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New concepts of pentapartitioned neutrosophic graphs and applications for determining safest paths and towns in response to COVID-19

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Abstract
Pentapartitioned neutrosophic sets are a generalization of the single-valued and quadri-partitioned single-valued neutrosophic sets, and utilizes five symbol-valued neutrosophic logic. In this paper, we introduce some novel concepts regarding pentapartitioned neutrosophic graphs (PPNGs), and emphasize the effectiveness at interpreting extremely heterogeneous data that are prevalent in our daily life, particularly data gathered from various different sources which are becoming increasingly common place in the current times. The applicability of the proposed PPNG is demonstrated by applying the PPNGs on a potential real-life scenario on responding to the spread of COVID-19, where PPNGs are used to determine the safest path of travel and the safest place to stay to minimize the chances of getting infected. Both of this information have proven to be vital aspects in the efforts to combat the spread of the COVID-19 pandemic while providing the necessary support to the domestic economies, most of which are currently in recession due to the adverse effects brought upon by the pandemic. Hence, the PPNGs are applicable to all countries around the world and can be used under any circumstances such as pandemics or even in regular situations to optimize the travelling time and distance.

Keywords Fuzzy graph · Fuzzy set · Pentapartitioned neutrosophic set · Neutrosophic set · Pentapartitioned neutrosophic graphs · Neutrosophic graph · COVID-19 · Safest path to travel

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1 Introduction

Fuzzy set theory, which was firstly introduced by Zadeh (1965) is an important tool in dealing with vagueness, imprecise and incomplete information that are prevalent in many different fields such as engineering, medicine, social sciences, artificial intelligence, pattern recognition and image analysis. The easy applicability of fuzzy sets has led to its extension and generalization to models such as intuitionistic fuzzy sets, type 2 fuzzy sets, interval valued fuzzy sets, Pythagorean fuzzy sets and neutrosophic sets. Neutrosophic set proposed by Smarandache (1999) is a generalization of fuzzy set and intuitionistic fuzzy set. It has the capability to deal with incomplete, indeterminant and inconsistent information. Wang et al. (2010) subsequently proposed the concept of single-valued neutrosophic sets (SVNS) as a subclass of the neutrosophic set in which each of the truth, indeterminacy and falsity memberships assume values in the interval of [0, 1].

This paper pertains to the study of SVNS graphs and some of the important research related to the SVNS and other related models and applications will be expounded. Chatterjee et al. (2016a) defined quadripartitioned SVNS which comprises four membership functions namely the truth, falsity and indeterminacy which is split into unknown and contradiction, all of which assume values in the range of [0, 1]. Smarandache (2013) defined five-valued neutrosophic logic in which the indeterminacy membership degree is further refined into unknown, ignorance and contradiction. This work was subsequently extended to seven-valued logic and refined to n-valued neutrosophic logic in Smarandache (2013), whereby the indeterminacy membership is split into many types of truths, falsities and indeterminacies. Chatterjee et al. (2016b, 2020) studied the implementation of quadripartitioned SVNS on various decision-making operators and applications. Mallick and Pramanik (2020) proposed the concept of pentapartitioned neutrosophic set, which comprises the truth, contradiction, ignorance, unknown and falsity membership functions.

Many researchers have worked on neutrosophic sets, SVNS and their extensions in many different applications (Peng et al. 2014; Zhang et al. 2016; Tian et al. 2018; Wu et al. 2016; Karabašević et al. 2020; Tan 2021; Ye et al. 2020; Gulistan et al. 2019; Quek et al. 2018). Peng et al. (2014) and Zhang et al. (2016) proposed outranking approaches for multi-criteria decision-making (MCDM) problems for simplified neutrosophic sets and interval-valued neutrosophic sets, respectively. Tian et al. (2018) introduced a MCDM method based on generalized prioritized aggregation operators, while Wu et al. (2016) introduced the concept of cross-entropy and prioritized aggregation operators for simplified neutrosophic sets in MCDM problems. Karabašević et al. (2020) introduced a novel MCDM method based on TOPSIS on SVNS for selection of strategies for e-commerce development, while Tan (2021) introduced a new MCDM method for the SVNS model based on entropy for typhoon disaster assessment. Ye et al. (2020) introduced new correlation coefficients of consistency for neutrosophic multi-valued sets, Gulistan et al. (2019) proposed neutrosophic cubic Heronian mean operators using cosine similarity functions, whereas Quek et al. (2018) presented some new results pertaining to the graph theory for complex neutrosophic sets.

Graphs are the visual symbolization of objects and their relations. Graph theory is applicable to many fields and have been successfully applied in diverse areas such as medicine, engineering, computer science, biochemistry and operations research. The relations between objects may become vague and uncertain when some real-world problems are considered, and therefore fuzzy graphs are considered in such situations. Having Zadeh's fuzzy relation as a base, Kaufmann (1973) proposed the idea of fuzzy graphs. Various basic graph theory concepts such as cycles, paths, bridges and connectedness were defined by Rosenfeld (1975).

Morderson and Peng (1994) introduced some operations on fuzzy graphs, while Gani and Radha (2008), Akram and Davvaz (2012) and Karunambigai and Parvathi (2006) proposed the concept of regular fuzzy graphs, strong fuzzy graphs and intuitionistic fuzzy graphs, respectively. Akram and Dudek (2013) presented the concept of intuitionistic fuzzy hypergraphs and demonstrated the applications of this hypergraphs in real-life problems. Broumi et al. (2016a, b) introduced single-valued neutrosophic graphs and isolated single-valued neutrosophic graphs, respectively, while Naz et al. (2017) proposed several new operations for single-valued neutrosophic graphs and introduced the concept of regular single-valued neutrosophic graphs.

Broumi et al. (2016c) introduced bipolar single-valued neutrosophic graphs and defined the concept of strong, complex and regular bipolar single-valued neutrosophic graphs. Broumi et al. (2016d) presented some new results on bipolar single-valued neutrosophic graphs, whereas Hassan et al. (2017) introduced some special types of bipolar neutrosophic graphs. Dey et al. (2018) introduced several new concepts on vertex and edge coloring of simple vague graphs, and Naz et al. (2018) introduced some important concepts related to the energy and Laplacian energy for single-valued neutrosophic graphs. In recent years, fuzzy graphs have been extended to Pythagorean graphs, picture fuzzy graphs, spherical fuzzy graphs, etc. Numerous concepts pertaining to these graphs such as the energy of the graphs and coloring of these graphs have been introduced in Zuo et al. (2019), Akram et al. (2020a, b), Akram and Naz (2018), Mohamed and Ali (2020), Akram and Khan (2020) and Ajay and Chellamani (2020). Zuo et al. (2019) proposed new concepts related to picture fuzzy graphs, Akram et al. (2020a) introduced the concept of spherical fuzzy graphs. Akram and Naz (2018) studied the energy of Pythagorean fuzzy graphs, while Mohamed and Ali (2020) presented a study on the energy of spherical fuzzy graphs. Akram, Dar and Naz (2020b) introduced the concept of Pythagorean Dombi fuzzy graphs, while Akram and Khan (2020) proposed the notion of complex Pythagorean Dombi fuzzy graphs. Pythagorean neutrosophic graphs were presented by Ajay and Chellamani (2020) by combining the idea of fuzzy graphs and Pythagorean neutrosophic set.

In this paper, we present the concept of pentapartitioned neutrosophic graphs, explore the fundamental properties of this newly introduced graph and demonstrate the applicability of this graph by applying it in a MCDM problem. On the other hand, in the current state of the increasing spread of the COVID-19 pandemic, transport navigation systems (e.g., WAZE, Google Maps, or any customized navigation systems) requires an unprecedentedly sophisticated way of navigating the drivers through the most optimal roads, which considers not only the conventional parameters such as distance, traffic congestion, and road conditions but also many new pandemic related parameters, such as the number of reported COVID-19 cases (Khalifa et al. 2021; Eroğlu and Şahin 2020). Such complexity in consideration enables the creation of better mobile applications of traffic navigation systems in response to the current pandemic situation. With the abundance of information being considered, this gives rise to five major types of situations for a given town, all of which are as described below:

(i) All types of information unanimously affirm that a town is safe: No reported COVID-19 cases, stringent government intervention, and full adherence to the disease prevention rules imposed by the government.
(ii) Contradicting information exists on the safety of a town: No reported COVID-19 cases, but no adherence to rules.
(iii) Denied access to information for a town: The government refuses to reveal the particulars of a town.

(iv) Information is unknown for a town.
(v) All types of information unanimously affirm that a town is dangerous.

As for a given road, the five major types of situations that may be encountered are as follows.

(i) All types of information unanimously affirm that a road is safe and efficient: No reported COVID-19 cases, stringent government intervention, and full adherence to the disease prevention rules imposed by the government in both the two towns connecting at each of its ends. In addition, the road is short on distance, has smooth traffic, and the best road conditions.
(ii) Contradicting information exists on a road's safety or efficiency.
(iii) Denied access to information for a road.
(iv) Information is unknown for a road.
(v) All types of information unanimously affirm that a road is dangerous.

Therefore, *pentapartitioned neutrosophic sets* (PPNSs) and *pentapartitioned neutrosophic graphs* (PPNGs) prove to be more suitable compared to all the previous derivatives of fuzzy systems to model and represent such as a vast group of information. This is because PPNS and PPNG as their names suggest is able to consider all the five aforementioned major types of situations as individual memberships. Therefore, PPNS and PPNG can consider a much greater variety of information as independent entities and process them accordingly.

The remainder of this paper is organized as follows. Section 2 covers some of the important definitions and terminologies which form the background of this study are recapitulated. In Sect. 3, the newly introduced concept of PPNG is introduced and some of the important properties of this concept are also presented and discussed. Section 4 demonstrates the applicability of the PPNGs by applying it to a MCDM problem on the spread of COVID-19, specifically on determining the safest path to travel and the safest place to stay to minimize the chances of getting infected. Concluding remarks are presented in Sect. 5, followed by the acknowledgements and list of references.

2 Preliminaries

This section recapitulates some important concepts pertaining to the theory of SVNSs and the concept of PPNSs from which the concept of PPNGs is derived. We refer the readers to Smarandache (1999) and Wang et al. 2010) for further details pertaining to the NS and SVNS theory, respectively.

The single-valued neutrosophic set (SVNS) model introduced in Wang et al. (2010) is one of the most well-known and commonly used models among the neutrosophic models. It is a special case of the classical neutrosophic set in which the range of each of the three membership functions, namely the truth, indeterminacy and falsity membership functions lie in the standard unit of interval of [0, 1], instead of the non-standard interval of $]^{-}0, 1^{+}[$, thereby making it compatible with traditional fuzzy sets and other fuzzy based models in literature. The formal definition of the classical NS introduced by Smarandache (1999) is given below.

Let U be a universe of discourse, with a class of elements in U denoted by x.

Definition 2.1. Smarandache 1999) A neutrosophic set A is an object having the form $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in U\}$, where the functions $T, I, F : U \to]^{-}0, 1^{+}[$ denote the truth, indeterminacy, and falsity membership functions, respectively, of the element $x \in U$

with respect to A. The membership functions must satisfy the condition ${}^{-}0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3^{+}$.

