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Location selection of electric vehicles charging stations by using the spherical fuzzy CPT–CoCoSo and D-CRITIC method

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Abstract
Location selection of electric vehicle charging stations (LSEVCS) is a complex multi-attribute group decision-making (MAGDM) problem involving multiple experts and multiple conflicting attributes. Spherical fuzzy sets (SFSs) can deeply excavate fuzziness and uncertainty in MAGDM. In this paper, we first propose some new spherical fuzzy distance measures based on Dice and Jaccard indexes to detect the differences between SFSs or inputs. Secondly, considering risk preferences of decision makers, we integrate cumulative prospect theory (CPT) and combined compromise solutions (CoCoSo) method to develop a spherical fuzzy CoCoSo based on CPT (SF-CPT–CoCoSo) model for settling MAGDM issues. At the same time, we extend the improved CRiteria Importance Through Intercriteria Correlation (CRITIC) method, called the distance correlation-based CRITIC (D-CRITIC) method, to reasonably obtain unknown attribute weights under SFSs. Finally, this paper applies the developed model for LSEVCS to verify its practicability. Moreover, sensitivity analysis and comparative discussion with existing methods further demonstrate the robustness and effectiveness of the SF-CPT–CoCoSo model.

Keywords Multiple attribute group decision making · Spherical fuzzy sets · CoCoSo method · Cumulative prospect theory · D-CRITIC method · Location of electric vehicle charging stations

Mathematics Subject Classification 90B50 · 91B06

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1 Introduction

In the electric vehicle (EV) industry chain, EV charging station is an indispensable and important part. Reasonable assessment of EV charging stations plays a great role in promoting the market development of EV. As one of the popular MAGDM problems, the study on LSEVCS has become a hot issue in the decision-making field (Liu et al. 2019; Karasan et al. 2020; Wei et al. 2022, 2021; Rani and Mishra 2021; Feng et al. 2021). However, in the face of complex and changeable environment and many uncertainties of human cognitive ability, efficient acquisition of evaluation information is a thorny problem faced by MAGDM, especially the LSEVCS. In 1965, an efficacious tool for dealing with MAGDM, called fuzzy sets (FSs), was initiated by Zadeh (1965), which made up for inadequacy of using crisp numbers in describing the vagueness of things. In the past decades, FSs have facilitated the advancement of various branches and derived many new versions (Frini 2017; Salah and Moselhi 2016; Chen et al. 2014; Zadeh 1975; Abdullah et al. 2014; Tao et al. 2015).

As a new enhanced version of FSs, SFSs were presented by Mahmood et al. (2019), which expressed the uncertainty of things more extensively through four dimensions: membership degree (MD), non-membership degree (N-MD), abstinence degree (AD) as well as rejection degree (RD). Compared with intuitionistic FSs (IFSs) (Atanassov 1986), Pythagorean FSs (PYFSs) (Yager 2014), picture FSs (PFSs) (Cuong 2014), etc., SFSs required that the sum of squares of MD, N-MD and AD could not exceed 1, and provided more flexible scope for the four dimensions. In recent years, the study of SFSs to deal with uncertain problems has attracted many scholars' attentions. Mahmood et al. (2021a) designed similarity measures based on SFSs in settling medical assessment problems. Nguyen et al. (2022) developed a spherical fuzzy multi-attribute decision-making (MADM) method to supplier option of renewable energy wind-driven generator. Ashraf and Abdullah (2020) designed some novel approaches for dealing with COVID-19 in SFSs. Zhang et al. (2022a) developed spherical fuzzy GRA approach with CPT for emergency supply supplier selection. Ashraf et al. (2019a) designed spherical fuzzy distance measure to analyze the environmental factors affecting children's development. Aydogdu and Gul (2020) developed a new entropy proposition of SFSs and its application to MADM. Ayyildiz and Gumus (2020) combined AHP and WASPAS method for location of gas station in spherical fuzzy environment. Zhang et al. (2022b) presented an improved TOPSIS on the basis of CPT under SFSs for residential location. Donyatalab et al. (2020) established linear assignment model to settle MAGDM issues in SFSs. Gul (2020) extended the DEMATEL method to SFSs. Gundogdu (2020) put forward MULTIMOORA method under SFSs. Gundogdu and Kahraman (2019a) developed VIKOR method by using SFSs for warehouse site selection. Jin et al. (2019) came up with logarithmic aggregation operators and entropy under SFSs for decision support systems. Zhang et al. (2022c) presented Dombi power Heronian mean operators based on SFSs for enterprise resource planning systems selection. Wei et al. (2019) utilized cosine function to introduce the similarity measure under SFSs. Khan et al. (2020) proposed spherical fuzzy similarity and DIMs in selecting mega projects.

The CoCoSo method was originally introduced by Yazdani et al. (2019). This method provides a combinatorial decision algorithm based on compromise attitude and aggregation strategy, which can evaluate and select the scheme comprehensively. Because of its accurate calculation and easy operation, the CoCoSo method has been widely used to tackle uncertainty problems. For instance, Turskis et al. (2022) extended the CoCoSo method to m-generalized q-neutrosophic sets for MADM problems. Peng et al. (2022) proposed hesitant fuzzy soft CoCoSo approach for IoE companies' evaluation. Mandal and Khan (2022) established cloud model-based CoCoSo method for trusted cloud service provider selection. Peng and Luo (2021) introduced CoCoSo method with picture fuzzy information to China's stock market bubble alarm. Peng et al. (2020) presented CoCoSo and CRITIC approach for MADM problems in PYFSs. Peng and Smarandache (2020) came up with neutrosophic soft CoCoSo approach for solving evaluation of safety issues. Mi and Liao (2020) introduced the CoCoSo approach by stochastic information for renewable energy investments. However, the above decision-making processes often assume that DMs make judgments under completely rational conditions without considering DMs' risk psychology.

Generally, in the face of many uncertain factors, peoples' judgment and decision-making behavior are not completely rational, but greatly affected by personal preference, risk attitude and so on. Hence, in 1979, Kahneman and Tversky (1979) put forward the prospect theory (PT) on the basis of bounded rationality hypothesis to explain many phenomena that expected utility theory could not. Subsequently, Tversky and Kahneman (1992) modified PT and further proposed CPT. CPT satisfied random dominance, which could well capture DMs' psychological senses and fully reflected the subjective risk appetite of DMs. At present, the research of risk MADM methods based on CPT has become a new hot topic in decision system. For example, Liao et al. (2021) proposed the CPT-MABAC method for MAGDM based on probabilistic hesitant FSs. Zhao et al. (2021) established CPT-TODIM approach for network security service provider selection in bipolar FSs, Jiang et al. (2022) presented picture fuzzy EDAS method based on CPT. Wang et al. (2020) proposed a set emergency decision-making method with CPT in hesitant FSs. Fu et al. (2020) presented improved FMEA approach based on CPT to the railway train risk prioritization. Liao et al. (2020) integrated the CPT and CoCoSo (CPT–CoCoSo) method for cold chain logistics distribution center selection under PYFSs. However, up to now, the research of spherical fuzzy MAGDM method based on CPT is not much. More importantly, the CPT–CoCoSo approach has not been proposed to solve the uncertain issues under SFSs.

For MAGDM problems, attribute weights often have an important influence on results. The objective weight methods obtain weight information by measuring objective relations among attributes in original decision matrix without any subjective judgment of DMs, which can effectively eliminate DMs' personal bias against attribute set and enhance the rationality of weight information. As a common objective weight method, CRITIC (Diakoulaki et al. 1995) method not only characterizes the contrast intensity of each attribute through standard deviation, but also models the conflicting relationships between attributes by Pearson correlation coefficient. In view of these advantages, CRITIC has been applied to many practical problems (Mohamadghasemi et al. 2020; Zafar et al. 2021; Wang et al. 2022; Kahraman et al. 2022).

Nonetheless, Krishnan et al. (2021) found that the original CRITIC method could only capture linear rather than nonlinear relationships between attributes by using Pearson correlation, which may lead to disputes over the validity of the final weight information. Therefore, an improved CRITIC method, called D-CRITIC (Krishnan et al. 2021) method, can overcome the above problems. It replaces Pearson correlation with distance correlation to arbitrarily capture the nonlinear relationship between attributes, making weight information more reasonable. However, as a new and effective objective weight method, D-CRITIC, has not received enough attention. Especially, it has not been extended to fuzzy environment. In addition, Dice distance and Jaccard distance, as the complement of Dice and Jaccard indexes, respectively, can effectively capture the difference or deviation between sets or arguments. They have been extended to some fuzzy environments, such as intuitionistic multiplicative set (Luo et al. 2019), PYFSs (Huang et al. 2020) and interval-valued SFSs (Jin et al. 2021). But there is no relevant research on Dice distance and Jaccard distance under SFSs.

Therefore, stimulated by the above researches, this paper first proposes some Dice and Jaccard DIMs to capture the differences between SFSs or arguments. Secondly, we integrate the DMs' mental behavior into the decision-making process to establish an SF-CPT–CoCoSo model for MAGDM. In the meantime, the D-CRITIC method is extended for the first time to SFSs in obtaining unknown attribute weights. Finally, we use the developed model for LSEVCS to illustrate the applicability of the SF-CPT–CoCoSo model and carry out sensitivity analysis as well as comparative study to demonstrate the robustness, effectiveness and superiority of the proposed model. The proposed model integrates the subjective psychological factors of DMs into the decision-making process, and extends D-CRITIC method by the proposed DIMs to objectively obtain the unknown attribute weight information under SFSs, which makes the decision-making results more in line with the actual needs.

The main motivations of this paper are as follows. (1) Faced with complex and uncertain decision-making environment, SFSs are superior to other FSs (such as IFSs, PYFSs and PFSs) in expressing the universality and depth of things. (2) Dice and Jaccard distances are effective tools for measuring differences between sets or arguments, but they have not been discussed in SFSs. (3) CoCoSo method sorts schemes based on compromise attitude and aggregation strategy. It is a popular method to deal with uncertain problems because of its simple calculation and easy operation. As an effective behavior description model, CPT can fully simulate DMs' psychological behaviors when facing risks. The fusion of CoCoSo method and CPT (CPT–CoCoSo method) can scientifically evaluate and rank schemes under the premise of fully considering DMs' psychological behavior. However, up to now, the CPT–CoCoSo method has not been extended under SFSs to solve uncertain problems. (4) As a modified type of CRITIC method, D-CRITIC method can effectively model the nonlinear relations among attributes via distance correlation to obtain the attribute weights reasonably. However, the application of D-CRITIC approach in fuzzy environment is still a gap. (5) LSEVCS is a hot topic in MAGDM, which has attracted the attention of many scholars. However, most existing methods for LSEVCS often assume that DMs are completely rational and do not consider their psychological behavior characteristics. Following the above reasons, this paper first presents some novel DIMs by Dice and Jaccard indexes. Secondly, SF-CPT–CoCoSo approach is developed for solving MAGDM issues, and D-CRITIC method is extended to obtain attribute weights in SFSs. Finally, the established model is applied to LSEVCS to demonstrate its validity. Furthermore, sensitivity analysis and comparative discussion with existing methods further prove the robustness, effectiveness and superiority of the SF-CPT–CoCoSo model.

The contributions of this article are: (1) to propose some novel spherical fuzzy DIMs, including spherical fuzzy Dice distance, generalized spherical fuzzy Dice distance, spherical fuzzy Jaccard distance, generalized spherical fuzzy Jaccard distance and their weighted forms. (2) To establish an SF-CPT–CoCoSo model in settling MAGDM issues. The established model employs SFSs to express the potential ambiguity in the evaluation information of DMs and uses CoCoSo method to effectively evaluate and rank schemes. Furthermore, CPT is used to simulate DMs' psychological perception for gains and losses in the evaluation process. (3) To extend the D-CRITIC method to acquire unknown attribute weights in spherical fuzzy environment for the first time. (4) To utilize the established method to solve LSEVCS and to illustrate the applicability for the SF-CPT–CoCoSo model. Meanwhile, the robustness, validity and advantages of the established model are proved by sensitivity analysis and further comparison. (5) To afford DMs more choices in settling MAGDM issues and to also offer some reference on the extension of the CPT–CoCoSo method in other decision-making environments.

The remainder of the paper is scheduled as follows: the primary knowledge of SFSs, the basic ideas of the CoCoSo method, D-CRITIC method and CPT are reviewed in part 2. Some new spherical fuzzy DIMs are proposed in part 3. The general steps for integrating D-CRITIC method and SF-CPT–CoCoSo model are presented in part 4. The proposed method is applied in settling LSEVCS to attest its availability in part 5. This paper is ended with some conclusions in part 6.

2 Preliminaries

In this part, some essential knowledge of SFSs and the general steps for CoCoSo method, D-CRITIC method and CPT will be reviewed.

2.1 Spherical fuzzy sets

Definition 1 (Mahmood et al. 2019): Let $\ddot{O}$ be a universe of discourse, then the SFS $\overleftrightarrow{F}$ on $\ddot{O}$ is defined as where $\xi_{\overleftrightarrow{F}} : \ddot{O} \to [0, 1]$, $\vartheta_{\overleftrightarrow{F}} : \ddot{O} \to [0, 1]$, $\tau_{\overleftrightarrow{F}} : \ddot{O} \to [0, 1]$ and $0 \leq \xi^2_{\overleftrightarrow{F}}(\ddot{o}) + \vartheta^2_{\overleftrightarrow{F}}(\ddot{o}) + \tau^2_{\overleftrightarrow{F}}(\ddot{o}) \leq 1, \forall \ddot{o} \in \ddot{O}$. At the same time, for each $\ddot{o}$, the numbers $\xi_{\overleftrightarrow{F}}(\ddot{o})$, $\vartheta_{\overleftrightarrow{F}}(\ddot{o})$ and $\tau_{\overleftrightarrow{F}}(\ddot{o})$ are MD, N-MD and AD of $\ddot{o}$ to $\overleftrightarrow{F}$,respectively, and $\varsigma_{\overleftrightarrow{F}}(\ddot{o}) = \sqrt{1 - \xi^2_{\overleftrightarrow{F}}(\ddot{o}) - \vartheta^2_{\overleftrightarrow{F}}(\ddot{o}) - \tau^2_{\overleftrightarrow{F}}(\ddot{o})}$ denotes the refusal degree. The triplet $\overleftrightarrow{m} = (\xi_{\overleftrightarrow{m}}, \vartheta_{\overleftrightarrow{m}}, \tau_{\overleftrightarrow{m}})$ is called a spherical fuzzy number (SFN).

