\mathcal{A}_i, \beta^-) \tag{21}$$

$$= \frac{1}{4}\sum_{j=1}^{v} w_j D\Big(r_{ij}, \beta_j^-\Big) \tag{22}$$

$$= \frac{1}{4}\sum_{j=1}^{v} w_j \Big(|\xi_{ij}^L - 0| + |\xi_{ij}^R - 0| + |\chi_{ij}^L - 1| + |\chi_{ij}^R - 1| + |\pi_{ij}^L - 0| + |\pi_{ij}^L - 0|\Big)$$

$$= \frac{1}{4}\sum_{j=1}^{v} w_j \Big(2 + \xi_{ij}^L + \xi_{ij}^R + -(\chi_{ij}^L + \chi_{ij}^R) + 1 - \xi_{ij}^L - \xi_{ij}^R + 1 - \chi_{ij}^L - \chi_{ij}^R\Big)$$

$$= \frac{1}{2}\sum_{j=1}^{v} w_j \Big(2 - (\chi_{ij}^L + \chi_{ij}^R)\Big). \tag{23}$$

Table 11 Aggregate value of Q-ROF information

	$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}_3$	$\mathcal{G}_4$
$\mathcal{A}_1$	(0.3004, 0.2828, 0.3532)	(0.5324, 0.2169, 0.2507)	(0.4142, 0.3323, 0.2535)	(0.4000, 0.2449, 0.3551)
$\mathcal{A}_2$	(0.4467, 0.3000, 0.2533)	(0.5053, 0.2462, 0.2669)	(0.6536, 0.1732, 0.1732)	(0.3751, 0.3194, 0.3055)
$\mathcal{A}_3$	(0.4262, 0.3249, 0.2483)	(0.3235, 0.3249, 0.3516)	(0.4998, 0.1951, 0.3051)	(0.4467, 0.2000, 0.3533)
$\mathcal{A}_4$	(0.3716, 0.2449, 0.3835)	(0.3213, 0.3249, 0.3538)	(0.4467, 0.2259, 0.4285)	(0.3004, 0.3464, 0.3532)
$\mathcal{A}_5$	(0.5735, 0.2000, 0.2265)	(0.5528, 0.2766, 0.1706)	(0.3456, 0.3270, 0.3274)	(0.3951, 0.2753, 0.3296)

Table 12 Aggregated values of the alternatives for Q-IVROF information

	$\mathcal{G}_1$	$\mathcal{G}_2$
$\mathcal{A}_1$	([0.2744, 0.3751], [0.2449, 0.3464], [0.2785, 0.4807])	([0.4319, 0.5324], [0.1625, 0.2670], [0.2005, 0.4156])
$\mathcal{A}_2$	([0.3456, 0.4467], [0.2449, 0.3464], [0.2069, 0.4095])	([0.4264, 0.5276], [0.2144, 0.3224], [0.1500, 0.3592])
$\mathcal{A}_3$	([0.3441, 0.4467], [0.2656, 0.3669], [0.1864, 0.3903])	([0.2661, 0.7024], [0.2297, 0.2656], [0.0320, 0.5042])
$\mathcal{A}_4$	([0.3000, 0.4000], [0.2000, 0.3000], [0.3000, 0.5000])	([0.2714, 0.4000], [0.3067, 0.4102], [0.1898, 0.4219])
$\mathcal{A}_5$	([0.4949, 0.5962], [0.1414, 0.2449], [0.1589, 0.3637])	([0.4523, 0.5528], [0.2259, 0.3270], [0.1202, 0.3218])
	$\mathcal{G}_3$	$\mathcal{G}_4$
	([0.3645, 0.4652], [0.2814, 0.3837], [0.1511, 0.3541])	([0.3000, 0.4000], [0.2000, 0.3000], [0.3000, 0.5000])
	([0.5528, 0.6536], [0.1414, 0.2449], [0.1015, 0.3058])	([0.2744, 0.3751], [0.2828, 0.3873], [0.2376, 0.4428])
	([0.3983, 0.4998], [0.1866, 0.2942], [0.2060, 0.4151])	([0.3519, 0.4523], [0.1741, 0.2766], [0.2711, 0.4740])
	([0.3188, 0.4205], [0.1741, 0.2766], [0.3029, 0.5071])	([0.2447, 0.3456], [0.2828, 0.3873], [0.2671, 0.4725])
	([0.2447, 0.3456], [0.2462, 0.3497], [0.3047, 0.5091])	([0.3072, 0.4084], [0.2462, 0.3571], [0.2345, 0.5193])

Step 4: Calculate the closeness coefficient of each alternative by the following equation:

$$C_i(\mathcal{A}_i) = \frac{D(\mathcal{A}_i, \beta^-)}{D(\mathcal{A}_i, \beta^+) + D(\mathcal{A}_i, \beta^-)}$$
$$i = 1, 2, \ldots, v. \tag{24}$$

Step 5: Rank all the alternatives $\mathcal{A}_i$ $(i = 1, 2, \ldots, u)$ based on the values of the closeness coefficient, higher value of closeness coefficient $C_i(\mathcal{A}_i)$ imply the better choice alternative.