Definition 2.2. Smarandache 1999) A neutrosophic set A is contained in another neutrosophic set B, if $T_A(x) \leq T_B(x), I_A(x) \geq I_B(x)$, and $F_A(x) \geq F_B(x)$, for all $x \in U$. This relationship is denoted as $A \subseteq B$.

The SVNS is a specific form of the NS with values of the membership functions defined in the standard interval of [0, 1]. The formal definition of the SVNS is presented below, and this is followed by the definitions of some of the important concepts and set theoretic operations of the SVNS.

Definition 2.3. Wang et al. 2010) A SVNS A over a universe Y is given by $A = \{(y, T_A(y), I_A(y), F_A(y)) : y \in Y\}$, where $T_A(y), I_A(y), F_A(y) \in [0, 1]$ signify truth, indeterminacy and false membership of each $y \in A$ respectively, and $T_A(y), I_A(y), F_A(y)$ satisfy $0 \leq T_A(y) + I_A(y) + F_A(y) \leq 3$.

For a SVNS N in Y, the triplet $(T_A(y), I_A(y), F_A(y))$ is called a single-valued neutrosophic number (SVNN). For the sake of convenience, we simply let $y = \left(T_y, I_y, F_y\right)$ to represent a SVNN as an element in the SVNS A.

Definition 2.4. Wang et al. 2010) Let A and B be two SVNSs over a universe Y.

(i) A is contained in B, if $T_A(y) \leq T_B(y), I_A(y) \geq I_B(y)$, and $F_A(y) \geq F_B(y)$, for all $y \in Y$. This relationship is denoted as $A \subseteq B$.
(ii) A and B are said to be equal if $A \subseteq B$ and $B \subseteq A$.
(iii) $A^c = (y, (F_A(y), 1 - I_A(y), T_A(y)))$, for all $y \in Y$.
(iv) $A \cup B = (y, (\max(T_A, T_B), \min(I_A, I_B), \min(F_A, F_B)))$, for all $y \in Y$.
(v) $A \cap B = (y, (\min(T_A, T_B), \max(I_A, I_B), \max(F_A, F_B)))$, for all $y \in Y$.

Definition 2.5. Wang et al. 2010) Let $x = (T_x, I_x, F_x)$ and $y = \left(T_y, I_y, F_y\right)$ be two SVNNs. The operations for SVNNs can be defined as follows:

(i) $x \oplus y = \left(T_x + T_y - T_x * T_y, I_x * I_y, F_x * F_y\right)$
(ii) $x \oplus y = \left(T_x * T_y, I_x + I_y - I_x * I_y, F_x + F_y - F_x * F_y\right)$
(iii) $\lambda x = \left(1 - (1 - T_x)^{\lambda}, (I_x)^{\lambda}, (F_x)^{\lambda}\right)$, where $\lambda > 0$
(iv) $x^{\lambda} = \left((T_x)^{\lambda}, 1 - (1 - I_x)^{\lambda}, 1 - (1 - F_x)^{\lambda}\right)$, where $\lambda > 0$.

Definition 2.6. Majumdar and Samanta 2014) Let A and B be two SVNSs over a finite universe $Y = \{y_1, y_2, \ldots, y_n\}$. Then the various distance measures between A and B are defined as follows:

(i) The Hamming distance between A and B are defined as:

$$d_{\mathrm{H}}(A, B) = \sum_{i=1}^{n}\{|T_A(y_i) - T_B(y_i)| + |I_A(y_i) - I_B(y_i)| + |F_A(y_i) - F_B(y_i)|\} \quad (1)$$

(ii) The normalized Hamming distance between A and B are defined as:

$$d_{\mathrm{H}}^{N}(A, B) = \frac{1}{3n}\sum_{i=1}^{n}\{|T_A(y_i) - T_B(y_i)| + |I_A(y_i) - I_B(y_i)| + |F_A(y_i) - F_B(y_i)|\} \quad (2)$$

(iii) The Euclidean distance between A and B are defined as:

$$d_{\mathrm{E}}(A, B) = \sqrt{\sum_{i=1}^{n}\left\{(T_A(y_i) - T_B(y_i))^2 + (I_A(y_i) - I_B(y_i))^2 + (F_A(y_i) - F_B(y_i))^2\right\}} \tag{3}$$

(iv) The normalized Euclidean distance between A and B are defined as:

$$d_{\mathrm{E}}^{N}(A, B) = \sqrt{\frac{1}{3n}\sum_{i=1}^{n}\left\{(T_A(y_i) - T_B(y_i))^2 + (I_A(y_i) - I_B(y_i))^2 + (F_A(y_i) - F_B(y_i))^2\right\}} \tag{4}$$

Definition 2.7. Broumi et al. 2016a) Let V be a set. Let $E \subseteq \{\{u, v\} : u, v \in V \text{ with } u \neq v\}$. Let A be a SVNS on V and B be a SVNS on E with $T_B(u, v) \leq \min\{T_A(u), T_A(v)\}$, $I_B(u, v) \geq \max\{I_A(u), I_A(v)\}$ and $F_B(u, v) \geq \max\{F_A(u), F_A(v)\}$ for all $\{u, v\} \in E$. Then $G = (A, B, V, E)$ is said to be a single-valued neutrosophic graph.

Remark. To avoid too many brackets in notation, we simply denote $T_B(u, v) = T_B(\{u, v\})$ in this entire paper and this also applies to all other membership functions defined similarly on E.

Definition 2.8. Mallick and Pramanik 2020) A pentapartitioned neutrosophic set (PPNS) A on Y is defined as:

$$A = \{(y, \mathfrak{t}_A(y), \mathfrak{c}_A(y), \mathfrak{g}_A(y), \mathfrak{u}_A(y), \mathfrak{f}_A(y)) : y \in Y\},$$

where $\mathfrak{t}_A(y), \mathfrak{c}_A(y), \mathfrak{g}_A(y), \mathfrak{u}_A(y), \mathfrak{f}_A(y) \in [0, 1]$ signify the truth, contradiction, ignorance, unknown and falsity membership functions of each $y \in A$, respectively. The membership functions $\mathfrak{t}_A, \mathfrak{c}_A, \mathfrak{g}_A, \mathfrak{u}_A, \mathfrak{f}_A$ must satisfy the condition $0 \leq \mathfrak{t}_A(y), \mathfrak{c}_A(y), \mathfrak{g}_A(y), \mathfrak{u}_A(y), \mathfrak{f}_A(y) \leq 5$.

3 Pentapartitioned neutrosophic graphs

In this section, we present the proposed concept of pentapartitioned neutrosophic graphs (PPNGs). The definitions of SVNS, single-valued neutrosophic graphs and PPNS given in Definitions 2.3, 2.7 and 2.8, respectively will be used to develop the formal definition of PPNGs. The important components pertaining to this concept will be subsequently presented and expounded.

Definition 3.1. Let V be a set. Let $E \subseteq \{\{u, v\} : u, v \in V \text{ with } u \neq v\}$. Let A be a PPNS on V, and B be a PPNS on E, with $\mathfrak{t}_B(u, v) \leq \min\{\mathfrak{t}_A(u), \mathfrak{t}_A(v)\}$, $\mathfrak{c}_B(u, v) \geq \max\{\mathfrak{c}_A(u), \mathfrak{c}_A(v)\}$, $\mathfrak{g}_B(u, v) \geq \max\{\mathfrak{g}_A(u), \mathfrak{g}_A(v)\}$, $\mathfrak{u}_B(u, v) \geq \max\{\mathfrak{u}_A(u), \mathfrak{u}_A(v)\}$ and $\mathfrak{f}_B(u, v) \geq \max\{\mathfrak{f}_A(u), \mathfrak{f}_A(v)\}$ for all $\{u, v\} \in E$. Then we have the following:

(i) $G = (A, B, V, E)$ is said to be a *pentapartitioned neutrosophic graph* (PPNG).
(ii) Each $v \in V$ is said to be a *vertex* of G.
(iii) Each $\{u, v\} \in E$ is said to be an *edge* of G.

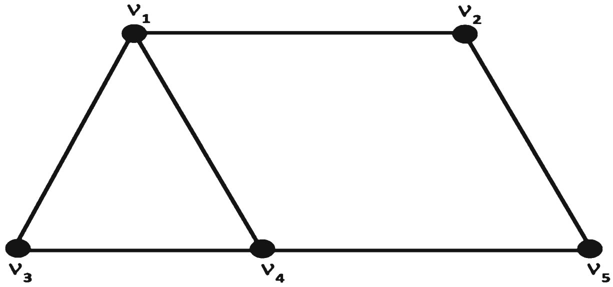

Fig. 1 A graphical representation of the PPNG G

Notation 3.1.1 Let $G = (A, B, V, E)$ be a PPNG. Denote $\mathbf{m}_A : V \to [0,1]^5$, where $\mathbf{m}_A(v) = (\mathfrak{t}_A(v), \mathfrak{c}_A(v), \mathfrak{g}_A(v), \mathfrak{u}_A(v), \mathfrak{f}_A(v))$ for all $v \in V$. Denote $\mathbf{m}_B : E \to [0,1]^5$, where $\mathbf{m}_B(u,v) = (\mathfrak{t}_B(u,v), \mathfrak{c}_B(u,v), \mathfrak{g}_B(u,v), \mathfrak{u}_B(u,v), \mathfrak{f}_B(u,v))$ for all $\{u,v\} \in E$.

Example 3.2 Let $G = (A, B, V, E)$ be a PPNG with $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{\{v_1,v_2\}, \{v_1,v_3\}, \{v_1,v_4\}, \{v_2,v_5\}, \{v_4,v_5\}\}$. Then we have the following (Fig. 1):

$$\mathbf{m}_A(v_1) = (0.5, 0.6, 0.8, 0.9, 0.7),\ \mathbf{m}_B(v_1,v_2) = (0.3, 0.6, 0.9, 0.9, 0.8),$$
$$\mathbf{m}_A(v_2) = (0.4, 0.3, 0.6, 0.5, 0.8),$$
$$\mathbf{m}_B(v_1,v_3) = (0.4, 0.7, 0.8, 0.9, 0.8), \mathbf{m}_A(v_3) = (0.9, 0.7, 0.5, 0.3, 0.2),$$
$$\mathbf{m}_B(v_1,v_4) = (0.5, 0.8, 0.9, 0.9, 0.7), \mathbf{m}_A(v_4) = (0.8, 0.5, 0.4, 0.6, 0.5),$$
$$\mathbf{m}_B(v_2,v_5) = (0.4, 0.5, 0.6, 0.5, 0.8), \mathbf{m}_A(v_5) = (0.7, 0.4, 0.3, 0.2, 0.3),$$
$$\mathbf{m}_B(v_4,v_5) = (0.6, 0.6, 0.4, 0.6, 0.6), \mathbf{m}_B(v_3,v_4) = (0.3, 0.9, 0.8, 0.6, 0.5).$$

Definition 3.3. Let $G = (A, B, V, E)$ and $H = \left(A', B', V', E'\right)$ be two PPNGs that satisfies the following conditions:

(i) $V' \subseteq V$
(ii) $E' \subseteq \left\{\{v, w\} : v, w \in V' \text{ with } u \neq v\right\}$
(iii) $\mathbf{m}_{A'}(v) = \mathbf{m}_A(v)$, for all $v \in V' \subseteq V$
(iv) $\mathbf{m}_{B'}(v,w) = \mathbf{m}_B(v,w)$, for all $\{v,w\} \in E' \subseteq E$.