Definition 2 (Gundogdu and Kahraman 2019b; Sharaf 2021): Suppose there are two SFNs $\overleftrightarrow{m} = (\xi_{\overleftrightarrow{m}}, \vartheta_{\overleftrightarrow{m}}, \tau_{\overleftrightarrow{m}})$ and $\overleftrightarrow{n} = (\xi_{\overleftrightarrow{n}}, \vartheta_{\overleftrightarrow{n}}, \tau_{\overleftrightarrow{n}})$, respectively, then:

$$\overleftrightarrow{m}^c = (\vartheta_{\overleftrightarrow{m}}, \xi_{\overleftrightarrow{m}}, \tau_{\overleftrightarrow{m}})\ \overleftrightarrow{m}^c \text{ denotes the complement of } \overleftrightarrow{m}, \tag{2}$$

$$\overleftrightarrow{m} \oplus \overleftrightarrow{n} = \begin{pmatrix} (\xi^2_{\overleftrightarrow{m}} + \xi^2_{\overleftrightarrow{n}} - \xi^2_{\overleftrightarrow{m}}\xi^2_{\overleftrightarrow{n}})^{1/2}, & \vartheta_{\overleftrightarrow{m}}\vartheta_{\overleftrightarrow{n}}, \\ ((1 - \xi^2_{\overleftrightarrow{n}})\tau^2_{\overleftrightarrow{m}} + (1 - \xi^2_{\overleftrightarrow{m}})\tau^2_{\overleftrightarrow{n}} - \tau^2_{\overleftrightarrow{m}}\tau^2_{\overleftrightarrow{n}})^{1/2} & \end{pmatrix}, \tag{3}$$

$$\overleftrightarrow{m} \otimes \overleftrightarrow{n} = \begin{pmatrix} \xi_{\overleftrightarrow{m}}\xi_{\overleftrightarrow{n}}, & (\vartheta^2_{\overleftrightarrow{m}} + \vartheta^2_{\overleftrightarrow{n}} - \vartheta^2_{\overleftrightarrow{m}}\vartheta^2_{\overleftrightarrow{n}})^{1/2}, \\ ((1 - \vartheta^2_{\overleftrightarrow{n}})\tau^2_{\overleftrightarrow{m}} + (1 - \vartheta^2_{\overleftrightarrow{m}})\tau^2_{\overleftrightarrow{n}} - \tau^2_{\overleftrightarrow{m}}\tau^2_{\overleftrightarrow{n}})^{1/2} & \end{pmatrix}, \tag{4}$$

$$\sigma \cdot \overleftrightarrow{m} = \begin{pmatrix} (1 - (1 - \xi^2_{\overleftrightarrow{m}})^{\sigma})^{1/2}, & \vartheta^{\sigma}_{\overleftrightarrow{m}}, \\ ((1 - \xi^2_{\overleftrightarrow{m}})^{\sigma} - (1 - \xi^2_{\overleftrightarrow{m}} - \tau^2_{\overleftrightarrow{m}})^{\sigma})^{1/2} & \end{pmatrix}, \quad \sigma > 0, \tag{5}$$

$$\overleftrightarrow{m}^{\sigma} = \begin{pmatrix} \xi^{\sigma}_{\overleftrightarrow{m}}, & (1 - (1 - \vartheta^2_{\overleftrightarrow{m}})^{\sigma})^{1/2}, \\ ((1 - \vartheta^2_{\overleftrightarrow{m}})^{\sigma} - (1 - \vartheta^2_{\overleftrightarrow{m}} - \tau^2_{\overleftrightarrow{m}})^{\sigma})^{1/2} & \end{pmatrix}, \quad \sigma > 0. \tag{6}$$

Definition 3 Sharaf (2021): Let $\overleftrightarrow{m} = (\xi_{\overleftrightarrow{m}}, \vartheta_{\overleftrightarrow{m}}, \tau_{\overleftrightarrow{m}})$ be an SFN, then the score function $\mathrm{Sc}(\overleftrightarrow{m})$ and accuracy function $\mathrm{A}c(\overleftrightarrow{m})$ are given as:

$$\mathrm{Sc}(\overleftrightarrow{m}) = (\xi_{\overleftrightarrow{m}} - \tau_{\overleftrightarrow{m}})^2 - (\vartheta_{\overleftrightarrow{m}} - \tau_{\overleftrightarrow{m}})^2, \tag{7}$$

$$\mathrm{A}c(\overleftrightarrow{m}) = \xi^2_{\overleftrightarrow{m}} + \vartheta^2_{\overleftrightarrow{m}} + \tau^2_{\overleftrightarrow{m}}. \tag{8}$$

Note that: $\overset{\leftrightarrow}{m} < \overset{\leftrightarrow}{n}$ if and only if

i. $\mathrm{Sc}(\overset{\leftrightarrow}{m}) < \mathrm{Sc}(\overset{\leftrightarrow}{n})$ or
ii. $\mathrm{Sc}(\overset{\leftrightarrow}{m}) = \mathrm{Sc}(\overset{\leftrightarrow}{n})$ and $\mathrm{Ac}(\overset{\leftrightarrow}{m}) < \mathrm{Ac}(\overset{\leftrightarrow}{n})$.

Definition 4 (Gundogdu and Kahraman 2019b): Let $\overset{\leftrightarrow}{m}_c = (\xi_{\overset{\leftrightarrow}{m}_c}, \vartheta_{\overset{\leftrightarrow}{m}_c}, \tau_{\overset{\leftrightarrow}{m}_c})(c = 1, 2, \ldots p)$ be a set of SFNs and $\overset{\leftrightarrow}{\omega} = (\overset{\leftrightarrow}{\omega}_1, \overset{\leftrightarrow}{\omega}_2, \ldots, \overset{\leftrightarrow}{\omega}_p)^{\mathrm{T}}$ the corresponding weight vector, with $\overset{\leftrightarrow}{\omega}_c \in [0, 1]$, $\sum_{c=1}^{p} \overset{\leftrightarrow}{\omega}_c = 1$.Then, spherical weighted arithmetic mean (SWAM) operator is defined as:

$$\begin{aligned} \mathrm{SWAM}_{\overset{\leftrightarrow}{\omega}}(\overset{\leftrightarrow}{m}_1, \overset{\leftrightarrow}{m}_2, \ldots \overset{\leftrightarrow}{m}_d) &= \overset{\leftrightarrow}{\omega}_1 \overset{\leftrightarrow}{m}_1 + \overset{\leftrightarrow}{\omega}_2 \overset{\leftrightarrow}{m}_2 + \cdots + \overset{\leftrightarrow}{\omega}_p \overset{\leftrightarrow}{m}_p \\ &= \begin{pmatrix} [1 - \prod_{c=1}^{p} (1 - \xi^2_{\overset{\leftrightarrow}{m}_c})^{\overset{\leftrightarrow}{\omega}_c}]^{1/2}, \prod_{c=1}^{p} \vartheta_{\overset{\leftrightarrow}{m}_c}^{\overset{\leftrightarrow}{\omega}_c}, \\ [\prod_{c=1}^{p} (1 - \xi^2_{\overset{\leftrightarrow}{m}_c})^{\overset{\leftrightarrow}{\omega}_c} - \prod_{c=1}^{p} (1 - \xi^2_{\overset{\leftrightarrow}{m}_c} - \tau^2_{\overset{\leftrightarrow}{m}_c})^{\overset{\leftrightarrow}{\omega}_c}]^{1/2} \end{pmatrix}. \end{aligned} \tag{9}$$

Definition 5 (Gundogdu and Kahraman 2019b): Let $\overset{\leftrightarrow}{m}_c = (\xi_{\overset{\leftrightarrow}{m}_c}, \vartheta_{\overset{\leftrightarrow}{m}_c}, \tau_{\overset{\leftrightarrow}{m}_c})(c = 1, 2, \ldots p)$ be a set of SFNs and $\overset{\leftrightarrow}{\omega} = (\overset{\leftrightarrow}{\omega}_1, \overset{\leftrightarrow}{\omega}_2, \ldots, \overset{\leftrightarrow}{\omega}_p)^{\mathrm{T}}$ the corresponding weight vector, with $\overset{\leftrightarrow}{\omega}_c \in [0, 1]$, $\sum_{c=1}^{p} \overset{\leftrightarrow}{\omega}_c = 1$.Then the spherical weighted geometric mean (SWGM) operator is defined as:

$$\begin{aligned} \mathrm{SWGM}_{\overset{\leftrightarrow}{\omega}}(\overset{\leftrightarrow}{m}_1, \overset{\leftrightarrow}{m}_2, \ldots, \overset{\leftrightarrow}{m}_p) &= \overset{\leftrightarrow}{m}_1^{\overset{\leftrightarrow}{\omega}_1} + \overset{\leftrightarrow}{m}_2^{\overset{\leftrightarrow}{\omega}_2} + \cdots + \overset{\leftrightarrow}{m}_p^{\overset{\leftrightarrow}{\omega}_p} \\ &= \begin{pmatrix} \prod_{c=1}^{p} \xi_{\overset{\leftrightarrow}{m}_c}^{\overset{\leftrightarrow}{\omega}_c}, \quad \left[1 - \prod_{c=1}^{p} (1 - \vartheta^2_{\overset{\leftrightarrow}{m}_c})^{\overset{\leftrightarrow}{\omega}_c}\right]^{1/2}, \\ \left[\prod_{c=1}^{p} (1 - \vartheta^2_{\overset{\leftrightarrow}{m}_c})^{\overset{\leftrightarrow}{\omega}_c} - \prod_{c=1}^{p} (1 - \vartheta^2_{\overset{\leftrightarrow}{m}_c} - \tau^2_{\overset{\leftrightarrow}{m}_c})^{\overset{\leftrightarrow}{\omega}_c}]^{1/2}\right] \end{pmatrix}. \end{aligned} \tag{10}$$

2.2 CoCoSo method

The CoCoSo method (Yazdani et al. 2019) provides a compendium of compromise solutions based on an integrated additive weighting and is exponentially weighted. This method effectively utilizes the multiplication rules and weighted powers in the aggregation process and reasonably combines some aggregation strategies and compromise attitudes to obtain valuable decision results. The general steps are shown as follows.

About an MADM problem, the initial decision matrix from DMs is given as follows:

$$(y_{cd})_{p \times q} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1q} \\ y_{21} & y_{22} & \cdots & y_{2q} \\ \vdots & \vdots & \vdots & \vdots \\ y_{p1} & y_{p2} & \cdots & y_{pq} \end{bmatrix}, \quad c = 1, 2, \ldots, p; \; d = 1, 2, \ldots, q. \tag{11}$$

$\overset{\leftrightarrow}{\partial} = (\overset{\leftrightarrow}{\partial}_1, \overset{\leftrightarrow}{\partial}_2, \ldots, \overset{\leftrightarrow}{\partial}_q)$ indicates the weight vector of attributes.

Step 1. Normalize all attribute values by (12):

$$y^*_{cd} = \begin{cases} \dfrac{y_{cd} - \min\limits_c y_{cd}}{\max\limits_c y_{cd} - \min\limits_c y_{cd}}, & \text{for the benefit attributes.} \\ \dfrac{\max\limits_c y_{cd} - y_{cd}}{\max\limits_c y_{cd} - \min\limits_c y_{cd}}, & \text{for the cost attributes.} \end{cases} \tag{12}$$

Step 2. Utilize the weighted sum measure (WSM) and weighted product measure (WPM) to calculate the weighted sum comparability sequence S_c and the power weighted comparability sequence P_c (Yazdani et al. 2019; Liao et al. 2020):

$$S_c = \sum_{d=1}^{q} (\overleftrightarrow{\partial}_d \cdot y^*_{cd}), \quad c = 1, 2, \ldots, p. \tag{13}$$

$$P_c = \sum_{d=1}^{q} (y^*_{cd})^{\overleftrightarrow{\partial}_d}, \quad c = 1, 2, \ldots, p. \tag{14}$$

Step 3. Compute the three appraisal score strategies with (15)–(17) to measure the relative importance of alternatives (Yazdani et al. 2019):

$$Z^1_c = \frac{P_c + S_c}{\sum_{c=1}^{p} (P_c + S_c)}, \tag{15}$$

$$Z^2_c = \frac{S_c}{\min\limits_c S_c} + \frac{P_c}{\min\limits_c P_c}, \tag{16}$$

$$Z^3_c = \frac{\hat{\beta} S_c + (1 - \hat{\beta}) P_c}{(\hat{\beta} \max\limits_c S_c + (1 - \hat{\beta}) \max\limits_c P_c)}. \tag{17}$$

Z^1_c denotes the arithmetic average of the sum of WSM and WPM scores, Z^2_c indicates a sum of the relative score of WSM and WPM compared to the optimal, and Z^3_c represents the balanced compromise of the WSM and WPM scores. $\hat{\beta}$ indicates compromise coefficient and $0 \leq \hat{\beta} \leq 1$.

Step 4. Sort the alternatives based on Z_c, the greater the value of Z_c is, the better the alternative will be.

$$Z_c = (Z^1_c \cdot Z^2_c \cdot Z^3_c) + \frac{1}{3}(Z^1_c + Z^2_c + Z^3_c). \tag{18}$$

2.3 D-CRITIC method

As a modified form of the original CRITIC approach, D-CRITIC was proposed by Krishnan et al. (2021). This method integrates distance correlation into the CRITIC method to capture the nonlinear relationship between attributes and makes up for the deficiency of the CRITIC method in achieving this purpose. It can simulate the conflict relation between attributes more reliably to obtain attribute weight effectively. Assuming that the initial decision matrix is given in (11), its general steps are as follows:

Step 1. Normalize the decision matrix with (12).

Step 2. Compute the standard deviation for each attribute (19):

$$\zeta_d = \sqrt{\frac{\sum_{c=1}^{p} (y^*_{cd} - \overline{y}_d)^2}{p - 1}}, \quad d = 1, 2, \ldots, q. \tag{19}$$

Here, ζ_d and $\overline{y}_d$ represent the standard deviation and mean value under the dth attribute, respectively.

Step 3. Figure out the distance correlation of every pair of attributes. This is the main difference between the D-CRITIC method and the original CRITIC method. The CRITIC method uses Pearson correlation to measure the relationship between attributes, which could lead to some unreasonable results because two attributes where Pearson correlation is zero may not be completely independent. Therefore, D-CRITIC method improves this shortcoming by modeling the conflict relationship between attributes with distance correlation (Szekely et al. 2007) to obtain the final attribute weight rationally. The distance correlation of attributes $\overset{\leftrightarrow}{\Re}_d$ and $\overset{\leftrightarrow}{\Re}_f$ is computed as follows:

$$d\mathrm{Cor}\left(\overset{\leftrightarrow}{\Re}_d, \overset{\leftrightarrow}{\Re}_f\right) = \frac{d\mathrm{Cov}\left(\overset{\leftrightarrow}{\Re}_d, \overset{\leftrightarrow}{\Re}_f\right)}{\sqrt{d\mathrm{Var}(\overset{\leftrightarrow}{\Re}_d) \cdot d\mathrm{Var}(\overset{\leftrightarrow}{\Re}_f)}}, \quad d, f = 1, 2, \ldots, q, \tag{20}$$

where $d\mathrm{Cov}\left(\overset{\leftrightarrow}{\Re}_d, \overset{\leftrightarrow}{\Re}_f\right)$ indicates the distance covariance between $\overset{\leftrightarrow}{\Re}_d$ and $\overset{\leftrightarrow}{\Re}_f$, and $d\mathrm{Var}(\overset{\leftrightarrow}{\Re}_d) = d\mathrm{Cov}\left(\overset{\leftrightarrow}{\Re}_d, \overset{\leftrightarrow}{\Re}_d\right)$ and $d\mathrm{Var}(\overset{\leftrightarrow}{\Re}_f) = d\mathrm{Cov}\left(\overset{\leftrightarrow}{\Re}_f, \overset{\leftrightarrow}{\Re}_f\right)$ are the distance variance of $\overset{\leftrightarrow}{\Re}_d$ and $\overset{\leftrightarrow}{\Re}_f$, respectively. The detailed calculation process is shown below:

(1) Construct the distance matrix for each attribute $\overset{\leftrightarrow}{\Re}_d$ based on all the alternatives by (21):

$$\overset{\leftrightarrow}{A}^d = (\overset{\leftrightarrow}{a}{}^d_{ce})_{p\times p} = (y^*_{cd} - y^*_{ed})_{p\times p}, \quad c, e = 1, 2, \ldots, p; \quad d = 1, 2, \ldots, q. \tag{21}$$

Here, A^d stands for the distance matrix about attribute $\overset{\leftrightarrow}{\Re}_d$.