In this method, we give some main calculating results. In this direction, applying step by 1-4, at the periods t_m $(m = 1, 2, \ldots, p)$, we obtain the measures of each option from UIFPIA and UIFNIA. Finally, we are finding all the closeness coefficients of each options A_i as follows $C(\mathcal{A}_1) = 0.5406$, $C(\mathcal{A}_2) = 0.5587$, $C(\mathcal{A}_3) = 0.5687$, $C(\mathcal{A}_4) = 0.5153$ and $C(\mathcal{A}_5) = 0.5630$. Then ordering of the alternative is $\mathcal{A}_3 \succ \mathcal{A}_5 \succ \mathcal{A}_2 \succ \mathcal{A}_1 \succ \mathcal{A}_4$, and best alternative is $\mathcal{A}_3$.

From the above-using operator's output, when we used the DCQROFWA operator, then ordering of the alternative is $\mathcal{A}_3 \succ \mathcal{A}_5 \succ \mathcal{A}_1 \succ \mathcal{A}_4 \succ \mathcal{A}_2$, and when used UDCQROFWA operator then ranking is as follows $\mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_2 \succ \mathcal{A}_4$. Here, $\mathcal{A}_3$ is the most favourable alternative in both CQROF and CIVQROF environments. But, in comparative results, the ranking order shows as $\mathcal{A}_3 \succ \mathcal{A}_5 \succ \mathcal{A}_2 \succ \mathcal{A}_1 \succ \mathcal{A}_4$ and $\mathcal{A}_3 \succ \mathcal{A}_5 \succ \mathcal{A}_2 \succ \mathcal{A}_1 \succ \mathcal{A}_4$ for using the two operators DIFWA and UDIFWA, and the best alternative is $\mathcal{A}_3$. Although ranking order is different but desirable option is always the same as $\mathcal{A}_3$. Our proposed methods in complex q-rung Orthopair fuzzy environment used more fuzzy information than comparative methods in intuitionistic fuzzy environment. Hence, our result is optimal.

Here, we listed some advantages of the proposed method:

(1) The proposed approach in this paper express a wider range of fuzzy information, and they used the sum of membership and non-membership grade in real as well as in imaginary part is greater than one. So there is a huge scope of application in our complex real-life problems. In Today's complex decision-making environment, we effectively express fuzzy decision making information and avoid a lot of information distortion. Our methods are more general, which are justified in the following points.
(2) If the parameter $q = 1$, then the proposed DCQROFWA operator converted into DCIFWA operator is a special case of the sponcord work.
(3) If the parameter $q = 2$, then the proposed DCQROFWA operator converted into DCPYFWA operator is a special case of the proposed work.
(4) If the parameter $q = 1$, the proposed UDCQROFWA operator converted into UDCIFWA operator is a special case of the proposed work.
(5) If the parameter $q = 2$, the proposed UDCQROFWA operator converted into UDCPYFWA operator, is a special case of the proposed work.
(6) If the parameter $q = 1$ and imaginary part zero, then the proposed DCQROFWA operator converted into DIFWA operator is a special case of the proposed work.
(7) If the parameter $q = 2$ and imaginary part zero, then the proposed DCQROFWA operator converted into DPYFWA operator is a special case of the proposed work.
(8) If the parameter $q = 1$ and imaginary part zero, then the proposed UDCQROFWA operator converted into UDIFWA operator is a special case of the proposed work.
(9) If the parameter $q = 2$ and imaginary part zero, then the proposed UDCQROFWA operator converted into UDPYFWA operator is a special case of the proposed work.

(10) If we choose the parameter $q = 1$ and imaginary part zero, then the proposed CQROFWA operator converted into intuitionistic fuzzy set (Attanassov 1986), is a special case of the proposed work.
(11) If the parameter q and imaginary part zero, then the proposed CQROFWA operator converted into q-rung Orthopair fuzzy sets (Yager 2014), is a special case of the proposed work.

Thus, our proposed methods in a complex q-rung Orthopair fuzzy environment have adopted more fuzzy information than other fuzzy frames. Hence, our result is optimal.

11 Conclusions

In this manuscript, we have studied the dynamic complex q-rung Orthopair fuzzy multiple attributes decision making (DCQROFMADM) models where all the attribute values are used in CQROFNs or CIVQROFNs information. Here, we considered some dynamic new averaging aggregation functions such as the dynamic CQROF weighted averaging (DCQROFWA) operator and uncertain dynamic CIVQROF weighted averaging (UDQROFWA) operator to aggregate the dynamic or uncertain dynamic CQROF information. We have developed two procedures to solve DMADM problems: one procedure has been developed based on the DCQROFWA and CQROFWA operators to solve the DCQROF-MADM problems where attribute information used in CQROFNs are collected at distinct periods, and another one is constructed based on the UDQROFWA and CIVQROFWA operators to solve uncertain DMADM problems where all attribute information use CIVQROFNs collected at multiple periods. For the efficiency of the proposed approach, a numerical example is discussed to the potential software selection problems. The sensitivity analysis is performed to show the flexibility of the study on the decision-making method for various values of q. Later, a comparative study was performed with the former results to check the effectiveness of the proposed method. The main advantage of this proposed approach is that the technique is more general to accumulate QROF and CQROF information than others. The proposed method can be applied for the future development of dynamic decision-making methods such as dynamic personal selection, medical diagnosis for management of the COVID-19 situation, dynamic investment method, active social media management, and dynamic evaluation of military system management, complex fuzzy dynamic decision-making with GRA method, AHP based dynamic decision making in complex fuzzy environment.

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Declarations

Conflict of Interest There is no conflict of interest between the authors and the institute where the work has been carried out.

Ethical Approval The article does not contain any studies with human participants or animals performed by any of the authors.

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