Then, H is said to be a *partial pentapartitioned neutrosophic graph* (partial-PPNG) of G.

Definition 3.4. Let $G = (A, B, V, E)$ and $H = \left(A', B', V', E'\right)$ be two PPNGs that satisfies the following conditions:

(i) $V' \subseteq V$
(ii) $E' \subseteq \left\{\{v, w\} : v, w \in V' \text{ with } u \neq v\right\}$
(iii) $\mathfrak{t}_{A'}(v) \leq \mathfrak{t}_A(v)$, $\mathfrak{c}_{A'}(v) \geq \mathfrak{c}_A(v)$, $\mathfrak{g}_{A'}(v) \geq \mathfrak{g}_A(v)$, $\mathfrak{u}_{A'}(v) \geq \mathfrak{u}_A(v)$, $\mathfrak{f}_{A'}(v) \geq \mathfrak{f}_A(v)$ for all $v \in V' \subseteq V$
(iv) $\mathfrak{t}_{B'}(v,w) \leq \mathfrak{t}_B(v,w)$, $\mathfrak{c}_{B'}(v,w) \geq \mathfrak{c}_B(v,w)$, $\mathfrak{g}_{B'}(v,w) \geq \mathfrak{g}_B(v,w)$, $\mathfrak{u}_{B'}(v,w) \geq \mathfrak{u}_B(v,w)$, $\mathfrak{f}_{B'}(v,w) \geq \mathfrak{f}_B(v,w)$, for all $\{v,w\} \in E' \subseteq E$.

Then, H is said to be a *pentapartitioned neutrosophic subgraph* (PPNSG) of G.

Example 3.5. Let G_1 be a PPNG and H_1, H_2 be the partial-PPNG and PPNSG of G_1, respectively. The graphical representation of G_1, H_1 and H_2 are shown in Figs. 2, 3 and 4, respectively. Here H_2 is a PPNSG of G_1 but not a partial-PPNG of G_1.

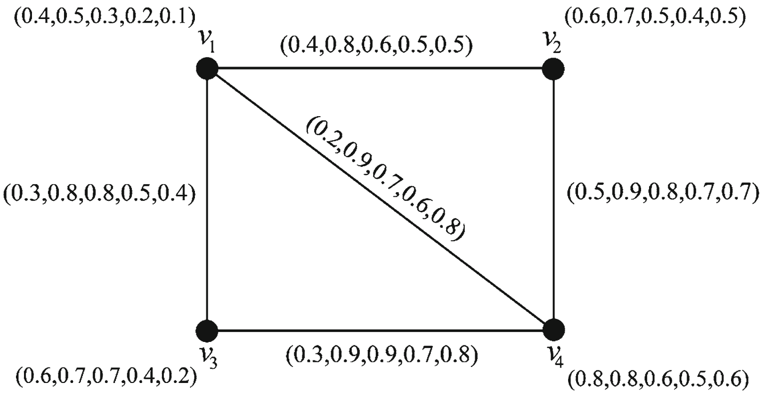

Fig. 2 Graphical representation of the PPNG G_1

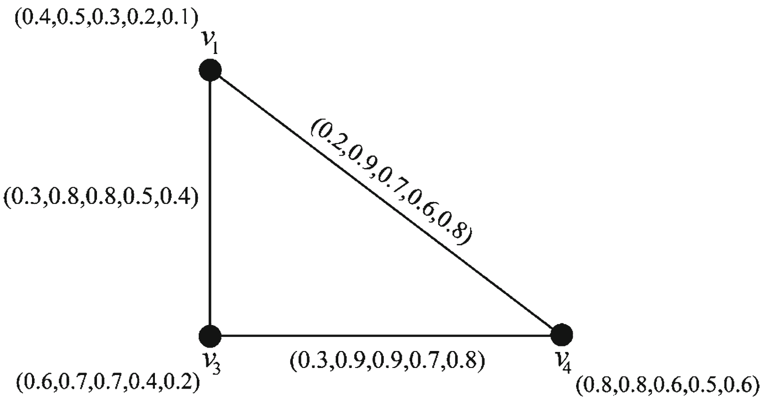

Fig. 3 Graphical representation of H_1

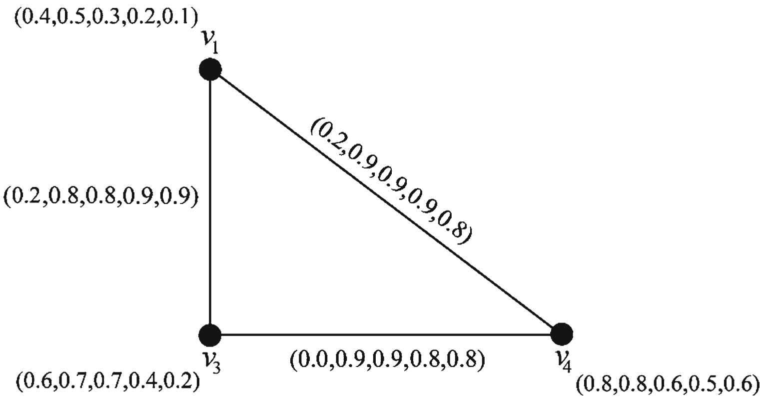

Fig. 4 Graphical representation of H_2

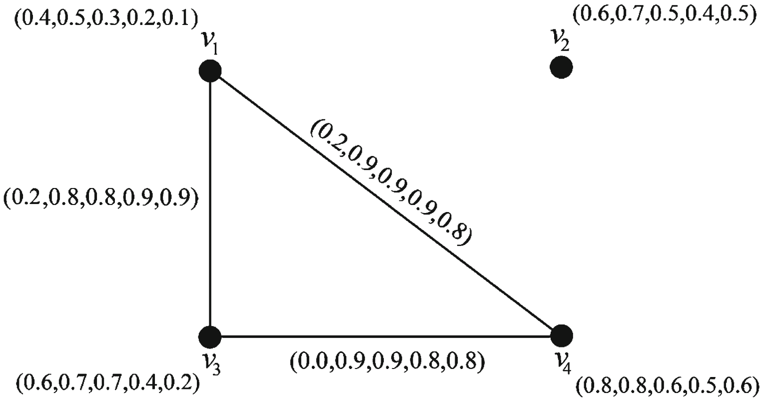

Fig. 5 Example of a PPNG with isolated vertex

Definition 3.6. Let $G = (A, B, V, E)$ be a PPNG. Let $v_1, v_2, \ldots, v_n \in V$, with $\{v_{i-1}, v_i\} \in E$ for all $1 < i \leq n$, and with $v_i \neq v_j$ for all $1 \leq i < j \leq n$. Then we have the following:

(i) $P = (v_1, v_2, \ldots, v_n)$ is said to be a *pentapartitioned neutrosophic path* (PPNP) in G.
(ii) For each i, $\{v_{i-1}, v_i\}$ is said to be an *edge* of P.
(iii) n is said to be the *length* of P.

Definition 3.7. Let $G = (A, B, V, E)$ be a PPNG. Then G is said to be *connected* if there exists at least one $\{v, w\} \in E$ for all $v \in V$.

Definition 3.8. Let $G = (A, B, V, E)$ be a PPNG. Then $v \in V$ is said to be *isolated* if $\{v, w\} \notin E$ for all $w \in V \setminus \{v\}$.

In Fig. 5, the PPNG has v_1 as the isolated vertex.

Definition 3.9. Let $G = (A, B, V, E)$ be a PPNG and let $v \in V$. The *degree* of v, written as $\mathbf{d}(v)$, is defined as $\mathbf{d}(v) = \sum_{u \in V \text{with} \{v,w\} \in E} \mathbf{m}_B(v, w)$.

Remark 3.9.1. It follows that $\mathbf{d}(v) \in [0, 1]^5$.

Example 3.10. In Fig. 3 under Eg. 3.5, the degree of the vertices are as follows.

$$\mathbf{d}(v_1) = (0.5, 1.7, 1.5, 1.1, 1.2), \mathbf{d}(v_3) = (0.6, 1.7, 1.7, 1.2, 1.2),$$

$$\mathbf{d}(v_4) = (0.5, 1.8, 1.6, 1.3, 1.6).$$

Definition 3.11. Let $G = (A, B, V, E)$ be a PPNG and let $P = (v_1, v_2, \ldots, v_n)$ be a PPNP in G. The *strength* of P, denoted as $\mathbf{s}(P)$, is defined as:

$$\mathbf{s}(P) = \big(s_{\mathfrak{t}}(P), s_{\mathfrak{c}}(P), s_{\mathfrak{g}}(P), s_{\mathfrak{u}}(P), s_{\mathfrak{f}}(P)\big),$$

where $s_{\mathfrak{t}}(P) = \min\{\mathfrak{t}_B(v_{i-1}, v_i) : 1 < i \leq n\}$, $s_{\mathfrak{c}}(P) = \max\{\mathfrak{c}_B(v_{i-1}, v_i) : 1 < i \leq n\}$, $s_{\mathfrak{g}}(P) = \max\{\mathfrak{g}_B(v_{i-1}, v_i) : 1 < i \leq n\}$, $s_{\mathfrak{u}}(P) = \max\{\mathfrak{u}_B(v_{i-1}, v_i) : 1 < i \leq n\}$ and $s_{\mathfrak{f}}(P) = \max\{\mathfrak{f}_B(v_{i-1}, v_i) : 1 < i \leq n\}$.

Moreover, the *strength of connectedness* among the vertices $a, b \in V$ in G, denoted as $\mathbf{r}_G(a, b)$, is defined as:

$$\mathbf{r}_G(a, b) = \big(r_{\mathfrak{t},G}(a, b), r_{\mathfrak{c},G}(a, b), r_{\mathfrak{g},G}(a, b), r_{\mathfrak{u},G}(a, b), r_{\mathfrak{f},G}(a, b)\big),$$

where:

$$r_{\mathfrak{t},G}(a,b)=\max\left\{s_{\mathfrak{t}}(P):P=\left(v_1,v_2,\ldots,v_{n_P}\right)\text{ in }G\text{ with }v_1=a\text{ and }v_{n_P}=b\right\},$$
$$r_{\mathfrak{c},G}(a,b)=\min\left\{s_{\mathfrak{c}}(P):P=\left(v_1,v_2,\ldots,v_{n_P}\right)\text{ in }G\text{ with }v_1=a\text{ and }v_{n_P}=b\right\},$$
$$r_{\mathfrak{g},G}(a,b)=\min\left\{s_{\mathfrak{g}}(P):P=\left(v_1,v_2,\ldots,v_{n_P}\right)\text{ in }G\text{ with }v_1=a\text{ and }v_{n_P}=b\right\},$$
$$r_{\mathfrak{u},G}(a,b)=\min\left\{s_{\mathfrak{u}}(P):P=\left(v_1,v_2,\ldots,v_{n_P}\right)\text{ in }G\text{ with }v_1=a\text{ and }v_{n_P}=b\right\},$$
$$r_{\mathfrak{f},G}(a,b)=\min\left\{s_{\mathfrak{f}}(P):P=\left(v_1,v_2,\ldots,v_{n_P}\right)\text{ in }G\text{ with }v_1=a\text{ and }v_{n_P}=b\right\}.$$

Definition 3.12. Let $G=(A,B,V,E)$ be a PPNG and let $\{v,w\}$ be an edge in G. Denote $G'_{\{v,w\}}$ as the partial-PPNG of G, in which $G'_{\{v,w\}}=\left(A',B',V',E'\right)$ with $V^{'}=V$ and $E^{'}=E-\{\{v,w\}\}$. Then, $\{v,w\}$ is said to be a *pentapartitioned neutrosophic bridge* (PPNB) in G if at least one of the following conditions holds for some $a,b\in V$.