(2) Obtain the double-centered matrix B^d for attribute $\overset{\leftrightarrow}{\Re}_d$ with (22):

$$\begin{aligned} \overset{\leftrightarrow}{B}^d &= (\overset{\leftrightarrow}{b}{}^d_{ce})_{p\times p} \\ \overset{\leftrightarrow}{b}{}^d_{ce} &= \overset{\leftrightarrow}{a}{}^d_{ce} - \frac{1}{p}\sum_{e=1}^{p} \overset{\leftrightarrow}{a}{}^d_{ce} - \frac{1}{p}\sum_{c=1}^{p} \overset{\leftrightarrow}{a}{}^d_{ce} + \frac{1}{p^2}\sum_{c=1}^{p}\sum_{e=1}^{p} \overset{\leftrightarrow}{a}{}^d_{ce}, \quad d = 1, 2, \ldots, q, \end{aligned} \tag{22}$$

where $\frac{1}{p}\sum_{e=1}^{p} \overset{\leftrightarrow}{a}{}^d_{ce}$ and $\frac{1}{p}\sum_{c=1}^{p} \overset{\leftrightarrow}{a}{}^d_{ce}$ denote the average of the cth row and e th column of A^d, respectively, and $\frac{1}{p^2}\sum_{c=1}^{p}\sum_{e=1}^{p} \overset{\leftrightarrow}{a}{}^d_{ce}$ represents the total average of A^d.

(3) Determine the distance covariance of attributes $\overset{\leftrightarrow}{\Re}_d$ and $\overset{\leftrightarrow}{\Re}_f$ by (23):

$$d\mathrm{Cov}\left(\overset{\leftrightarrow}{\Re}_d, \overset{\leftrightarrow}{\Re}_f\right) = \sqrt{\frac{\sum_{c=1}^{p}\sum_{e=1}^{p} \overset{\leftrightarrow}{b}{}^d_{ce} \cdot \overset{\leftrightarrow}{b}{}^f_{ce}}{p^2}}, \quad d, f = 1, 2, \ldots, q. \tag{23}$$

(4) Calculate the distance variance for attribute $\overset{\leftrightarrow}{\Re}_d$ depending on (24):

$$d\mathrm{Var}(\overset{\leftrightarrow}{\Re}_d) = d\mathrm{Cov}(\overset{\leftrightarrow}{\Re}_d, \overset{\leftrightarrow}{\Re}_d), \quad d = 1, 2, \ldots, q. \tag{24}$$

(5) Obtain the distance correlation between attributes based on distance covariance and distance variance of attribute.

Step 4. Compute the information content for each attribute by (25):

$$I_d = \zeta_d \cdot \sum_{f=1}^{q} \left(1 - d\mathrm{Cor}(\overset{\leftrightarrow}{\Re}_d, \overset{\leftrightarrow}{\Re}_f)\right), \quad d = 1, 2, \ldots, q. \tag{25}$$

Step 5. Calculate the objective weights by (26):

$$\overset{\leftrightarrow}{\partial}_d = \frac{I_d}{\sum_{d=1}^{q} I_d}, \quad d = 1, 2, \ldots, q, \tag{26}$$

where $\overset{\leftrightarrow}{\partial}_d$ indicates the objective weight about attribute $\overset{\leftrightarrow}{\Re}_d$.

2.4 Cumulative prospect theory

CPT is an effective model to describe the risk preference of DMs, which was presented by Tversky and Kahneman (1992). As an improved form of PT (Kahneman and Tversky 1979), CPT replaces decision weight in PT with cumulative decision weight, which can effectively overcome the phenomenon that PT violates random predominance. To describe the mathematical formula of CPT, consider that k results $\overset{\leftrightarrow}{x}$ are sorted by ascending order (i.e., $\overset{\leftrightarrow}{x}_1 \leq \cdots \leq \overset{\leftrightarrow}{x}_h \leq 0 \leq \overset{\leftrightarrow}{x}_{h+1} \leq \cdots \leq \overset{\leftrightarrow}{x}_k$), where $\overset{\leftrightarrow}{\diamondsuit}_c$ is the probability of result $\overset{\leftrightarrow}{x}_c$, and the prospect value $\overset{\leftrightarrow}{V}$ is defined as:

$$\overset{\leftrightarrow}{V} = \sum_{c=1}^{h} \overset{\leftrightarrow}{v}(\overset{\leftrightarrow}{x}_c) \cdot \eta_c^- + \sum_{c=h+1}^{k} \overset{\leftrightarrow}{v}(\overset{\leftrightarrow}{x}_c) \cdot \eta_c^+, \tag{27}$$

where $\overset{\leftrightarrow}{v}(\overset{\leftrightarrow}{x}_c)$ indicates the value function, which can fully reflect DMs' risk attitude and subjective preference when facing the gain and loss. It takes the difference between the index result and the decision reference point as the basis for decision making, rather than the absolute value of the index result, which conforms with the psychology and actual situation of DMs. The specific expression is as follows:

$$\overset{\leftrightarrow}{v}(\overset{\leftrightarrow}{x}_c) = \begin{cases} (\overset{\leftrightarrow}{x}_c)^{\alpha}, & \overset{\leftrightarrow}{x}_c \geq 0 \\ -\lambda(-\overset{\leftrightarrow}{x}_c)^{\beta}, & \overset{\leftrightarrow}{x}_c < 0 \end{cases}, \tag{28}$$

where $\overset{\leftrightarrow}{x}_c$ represents the difference with respect to the reference point, and $\overset{\leftrightarrow}{x}_c \geq 0$ indicates the gain, whereas $\overset{\leftrightarrow}{x}_c < 0$ means the loss.α, β are the risk attitude coefficients of DMs and mean the preference degrees in the region of gain and loss.λ denotes the coefficient of loss aversion that is more sensitive to loss than gain.

η_c^+ and η_c^- represent probability weighting function for gain and loss, respectively, defined as the following:

$$\eta_c^- = \begin{cases} \kappa^-(\overset{\leftrightarrow}{\diamondsuit}_c), & \text{if } c = 1 \\ \kappa^-(\overset{\leftrightarrow}{\diamondsuit}_1 + \cdots + \overset{\leftrightarrow}{\diamondsuit}_c) - \kappa^-(\overset{\leftrightarrow}{\diamondsuit}_1 + \cdots + \overset{\leftrightarrow}{\diamondsuit}_{c-1}), & \text{if } 2 \leq c \leq h \end{cases}, \tag{29}$$

$$\eta_c^+ = \begin{cases} \kappa^+(\overset{\leftrightarrow}{\diamondsuit}_c), & \text{if } c = k \\ \kappa^+(\overset{\leftrightarrow}{\diamondsuit}_c + \cdots + \overset{\leftrightarrow}{\diamondsuit}_k) - \kappa^+(\overset{\leftrightarrow}{\diamondsuit}_{c+1} + \cdots + \overset{\leftrightarrow}{\diamondsuit}_k), & \text{if } h+1 \leq c \leq k-1 \end{cases} \tag{30}$$

and

$$\kappa^+(\overset{\leftrightarrow}{\diamondsuit}) = \frac{\overset{\leftrightarrow}{\diamondsuit}^{\gamma}}{(\overset{\leftrightarrow}{\diamondsuit}^{\gamma} + (1 - \overset{\leftrightarrow}{\diamondsuit})^{\gamma})^{\frac{1}{\gamma}}}$$

$$\kappa^{-}(\overset{\leftrightarrow}{\Diamond}) = \frac{\overset{\leftrightarrow}{\Diamond}^{\delta}}{(\overset{\leftrightarrow}{\Diamond}^{\delta} + (1 - \overset{\leftrightarrow}{\Diamond})^{\delta})^{\frac{1}{\delta}}}, \tag{31}$$

where γ, δ express the curvature for probability weighting function of gain and loss and reflect DMs' different risk attitudes toward gain and loss.

3 Some novel distance measures for spherical fuzzy sets

Dice and Jaccard indexes (Dice 1945; Jaccard 1901) can efficaciously describe the degree of similarity between sets or inputs, which have been widely extended in many decision-making environments (Wang et al. 2021, 2019a; Ali and Mahmood 2020; Jan et al. 2020; Mahmood et al. 2021b; Wei and Gao 2018). Based on existing results, in this section we present some new spherical fuzzy DIMs by Dice and Jaccard indexes to detect the deviation degree between SFSs or inputs and avoid the loss of information.

3.1 Some Dice and Jaccard distance measures for spherical fuzzy sets

In the subsection, we shall introduce some Dice and Jaccard DIMs and their weighted forms in spherical fuzzy environment.

Definition 6 Let $\overset{\leftrightarrow}{F}_1 = \{< \ddot{o}, (\xi_{\overset{\leftrightarrow}{F}_1}(\ddot{o}), \vartheta_{\overset{\leftrightarrow}{F}_1}(\ddot{o}), \tau_{\overset{\leftrightarrow}{F}_1}(\ddot{o})) | \ddot{o} \in \ddot{O}\}$ and $\overset{\leftrightarrow}{F}_2 = \{< \ddot{o}, (\xi_{\overset{\leftrightarrow}{F}_2}(\ddot{o}), \vartheta_{\overset{\leftrightarrow}{F}_2}(\ddot{o}), \tau_{\overset{\leftrightarrow}{F}_2}(\ddot{o})) | \ddot{o} \in \ddot{O}\}$ be two SFSs on $\ddot{O} = \{\ddot{o}_1, \ddot{o}_2, \cdots, \ddot{o}_p\}$, then Dice distance measure between SFSs $\overset{\leftrightarrow}{F}_1$ and $\overset{\leftrightarrow}{F}_2$ is given as

$$d_{\text{Dice}}^{1}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \frac{1}{p} \sum_{c=1}^{p} \frac{2\overset{\leftrightarrow}{\Lambda}_c}{\overset{\leftrightarrow}{\mathrm{M}}_c + \overset{\leftrightarrow}{\mathrm{N}}_c}, \tag{32}$$

where $\overset{\leftrightarrow}{\Lambda}_c = \xi^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\xi^2_{F_2}(\ddot{o}_c) + \vartheta^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\vartheta^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \tau^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\tau^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)$, $\overset{\leftrightarrow}{\mathrm{M}}_c = \xi^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) + \vartheta^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) + \tau^4_{F_1}(\ddot{o}_c)$, $\overset{\leftrightarrow}{\mathrm{N}}_c = \xi^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \vartheta^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \tau^4_{F_2}(\ddot{o}_c)$, and d_{Dice}^{1} also satisfies the following conditions:

(1) $d_{\text{Dice}}^{1}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = d_{\text{Dice}}^{1}(\overset{\leftrightarrow}{F}_2, \overset{\leftrightarrow}{F}_1)$;
(2) $0 \le d_{\text{Dice}}^{1}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) \le 1$;
(3) $d_{\text{Dice}}^{1}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 0$ if $\overset{\leftrightarrow}{F}_1 = \overset{\leftrightarrow}{F}_2$.

Proof

(1) Based on Definition 6, $d_{\text{Dice}}^{1}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = d_{\text{Dice}}^{1}(\overset{\leftrightarrow}{F}_2, \overset{\leftrightarrow}{F}_1)$ is obvious.
(2) Since $a^2 + b^2 \ge 2ab$, we have

$$\overset{\leftrightarrow}{\mathrm{M}}_c + \overset{\leftrightarrow}{\mathrm{N}}_c = \xi^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) + \vartheta^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) + \tau^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) + \xi^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \vartheta^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \tau^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) \ge 2\Big(\xi^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\xi^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)$$

$$+ \vartheta^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\vartheta^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \tau^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\tau^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\Big) = 2\overset{\leftrightarrow}{\Lambda}_c \Rightarrow 0 \le \frac{2\overset{\leftrightarrow}{\Lambda}_c}{\overset{\leftrightarrow}{\mathrm{M}}_c + \overset{\leftrightarrow}{\mathrm{N}}_c} \le 1 \Rightarrow 0 \le \frac{1}{p}\sum_{c=1}^{p} \frac{2\overset{\leftrightarrow}{\Lambda}_c}{\overset{\leftrightarrow}{\mathrm{M}}_c + \overset{\leftrightarrow}{\mathrm{N}}_c} \le 1$$

$$\Rightarrow 0 \le 1 - \frac{1}{p}\sum_{c=1}^{p} \frac{2\overset{\leftrightarrow}{\Lambda}_c}{\overset{\leftrightarrow}{\mathrm{M}}_c + \overset{\leftrightarrow}{\mathrm{N}}_c} \le 1,$$

so $0 \leq d^1_{Dice}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2) \leq 1$.

(3) When $\overleftrightarrow{F}_1 = \overleftrightarrow{F}_2$, we can get $\xi_{\overleftrightarrow{F}_1}(\ddot{o}_c) = \xi_{\overleftrightarrow{F}_2}(\ddot{o}_c), \vartheta_{\overleftrightarrow{F}_1}(\ddot{o}_c) = \vartheta_{\overleftrightarrow{F}_2}(\ddot{o}_c), \tau_{\overleftrightarrow{F}_1}(\ddot{o}_c) = \tau_{\overleftrightarrow{F}_2}(\ddot{o}_c)$, $c = 1, 2, \ldots p$. Therefore, $d^1_{\mathrm{Dice}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2) = 1 - \frac{1}{p}\sum_{c=1}^{p} \frac{2\overleftrightarrow{\mathrm{M}}_c}{2\overleftrightarrow{\mathrm{M}}_c} = 1 - \frac{1}{p}\sum_{c=1}^{p} \frac{2\overleftrightarrow{\mathrm{N}}_c}{2\overleftrightarrow{\mathrm{N}}_c} = 0$.

In practical decision making, elements in SFSs may have different degrees of importance, so the weighted Dice distance measure is given below:

$$d^1_{\overleftrightarrow{\partial}\,\mathrm{Dice}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2) = 1 - \sum_{c=1}^{p} \overleftrightarrow{\partial}_c \frac{2\overleftrightarrow{\Lambda}_c}{\overleftrightarrow{\mathrm{M}}_c + \overleftrightarrow{\mathrm{N}}_c}, \tag{33}$$

where $\overleftrightarrow{\partial} = (\overleftrightarrow{\partial}_1, \overleftrightarrow{\partial}_2, \ldots, \overleftrightarrow{\partial}_p)$ denote the weight vector of elements, with $\overleftrightarrow{\partial}_c \geq 0$ and $\sum_{c=1}^{p} \overleftrightarrow{\partial}_c = 1$. When $\overleftrightarrow{\partial} = (\frac{1}{p}, \frac{1}{p}, \ldots, \frac{1}{p})$, then the weighted Dice distance measure becomes Dice distance measure.