(i) $r_{\mathfrak{t},G'_{\{v,w\}}}(a,b)<r_{\mathfrak{t},G}(a,b)$
(ii) $r_{\mathfrak{c},G'_{\{v,w\}}}(a,b)>r_{\mathfrak{c},G}(a,b)$
(iii) $r_{\mathfrak{g},G'_{\{v,w\}}}(a,b)>r_{\mathfrak{g},G}(a,b)$
(iv) $r_{\mathfrak{u},G'_{\{v,w\}}}(a,b)>r_{\mathfrak{u},G}(a,b)$
(v) $r_{\mathfrak{f},G'_{\{v,w\}}}(a,b)>r_{\mathfrak{f},G}(a,b)$

In particular, if all of the conditions (i)–(v) are true for some $a,b\in V$, then $\{v,w\}$ is said to be a *strong pentapartitioned neutrosophic bridge* (strong-PPNB) in G.

Remark 3.12.1. The behavior of A' and B' follows that of Definition 3.3.

Definition 3.13. Let $G=(A,B,V,E)$ be a PPNG and let v be a vertex in G. Denote G'_v as the partial-PPNG of G in which $G'_v=\left(A',B',V',E'\right)$ with $V^{'}=V-\{v\}$ and $E^{'}=E-\{\{a,v\}:a\in V-\{v\}\}$. Then v is said to be a *pentapartitioned neutrosophic cut vertex* (PPNCV) in G if at least one of the following conditions holds for some $a,b\in V$.

(i) $r_{\mathfrak{t},G'_v}(a,b)<r_{\mathfrak{t},G}(a,b)$
(ii) $r_{\mathfrak{c},G'_v}(a,b)>r_{\mathfrak{c},G}(a,b)$
(iii) $r_{\mathfrak{g},G'_v}(a,b)>r_{\mathfrak{g},G}(a,b)$
(iv) $r_{\mathfrak{u},G'_v}(a,b)>r_{\mathfrak{u},G}(a,b)$
(v) $r_{\mathfrak{f},G'_v}(a,b)>r_{\mathfrak{f},G}(a,b)$

In particular, if all of the conditions (i) to (v) are true for some $a,b\in V$, then v is said to be a *strong pentapartitioned neutrosophic bridge* (strong-PPNCV) in G.

4 Application of the pentapartitioned neutrosophic graphs

In this section, the utility, applicability and practicality of our proposed PPNGs are demonstrated by applying the concept of PPNGs to a MCDM problem related to the COVID-19 pandemic. Specifically, PPNGS will be used to determine the safest path of travel and the safest place to stay in order to minimize the chances of getting infected by COVID-19.

4.1 The scenario and datasets

4.1.1 The towns and the roads of connection

In this example, consider a district which consists of 6 towns denoted by $v_1, v_2, v_3, v_4, v_5, v_6$. The towns are connected by roads as shown in Fig. 6. However, Fig. 6 only shows the manner of connection and henceforth the lengths of those roads are not shown to scale in this figure.

4.1.2 The raw input variables considered

In this example, consider the following information gathered for each of the 6 towns, obtained from various sources such as governmental reports, check-in registrations and online maps, as follows:

$n(v)$: number of new COVID-19 cases reported in the most recent 24 h.

$m(v)$: maximum ICU occupancy ratio in the town during the most recent 24 h, from 0 to 1.

$p(v)$: number of traffic that has checked-in through entering the town in the most recent 1 h,

if no check-in is enforced, then $p(v)$ is assigned to be $+\infty$.

$q(v)$: number of check-ins by customers in all shops in the town in the most recent 1 h,

if no check-in is enforced, then $q(v)$ is assigned to be $+\infty$.

$s(v)$: number of violations of disease prevention rules reported in the most recent 24 h.

$d(v)$: population density of the town, in terms of the number of headcounts per km^2 (thus it can always be known).

In addition, for each town v, w that are directly connected by a road, the following information are further considered:

$l(v, w)$: length of the road connecting the two towns, in km (thus it can always be known).

$u(v, w)$: average number of vehicles per 100 m in the most recent 1 h.

$a(v, w)$: average speed of vehicles in the most recent 1 h.

Each of $n(v), m(v), p(v), q(v), s(v), u(v, w), a(v, w)$ are assigned to be -1 if unknown, -2 if denied access. The maximum operator, as in $\max\{n(v), 0\}$ for example, will be deployed to separate the values of -1 (absent data) and -2 (data denied access) from known data.

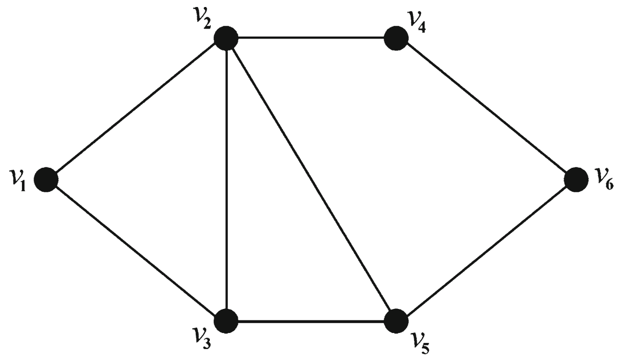

Fig. 6 The connection of towns for the scenarios presented in this scenario

4.1.3 The dataset for the variables involved

In this example, suppose the following datasets given in Tables 1 and 2 are obtained at a given instant, for which decisions are to be made.

4.2 Formation of PPNG and the score function

A PPNG $G = (A, B, V, E)$ is to be constructed, firstly by assigning V and E to all the towns and paths respectively. Therefore, such choices of V and E apply to all users regardless of their needs.

The next step of forming G is to choose function $\varphi : (\mathbb{R} \cup \{+\infty\})^6 \to [0,1]^5$ and $\psi : (\mathbb{R} \cup \{+\infty\})^{15} \to [0,1]^5$ called the *fuzzification functions* for V and E, respectively. These are to construct A and B by assigning the following:

(i) $\mathbf{m}_A(v) = \left[\mathbf{m}_A(v)_i\right]_5 = \varphi(n(v), m(v), p(v), q(v), s(v), d(v)) = (\mathfrak{t}_A(v), \mathfrak{c}_A(v), \mathfrak{g}_A(v), \mathfrak{u}_A(v), \mathfrak{f}_A(v))$ for all $v \in V$, and

(ii) $\mathbf{m}_B(v, w) = \left[\mathbf{m}_B(v, w)_i\right]_5 = \psi\begin{pmatrix} n(v), m(v), p(v), q(v), s(v), d(v), \\ n(w), m(w), p(w), q(w), s(w), d(w), \\ l(v, w), u(v, w), a(v, w) \end{pmatrix}$

$= (\mathfrak{t}_B(v, w), \mathfrak{c}_B(v, w), \mathfrak{g}_B(v, w), \mathfrak{u}_B(v, w), \mathfrak{f}_B(v, w))$ for all $\{v, w\} \in E$.

Unlike V and E however, such choices of φ and ψ will however depend on the preference of the user, as long as the following conditions are satisfied for all $\{v, w\} \in E$:

Table 1 Data for each town v

v	$n(v)$	$m(v)$	$p(v)$	$q(v)$	$s(v)$	$d(v)$
v_1	0	0	10	15	– 1	4200
v_2	0	80	10	360	– 2	6000
v_3	30	– 2	70	127	2	6000
v_4	167	93	249	1404	43	12,000
v_5	5	23	– 2	43	– 1	4000
v_6	4	26	12	192	1	7000

Table 2 Data for each road $\{v, w\}$

v, w	$l(v, w)$	$u(v, w)$	$a(v, w)$
v_1, v_2	5	6	60
v_1, v_3	13	37	70
v_2, v_3	26	6	50
v_2, v_4	6	76	40
v_2, v_5	31	– 1	– 1
v_3, v_5	8	57	5
v_4, v_6	9	56	50
v_5, v_6	13	5	60

(i) $\mathbf{m}_B(v, w)_1 \leq \min\{\mathbf{m}_A(v)_1, \mathbf{m}_A(w)_1\}$
(ii) $\mathbf{m}_B(v, w)_i \geq \max\{\mathbf{m}_A(v)_i, \mathbf{m}_A(w)_i\}$ for all $i = 2, 3, 4, 5$.

Consequently, each user may use a different A and B subject to their needs.

Moreover, in comparing the safety among all paths $P \in \mathrm{PPNP}(G)$, a score function $\sigma : [0, 1]^5 \to \mathbb{R}$ will be necessary to act on $\mathbf{s}(P)$ because there are 5 entries in $\mathbf{s}(P)$ to be dealt with. With that σ, path P_1 will be regarded as relatively safer and/or more efficient to travel along compared to path P_2 whenever $\sigma(\mathbf{s}(P_1)) > \sigma(\mathbf{s}(P_2))$. Moreover, that σ will also be used to compare the safety of staying among all towns $v \in V$, for which town v_1 will be regarded as being relatively safer to stay in compared to town v_2 whenever $\sigma(v_1) > \sigma(v_2)$. Such choices of σ is likewise dependent on the needs of the user.

4.3 Definition of fuzzification function

Thus, in the context of the scenario, let the fuzzification functions be defined as in Sects. 4.3.1 and 4.3.2, with the constants $\{c_i | 1 \leq i \leq 18\} \in \mathbb{R}_0^+$ and $\{s_1, s_2\} \in \mathbb{R}^+$ chosen as appropriate by the user, as demonstrated in Sect. 4.4. On the other hand, $s_0 \in \mathbb{R}^+$ are chosen (either manually or by a program) as an appropriate normalization parameter such that:

$$\max\left\{\begin{array}{c} \mathrm{mean}(\{\rho(v) : v \in V\} \cup \{\varrho(v, w) : \{v, w\} \in E\})| \\ (\rho, \varrho) \in \{(\mathfrak{t}_A, \mathfrak{t}_B), (\mathfrak{c}_A, \mathfrak{c}_B), (\mathfrak{g}_A, \mathfrak{g}_B), (\mathfrak{u}_A, \mathfrak{u}_B), (\mathfrak{f}_A, \mathfrak{f}_B)\} \end{array}\right\} \approx 0.5$$

to enable the most prominent membership value out of the truth, contradiction, ignorance, unknown and falsity membership functions to receive the most attention, without causing oversaturation of fuzzified values (e.g., when most of the fuzzified values are getting too close to 1).

4.3.1 Fuzzification function φ

$\varphi(n(v), m(v), p(v), q(v), s(v), d(v)) = (\mathfrak{t}_A(v), \mathfrak{c}_A(v), \mathfrak{g}_A(v), \mathfrak{u}_A(v), \mathfrak{f}_A(v))$, where:

(i) $\mathfrak{t}_A(v) = 1 - \tanh(s_0 c_1 d(v) \max\{n(v), 0\})$
Thus, the lower the number of COVID-19 cases in a town, the safer the town. The variable $d(v)$ serves to accentuate the deviation of the value from 0 according to the population density—the denser the population, the more prominent the outcomes are.

(ii) $$\mathfrak{c}_A(v) = \min\left\{\begin{array}{c} |\tanh(s_0 c_2 d(v) \max\{n(v), 0\}) - \tanh(s_0 c_3 d(v) \max\{m(v), 0\})| \\ +\tanh(s_0 c_4 \max\{s(v), 0\}), \\ 1 \end{array}\right\}$$

When there is a lot of new COVID-19 cases but low ICU occupancy, or few COVID-19 cases but higher ICU occupancy, it is deemed that the data obtained are contradictory as when the number of COVID-19 cases is small, low ICU occupancy is expected. The violations of disease prevention rules reported possesses two mutually opposite perceptions. One may conclude that it is unsafe due to the presence of violation, another may conclude that it is safe as it shows that the laws are strictly enforced. Likewise, the role of the maximum in $\max\{n(v), 0\}$ is to separate the values of -1 (absent data) and -2 (data denied access) for special consideration. The variable $d(v)$ serves to accentuate the deviation of the value from 0 according to the population density, but it does not apply to the number of violations.

(iii) $$\mathfrak{g}_A(v) = \tanh\left(s_0 c_5 d(v)\begin{pmatrix} \text{boolean}(n(v) = -2) \\ +\text{boolean}(m(v) = -2) \\ +\text{boolean}(p(v) = -2) \\ +\text{boolean}(q(v) = -2) \\ +\text{boolean}(s(v) = -2) \end{pmatrix}\right)$$

Thus, the total number of information denied accesses are added, and $d(v)$ serves to accentuate the deviation of the value from 0 according to the population density.

(iv) $$\mathfrak{u}_A(v) = \tanh\left(s_0 c_6 d(v)\begin{pmatrix} \text{boolean}(n(v) = -1) \\ +\text{boolean}(m(v) = -1) \\ +\text{boolean}(p(v) = -1) \\ +\text{boolean}(q(v) = -1) \\ +\text{boolean}(s(v) = -1) \end{pmatrix}\right)$$

The total number of unknown information is added, and $d(v)$ serves to accentuate the deviation of the value from 0 according to the population density.

(v) $$\mathfrak{f}_A(v) = \tanh\left(s_0 c_7 d(v)\max\{n(v), 0\}\begin{pmatrix} c_8\max\{m(v) + c_9, 0\} \\ +c_{10}(\max\{p(v), 0\} + \max\{c_{11}s(v), 0\}) \\ +c_{12}(\max\{q(v), 0\} + \max\{c_{13}s(v), 0\}) \end{pmatrix}\right)$$

The higher the number of COVID-19 cases, the higher the danger level. Such danger is further accentuated when the ratio of ICU occupancy is high, number of check-ins is high, or the number of violations is high, and $d(v)$ serves to further accentuate the deviation of the value from 0 according to the population density.

4.3.2 Fuzzification function ψ

Likewise, define.

$$\psi\begin{pmatrix} n(v), m(v), p(v), q(v), s(v), d(v), \\ n(w), m(w), p(w), q(w), s(w), d(w), \\ l(v,w), u(v,w), a(v,w) \end{pmatrix} = (\mathfrak{t}_B(v,w), \mathfrak{c}_B(v,w), \mathfrak{g}_B(v,w), \mathfrak{u}_B(v,w), \mathfrak{f}_B(v,w)),$$

where:

(i) $$\mathfrak{t}_B(v,w) = \min\left\{\begin{matrix} \mathfrak{t}_A(v), \mathfrak{t}_A(w), \\ \tanh(s_0 c_{14}\min\{d(v), d(w)\}a(v,w)) \end{matrix}\right\}$$

The higher the travel speed of a car on a road *and* the safer both the adjacent towns of the road, the safer and more efficient is that road itself.

(ii) $$\mathfrak{c}_B(v,w) = \max\left\{\begin{matrix} \mathfrak{c}_A(v), \mathfrak{c}_A(w), \\ \tanh\left(s_0 c_{15}\max\{d(v), d(w)\}\left|\frac{a(v,w)u(v,w)}{s_1} + \frac{s_1}{a(v,w)u(v,w)} - 2\right|\right) \end{matrix}\right\}$$

It is usually observed that cars move faster when the roads are open and move slower when the roads are congested. Thus, when cars move fast on a congested road, or cars move slowly on an open road, the amount of contradiction will be higher.

(iii) $$\mathfrak{g}_B(v,w) = \max\left\{\begin{matrix} \mathfrak{g}_A(v), \mathfrak{g}_A(w), \\ \tanh\left(s_0 c_{16}\max\{d(v), d(w)\}\begin{pmatrix} \text{boolean}(u(v,w) = -2) \\ +\text{boolean}(a(v,w) = -2) \end{pmatrix}\right) \end{matrix}\right\}$$

(iv) $$\mathfrak{u}_B(v,w) = \max\left\{\begin{matrix} \mathfrak{u}_A(v), \mathfrak{u}_A(w), \\ \tanh\left(s_0 c_{17}\max\{d(v), d(w)\}\begin{pmatrix} \text{boolean}(u(v,w) = -1) \\ +\text{boolean}(a(v,w) = -1) \end{pmatrix}\right) \end{matrix}\right\}$$

(v) $\mathfrak{f}_B(v,w) = \max\left\{\begin{matrix} \mathfrak{f}_A(v), \mathfrak{f}_A(w), \\ \tanh\left(s_0 c_{18} \max\{d(v), d(w)\}\left(s_2 l(v,w) + \frac{u(v,w)}{s_2}\right)\right) \end{matrix}\right\}$

When *any* of the adjacent towns are deemed dangerous or the road itself is deemed too long and too congested, then the riskier and the less efficient it is to travel along that road.

In particular, $\{c_i | 1 \le i \le 18\} \in \mathbb{R}_0^+$ denotes the relative sensitivities among all the inputs, $\{s_1, s_2\} \in \mathbb{R}^+$ are chosen depending on the traffic pattern (e.g., the usual speed of travel as determined by the speed limit) for a given road. Thus, a higher value of the corresponding c_i must be chosen by the user if more attention is to be given for an input.

In Sects. 4.4 and 4.5, three specific examples of adopting the fuzzification procedure will be discussed, whereby each is assigned a different $\{c_i | 1 \le i \le 18\}$ to suit their respective needs.

4.4 Illustrative example 1: choosing a path to travel

Suppose we have three groups of people who are currently situated at town v_1. They would like to travel from town v_1 to town v_6. The three groups of people are: Riders from a delivery company who need to deliver goods bought online to customers, a medical team which deals with COVID-19 patients, and a family on a trip. Therefore, each team must choose the safest and most efficient path to travel from v_1 to v_6.

For the delivery company, the truth and false membership functions carry greater importance than the rest as the delivery team only stays on the path for a very short period as they are tasked to deliver their goods within a stipulated amount of time. Hence, the delivery team cannot afford to give much attention to contradicting or absent information and must make a quick decision based on whatever information that is certain. On the other hand, a delivery company must pay the utmost attention to the road length and the traffic conditions. Thus, in this example, let the choice of fuzzification function and the score function for the delivery team be defined as follows:

$$\varphi_{\mathrm{d}}(n(v), m(v), p(v), q(v), s(v), d(v)) = \left(\mathfrak{t}_{A_{\mathrm{d}}}(v), \mathfrak{c}_{A_{\mathrm{d}}}(v), \mathfrak{g}_{A_{\mathrm{d}}}(v), \mathfrak{u}_{A_{\mathrm{d}}}(v), \mathfrak{f}_{A_{\mathrm{d}}}(v)\right),$$

where

$$\mathfrak{t}_{A_{\mathrm{d}}}(v) = 1 - \tanh\left(\frac{s_0 d(v) \max\{n(v), 0\}}{4000}\right),$$

$$\mathfrak{c}_{A_{\mathrm{d}}}(v) = \min\left\{\begin{matrix} \left|\tanh\left(\frac{s_0 d(v)\max\{n(v),0\}}{1000000}\right) - \tanh\left(\frac{s_0 d(v)\max\{m(v),0\}}{1000000}\right)\right| \\ +\tanh\left(\frac{s_0 \max\{s(v),0\}}{1000000}\right), \\ 1 \end{matrix}\right\},$$

$$\mathfrak{g}_{A_{\mathrm{d}}}(v) = \tanh\left(\frac{s_0 d(v)}{1000000}\begin{pmatrix} \text{boolean}(n(v) = -2) \\ +\text{boolean}(m(v) = -2) \\ +\text{boolean}(p(v) = -2) \\ +\text{boolean}(q(v) = -2) \\ +\text{boolean}(s(v) = -2) \end{pmatrix}\right),$$

$$\mathfrak{u}_{A_\mathrm{d}}(v)=\tanh\left(\frac{s_0 d(v)}{1000000}\begin{pmatrix}\text{boolean}(n(v)=-1)\\+\text{boolean}(m(v)=-1)\\+\text{boolean}(p(v)=-1)\\+\text{boolean}(q(v)=-1)\\+\text{boolean}(s(v)=-1)\end{pmatrix}\right), \text{ and}$$

$$\mathfrak{f}_{A_\mathrm{d}}(v)=\tanh\left(\frac{s_0 d(v)}{4000}\max\{n(v),0\}\begin{pmatrix}\frac{\max\{m(v)+10,0\}}{2}\\+\frac{(\max\{p(v),0\}+\max\{10s(v),0\})}{50}\\+\frac{(\max\{q(v),0\}+\max\{10s(v),0\})}{200}\end{pmatrix}\right);$$

whereas

$$\psi_\mathrm{d}\begin{pmatrix}n(v),m(v),p(v),q(v),s(v),d(v),\\n(w),m(w),p(w),q(w),s(w),d(w),\\l(v,w),u(v,w),a(v,w)\end{pmatrix}$$
$$=\left(\mathfrak{t}_{B_\mathrm{d}}(v,w),\mathfrak{c}_{B_\mathrm{d}}(v,w),\mathfrak{g}_{B_\mathrm{d}}(v,w),\mathfrak{u}_{B_\mathrm{d}}(v,w),\mathfrak{f}_{B_\mathrm{d}}(v,w)\right),$$

where

$$\mathfrak{t}_{B_\mathrm{d}}(v,w)=\min\left\{\mathfrak{t}_{A_\mathrm{d}}(v),\mathfrak{t}_{A_\mathrm{d}}(w),\tanh\left(s_0\left(\frac{\min\{d(v),d(w)\}}{40}\right)(a(v,w))\right)\right\},$$