Clearly, the weighted Dice distance measure satisfies the properties in Definition 6. In the following, we shall give some other forms for Dice distance measure:

$$d^2_{\mathrm{Dice}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2) = 1 - \frac{\sum_{c=1}^{p} 2\overleftrightarrow{\Lambda}_c}{\sum_{c=1}^{p} \overleftrightarrow{\mathrm{M}}_c \sum_{c=1}^{p} \overleftrightarrow{\mathrm{N}}_c}, \tag{34}$$

$$d^3_{\mathrm{Dice}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2) = 1 - \frac{1}{p}\sum_{c=1}^{p} \frac{2\left(\overleftrightarrow{\Lambda}_c + \varsigma^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\varsigma^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)\right)}{\overleftrightarrow{\mathrm{M}}_c + \varsigma^4_{\overleftrightarrow{F}_1}(\ddot{o}_c) + \overleftrightarrow{\mathrm{N}}_c + \varsigma^4_{\overleftrightarrow{F}_2}(\ddot{o}_c)}, \tag{35}$$

$$d^4_{\mathrm{Dice}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2) = 1 - \frac{\sum_{c=1}^{p} 2\left(\overleftrightarrow{\Lambda}_c + \varsigma^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\varsigma^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)\right)}{\sum_{c=1}^{p}\left(\overleftrightarrow{\mathrm{M}}_c + \varsigma^4_{\overleftrightarrow{F}_1}(\ddot{o}_c)\right) + \sum_{c=1}^{p}\left(\overleftrightarrow{\mathrm{N}}_c + \varsigma^4_{\overleftrightarrow{F}_2}(\ddot{o}_c)\right)}, \tag{36}$$

where $\varsigma_{\overleftrightarrow{F}_1}(\ddot{o}_c)$ and $\varsigma_{\overleftrightarrow{F}_2}(\ddot{o}_c)$ represent the RD of elements in SFSs $\overleftrightarrow{F}_1$ and $\overleftrightarrow{F}_2$, respectively. Accordingly, their weighted forms are as follows:

$$d^2_{\overleftrightarrow{\partial}\,\mathrm{Dice}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2) = 1 - \frac{\sum_{c=1}^{p} 2\overleftrightarrow{\partial}_c\overleftrightarrow{\Lambda}_c}{\sum_{c=1}^{p} \overleftrightarrow{\partial}_c\overleftrightarrow{\mathrm{M}}_c + \sum_{c=1}^{p} \overleftrightarrow{\partial}_c\overleftrightarrow{\mathrm{N}}_c}, \tag{37}$$

$$d^3_{\overleftrightarrow{\partial}\,\mathrm{Dice}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2) = 1 - \sum_{c=1}^{p} \overleftrightarrow{\partial}_c \frac{2\left(\overleftrightarrow{\Lambda}_c + \varsigma^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\varsigma^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)\right)}{\overleftrightarrow{\mathrm{M}}_c + \varsigma^4_{\overleftrightarrow{F}_1}(\ddot{o}_c) + \overleftrightarrow{\mathrm{N}}_c + \varsigma^4_{\overleftrightarrow{F}_2}(\ddot{o}_c)}, \tag{38}$$

$$d^4_{\overleftrightarrow{\partial}\,\mathrm{Dice}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2) = 1 - \frac{\sum_{c=1}^{p} 2\overleftrightarrow{\partial}_c\left(\overleftrightarrow{\Lambda}_c + \varsigma^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\varsigma^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)\right)}{\sum_{c=1}^{p} \overleftrightarrow{\partial}_c\left(\overleftrightarrow{\mathrm{M}}_c + \varsigma^4_{\overleftrightarrow{F}_1}(\ddot{o}_c)\right) + \sum_{c=1}^{p} \overleftrightarrow{\partial}_c\left(\overleftrightarrow{\mathrm{N}}_c + \varsigma^4_{\overleftrightarrow{F}_2}(\ddot{o}_c)\right)}, \tag{39}$$

where $\overleftrightarrow{\partial}=(\overleftrightarrow{\partial}_1,\overleftrightarrow{\partial}_2,\ldots,\overleftrightarrow{\partial}_p)$ denote the weight vector of elements, with $\overleftrightarrow{\partial}_c\geq 0$ and $\sum_{c=1}^{p}\overleftrightarrow{\partial}_c=1$, $\varsigma_{\overleftrightarrow{F}_1}(\ddot{o}_c)$ and $\varsigma_{\overleftrightarrow{F}_2}(\ddot{o}_c)$ represent the rejection degree of elements in SFSs $\overleftrightarrow{F}_1$ and $\overleftrightarrow{F}_2$, respectively.

Furthermore, Dice DIMs $d^2_{\text{Dice}}(\overleftrightarrow{F}_1,\overleftrightarrow{F}_2)$,$d^3_{\text{Dice}}(\overleftrightarrow{F}_1,\overleftrightarrow{F}_2)$,$d^4_{\text{Dice}}(\overleftrightarrow{F}_1,\overleftrightarrow{F}_2)$ and their weighted forms also satisfy the properties in Definition 6.

Definition 7 Let $\overleftrightarrow{F}_1=\{<\ddot{o},(\xi_{\overleftrightarrow{F}_1}(\ddot{o}),\vartheta_{\overleftrightarrow{F}_1}(\ddot{o}),\tau_{\overleftrightarrow{F}_1}(\ddot{o}))|\ddot{o}\in\ddot{O}\}$ and $\overleftrightarrow{F}_2=\{<\ddot{o},(\xi_{\overleftrightarrow{F}_2}(\ddot{o}),\vartheta_{\overleftrightarrow{F}_2}(\ddot{o}),\tau_{\overleftrightarrow{F}_2}(\ddot{o}))|\ddot{o}\in\ddot{O}\}$ be two SFSs on $\ddot{O}=\{\ddot{o}_1,\ddot{o}_2,\ldots,\ddot{o}_p\}$, then Jaccard distance measure between SFSs $\overleftrightarrow{F}_1$ and $\overleftrightarrow{F}_2$ is given as

$$d^1_{\text{Jaccard}}(\overleftrightarrow{F}_1,\overleftrightarrow{F}_2)=1-\frac{1}{p}\sum_{c=1}^{p}\frac{\overleftrightarrow{\Lambda}_c}{\overleftrightarrow{\mathrm{M}}_c+\overleftrightarrow{\mathrm{N}}_c-\overleftrightarrow{\Lambda}_c},\tag{40}$$

where $\overleftrightarrow{\Lambda}_c=\xi^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\xi^2_{F_2}(\ddot{o}_c)+\vartheta^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\vartheta^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)+\tau^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\tau^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)$,$\overleftrightarrow{\mathrm{M}}_c=\xi^4_{\overleftrightarrow{F}_1}(\ddot{o}_c)+\vartheta^4_{\overleftrightarrow{F}_1}(\ddot{o}_c)+\tau^4_{F_1}(\ddot{o}_c)$, $\overleftrightarrow{\mathrm{N}}_c=\xi^4_{\overleftrightarrow{F}_2}(\ddot{o}_c)+\vartheta^4_{\overleftrightarrow{F}_2}(\ddot{o}_c)+\tau^4_{F_2}(\ddot{o}_c)$, and d^1_{Jaccard} also satisfies the following conditions:

(1) $d^1_{\text{Jaccrad}}(\overleftrightarrow{F}_1,\overleftrightarrow{F}_2)=d^1_{\text{Jaccard}}(\overleftrightarrow{F}_2,\overleftrightarrow{F}_1)$;
(2) $0\leq d^1_{\text{Jaccard}}(\overleftrightarrow{F}_1,\overleftrightarrow{F}_2)\leq 1$;
(3) $d^1_{\text{Jaccard}}(\overleftrightarrow{F}_1,\overleftrightarrow{F}_2)=0$ if $\overleftrightarrow{F}_1=\overleftrightarrow{F}_2$.

Proof

(1) Based on Definition 7, $d^1_{\text{Jaccard}}(\overleftrightarrow{F}_1,\overleftrightarrow{F}_2)=d^1_{\text{Jaccard}}(\overleftrightarrow{F}_2,\overleftrightarrow{F}_1)$ is obvious.
(2) Since $a^2+b^2\geq 2ab$, we have

$$\overleftrightarrow{\mathrm{M}}_c+\overleftrightarrow{\mathrm{N}}_c-\overleftrightarrow{\Lambda}_c=\xi^4_{\overleftrightarrow{F}_1}(\ddot{o}_c)+\vartheta^4_{\overleftrightarrow{F}_1}(\ddot{o}_c)+\tau^4_{F_1}(\ddot{o}_c)+\xi^4_{\overleftrightarrow{F}_2}(\ddot{o}_c)+\vartheta^4_{\overleftrightarrow{F}_2}(\ddot{o}_c)+\tau^4_{F_2}(\ddot{o}_c)-\Big(\xi^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\xi^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)+\vartheta^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\vartheta^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)$$
$$+\tau^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\tau^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)\Big)\geq 2\Big(\xi^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\xi^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)+\vartheta^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\vartheta^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)+\tau^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\tau^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)\Big)-\Big(\xi^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\xi^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)+\vartheta^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\vartheta^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)$$
$$+\tau^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\tau^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)\Big)=\xi^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\xi^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)+\vartheta^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\vartheta^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)+\tau^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\tau^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)=\overleftrightarrow{\Lambda}_c\Rightarrow 0\leq\frac{\overleftrightarrow{\Lambda}_c}{\overleftrightarrow{\mathrm{M}}_c+\overleftrightarrow{\mathrm{N}}_c-\overleftrightarrow{\Lambda}_c}\leq 1\Rightarrow$$
$$0\leq\frac{1}{p}\sum_{c=1}^{p}\frac{\overleftrightarrow{\Lambda}_c}{\overleftrightarrow{\mathrm{M}}_c+\overleftrightarrow{\mathrm{N}}_c-\overleftrightarrow{\Lambda}_c}\leq 1\Rightarrow 0\leq 1-\frac{1}{p}\sum_{c=1}^{p}\frac{\overleftrightarrow{\Lambda}_c}{\overleftrightarrow{\mathrm{M}}_c+\overleftrightarrow{\mathrm{N}}_c-\overleftrightarrow{\Lambda}_c}\leq 1.$$

So, $0\leq d^1_{\text{Jaccard}}(\overleftrightarrow{F}_1,\overleftrightarrow{F}_2)\leq 1$.
(3) When $\overleftrightarrow{F}_1=\overleftrightarrow{F}_2$, we can get $\xi_{\overleftrightarrow{F}_1}(\ddot{o}_c)=\xi_{\overleftrightarrow{F}_2}(\ddot{o}_c),\vartheta_{\overleftrightarrow{F}_1}(\ddot{o}_c)=\vartheta_{\overleftrightarrow{F}_2}(\ddot{o}_c),\tau_{\overleftrightarrow{F}_1}(\ddot{o}_c)=\tau_{\overleftrightarrow{F}_2}(\ddot{o}_c)$, $c=1,2,\ldots p$. Therefore, $d^1_{\text{Jaccard}}(\overleftrightarrow{F}_1,\overleftrightarrow{F}_2)=1-\frac{1}{p}\sum_{c=1}^{p}\frac{\overleftrightarrow{\Lambda}_c}{\overleftrightarrow{\Lambda}_c}=0$.

In practical decision making, elements in SFSs may have different degrees of importance, so the weighted Jaccard distance measure is given below:

$$d^1_{\overleftrightarrow{\partial}\,\text{Jaccard}}(\overleftrightarrow{F}_1,\overleftrightarrow{F}_2)=1-\sum_{c=1}^{p}\overleftrightarrow{\partial}_c\,\frac{\overleftrightarrow{\Lambda}_c}{\overleftrightarrow{\mathrm{M}}_c+\overleftrightarrow{\mathrm{N}}_c-\overleftrightarrow{\Lambda}_c},\tag{41}$$

where $\overleftrightarrow{\partial} = (\overleftrightarrow{\partial}_1, \overleftrightarrow{\partial}_2, \ldots, \overleftrightarrow{\partial}_p)$ denote the weight vector of elements, with $\overleftrightarrow{\partial}_c \geq 0$ and $\sum_{c=1}^{p} \overleftrightarrow{\partial}_c = 1$. When $\overleftrightarrow{\partial} = (\frac{1}{p}, \frac{1}{p}, \ldots, \frac{1}{p})$, then the weighted Jaccard distance measure becomes Jaccard distance measure.

Clearly, the weighted Jaccard distance measure satisfies the properties in Definition 7. In the following, we shall give some other forms for Jaccard distance measure.

$$d^2_{\text{Jaccard}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2) = 1 - \frac{\sum_{c=1}^{p} \overleftrightarrow{\Lambda}_c}{\sum_{c=1}^{p} \overleftrightarrow{\mathrm{M}}_c + \sum_{c=1}^{p} \overleftrightarrow{\mathrm{N}}_c - \sum_{c=1}^{p} \overleftrightarrow{\Lambda}_c}, \tag{42}$$

$$d^3_{\text{Jaccard}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2) = 1 - \frac{1}{p}\sum_{c=1}^{p} \frac{\overleftrightarrow{\Lambda}_c + \varsigma^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\varsigma^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)}{\overleftrightarrow{\mathrm{M}}_c + \varsigma^4_{\overleftrightarrow{F}_1}(\ddot{o}_c) + \overleftrightarrow{\mathrm{N}}_c + \varsigma^4_{\overleftrightarrow{F}_2}(\ddot{o}_c) - \left(\overleftrightarrow{\Lambda}_c + \varsigma^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\varsigma^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)\right)}, \tag{43}$$

$$d^4_{\text{Jaccard}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2) = 1 - \frac{\sum_{c=1}^{p}\left(\overleftrightarrow{\Lambda}_c + \varsigma^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\varsigma^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)\right)}{\sum_{c=1}^{p}\left(\overleftrightarrow{\mathrm{M}}_c + \varsigma^4_{\overleftrightarrow{F}_1}(\ddot{o}_c)\right) + \sum_{c=1}^{p}\left(\overleftrightarrow{\mathrm{N}}_c + \varsigma^4_{\overleftrightarrow{F}_1}(\ddot{o}_c)\right) - \sum_{c=1}^{p}\left(\overleftrightarrow{\Lambda}_c + \varsigma^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\varsigma^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)\right)}, \tag{44}$$

where $\varsigma_{\overleftrightarrow{F}_1}(\ddot{o}_c)$ and $\varsigma_{\ddot{F}_2}(\ddot{o}_c)$ represent the RD of elements in SFSs $\overleftrightarrow{F}_1$ and $\overleftrightarrow{F}_2$, respectively. Accordingly, their weighted forms are as follows:

$$d^2_{\overleftrightarrow{\partial}\,\text{Jaccard}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2) = 1 - \frac{\sum_{c=1}^{p} \overleftrightarrow{\partial}_c \Lambda_c}{\sum_{c=1}^{p} \overleftrightarrow{\partial}_c \overleftrightarrow{\mathrm{M}}_c + \sum_{c=1}^{p} \overleftrightarrow{\partial}_c \overleftrightarrow{\mathrm{N}}_c - \sum_{c=1}^{p} \overleftrightarrow{\partial}_c \overleftrightarrow{\Lambda}_c}, \tag{45}$$

$$d^3_{\overleftrightarrow{\partial}\,\text{Jaccard}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2) = 1 - \sum_{c=1}^{p} \overleftrightarrow{\partial}_c \frac{\overleftrightarrow{\Lambda}_c + \varsigma^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\varsigma^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)}{\overleftrightarrow{\mathrm{M}}_c + \overleftrightarrow{\mathrm{N}}_c - \left(\overleftrightarrow{\Lambda}_c + \varsigma^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\varsigma^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)\right)}, \tag{46}$$

$$d^4_{\overleftrightarrow{\partial}\,\text{Jaccard}}(F_1, F_2) = 1 - \frac{\sum_{c=1}^{p} \overleftrightarrow{\partial}_c \left(\overleftrightarrow{\Lambda}_c + \varsigma^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\varsigma^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)\right)}{\sum_{c=1}^{p} \overleftrightarrow{\partial}_c \overleftrightarrow{\mathrm{M}}_c + \sum_{c=1}^{p} \overleftrightarrow{\partial}_c \overleftrightarrow{\mathrm{N}}_c - \sum_{c=1}^{p} \overleftrightarrow{\partial}_c \left(\overleftrightarrow{\Lambda}_c + \varsigma^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\varsigma^2_{\overleftrightarrow{F}_2}(\ddot{o}_c)\right)}, \tag{47}$$

where $\overleftrightarrow{\partial} = (\overleftrightarrow{\partial}_1, \overleftrightarrow{\partial}_2, \ldots, \overleftrightarrow{\partial}_p)$ denote the weight vector of elements, with $\overleftrightarrow{\partial}_c \geq 0$ and $\sum_{c=1}^{p} \overleftrightarrow{\partial}_c = 1$, and $\varsigma_{\overleftrightarrow{F}_1}(\ddot{o}_c)$ and $\varsigma_{\overleftrightarrow{F}_2}(\ddot{o}_c)$ represent the rejection degree of elements in SFSs $\overleftrightarrow{F}_1$ and $\overleftrightarrow{F}_2$ respectively.