$$\mathfrak{c}_{B_\mathrm{d}}(v,w)=\max\left\{\begin{array}{c}\mathfrak{c}_{A_\mathrm{d}}(v),\mathfrak{c}_{A_\mathrm{d}}(w),\\\tanh\left(\begin{array}{c}s_0\left(\frac{\max\{d(v),d(w)\}}{1000000}\right)\times\\\left|\frac{\max\{a(v,w),0\}\max\{u(v,w),0\}}{300}+\frac{300}{\max\{a(v,w),0\}\max\{u(v,w),0\}}-2\right|\end{array}\right)\end{array}\right\},$$

$$\mathfrak{g}_{B_\mathrm{d}}(v,w)=\max\left\{\begin{array}{c}\mathfrak{g}_{A_\mathrm{d}}(v),\mathfrak{g}_{A_\mathrm{d}}(w),\\\tanh\left(s_0\left(\frac{\max\{d(v),d(w)\}}{1000000}\right)\begin{pmatrix}\text{boolean}(u(v,w)=-2)\\+\text{boolean}(a(v,w)=-2)\end{pmatrix}\right)\end{array}\right\},$$

$$\mathfrak{u}_{B_\mathrm{d}}(v,w)=\max\left\{\begin{array}{c}\mathfrak{u}_{A_\mathrm{d}}(v),\mathfrak{u}_{A_\mathrm{d}}(w),\\\tanh\left(s_0\left(\frac{\max\{d(v),d(w)\}}{1000000}\right)\begin{pmatrix}\text{boolean}(u(v,w)=-1)\\+\text{boolean}(a(v,w)=-1)\end{pmatrix}\right)\end{array}\right\},$$

$$\mathfrak{f}_{B_\mathrm{d}}(v,w)$$
$$=\max\left\{\mathfrak{f}_{A_\mathrm{d}}(v),\mathfrak{f}_{A_\mathrm{d}}(w),\tanh\left(s_0\left(\frac{\max\{d(v),d(w)\}}{10}\right)(l(v,w)+\max\{u(v,w),0\})\right)\right\},$$

and with the score function

$$\sigma_\mathrm{d}(\mathfrak{t},\mathfrak{c},\mathfrak{g},\mathfrak{u},\mathfrak{f})=2\mathfrak{t}-\mathfrak{c}-\mathfrak{g}-\mathfrak{u}-2\mathfrak{f}.$$

Thus, a relatively low sensitivity of $\frac{1}{1000000}$ is assigned for $\{c_2,c_3,c_4,c_5,c_6,c_{15},c_{16},c_{17}\}$ compared to the other elements in the set $\{c_i|1\le i\le 18\}$, which results in a constantly near-zero values for $\left\{\mathfrak{c}_{A_\mathrm{d}}(v),\mathfrak{g}_{A_\mathrm{d}}(v),\mathfrak{u}_{A_\mathrm{d}}(v),\mathfrak{c}_{B_\mathrm{d}}(v),\mathfrak{g}_{B_\mathrm{d}}(v),\mathfrak{u}_{B_\mathrm{d}}(v)\right\}$ as the contradiction and absence of information are deemed not very important for quick delivery. On the other hand, a relatively high sensitivity value of $\frac{1}{10}$ is assigned for c_{18} compared to the other elements in $\{c_i|1\le i\le 18\}$. This is because the traffic conditions and the length of travel are deemed very important in this case.

For the medical team, the contradiction, ignorance, and the unknown membership functions carry much more importance compared to the others, whether in a town or on a road. This is because, when there is absence or contradiction of information, it is difficult for the medical team to make their decisions. On the other hand, the truth and false membership functions would carry lesser weight because the medical team can handle a known situation very well with their knowledge and medical skills, although a safer and more efficient route will still be favorable. Moreover, the medical team are well prepared even when the reported number of Covid-19 cases are high. This results in much less attention given to the reported number of Covid-19 cases and ICU occupancy upon the fuzzification. For the route of travel, the medical team may give less attention as they will be given priority. Thus, in this example, let the choice of fuzzification function and the score function for the medical team be defined as follows:

$$\varphi_{\mathrm{m}}(n(v), m(v), p(v), q(v), s(v), d(v)) = \left(\mathfrak{t}_{A_{\mathrm{m}}}(v), \mathfrak{c}_{A_{\mathrm{m}}}(v), \mathfrak{g}_{A_{\mathrm{m}}}(v), \mathfrak{u}_{A_{\mathrm{m}}}(v), \mathfrak{f}_{A_{\mathrm{m}}}(v)\right),$$

where

$$\mathfrak{t}_{A_{\mathrm{m}}}(v) = 1 - \tanh\left(\frac{s_0 d(v)\max\{n(v), 0\}}{1000000}\right),$$

$$\mathfrak{c}_{A_{\mathrm{m}}}(v) = \min\left\{\begin{array}{c} \left|\tanh\left(\frac{s_0 d(v)\max\{n(v),0\}}{1000}\right) - \tanh\left(\frac{s_0 d(v)\max\{m(v),0\}}{1000}\right)\right| \\ +\tanh\left(\frac{s_0 \max\{s(v),0\}}{10}\right), \\ 1 \end{array}\right\},$$

$$\mathfrak{g}_{A_{\mathrm{m}}}(v) = \tanh\left(\frac{s_0 d(v)}{10}\left(\begin{array}{c} \mathrm{boolean}(n(v) = -2) \\ +\mathrm{boolean}(m(v) = -2) \\ +\mathrm{boolean}(p(v) = -2) \\ +\mathrm{boolean}(q(v) = -2) \\ +\mathrm{boolean}(s(v) = -2) \end{array}\right)\right),$$

$$\mathfrak{u}_{A_{\mathrm{m}}}(v) = \tanh\left(\frac{s_0 d(v)}{10}\left(\begin{array}{c} \mathrm{boolean}(n(v) = -1) \\ +\mathrm{boolean}(m(v) = -1) \\ +\mathrm{boolean}(p(v) = -1) \\ +\mathrm{boolean}(q(v) = -1) \\ +\mathrm{boolean}(s(v) = -1) \end{array}\right)\right),$$

$$\mathfrak{f}_{A_{\mathrm{m}}}(v) = \tanh\left(\frac{s_0 d(v)}{1000000}\max\{n(v), 0\}\left(\begin{array}{c} \frac{\max\{m(v)+10,0\}}{1000000} \\ +\frac{(\max\{p(v),0\}+\max\{10s(v),0\})}{50} \\ +\frac{(\max\{q(v),0\}+\max\{10s(v),0\})}{200} \end{array}\right)\right),$$

whereas

$$\begin{aligned} &\psi_{\mathrm{m}}\left(\begin{array}{c} n(v), m(v), p(v), q(v), s(v), d(v), \\ n(w), m(w), p(w), q(w), s(w), d(w), \\ l(v, w), u(v, w), a(v, w) \end{array}\right) \\ &= \left(\mathfrak{t}_{B_{\mathrm{m}}}(v, w), \mathfrak{c}_{B_{\mathrm{m}}}(v, w), \mathfrak{g}_{B_{\mathrm{m}}}(v, w), \mathfrak{u}_{B_{\mathrm{m}}}(v, w), \mathfrak{f}_{B_{\mathrm{m}}}(v, w)\right), \end{aligned}$$

where

$$\mathfrak{t}_{B_{\mathrm{m}}}(v, w) = \min\left\{\mathfrak{t}_{A_{\mathrm{m}}}(v), \mathfrak{t}_{A_{\mathrm{m}}}(w), \tanh\left(s_0\left(\frac{\min\{d(v), d(w)\}}{10000}\right)a(v, w)\right)\right\},$$

$$\mathfrak{c}_{B_{\mathrm{m}}}(v, w) = \max\left\{\begin{array}{c} \mathfrak{c}_{A_{\mathrm{m}}}(v), \mathfrak{c}_{A_{\mathrm{m}}}(w), \\ \tanh\left(s_0\left(\frac{\max\{d(v),d(w)\}}{10000}\right)\left|\frac{a(v,w)u(v,w)}{300} + \frac{300}{a(v,w)u(v,w)} - 2\right|\right) \end{array}\right\},$$

$$\mathfrak{g}_{B_{\mathrm{m}}}(v,w)=\max\left\{\begin{array}{c}\mathfrak{g}_{A_{\mathrm{m}}}(v),\mathfrak{g}_{A_{\mathrm{m}}}(w),\\ \tanh\left(s_0\left(\frac{\max\{d(v),d(w)\}}{1000}\right)\left(\begin{array}{c}\text{boolean}(u(v,w)=-2)\\ +\text{boolean}(a(v,w)=-2)\end{array}\right)\right)\end{array}\right\},$$

$$\mathfrak{u}_{B_{\mathrm{m}}}(v,w)=\max\left\{\begin{array}{c}\mathfrak{u}_{A_{\mathrm{m}}}(v),\mathfrak{u}_{A_{\mathrm{m}}}(w),\\ \tanh\left(s_0\left(\frac{\max\{d(v),d(w)\}}{1000}\right)\left(\begin{array}{c}\text{boolean}(u(v,w)=-1)\\ +\text{boolean}(a(v,w)=-1)\end{array}\right)\right)\end{array}\right\},$$

$$\mathfrak{f}_{B_{\mathrm{m}}}(v,w)=\max\left\{\mathfrak{f}_{A_{\mathrm{m}}}(v),\mathfrak{f}_{A_{\mathrm{m}}}(w),\tanh\left(s_0\left(\frac{\max\{d(v),d(w)\}}{50000}\right)(l(v,w)+u(v,w))\right)\right\};$$

and with the score function $\sigma_{\mathrm{m}}(\mathfrak{t},\mathfrak{c},\mathfrak{g},\mathfrak{u},\mathfrak{f})=\mathfrak{t}-2\mathfrak{c}-2\mathfrak{g}-2\mathfrak{u}-\mathfrak{f}$.