Furthermore, Jaccard DIMs $d^2_{\text{Jaccard}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2)$, $d^3_{\text{Jaccard}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2)$, $d^4_{\text{Jaccard}}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2)$ and their weighted forms also satisfy the properties in Definition 7.

3.2 Some generalized Dice and Jaccard distance measures for spherical fuzzy sets

In the subsection, we shall introduce some generalized Dice and Jaccard DIMs and their weighted forms in spherical fuzzy environment.

Definition 8 Let $\overset{\leftrightarrow}{F}_1 = \{< \ddot{o}, (\xi_{\overset{\leftrightarrow}{F}_1}(\ddot{o}), \vartheta_{\overset{\leftrightarrow}{F}_1}(\ddot{o}), \tau_{\overset{\leftrightarrow}{F}_1}(\ddot{o})) | \ddot{o} \in \ddot{O}\}$ and $\overset{\leftrightarrow}{F}_2 = \{< \ddot{o}, (\xi_{\overset{\leftrightarrow}{F}_2}(\ddot{o}), \vartheta_{\overset{\leftrightarrow}{F}_2}(\ddot{o}), \tau_{\overset{\leftrightarrow}{F}_2}(\ddot{o})) | \ddot{o} \in \ddot{O}\}$ be two SFSs on $\ddot{O} = \{\ddot{o}_1, \ddot{o}_2, \ldots, \ddot{o}_p\}$, then some generalized Dice DIMs between SFSs $\overset{\leftrightarrow}{F}_1$ and $\overset{\leftrightarrow}{F}_2$ are given as:

$$d^1_{GDice}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \frac{1}{p}\sum_{c=1}^{p} \frac{\overset{\leftrightarrow}{\Lambda}_c}{\ell \overset{\leftrightarrow}{\mathrm{M}}_c + (1-\ell)\overset{\leftrightarrow}{\mathrm{N}}_c}, \tag{48}$$

$$d^2_{GDice}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \frac{\sum_{c=1}^{p} \overset{\leftrightarrow}{\Lambda}_c}{\ell \sum_{c=1}^{p} \overset{\leftrightarrow}{\mathrm{M}}_c + (1-\ell)\sum_{c=1}^{p} \overset{\leftrightarrow}{\mathrm{N}}_c}, \tag{49}$$

$$d^3_{GDice}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \frac{1}{p}\sum_{c=1}^{p} \frac{\left(\overset{\leftrightarrow}{\Lambda}_c + \varsigma^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\varsigma^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\right)}{\ell\left(\overset{\leftrightarrow}{\mathrm{M}}_c + \varsigma^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\right) + (1-\ell)\left(\overset{\leftrightarrow}{\mathrm{N}}_c + \varsigma^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\right)}, \tag{50}$$

$$d^4_{GDice}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \frac{\sum_{c=1}^{p}\left(\overset{\leftrightarrow}{\Lambda}_c + \varsigma^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\varsigma^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\right)}{\ell\sum_{c=1}^{p}\left(\overset{\leftrightarrow}{\mathrm{M}}_c + \varsigma^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\right) + (1-\ell)\sum_{c=1}^{p}\left(\overset{\leftrightarrow}{\mathrm{N}}_c + \varsigma^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\right)}, \tag{51}$$

where $\overset{\leftrightarrow}{\Lambda}_c = \xi^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\xi^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \vartheta^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\vartheta^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \tau^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\tau^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)$, $\overset{\leftrightarrow}{\mathrm{M}}_c = \xi^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) + \vartheta^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) + \tau^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)$, $\overset{\leftrightarrow}{\mathrm{N}}_c = \xi^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \vartheta^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \tau^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)$, $\varsigma_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) = \sqrt{1 - \xi^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) + \vartheta^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) + \tau^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)}$ and $\varsigma_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) = \sqrt{1 - \xi^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \vartheta^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \tau^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)}$ indicate RD in SFSs $\overset{\leftrightarrow}{F}_1$ and $\overset{\leftrightarrow}{F}_2$ on $\ddot{O}$, respectively, $0 \le \ell \le 1$. Accordingly, some weighted generalized Dice DIMs between SFSs $\overset{\leftrightarrow}{F}_1$ and $\overset{\leftrightarrow}{F}_2$ are given as:

$$d^1_{\overset{\leftrightarrow}{\partial} GDice}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c \frac{\overset{\leftrightarrow}{\Lambda}_c}{\ell \overset{\leftrightarrow}{\mathrm{M}}_c + (1-\ell)\overset{\leftrightarrow}{\mathrm{N}}_c}, \tag{52}$$

$$d^2_{\overset{\leftrightarrow}{\partial} GDice}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \frac{\sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c \overset{\leftrightarrow}{\Lambda}_c}{\ell \sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c \overset{\leftrightarrow}{\mathrm{M}}_c + (1-\ell)\sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c \overset{\leftrightarrow}{\mathrm{N}}_c}, \tag{53}$$

$$d^3_{\overset{\leftrightarrow}{\partial} GDice}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c \frac{\left(\overset{\leftrightarrow}{\Lambda}_c + \varsigma^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\varsigma^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\right)}{\ell\left(\overset{\leftrightarrow}{\mathrm{M}}_c + \varsigma^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\right) + (1-\ell)\left(\overset{\leftrightarrow}{\mathrm{N}}_c + \varsigma^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\right)}, \tag{54}$$

$$d^4_{\overset{\leftrightarrow}{\partial} GDice}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \frac{\sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c \left(\overset{\leftrightarrow}{\Lambda}_c + \varsigma^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\varsigma^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\right)}{\ell\sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c \left(\overset{\leftrightarrow}{\mathrm{M}}_c + \varsigma^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\right) + (1-\ell)\sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c \left(\overset{\leftrightarrow}{\mathrm{N}}_c + \varsigma^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\right)}, \tag{55}$$

where $\overset{\leftrightarrow}{\partial} = (\overset{\leftrightarrow}{\partial}_1, \overset{\leftrightarrow}{\partial}_2, \ldots, \overset{\leftrightarrow}{\partial}_p)$ denote the weight vector of elements, with $\overset{\leftrightarrow}{\partial}_c \ge 0$ and $\sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c = 1$.

When $\ell = 0.5$, the generalized Dice DIMs (48–51) correspondingly change into Dice DIMs (32) and (34–36), and the weighted generalized Dice DIMs (52–55) correspondingly change into the weighted Dice DIMs (33) and (37–39).

Definition 9 Let $\overset{\leftrightarrow}{F}_1 = \{< \ddot{o}, (\xi_{\overset{\leftrightarrow}{F}_1}(\ddot{o}), \vartheta_{\overset{\leftrightarrow}{F}_1}(\ddot{o}), \tau_{\overset{\leftrightarrow}{F}_1}(\ddot{o}))|\ddot{o} \in \ddot{O}\}$ and $\overset{\leftrightarrow}{F}_2 = \{< \ddot{o}, (\xi_{\overset{\leftrightarrow}{F}_2}(\ddot{o}), \vartheta_{\overset{\leftrightarrow}{F}_2}(\ddot{o}), \tau_{\overset{\leftrightarrow}{F}_2}(\ddot{o}))|\ddot{o} \in \ddot{O}\}$ be two SFSs on $\ddot{O} = \{\ddot{o}_1, \ddot{o}_2, \ldots, \ddot{o}_p\}$, then some generalized Jaccard DIMs between SFSs $\overset{\leftrightarrow}{F}_1$ and $\overset{\leftrightarrow}{F}_2$ are given as:

$$d^1_{GJaccard}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \frac{1}{p}\sum_{c=1}^{p} \frac{\overset{\leftrightarrow}{\Lambda}_c}{\ell \overset{\leftrightarrow}{\mathrm{M}}_c + \hbar \overset{\leftrightarrow}{\mathrm{N}}_c + (1-\ell-\hbar) \overset{\leftrightarrow}{\Lambda}_c}, \tag{56}$$

$$d^2_{GJaccard}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \frac{\sum_{c=1}^{p} \overset{\leftrightarrow}{\Lambda}_c}{\ell \sum_{c=1}^{p} \overset{\leftrightarrow}{\mathrm{M}}_c + \hbar \sum_{c=1}^{p} \overset{\leftrightarrow}{\mathrm{N}}_c + (1-\ell-\hbar) \sum_{c=1}^{p} \overset{\leftrightarrow}{\Lambda}_c}, \tag{57}$$

$$d^3_{GJaccard}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \frac{1}{p}\sum_{c=1}^{p} \frac{\overset{\leftrightarrow}{\Lambda}_c + \varsigma^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\varsigma^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)}{\ell\left(\overset{\leftrightarrow}{\mathrm{M}}_c + \varsigma^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\right) + \hbar\left(\overset{\leftrightarrow}{\mathrm{N}}_c + \varsigma^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\right) + (1-\ell-\hbar)\left(\overset{\leftrightarrow}{\Lambda}_c + \varsigma^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\varsigma^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\right)}, \tag{58}$$

$$d^4_{GJaccard}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \frac{\sum_{c=1}^{p}\left(\overset{\leftrightarrow}{\Lambda}_c + \varsigma^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\varsigma^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\right)}{\ell \sum_{c=1}^{p}\left(\overset{\leftrightarrow}{\mathrm{M}}_c + \varsigma^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\right) + \hbar \sum_{c=1}^{p}\left(\overset{\leftrightarrow}{\mathrm{N}}_c + \varsigma^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\right) + (1-\ell-\hbar)\sum_{c=1}^{p}\left(\overset{\leftrightarrow}{\Lambda}_c + \varsigma^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\varsigma^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\right)}, \tag{59}$$

where $\overset{\leftrightarrow}{\Lambda}_c = \xi^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\xi^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \vartheta^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\vartheta^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \tau^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\tau^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)$, $\overset{\leftrightarrow}{\mathrm{M}}_c = \xi^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) + \vartheta^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) + \tau^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)$, $\overset{\leftrightarrow}{\mathrm{N}}_c = \xi^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \vartheta^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \tau^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)$, $\varsigma_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) = \sqrt{1 - \xi^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) + \vartheta^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c) + \tau^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)}$ and $\varsigma_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) = \sqrt{1 - \xi^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \vartheta^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c) + \tau^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)}$ indicate RD in SFSs $\overset{\leftrightarrow}{F}_1$ and $\overset{\leftrightarrow}{F}_2$ on $\ddot{O}$, respectively, $0 \le \ell, \hbar \le 1$. Accordingly, some weighted generalized Jaccard DIMs between SFSs $\overset{\leftrightarrow}{F}_1$ and $\overset{\leftrightarrow}{F}_2$ are given as:

$$d^1_{\overset{\leftrightarrow}{\partial} GJaccard}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c \frac{\overset{\leftrightarrow}{\Lambda}_c}{\ell \overset{\leftrightarrow}{\mathrm{M}}_c + \hbar \overset{\leftrightarrow}{\mathrm{N}}_c + (1-\ell-\hbar) \overset{\leftrightarrow}{\Lambda}_c}, \tag{60}$$

$$d^2_{\overset{\leftrightarrow}{\partial} GJaccard}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \frac{\sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c \overset{\leftrightarrow}{\Lambda}_c}{\ell \sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c \overset{\leftrightarrow}{\mathrm{M}}_c + \hbar \sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c \overset{\leftrightarrow}{\mathrm{N}}_c + (1-\ell-\hbar) \sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c \overset{\leftrightarrow}{\Lambda}_c}, \tag{61}$$

$$d^3_{\overset{\leftrightarrow}{\partial} GJaccard}(\overset{\leftrightarrow}{F}_1, \overset{\leftrightarrow}{F}_2) = 1 - \sum_{c=1}^{p} \overset{\leftrightarrow}{\partial}_c \frac{\overset{\leftrightarrow}{\Lambda}_c + \varsigma^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\varsigma^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)}{\ell\left(\overset{\leftrightarrow}{\mathrm{M}}_c + \varsigma^4_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\right) + \hbar\left(\overset{\leftrightarrow}{\mathrm{N}}_c + \varsigma^4_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\right) + (1-\ell-\hbar)\left(\overset{\leftrightarrow}{\Lambda}_c + \varsigma^2_{\overset{\leftrightarrow}{F}_1}(\ddot{o}_c)\varsigma^2_{\overset{\leftrightarrow}{F}_2}(\ddot{o}_c)\right)}, \tag{62}$$

$$d^4_{\overleftrightarrow{\partial} GJaccard}(\overleftrightarrow{F}_1, \overleftrightarrow{F}_2)$$

$$= 1 - \frac{\sum_{c=1}^{p} \overleftrightarrow{\partial}_c \left(\overleftrightarrow{\Lambda}_c + \varsigma^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\varsigma^2_{\overleftrightarrow{F}_2}(\ddot{o}_c) \right)}{\ell \sum_{c=1}^{p} \overleftrightarrow{\partial}_c \left(\overleftrightarrow{M}_c + \varsigma^4_{\overleftrightarrow{F}_1}(\ddot{o}_c) \right) + \hbar \sum_{c=1}^{p} \overleftrightarrow{\partial}_c \left(\overleftrightarrow{N}_c + \varsigma^4_{\overleftrightarrow{F}_1}(\ddot{o}_c) \right) + (1 - \ell - \hbar) \sum_{c=1}^{p} \overleftrightarrow{\partial}_c \left(\overleftrightarrow{\Lambda}_c + \varsigma^2_{\overleftrightarrow{F}_1}(\ddot{o}_c)\varsigma^2_{\overleftrightarrow{F}_2}(\ddot{o}_c) \right)}, \tag{63}$$

where $\overleftrightarrow{\partial} = (\overleftrightarrow{\partial}_1, \overleftrightarrow{\partial}_2, \ldots, \overleftrightarrow{\partial}_p)$ denote the weight vector of elements, with $\overleftrightarrow{\partial}_c \geq 0$ and $\sum_{c=1}^{n} \overleftrightarrow{\partial}_c = 1$.