For the family on a trip, the false membership function carries greater importance, followed by the contradiction and ignorance membership functions. Moreover, the family on a trip will be very mindful of the reported number of COVID-19 cases. This results in much more attention given to the reported number of COVID-19 cases and ICU occupancy upon the fuzzification. Thus, in this example, let the choice of fuzzification function and the score function for the family on a trip to be defined as follows:

$$\varphi_{\mathrm{z}}(n(v),m(v),p(v),q(v),s(v),d(v))=\left(\mathfrak{t}_{A_{\mathrm{z}}}(v),\mathfrak{c}_{A_{\mathrm{z}}}(v),\mathfrak{g}_{A_{\mathrm{z}}}(v),\mathfrak{u}_{A_{\mathrm{z}}}(v),\mathfrak{f}_{A_{\mathrm{z}}}(v)\right),$$

where

$$\mathfrak{t}_{A_{\mathrm{z}}}(v)=1-\tanh\left(\frac{s_0 d(v)\max\{n(v),0\}}{10}\right),$$

$$\mathfrak{c}_{A_{\mathrm{z}}}(v)=\min\left\{\begin{array}{c}\left|\tanh\left(\frac{s_0 d(v)\max\{n(v),0\}}{200}\right)-\tanh\left(\frac{s_0 d(v)\max\{m(v),0\}}{40}\right)\right|\\ +\tanh\left(\frac{s_0\max\{s(v),0\}}{100}\right),\\ 1\end{array}\right\},$$

$$\mathfrak{g}_{A_{\mathrm{z}}}(v)=\tanh\left(\frac{s_0 d(v)}{1000}\left(\begin{array}{c}\text{boolean}(n(v)=-2)\\ +\text{boolean}(m(v)=-2)\\ +\text{boolean}(p(v)=-2)\\ +\text{boolean}(q(v)=-2)\\ +\text{boolean}(s(v)=-2)\end{array}\right)\right),$$

$$\mathfrak{u}_{A_{\mathrm{z}}}(v)=\tanh\left(\frac{s_0 d(v)}{1000}\left(\begin{array}{c}\text{boolean}(n(v)=-1)\\ +\text{boolean}(m(v)=-1)\\ +\text{boolean}(p(v)=-1)\\ +\text{boolean}(q(v)=-1)\\ +\text{boolean}(s(v)=-1)\end{array}\right)\right),$$

$$\mathfrak{f}_{A_{\mathrm{z}}}(v)=\tanh\left(\frac{s_0 d(v)}{40}\max\{n(v),0\}\left(\begin{array}{c}\frac{\max\{m(v)+10,0\}}{2}\\ +\frac{(\max\{p(v),0\}+\max\{10s(v),0\})}{50}\\ +\frac{(\max\{q(v),0\}+\max\{10s(v),0\})}{200}\end{array}\right)\right),$$

whereas

$$\psi_{\mathrm{z}}\left(\begin{array}{c}n(v),m(v),p(v),q(v),s(v),d(v),\\ n(w),m(w),p(w),q(w),s(w),d(w),\\ l(v,w),u(v,w),a(v,w)\end{array}\right)$$
$$=\left(\mathfrak{t}_{B_{\mathrm{z}}}(v,w),\mathfrak{c}_{B_{\mathrm{z}}}(v,w),\mathfrak{g}_{B_{\mathrm{z}}}(v,w),\mathfrak{u}_{B_{\mathrm{z}}}(v,w),\mathfrak{f}_{B_{\mathrm{z}}}(v,w)\right),$$

where

$$\mathfrak{t}_{B_z}(v,w)=\min\left\{\mathfrak{t}_{A_z}(v),\,\mathfrak{t}_{A_z}(w),\,\tanh\left(s_0\left(\frac{\min\{d(v),d(w)\}}{4000}\right)a(v,w)\right)\right\},$$

$$\mathfrak{c}_{B_z}(v,w)=\max\left\{\begin{array}{c}\mathfrak{c}_{A_z}(v),\,\mathfrak{c}_{A_z}(w),\\ \tanh\left(s_0\left(\frac{\max\{d(v),d(w)\}}{4000}\right)\left|\frac{a(v,w)u(v,w)}{300}+\frac{300}{a(v,w)u(v,w)}-2\right|\right)\end{array}\right\},$$

$$\mathfrak{g}_{B_z}(v,w)=\max\left\{\begin{array}{c}\mathfrak{g}_{A_z}(v),\,\mathfrak{g}_{A_z}(w),\\ \tanh\left(s_0\left(\frac{\max\{d(v),d(w)\}}{200}\right)\left(\begin{array}{c}\text{boolean}(u(v,w)=-2)\\ +\text{boolean}(a(v,w)=-2)\end{array}\right)\right)\end{array}\right\},$$

$$\mathfrak{u}_{B_z}(v,w)=\max\left\{\begin{array}{c}\mathfrak{u}_{A_z}(v),\,\mathfrak{u}_{A_z}(w),\\ \tanh\left(s_0\left(\frac{\max\{d(v),d(w)\}}{200}\right)\left(\begin{array}{c}\text{boolean}(u(v,w)=-1)\\ +\text{boolean}(a(v,w)=-1)\end{array}\right)\right)\end{array}\right\},$$

$$\mathfrak{f}_{B_z}(v,w)=\max\left\{\mathfrak{f}_{A_z}(v),\,\mathfrak{f}_{A_z}(w),\,\tanh\left(s_0\left(\frac{\max\{d(v),d(w)\}}{40000}\right)(l(v,w)+u(v,w))\right)\right\},$$

and with the score function $\sigma_z(\mathfrak{t},\mathfrak{c},\mathfrak{g},\mathfrak{u},\mathfrak{f})=\mathfrak{t}-\mathfrak{c}-2\mathfrak{g}-2\mathfrak{u}-3\mathfrak{f}$.

Remark: for all the aforementioned examples, it is worth emphasizing that s_0 is to be chosen using the formula given below:

$$\max\left\{\begin{array}{c}\text{mean}(\{\rho(v):v\in V\}\cup\{\varrho(v,w):\{v,w\}\in E\})|\\ (\rho,\varrho)\in\{(\mathfrak{t}_A,\mathfrak{t}_B),(\mathfrak{c}_A,\mathfrak{c}_B),(\mathfrak{g}_A,\mathfrak{g}_B),(\mathfrak{u}_A,\mathfrak{u}_B),(\mathfrak{f}_A,\mathfrak{f}_B)\}\end{array}\right\}\approx 0.5$$

4.4.1 Step 1: assignment of V and E

Firstly, for all the three teams assign $V=\{v_1,v_2,v_3,v_4,v_5,v_6\}$ and $E=\{\{v_1,v_2\},\{v_1,v_3\},\{v_2,v_3\},\{v_2,v_4\},\{v_2,v_5\},\{v_3,v_5\},\{v_4,v_6\},\{v_5,v_6\}\}$.

4.4.2 Step 2: assignment of A and B which leads to the formation of $G=(A,B,V,E)$

For the delivery team, the truth, contradiction, ignorance, unknown and falsity membership functions for A_{d} and B_{d} are calculated using φ_{d} on the raw data. The values of the membership functions and the safety of travelling between adjacent towns that are yielded are given in Tables 3 and 4, respectively. In this example, $s_0=0.0000244291178987957$ is chosen by

Table 3 Values of the membership functions of A_{d} for the delivery team

v	$\mathfrak{t}_{A_{\mathrm{d}}}(v)$	$\mathfrak{c}_{A_{\mathrm{d}}}(v)$	$\mathfrak{g}_{A_{\mathrm{d}}}(v)$	$\mathfrak{u}_{A_{\mathrm{d}}}(v)$	$\mathfrak{f}_{A_{\mathrm{d}}}(v)$
v_1	1.00000	0.00000	0.00000	0.00000	0.00000
v_2	1.00000	0.00001	0.00000	0.00000	0.00000
v_3	0.99890	0.00000	0.00000	0.00000	0.00720
v_4	0.98774	0.00002	0.00000	0.00000	0.72138
v_5	0.99988	0.00000	0.00000	0.00000	0.00205
v_6	0.99983	0.00000	0.00000	0.00000	0.00333

an iterative program.

Likewise, for the medical team, the truth, contradiction, ignorance, unknown and falsity membership functions of A_m and B_m are calculated using φ_m on the raw data. The values of the membership functions and the safety of travelling between adjacent towns that are yielded are given in Tables 5 and 6, respectively. In this example, $s_0 = 0.00156005814758476$ is chosen by an iterative program.

Table 4 Values of the membership functions of B_d for the delivery team

v, w	$\mathfrak{t}_{B_\mathrm{d}}(v, w)$	$\mathfrak{c}_{B_\mathrm{d}}(v, w)$	$\mathfrak{g}_{B_\mathrm{d}}(v, w)$	$\mathfrak{u}_{B_\mathrm{d}}(v, w)$	$\mathfrak{f}_{B_\mathrm{d}}(v, w)$
v_1, v_2	0.15299	0.00001	0.00000	0.00000	0.16016
v_1, v_3	0.17799	0.00000	0.00000	0.00000	0.62569
v_2, v_3	0.18154	0.00001	0.00000	0.00000	0.43816
v_2, v_4	0.14581	0.00002	0.00000	0.00000	0.98395
v_2, v_5	0.00000	1.00000	0.00000	0.00000	0.42622
v_3, v_5	0.01224	0.00000	0.00000	0.00000	0.74186
v_4, v_6	0.21096	0.00002	0.00000	0.00000	0.95702
v_5, v_6	0.14581	0.00000	0.00000	0.00000	0.29899

Table 5 Values of the membership functions of A_m for the medical team

v	$\mathfrak{t}_{A_\mathrm{m}}(v)$	$\mathfrak{c}_{A_\mathrm{m}}(v)$	$\mathfrak{g}_{A_\mathrm{m}}(v)$	$\mathfrak{u}_{A_\mathrm{m}}(v)$	$\mathfrak{f}_{A_\mathrm{m}}(v)$
v_1	1.00000	0.00000	0.00000	0.57585	0.00000
v_2	1.00000	0.63514	0.73406	0.00000	0.00000
v_3	0.99972	0.27437	0.73406	0.00000	0.00071
v_4	0.99687	0.06226	0.00000	0.00000	0.07112
v_5	0.99997	0.11153	0.55459	0.55459	0.00001
v_6	0.99996	0.23338	0.00000	0.00000	0.00006

Table 6 Values of the membership functions of B_m for the medical team

v, w	$\mathfrak{t}_{B_\mathrm{m}}(v, w)$	$\mathfrak{c}_{B_\mathrm{m}}(v, w)$	$\mathfrak{g}_{B_\mathrm{m}}(v, w)$	$\mathfrak{u}_{B_\mathrm{m}}(v, w)$	$\mathfrak{f}_{B_\mathrm{m}}(v, w)$
v_1, v_2	0.00039	0.63514	0.73406	0.57585	0.00010
v_1, v_3	0.00046	0.91888	0.73406	0.57585	0.00071
v_2, v_3	0.00047	0.63514	0.73406	0.00000	0.00071
v_2, v_4	0.00037	0.99911	0.73406	0.00000	0.07112
v_2, v_5	0.00000	1.00000	0.73406	0.55459	0.00029
v_3, v_5	0.00003	0.27437	0.73406	0.55459	0.00071
v_4, v_6	0.00055	0.99813	0.00000	0.00000	0.07112
v_5, v_6	0.00037	0.23338	0.55459	0.55459	0.00020

Table 7 Values of the membership functions of A_Z for the family on a trip

v	$\mathfrak{t}_{A_z}(v)$	$\mathfrak{c}_{A_z}(v)$	$\mathfrak{g}_{A_z}(v)$	$\mathfrak{u}_{A_z}(v)$	$\mathfrak{f}_{A_z}(v)$
v_1	1.00000	0.00000	0.00000	0.00012	0.00000
v_2	1.00000	0.33586	0.00017	0.00000	0.00000
v_3	0.51912	0.02620	0.00017	0.00000	0.69434
v_4	0.00002	0.38716	0.00000	0.00000	1.00000
v_5	0.94183	0.06396	0.00012	0.00012	0.23866
v_6	0.91865	0.12764	0.00000	0.00000	0.37690

Table 8 Values of the membership functions of B_Z for the family on a trip

v, w	$\mathfrak{t}_{B_z}(v, w)$	$\mathfrak{c}_{B_z}(v, w)$	$\mathfrak{g}_{B_z}(v, w)$	$\mathfrak{u}_{B_z}(v, w)$	$\mathfrak{f}_{B_z}(v, w)$
v_1, v_2	0.00183	0.33586	0.00017	0.00012	0.00005
v_1, v_3	0.00214	0.02620	0.00017	0.00012	0.69434
v_2, v_3	0.00218	0.33586	0.00017	0.00000	0.69434
v_2, v_4	0.00002	0.38716	0.00017	0.00000	1.00000
v_2, v_5	0.00000	1.00000	0.00017	0.00175	0.23866
v_3, v_5	0.00015	0.06396	0.00017	0.00012	0.69434
v_4, v_6	0.00002	0.38716	0.00000	0.00000	1.00000
v_5, v_6	0.00175	0.12764	0.00012	0.00012	0.37690

For the family on a trip, the truth, contradiction, ignorance, unknown and falsity membership functions of A_Z and B_Z are calculated using φ_Z on the raw data. The values of the membership functions and the safety of travelling between adjacent towns that are yielded are given in Tables 7 and 8, respectively. In this example, $s_0 = 0.0000290825859068550$ is chosen by an iterative program.