When $\ell = 0.5$ and $\hbar = 0.5$, the generalized Jaccard DIMs (56–59) correspondingly change into generalized Dice DIMs (48–51), the weighted generalized Jaccard DIMs (60–63) correspondingly change into weighted generalized Dice DIMs (52–55).

When $\ell = 1$ and $\hbar = 1$, the generalized Jaccard DIMs (56–59) correspondingly change into Jaccard DIMs (40) and (42–44), and the weighted generalized Jaccard DIMs (60–63) correspondingly change into weighted Jaccard DIMs (41) and (45–47).

4 The CPT–CoCoSo method with D-CRITIC for spherical fuzzy multi-attribute group decision-making problems

Suppose there are p alternatives $St = \{St_1, St_2, \ldots, St_p\}$,$q$ attributes $\overleftrightarrow{\Re} = \{\overleftrightarrow{\Re}_1, \overleftrightarrow{\Re}_2, \ldots, \overleftrightarrow{\Re}_q\}$ and k experts $HF = \{HF_1, HF_2, \ldots, HF_k\}$,$\overleftrightarrow{\nabla} = (\underset{1}{\overleftrightarrow{\nabla}}, \underset{2}{\overleftrightarrow{\nabla}}, \ldots, \underset{k}{\overleftrightarrow{\nabla}})^{\mathrm{T}}$ represents the weight vector of k experts, with $\underset{g}{\overleftrightarrow{\nabla}} \geq 0$, $\sum_{g=1}^{k} \underset{g}{\overleftrightarrow{\nabla}} = 1$, and attribute weights are unknown.

Let $X^g = (x^g_{cd})_{p \times q} = (\xi_{x^g_{cd}}, \vartheta_{x^g_{cd}}, \tau_{x^g_{cd}})_{p \times q}$ represents the decision matrix of the g th expert under SFSs, where $\xi_{x^g_{cd}}$, $\vartheta_{x^g_{cd}}$ and $\tau_{x^g_{cd}}$ denote MD, N-MD and AD of the gth expert respectively, with $\xi_{x^g_{cd}}, \vartheta_{x^g_{cd}}, \tau_{x^g_{cd}} \in [0, 1]$ and $0 \leq \xi^2_{x^g_{cd}} + \vartheta^2_{x^g_{cd}} + \tau^2_{x^g_{cd}} \leq 1$.

Next, we propose the CPT–CoCoSo method with D-CRITIC for MAGDM problems in SFSs.

Step 1. Construct assessment information from DMs utilizing the linguistic terms given in Table 1.

Table 1 Linguistic terms and their corresponding SFNs (Gundogdu and Kahraman 2019b)

Linguistic terms	(ξ, ϑ, τ)
Extremely very important (EVI)	(0.9, 0.1, 0.1)
Very important (VI)	(0.8, 0.2, 0.2)
Important (I)	(0.7, 0.3, 0.3)
Relative important (RI)	(0.6, 0.4, 0.4)
Average important (AI)	(0.5, 0.5, 0.5)
Slightly important (SI)	(0.4, 0.6, 0.4)
Not important (NI)	(0.3, 0.7, 0.3)
Very unimportant (VU)	(0.2, 0.8, 0.2)
Extremely very unimportant (EVU)	(0.1, 0.9, 0.1)

Step 2. Standardize all attribute values by means of (64):

$$U^g = (u_{cd}^g)_{p\times q}, \quad c = 1, 2, \ldots, p, \quad d = 1, 2, \ldots, q, \quad g = 1, 2, \ldots, k,$$

$$u_{cd}^g = (\tilde{\xi}_{u_{cd}^g}, \tilde{\vartheta}_{u_{cd}^g}, \tilde{\tau}_{u_{cd}^g}) = \begin{cases} x_{cd}^g = (\xi_{x_{cd}^g}, \vartheta_{x_{cd}^g}, \tau_{x_{cd}^g}), & \text{for benefit attribute} \\ (x_{cd}^g)^c = (\vartheta_{x_{cd}^g}, \xi_{x_{cd}^g}, \tau_{x_{cd}^g}), & \text{for cost attribute} \end{cases}. \tag{64}$$

Step 3. Aggregate the matrix $U^o = (u_{cd}^g)_{p\times q}(g = 1, 2 \ldots, k)$ to get the group decision matrix $W = (w_{cd})_{p\times q}$ by employing the SWAM operator.

$$w_{cd} = (\underset{w_{cd}}{\overset{\leftrightarrow}{\xi}}, \underset{w_{cd}}{\overset{\leftrightarrow}{\vartheta}}, \underset{w_{cd}}{\overset{\leftrightarrow}{\tau}}) = \text{SWAM}_{\overset{\leftrightarrow}{\nabla}}(u_{cd}^1, u_{cd}^2, \ldots, u_{cd}^k)$$

$$= \left(\begin{array}{c} \left[1 - \prod_{g=1}^{k} (1 - \tilde{\xi}^2_{u_{cd}^g})^{\overset{\leftrightarrow}{\nabla}_g}\right]^{1/2}, \\ \prod_{g=1}^{k} (\tilde{\vartheta}_{u_{cd}^g})^{\overset{\leftrightarrow}{\nabla}_g}, \left[\prod_{g=1}^{k} (1 - \tilde{\xi}^2_{u_{cd}^g})^{\overset{\leftrightarrow}{\nabla}_g} - \prod_{g=1}^{k} (1 - \tilde{\xi}^2_{u_{cd}^g} - \tilde{\tau}^2_{u_{cd}^g})^{\overset{\leftrightarrow}{\nabla}_g}\right]^{1/2} \end{array} \right). \tag{65}$$

Step 4. Get the attribute weights by the D-CRITIC method.

Step 4.1. Since the normalization of the decision matrices are completed in Step 2, then compute the average solution for each attribute by (66):

$$\overline{w}_d = \frac{1}{p} \bigoplus_{c=1}^{p} w_{cd}, \; d = 1, 2, \ldots, q. \tag{66}$$

Step 4.2. Determine the standard deviation about each attribute (67):

$$\upsilon_d = \sqrt{\frac{\sum_{c=1}^{p} (d(w_{cd}, \overline{w}_d))^2}{p-1}}, \quad d = 1, 2, \ldots, q, \tag{67}$$

where $d(w_{cd}, \overline{w}_d)$ indicates spherical fuzzy Dice distance measure calculated by (32).

Step 4.3. Calculate the distance correlation of every pair of attributes.

Step 4.3.1. Construct the distance matrix for each attribute based on all alternatives by (68):

$$R^d = (r_{ce}^d)_{p\times p} = (d(w_{cd}, w_{ed}))_{p\times p}, \quad c, e = 1, 2, \ldots, p; \quad d = 1, 2, \ldots, q. \tag{68}$$

Here, R^d stands for Dice distance matrix about attribute $\Re_d$, and $d(w_{cd}, w_{ed})$ denotes Dice distance between w_{cd} and w_{ed} computed by (32).

Step 4.3.2. Obtain the double-centered matrix T^d for attribute $\overset{\leftrightarrow}{\Re}_d$ with (69):

$$T^d = (t_{ce}^d)_{p\times p}$$

$$t_{ce}^d = r_{ce}^d - \frac{1}{p}\sum_{e=1}^{p} r_{ce}^d - \frac{1}{p}\sum_{c=1}^{p} r_{ce}^d + \frac{1}{p^2}\sum_{c=1}^{p}\sum_{e=1}^{p} r_{ce}^d, \quad d = 1, 2, \ldots, q, \tag{69}$$

where $\frac{1}{p}\sum_{e=1}^{p} r_{ce}^d$ and $\frac{1}{p}\sum_{c=1}^{p} r_{ce}^d$ denote the average of the c th row and d th column of distance matrix R^d, respectively, and $\frac{1}{p^2}\sum_{c=1}^{p}\sum_{e=1}^{p} r_{ce}^d$ represents the total average of R^d.

Step 4.3.3. Determine the distance covariance of attributes $\Re_d$ and $\Re_f$ by (70):

$$d\text{Cov}\left(\overset{\leftrightarrow}{\Re}_d, \overset{\leftrightarrow}{\Re}_f\right) = \sqrt{\frac{\sum_{c=1}^{p}\sum_{e=1}^{p} t_{ce}^d \cdot t_{ce}^f}{p^2}}, \quad d, f = 1, 2, \ldots, q. \tag{70}$$

Step 4.3.4. Calculate the distance variance for attribute $\overleftrightarrow{\Re}_d$ depending on (71):

$$d\mathrm{Var}(\overleftrightarrow{\Re}_d) = d\mathrm{Cov}(\overleftrightarrow{\Re}_d, \overleftrightarrow{\Re}_d), \quad d = 1, 2, \ldots, q. \tag{71}$$

Step 4.3.5. Obtain the distance correlation between attributes with (72):

$$d\mathrm{Cor}\left(\overleftrightarrow{\Re}_d, \overleftrightarrow{\Re}_f\right) = \frac{d\mathrm{Cov}\left(\overleftrightarrow{\Re}_d, \overleftrightarrow{\Re}_f\right)}{\sqrt{d\mathrm{Var}(\overleftrightarrow{\Re}_d) \cdot d\mathrm{Var}(\overleftrightarrow{\Re}_f)}}, \quad d, f = 1, 2, \ldots, q. \tag{72}$$

Step 4.4 Compute the information content $\mho_d$ for each attribute by (73):

$$\mho_d = \upsilon_d \cdot \sum_{f=1}^{q} \left(1 - d\mathrm{Cor}(\overleftrightarrow{\Re}_d, \overleftrightarrow{\Re}_f)\right), \quad d = 1, 2, \ldots, q. \tag{73}$$

Step 4.5 Get the attribute weights by (74):

$$\overleftrightarrow{\partial}_d = \frac{\mho_d}{\sum_{d=1}^{q} \mho_d}, \quad d = 1, 2, \ldots, q, \tag{74}$$

where $\overleftrightarrow{\partial}_d$ indicate weight about attribute $\overleftrightarrow{\Re}_d$.

Step 5. Determine the prospect matrix $\overleftrightarrow{L} = (\overleftrightarrow{l}_{cd})_{p\times q}$.

$$\overleftrightarrow{l}_{cd} = \begin{cases} (d(w_{cd}, \overline{w}_d))^{\delta}, & \text{if } \mathrm{Sc}(w_{cd}) \geq \mathrm{Sc}(\overline{w}_d) \\ -\phi(d(w_{cd}, \overline{w}_d))^{\hbar}, & \text{if } \mathrm{Sc}(w_{cd}) < \mathrm{Sc}(\overline{w}_d) \end{cases}, \tag{75}$$

where we take the average solution under each attribute as the reference point to establish the prospect matrix, $\overline{w}_d$ denotes the average solution about the dth attribute computed by (66), and $d(w_{cd}, \overline{w}_d)$ is the Dice distance measure between w_{cd} and $\overline{w}_d$ computed by (32). In addition, $\delta = 0.88$, $\hbar = 0.88$, $\phi = 2.25$ is based on Tversky and Kahneman (1992) and they have been accepted by most scholars.

Step 6. Figure out the probability weight matrix.

Put prospect value $\overleftrightarrow{l}_{cd}$ in ascending order as $l'_{c1} \leq \cdots \leq l'_{ch} \leq 0 \leq l'_{c(h+1)} \leq \cdots \leq l'_{cq}$, $c = 1, 2, \ldots, p$; l'_{cd} occurs with probability $\overleftrightarrow{\partial}_d$, then

$$\infty_{cd} = \begin{cases} \kappa^{-}(\overleftrightarrow{\partial}_h), & \text{if } h = 1 \\ \kappa^{-}(\overleftrightarrow{\partial}_1 + \cdots + \overleftrightarrow{\partial}_d) - \kappa^{-}(\overleftrightarrow{\partial}_1 + \cdots + \overleftrightarrow{\partial}_{d-1}), & \text{if } 2 \leq d \leq h \\ \kappa^{+}(\overleftrightarrow{\partial}_d), & \text{if } d = q \\ \kappa^{+}(\overleftrightarrow{\partial}_d + \cdots + \overleftrightarrow{\partial}_q) - \kappa^{+}(\overleftrightarrow{\partial}_{d+1} + \cdots + \overleftrightarrow{\partial}_q), & \text{if } h+1 \leq d \leq q-1 \end{cases}, \tag{76}$$

and

$$\kappa^{+}(\overleftrightarrow{\partial}) = \frac{\overleftrightarrow{\partial}^{\lambda}}{(\overleftrightarrow{\partial}^{\lambda} + (1 - \overleftrightarrow{\partial})^{\lambda})^{\frac{1}{\lambda}}}, \quad \kappa^{-}(\overleftrightarrow{\partial}) = \frac{\overleftrightarrow{\partial}^{\beta}}{(\overleftrightarrow{\partial}^{\beta} + (1 - \overleftrightarrow{\partial})^{\beta})^{\frac{1}{\beta}}}. \tag{77}$$

Here, $\lambda = 0.61$, $\beta = 0.69$ by experience of Tversky and Kahneman (1992).

Step 7. Normalize the prospect matrix and the probability weight matrix.

Because the distance of two SFNs range is from 0 and 1, then $-2.25 \leq \overset{\leftrightarrow}{l}_{cd} \leq 1$, normalizing the prospect matrix to make $\overset{\leftrightarrow}{l}_{cd}$ between 0 and 1 with (78):

$$l^*_{cd} = \frac{\overset{\leftrightarrow}{l}_{cd} - (-2.25)}{1 - (-2.25)}. \tag{78}$$

Furthermore, normalize the probability weight matrix with (79):

$$\infty^*_{cd} = \frac{\infty_{cd}}{\sum_{d=1}^{q} \infty_{cd}}. \tag{79}$$

Step 8. Calculate the prospect weighted sum sequence $\Theta_c(c = 1, 2, \ldots p)$ and the prospect weighted product sequence $\Omega_c(c = 1, 2, \ldots p)$ depending on (80) and (81):

$$\Theta_c = \sum_{d=1}^{q} \infty^*_{cd} \cdot l^*_{cd}, \tag{80}$$

$$\Omega_c = \sum_{d=1}^{q} \left(l^*_{cd}\right)^{\infty^*_{cd}}. \tag{81}$$

Step 9. Determine three score strategies of alternatives by (82–84):

$$L_c^1 = \frac{\Theta_c + \Omega_c}{\sum_{c=1}^{p} (\Theta_c + \Omega_c)}, \tag{82}$$

$$L_c^2 = \frac{\Theta_c}{\min_c \Theta_c} + \frac{\Omega_c}{\min_c \Omega_c}, \tag{83}$$

$$L_c^3 = \frac{\hat{\beta}\Theta_c + (1 - \hat{\beta})\Omega_c}{\hat{\beta} \max_c \Theta_c + (1 - \hat{\beta}) \max_c \Omega_c}, \tag{84}$$

where $\hat{\beta}$ indicates compromise coefficient,$0 \leq \hat{\beta} \leq 1$.

Step 10. Integrate the above three score strategies by (85), and sort all alternatives according to the value of L_c, thenthe maximum value of L_c is the best alternative (Fig. 1).

$$L_c = (L_c^1 \cdot L_c^2 \cdot L_c^3) + \frac{1}{3}(L_c^1 + L_c^2 + L_c^3). \tag{85}$$

To facilitate DMs to evaluate candidate schemes in spherical fuzzy environment, Gundogdu and Kahraman (2019b) put forward the linguistic terms for SFNs in Table 1. In this paper, we will utilize the above linguistic terms to invite experts for evaluating LSEVCS.

4.1 An example for spherical fuzzy multi-attribute group decision making

The fast development of economy has increased the rate of car purchases, but the problems of environmental pollution and excessive energy consumption caused by it have become increasingly prominent. As a new energy vehicle, EV has quickly entered into the public view with low emissions and low noise characteristics, and has become the main choice for the replacement of traditional fuel vehicle to reduce environmental pollution. However, reasonable LSEVCS is an important guarantee for promoting the development of the EV market and meeting the charging demands of EV users. Therefore, in this section we use

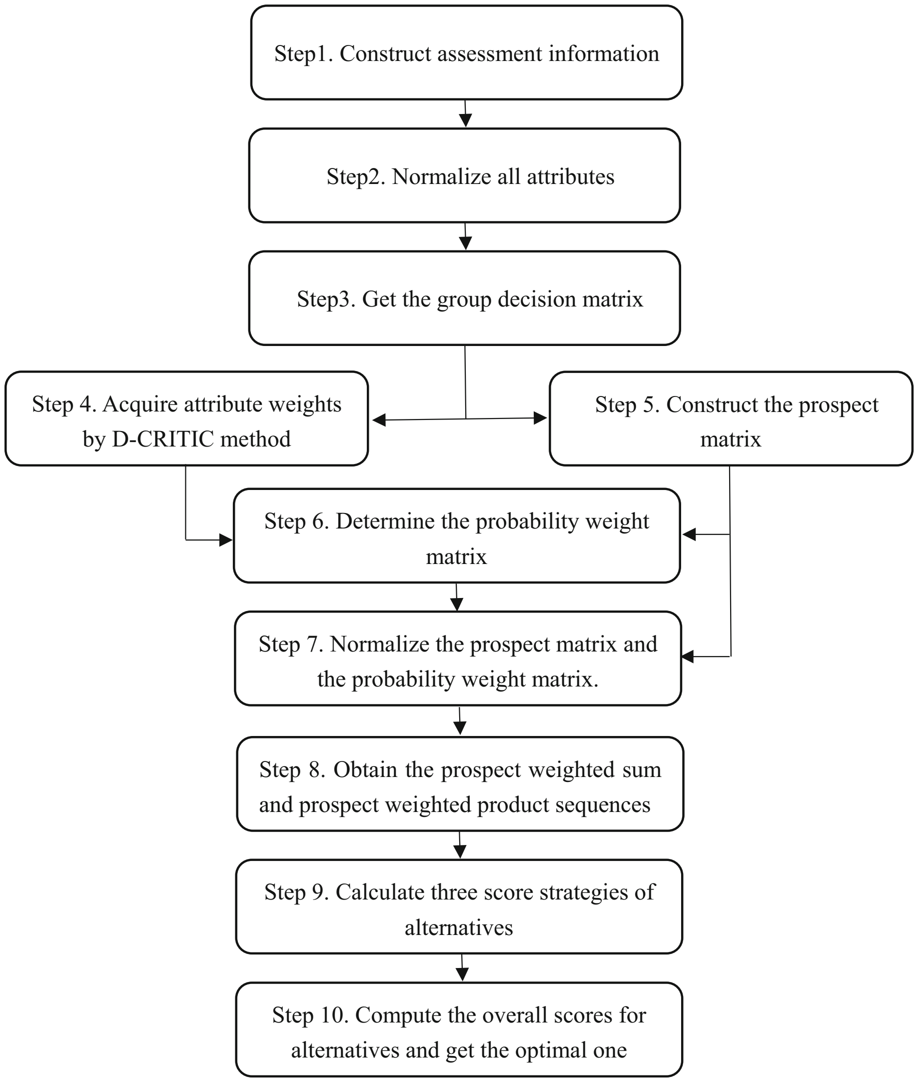

Fig. 1 The flowchart of the SF-CPT–CoCoSo method

the SF-CPT–CoCoSo model to deal with an example of LSEVCS with SFNs. Suppose that there are three experts (HF_1, HF_2, HF_3) to assess five possible EV charging stations $St_c(c = 1, \ldots, 5)$ through the following four attributes: $\overleftrightarrow{\Re}_1$ (traffic density), $\overleftrightarrow{\Re}_2$ (service level), $\overleftrightarrow{\Re}_3$ (charging demand), $\overleftrightarrow{\Re}_4$ (construction cost), and except for $\overleftrightarrow{\Re}_4$, the others are benefit attributes. $\overleftrightarrow{\nabla} = (0.20, 0.40, 0.40)$ represents expert's weights, but the attribute weights are unknown. The evaluation information from HF_1, HF_2, HF_3 are displayed in Table 2.

Step 1. The evaluation information of experts is given in Table 2, so Step 1 is completed.

Step 2. Normalize all attributes with (64) as shown in Tables 3, 4 and 5.

Table 2 Assessment information of experts

DMs	Alternatives	$\overleftrightarrow{\Re}_1$	$\overleftrightarrow{\Re}_2$	$\overleftrightarrow{\Re}_3$	$\overleftrightarrow{\Re}_4$
HF_1	St_1	AI	I	NI	SI
	St_2	VU	RI	RI	VI
	St_3	VU	NI	EVI	SI
	St_4	RI	AI	AI	EVI
	St_5	I	AI	SI	AI
HF_2	St_1	RI	NI	SI	AI
	St_2	VI	AI	AI	SI
	St_3	NI	NI	NI	I
	St_4	SI	SI	NI	NI
	St_5	RI	SI	I	RI
HF_3	St_1	I	NI	VU	RI
	St_2	RI	I	I	NI
	St_3	I	SI	NI	VU
	St_4	RI	NI	RI	AI
	St_5	AI	RI	SI	NI

Table 3 Normalized decision matrix from HF_1

Alternatives	$\overleftrightarrow{\Re}_1$	$\overleftrightarrow{\Re}_2$	$\overleftrightarrow{\Re}_3$	$\overleftrightarrow{\Re}_4$
St_1	(0.50, 0.50, 0.50)	(0.70, 0.30, 0.30)	(0.30, 0.70, 0.30)	(0.60, 0.40, 0.40)
St_2	(0.20, 0.80, 0.20)	(0.60, 0.40, 0.40)	(0.60, 0.40, 0.40)	(0.20, 0.80, 0.20)
St_3	(0.20, 0.80, 0.20)	(0.30, 0.70, 0.30)	(0.90, 0.10, 0.10)	(0.60, 0.40, 0.40)
St_4	(0.60, 0.40, 0.40)	(0.50, 0.50, 0.50)	(0.50, 0.50, 0.50)	(0.10, 0.90, 0.10)
St_5	(0.70, 0.30, 0.30)	(0.50, 0.50, 0.50)	(0.40, 0.60, 0.40)	(0.50, 0.50, 0.50)

Table 4 Normalized decision matrix of HF_2

Alternatives	$\overleftrightarrow{\Re}_1$	$\overleftrightarrow{\Re}_2$	$\overleftrightarrow{\Re}_3$	$\overleftrightarrow{\Re}_4$
St_1	(0.60, 0.40, 0.40)	(0.30, 0.70, 0.30)	(0.40, 0.60, 0.40)	(0.50, 0.50, 0.50)
St_2	(0.80, 0.20, 0.20)	(0.50, 0.50, 0.50)	(0.50, 0.50, 0.50)	(0.60, 0.40, 0.40)
St_3	(0.30, 0.70, 0.30)	(0.30, 0.70, 0.30)	(0.30, 0.70, 0.30)	(0.30, 0.70, 0.30)
St_4	(0.40, 0.60, 0.40)	(0.40, 0.60, 0.40)	(0.30, 0.70, 0.30)	(0.70, 0.30, 0.30)
St_5	(0.60, 0.40, 0.40)	(0.40, 0.60, 0.40)	(0.70, 0.30, 0.30)	(0.40, 0.60, 0.40)

Table 5 Normalized decision matrix of HF_3

Alternatives	$\overset{\leftrightarrow}{\Re}_1$	$\overset{\leftrightarrow}{\Re}_2$	$\overset{\leftrightarrow}{\Re}_3$	$\overset{\leftrightarrow}{\Re}_4$
St_1	(0.70, 0.30, 0.30)	(0.30, 0.70, 0.30)	(0.20, 0.80, 0.20)	(0.40, 0.60, 0.40)
St_2	(0.60, 0.40, 0.40)	(0.70, 0.30, 0.30)	(0.70, 0.30, 0.30)	(0.70, 0.30, 0.30)
St_3	(0.70, 0.30, 0.30)	(0.40, 0.60, 0.40)	(0.30, 0.70, 0.30)	(0.80, 0.20, 0.20)
St_4	(0.60, 0.40, 0.40)	(0.30, 0.70, 0.30)	(0.60, 0.40, 0.40)	(0.50, 0.50, 0.50)
St_5	(0.50, 0.50, 0.50)	(0.60, 0.40, 0.40)	(0.40, 0.60, 0.40)	(0.70, 0.30, 0.30)

Step 3. Aggregate individual assessment information to obtain group decision matrix by () (see Table 6).

Step 4. Calculate the attribute weights by the D-CRITIC approach with (66–74). The calculation processes are shown in Tables 7, 8, 9, 10 and 11.

Step 5. Determine the prospect matrix based on (75) as shown in Table 12 [where taking $\delta = 0.88$, $\hbar = 0.88$, $\phi = 2.25$ by experience of Tversky and Kahneman (1992)].

Step 6. Calculate the probability weight matrix with (76–77) shown in Table 13 [here taking$\lambda = 0.61$, $\beta = 0.69$ by experience of Tversky and Kahneman (1992)].

Step 7. Normalize the prospect matrix and probability weight matrix using (78–79) (see Tables 14, 15).

Table 6 The group decision matrix $W = (w_{et})_{5\times 4}$

Alternatives	$\overset{\leftrightarrow}{\Re}_1$	$\overset{\leftrightarrow}{\Re}_2$	$\overset{\leftrightarrow}{\Re}_3$	$\overset{\leftrightarrow}{\Re}_4$
St_1	(0.63, 0.37, 0.38)	(0.44, 0.59, 0.31)	(0.32, 0.69, 0.32)	(0.49, 0.51, 0.45)
St_2	(0.67, 0.35, 0.29)	(0.61, 0.39, 0.40)	(0.61, 0.39, 0.40)	(0.61, 0.41, 0.34)
St_3	(0.52, 0.51, 0.30)	(0.34, 0.66, 0.35)	(0.58, 0.47, 0.24)	(0.64, 0.38, 0.28)
St_4	(0.54, 0.47, 0.40)	(0.39, 0.62, 0.40)	(0.49, 0.52, 0.40)	(0.57, 0.46, 0.38)
St_5	(0.59, 0.41, 0.42)	(0.51, 0.49, 0.42)	(0.56, 0.45, 0.36)	(0.57, 0.44, 0.38)

Table 7 The average solution for each attribute

Average solution	$\overset{\leftrightarrow}{\Re}_1$	$\overset{\leftrightarrow}{\Re}_2$	$\overset{\leftrightarrow}{\Re}_3$	$\overset{\leftrightarrow}{\Re}_4$
$\overline{w}_d$	(0.59, 0.42, 0.36)	(0.48, 0.54, 0.38)	(0.53, 0.50, 0.35)	(0.58, 0.13, 0.37)

Table 8 Standard deviation of attributes

Standard deviation	$\overset{\leftrightarrow}{\Re}_1$	$\overset{\leftrightarrow}{\Re}_2$	$\overset{\leftrightarrow}{\Re}_3$	$\overset{\leftrightarrow}{\Re}_4$
υ_d	0.03384	0.07832	0.11200	0.16030

Table 9 Distance correlation matrix

Attributes	$\overleftrightarrow{\Re}_1$	$\overleftrightarrow{\Re}_2$	$\overleftrightarrow{\Re}_3$	$\overleftrightarrow{\Re}_4$
$\overleftrightarrow{\Re}_1$	1.00000	0.59529	0.22404	0.37011
$\overleftrightarrow{\Re}_2$	0.59529	1.00000	0.56770	0.22609
$\overleftrightarrow{\Re}_3$	0.22404	0.56770	1.00000	0.87974
$\overleftrightarrow{\Re}_4$	0.37011	0.22609	0.87974	1.00000

Table 10 Information content for each attribute

Information content	$\overleftrightarrow{\Re}_1$	$\overleftrightarrow{\Re}_2$	$\overleftrightarrow{\Re}_3$	$\overleftrightarrow{\Re}_4$
$\mho_d$	0.06062	0.12617	0.14880	0.24431

Table 11 Attribute weights $\overleftrightarrow{\partial}_d$

Attribute weight	$\overleftrightarrow{\Re}_1$	$\overleftrightarrow{\Re}_2$	$\overleftrightarrow{\Re}_3$	$\overleftrightarrow{\Re}_4$
$\overleftrightarrow{\partial}_d$	0.10454	0.21757	0.25659	0.42129

Table 12 Prospect matrix

Alternatives	$\overleftrightarrow{\Re}_1$	$\overleftrightarrow{\Re}_2$	$\overleftrightarrow{\Re}_3$	$\overleftrightarrow{\Re}_4$
St_1	0.01587	− 0.08243	− 0.58043	0.29707
St_2	0.05297	0.15697	0.08167	0.10722
St_3	− 0.16404	− 0.26069	0.03871	0.10509
St_4	− 0.08008	− 0.12576	− 0.04285	0.16262
St_5	− 0.02371	0.02565	0.01576	0.13784

Table 13 Probability weight matrix

Alternatives	$\overleftrightarrow{\Re}_1$	$\overleftrightarrow{\Re}_2$	$\overleftrightarrow{\Re}_3$	$\overleftrightarrow{\Re}_4$
St_1	0.05323	0.13973	0.29812	0.38076
St_2	0.29425	0.27165	0.20968	0.22443
St_3	0.07190	0.27020	0.13894	0.38076
St_4	0.07190	0.27020	0.16202	0.38076
St_5	0.17475	0.11532	0.20968	0.38076

Table 14 Normalized prospect matrix

Alternatives	$\overset{\leftrightarrow}{\Re}_1$	$\overset{\leftrightarrow}{\Re}_2$	$\overset{\leftrightarrow}{\Re}_3$	$\overset{\leftrightarrow}{\Re}_4$
St_1	0.69719	0.66694	0.51371	0.78371
St_2	0.70860	0.74061	0.71744	0.72530
St_3	0.64184	0.61210	0.70422	0.72464
St_4	0.66767	0.65361	0.67912	0.74234
St_5	0.68501	0.70020	0.69716	0.73472

Table 15 Normalized probability weight matrix

Alternatives	$\overset{\leftrightarrow}{\Re}_1$	$\overset{\leftrightarrow}{\Re}_2$	$\overset{\leftrightarrow}{\Re}_3$	$\overset{\leftrightarrow}{\Re}_4$
St_1	0.06106	0.16026	0.34195	0.43673
St_2	0.29425	0.27165	0.20968	0.22443
St_3	0.08343	0.31353	0.16122	0.44182
St_4	0.08126	0.30535	0.18309	0.43030
St_5	0.19846	0.13097	0.23813	0.43244

Table 16 Weighted sum sequence Θ_c and weight product sequence Ω_c

Alternatives	Θ_c	Ω_c
St_1	0.66739	3.61071
St_2	0.72290	3.68848
St_3	0.67915	3.63343
St_4	0.69761	3.65721
St_5	0.71139	3.67494

Step 8. Calculate the prospect weighted sum and the prospect weighted product sequences by (80–81) as displayed in Table 16.