We now form three PPNGs, namely $G_d = (A_d, B_d, V, E)$, $G_m = (A_m, B_m, V, E)$ and $G_z = (A_z, B_z, V, E)$ for the delivery team, the medical team, and the family on a trip, respectively.

4.4.3 Step 3: determining all the paths

In travelling from v_1 to v_6, all the possible paths are as follows:

$$P_1 = (v_1, v_2, v_4, v_6),\ P_2 = (v_1, v_2, v_5, v_6),$$
$$P_3 = (v_1, v_3, v_2, v_4, v_6),\ P_4 = (v_1, v_3, v_2, v_5, v_6),$$
$$P_5 = (v_1, v_2, v_3, v_5, v_6),\ P_6 = (v_1, v_3, v_5, v_2, v_4, v_6),\ P_7 = (v_1, v_3, v_5, v_6).$$

4.4.4 Step 4: calculating the score function for all the paths

The strength of each of the 7 paths will be computed from their corresponding PPNGs for each of the 3 teams, after which their own choices of score functions will be applied and the score function will then be determined. The score functions for all the 3 teams are given in Tables 9, 10 and 11, respectively.

Table 9 The strength and score functions for all the paths for the delivery team

P	$\mathrm{s_d}(P)$	$\sigma_\mathrm{d}(\mathrm{s_d}(P))$
P_1	(0.14581, 0.00001, 0.00000, 0.00000, 0.16016)	**– 0.02870**
P_2	(0.00000, 0.00000, 0.00000, 0.00000, 0.16016)	– 0.32032
P_3	(0.14581, 0.00000, 0.00000, 0.00000, 0.43816)	– 0.58470
P_4	(0.00000, 0.00000, 0.00000, 0.00000, 0.29899)	– 0.59798
P_5	(0.01224, 0.00000, 0.00000, 0.00000, 0.16016)	– 0.29584
P_6	(0.00000, 0.00000, 0.00000, 0.00000, 0.42622)	– 0.85245
P_7	(0.01224, 0.00000, 0.00000, 0.00000, 0.29899)	– 0.57351

Table 10 The strength and score functions for all the paths for the medical team

P	$\mathrm{s_m}(P)$	$\sigma_\mathrm{m}(\mathrm{s_m}(P))$
P_1	(0.00037,0.63514,0.00000,0.00000,0.00010)	– 1.27001
P_2	(0.00000,0.23338,0.55459,0.55459,0.00010)	– 2.68523
P_3	(0.00037,0.63514,0.00000,0.00000,0.00071)	– 1.27062
P_4	(0.00000,0.23338,0.55459,0.00000,0.00020)	– 1.57614
P_5	(0.00003,0.23338,0.55459,0.00000,0.00010)	– 1.57601
P_6	(0.00000,0.27437,0.00000,0.00000,0.00029)	**– 0.54903**
P_7	(0.00003,0.23338,0.55459,0.55459,0.00020)	– 2.68529

Table 11 The strength and score functions for all the paths for the family on a trip

P	$\mathrm{s_z}(P)$	$\sigma_\mathrm{z}(\mathrm{s_z}(P))$
P_1	(0.00002, 0.33586, 0.00000, 0.00000, 0.00005)	– 0.33599
P_2	(0.00000, 0.12764, 0.00012, 0.00012, 0.00005)	– 0.12825
P_3	(0.00002, 0.02620, 0.00000, 0.00000, 0.69434)	– 2.10921
P_4	(0.00000, 0.02620, 0.00012, 0.00000, 0.23866)	– 0.74242
P_5	(0.00015, 0.06396, 0.00012, 0.00000, 0.00005)	**– 0.06419**
P_6	(0.00000, 0.02620, 0.00000, 0.00000, 0.23866)	– 0.74218
P_7	(0.00015, 0.02620, 0.00012, 0.00012, 0.37690)	– 1.15722

4.4.5 Step 5 Conclusion on the best path to travel

Based on the score functions obtained in Step 4 above, the best path for the delivery team to travel along would be path $P_1 = (v_1, v_2, v_4, v_6)$, the best path for the medical team to travel along would be path $P_6 = (v_1, v_3, v_5, v_2, v_4, v_6)$, whereas the best path for the family on a trip to travel along would be path $P_5 = (v_1, v_2, v_3, v_5, v_6)$, in travelling from town v_1 to town v_6.

It is worth noting that the concept outlined in Definition 3.12 proves to be a much more appropriate model for this scenario compared to the mere multiplication of the membership functions of all the edges in a path. This is because the mere multiplication of values is always done under the assumption that all the events are independent which bares opposite resemblance with a real-life pandemic outbreak where an outbreak at a given place will potentially affect the safety of an entire path or even the whole town, thereby increasing the likelihood of occurrence of similar outbreaks.

4.5 Illustrative example 2: choosing a town to stay

Now, suppose the same three groups of people discussed in Sect. 4.3 wish to find a town to rest temporarily on their journey from town v_1 to town v_6 along the safest paths that were determined in Sect. 4.4.

4.5.1 Step 1: determination of vertices

Recall that in Sect. 4.4, the safest path to travel for the 3 teams are as given below:

Delivery team: path $P_1 = (v_1, v_2, v_4, v_6)$.

Medical team: path $P_6 = (v_1, v_3, v_5, v_2, v_4, v_6)$.

Family on a trip: path $P_5 = (v_1, v_2, v_3, v_5, v_6)$.

Therefore, the delivery team will choose the safest town out of $W_\mathrm{d} = \{v_2, v_4\}$, the medical team will choose the safest town out of $W_\mathrm{m} = \{v_3, v_5, v_2, v_4\}$, while the family on a trip will choose the safest town out of $W_\mathrm{z} = \{v_2, v_3, v_5\}$.

4.5.2 Step 2: calculating the score function on all the paths

Similar to Sect. 4.4.4, each team will apply their own choices of score functions on their own sets of towns. The values of the score functions for the safest town to stay temporarily for the 3 teams are given in Table 12.

4.5.3 Step 5: conclusion on the safest town to stay temporarily

Based on the values of the score functions obtained in Table 12, it can be concluded that the safest town for a temporary stay while travelling from town v_1 to town v_6 would be town v_4 for the medical team, and town v_2 for both the delivery team and the family on a trip.

5 Comparison with the previous works

Our PPNG model enables us to deal with 5 mutually distinct kinds of membership: truth, contradiction, ignorance, unknown, and falsity. Throughout our observation in the literature,

Table 12 The score functions for the safest town to stay temporarily in for the three teams

v	$\sigma_{\mathrm{d}}(v)$ for $v \in W_{\mathrm{d}}$	$\sigma_{\mathrm{m}}(v)$ for $v \in W_{\mathrm{m}}$	$\sigma_{\mathrm{z}}(v)$ for $v \in W_{\mathrm{z}}$
v_1	–	–	–
v_2	**1.99999**	− 1.73841	**0.66379**
v_3	–	− 1.01786	− 1.59045
v_4	0.53269	**0.80124**	–
v_5	–	− 1.44146	0.16142
v_6	–	–	–

there is yet to be any literature that consider the theory of PPNG as well as the practical applications of PPNG in real-life events. Therefore, all of the reputable works in literature that were observed have only 4 (mostly 3) distinct kinds of membership at most. Besides the advantage of having more types of membership functions, our work deploys a novel and dedicated, customized fuzzification procedure that was custom-made for the scenario. The function of every component of the formulas used in the fuzzification procedure has been justified in Sect. 4.3. Moreover, our fuzzification procedure grants the user an unprecedentedly great range of possibilities in adopting the model to suit their individual needs.

In contrast, all recent works encountered in literature were observed to directly start with the fuzzified input. This defeats the purpose of the demonstration of the application as it is a well-known fact that no data in real-life comes in the form of a fuzzy set or any of its derivatives, including the PPNGs proposed in this study. As a result, most (if not all) of the models introduced in the previous studies are general ones that were not specifically configured to deal with a specific scenario.

6 Conclusions

The concluding remarks and the significant contributions that were made by the work in this paper are summarized below:

(i) The pentapartitioned neutrosophic set is a generalization of the quadripartitioned neutrosophic sets and SVNS models. A novel concept called the pentapartitioned neutrosophic graph (PPNG) is defined and its fundamental properties are presented. Some of the important concepts related to the PPNGs such as the path, connectedness and tree of PPNGs are also defined, and the graphical representation of these concepts were presented alongside some examples.

(ii) Our proposed PPNG holds the distinction of being able to represent different aspects of the data such as independent entities and process them accordingly as it is able to represent the truth, contradiction, ignorance, unknown and falsity aspects of the data which makes it highly suitable to handle incomplete, imprecise and uncertain data.

(iii) The proposed concepts were applied to a real-life scenario pertaining to the ongoing Covid-19 pandemic.

(iv) The applicability of the proposed concepts was demonstrated by applying them to a real-life scenario pertaining to the on-going Covid-19 pandemic. In this example, the proposed PPNGs were used to determine the safest path of travel and the safest place to stay in order to minimize the chances of getting infected by Covid-19. These parameters have been proven to be vital aspects in the efforts to combat the spread of the Covid-19 pandemic while providing the necessary support to the domestic economies, most of which are currently in recession due to the adverse effects brought upon by the pandemic. It was proven that the novel concept of PPNGs that were introduced here can be used under any circumstances such as pandemics, emergencies or even in regular situations to optimize the travelling time and distance.

The future research directions with regards to this study are expounded below:

(i) To develop a full-fledged AI algorithm which a words extraction feature that would enable the system to take into consideration non-numerical data in addition to numerical data such as coordinates and distances. Such an AI algorithm will enable us to generate a PPNG using all sources of data gathered on the towns and roads, without having to rely on human intervention.
(ii) This improved PPNG will potentially provide a much clearer picture for the safety of an area, which has proven to be essential to the global population as Covid-19 lockdowns cannot be continuously implemented to the point of affecting the global economy.
(iii) The AI system developed in (i) would be further improved and expanded based on the concept of PPNG and applied to develop more efficient global transport navigation systems.

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Declarations

Conflict of interest The authors declare that there is no conflict of interests. This research does not involve any human or animal activities. All authors have checked and agreed the submission.

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