Step 9. Determine three score strategies of alternatives by (82–84) (where we take $\hat{\beta} = 0.5$) (see Table 17).

Table 17 Three score strategies of alternatives

Alternatives	L_c^1	L_c^2	L_c^3
St_1	0.19676	2.00000	0.96979
St_2	0.20289	2.10471	1.00000
St_3	0.19834	2.02392	0.97761
St_4	0.20028	2.05815	0.98718
St_5	0.20173	2.08371	0.99432

Table 18 Total scores of alternatives and sorting

Alternatives	The total score strategies L_c	Ranking
St_1	1.78086	$St_2 > St_5 > St_4 > St_3 > St_1$
St_2	1.85556	
St_3	1.79876	
St_4	1.82291	
St_5	1.84094	

Step 10. Obtain the total score for each alternative by (85) and sort all alternatives, as shown in Table 18.

4.2 Sensitivity analysis

In a realistic decision-making procedure, different DMs may have different subjective preferences for compromise coefficient $\hat{\beta}$, and different values of $\hat{\beta}$ may have some effect on decision results. Hence, we take 11 groups of values of $\hat{\beta}$ with a step size of 0.1 in the range of 0 to 1 to observe the influence of its change on the results (see Table 19).

By Figs. 2 and 3, we can draw the following conclusions:

(1) Taking different values of $\hat{\beta}$, the scores of alternative St_2 remain constant and are the largest among all alternatives. (2) Except for alternative St_2, the scores of other alternatives decreased with the increase of $\hat{\beta}$ value. (3) For 11 different sets of $\hat{\beta}$ values, the rankings of alternatives are always unchanged:$St_2 > St_5 > St_4 > St_3 > St_1$. The above analysis effectively shows that the developed model is insensitive to different values of compromise coefficient $\hat{\beta}$. In conclusion, the stability and dependability for SF-CPT–CoCoSo model in settling MAGDM issues is verified through the sensitivity investigation of LSEVCS.

Table 19 Sensitivity analysis of compromise coefficient $\hat{\beta}$ to alternative results

$\hat{\beta}$	St_1	St_2	St_3	St_4	St_5
$\hat{\beta} = 0.0$	1.78617	1.85556	1.80311	1.82544	1.84211
$\hat{\beta} = 0.1$	1.78548	1.85556	1.80254	1.82511	1.84195
$\hat{\beta} = 0.2$	1.78466	1.85556	1.80187	1.82472	1.84177
$\hat{\beta} = 0.3$	1.78366	1.85556	1.80105	1.82425	1.84155
$\hat{\beta} = 0.4$	1.78243	1.85556	1.80004	1.82366	1.84128
$\hat{\beta} = 0.5$	1.78086	1.85556	1.79876	1.82291	1.84094
$\hat{\beta} = 0.6$	1.77880	1.85556	1.79707	1.82193	1.84048
$\hat{\beta} = 0.7$	1.77599	1.85556	1.79477	1.82059	1.83986
$\hat{\beta} = 0.8$	1.77190	1.85556	1.79142	1.81864	1.83896
$\hat{\beta} = 0.9$	1.76541	1.85556	1.78611	1.81556	1.83754
$\hat{\beta} = 1.0$	1.75353	1.85556	1.77641	1.80993	1.83494

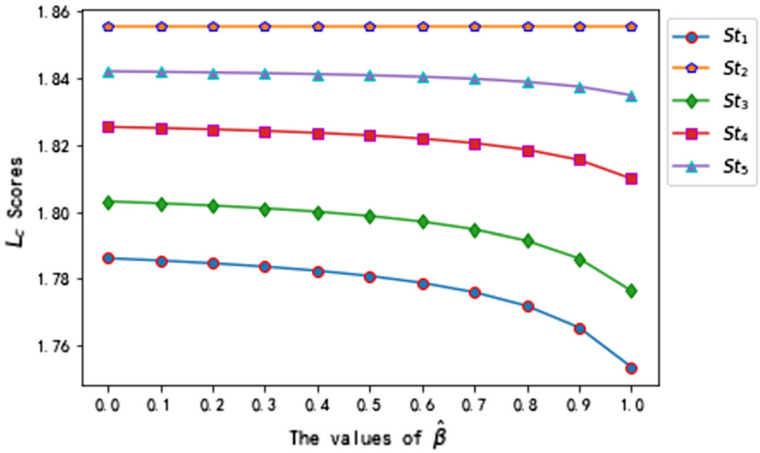

Fig. 2 Sensitivity analysis of compromise coefficient $\hat{\beta}$ to scores of alternatives

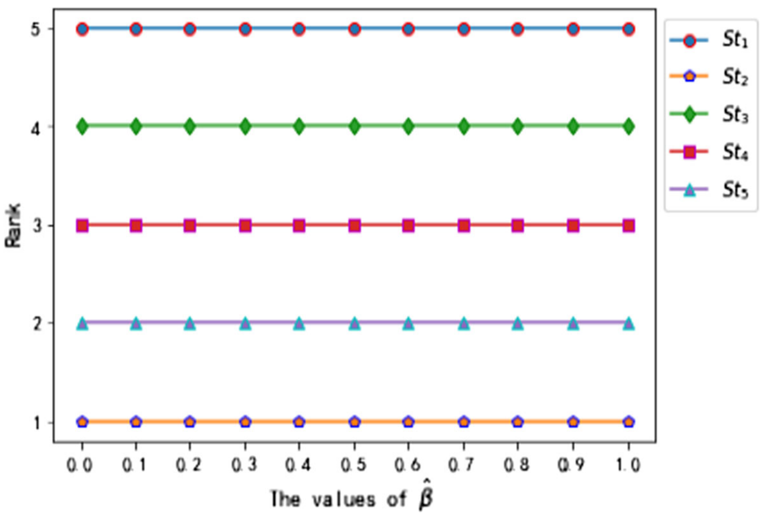

Fig. 3 Sensitivity analysis of compromise coefficient $\hat{\beta}$ to rankings of alternatives

4.3 Comparative analysis

4.3.1 Compare D-CRITIC method with some existing objective weighting methods

For this subpart, we employ some existing objective weight methods under SFSs [such as entropy method (Phi-Hung et al. 2022; Peng and Li 2022), CRITIC method (Ali 2021)] to calculate attribute weights, so as to illustrate the advantages of using D-CRITIC method in this paper. The calculation results are as follows:

As can be seen from Table 20, the ranking of attribute weights via using D-CRITIC method is obviously different from the entropy method and CRITIC method. The main

Table 20 Attribute weights and ranking under different objective weighting methods

Methods	$\overset{\leftrightarrow}{\Re}_1$	$\overset{\leftrightarrow}{\Re}_2$	$\overset{\leftrightarrow}{\Re}_3$	$\overset{\leftrightarrow}{\Re}_4$
Entropy method (Phi-Hung et al. 2022)	0.25017	0.25988	0.26171	0.22825
Ranking	3	2	1	4
Entropy method (Peng and Li 2022)	0.26333	0.23211	0.24574	0.25883
Ranking	1	4	3	2
CRITIC method (Ali 2021)	0.29325	0.16890	0.26410	0.27375
Ranking	1	4	3	2
D-CRITIC method in this paper	0.10454	0.21757	0.25659	0.42129
Ranking	4	3	2	1

reason is that entropy method only considers the contrast intensity to each attribute in the process of calculating the weight of attributes, whereas CRITIC method considers both the contrast intensity and the conflicting relationship held by each attribute. However, the CRITIC method has a flaw in appropriately capturing the conflicting relationship between attributes, because it only uses Pearson correlation to model the linear relationship between attributes. More precisely, two attributes with a zero Pearson correlation coefficient may not be completely independent. As an improved version of the CRITIC method, the D-CRITIC method introduces the idea of distance correlation on the basis of CRITIC method, which can effectively simulate the nonlinear relationship between attributes. Therefore, the D-CRITIC method is more reasonable and reliable for obtaining attribute weights.

4.3.2 Compare SF-CPT–CoCoSo with some spherical fuzzy operators

In this subsection, we compare the SF-CPT–CoCoSo model with SWAM (Gundogdu and Kahraman 2019b) operator, SWGM (Gundogdu and Kahraman 2019b) operator, spherical weighted averaging aggregation (SFNWAA) (Ashraf et al. 2019b) operator, spherical weighted geometric aggregation (SFNWGA) (Ashraf et al. 2019b) operator, spherical fuzzy weighted averaging interaction (SFWAI) (Ju et al. 2021) operator as well as spherical fuzzy weighted geometric interaction (SFWGI) (Ju et al. 2021) operator. The results are displayed in Tables 21, 22, 23, 24, 25 and 26. St_2 always remains the optimal alternative.

Table 21 SWAM operator and ranking

Alternatives	SWAM	Scores	Ranking
St_1	(0.46272, 0.55357, 0.38974)	– 0.02152	$St_2 > St_3 > St_5 > St_4 > St_1$
St_2	(0.61690, 0.39333, 0.36598)	0.06221	
St_3	(0.56869, 0.46727, 0.28837)	0.04658	
St_4	(0.51216, 0.50697, 0.39299)	0.00121	
St_5	(0.55880, 0.45094 0.38939)	0.02491	

Table 22 SWGM operator and ranking

Alternatives	SWGM	Scores	Ranking
St_1	(0.40207, 0.57505 0.37148)	– 0.04051	$St_2 > St_5 > St_1 > St_4 > St_3$
St_2	(0.55228, 0.46229, 0.36951)	0.02480	
St_3	(0.41815, 0.58802, 0.30894)	– 0.06595	
St_4	(0.41998, 0.58142, 0.36789)	– 0.04288	
St_5	(0.51543, 0.47794, 0.40452)	0.00691	

Table 23 SFNWAA operator and ranking

Alternatives	SFNWAA	Scores	Ranking
St_1	(0.46272, 0.55357, 0.35539)	0.51792	$St_2 > St_3 > St_5 > St_4 > St_1$
St_2	(0.61690, 0.39333, 0.33997)	0.62787	
St_3	(0.56869, 0.46727, 0.27585)	0.60852	
St_4	(0.51216, 0.50697, 0.34049)	0.55490	
St_5	(0.55880, 0.45094, 0.38195)	0.57531	

Table 24 SFNWGA operator and ranking

Alternatives	SFNWGA	Scores	Ranking
St_1	(0.40207, 0.60834, 0.35539)	0.47945	$St_2 > St_5 > St_3 > St_4 > St_1$
St_2	(0.55228, 0.47090, 0.33997)	0.58047	
St_3	(0.41815, 0.60407, 0.27585)	0.51274	
St_4	(0.41998, 0.59965, 0.34049)	0.49328	
St_5	(0.51543, 0.49537, 0.38195)	0.54604	

Table 25 SFWAI operator and ranking

Alternatives	SFWAI	Scores	Ranking
St_1	(0.46272, 0.57435, 0.38974)	0.36617	$St_2 > St_3 > St_5 > St_4 > St_1$
St_2	(0.61690, 0.42749, 0.36598)	0.53194	
St_3	(0.56869, 0.53468, 0.28837)	0.47719	
St_4	(0.51216, 0.54388, 0.39299)	0.40603	
St_5	(0.55880, 0.47588, 0.38939)	0.46709	

Table 26 SFWGI operator and ranking

Alternatives	SFWGI	Scores	Ranking
St_1	(0.43399, 0.60834, 0.37076)	0.34040	$St_2 > St_5 > St_3 > St_4 > St_1$
St_2	(0.58224, 0.47090, 0.36947)	0.49038	
St_3	(0.48146 0.60407, 0.30941)	0.38558	
St_4	(0.46634, 0.59965, 0.36810)	0.36120	
St_5	(0.53052, 0.49537, 0.40436)	0.43628	

Table 27 The closeness ratios and ranking of alternatives using the SF-TOPSIS method

	St_1	St_2	St_3	St_4	St_5
Closeness ratios	0.21682	0.85700	0.52072	0.42338	0.61473
Ranking	$St_2 > St_5 > St_3 > St_4 > St_1$				

Table 28 The scores and ranking of alternatives by the SF-CODAS method

		St_1	St_2	St_3	St_4	$St_5 St_5$
Assessment scores	– 0.32480		0.34118	0.08234	– 0.16152	0.06280
Ranking	$St_2 > St_3 > St_5 > St_4 > St_1$					

Table 29 The joint generalized scores and ranking of alternatives by the SF-WASPAS method

	St_1	St_2	St_3	St_4	St_5
Joint generalized scores	– 0.02850	0.06186	0.03105	– 0.00136	0.02446
Ranking	$St_2 > St_3 > St_5 > St_4 > St_1$				

Table 30 The scores and ranking of alternatives by the SF-CoCoSo method

	St_1	St_2	St_3	St_4	St_5
Joint generalized scores	1.69514	2.04242	1.90683	1.80333	1.91237
Ranking	$St_2 > St_5 > St_3 > St_4 > St_1$				

4.3.3 Comparison of the SF-CPT–CoCoSo method with some existing multi-attribute group decision-making approaches in spherical fuzzy sets

In this subpart, we employ the spherical fuzzy TOPSIS (SF-TOPSIS) (Kutlu Gündoğdu and Kahraman 2021) approach, spherical fuzzy CODAS (SF-CODAS) (Gundogdu and Kahraman 2019c) approach, spherical fuzzy WASPAS (SF-WASPAS) (Gundogdu and Kahraman 2019d) approach and spherical fuzzy CoCoSo (SF-CoCoSo) (Peng and Li 2022) approach to attest the legality for the developed method. In the light of the assessment information in Table 2 and attribute weights, the results are displayed in Tables 27, 28, 29 and 30, respectively.

Clearly, as you can see from Tables 27, 28, 29 and 30, alternative St_2 is always the best and alternative and St_1 is always the worst. Moreover, Table 31 shows the ranking for different approaches.

4.3.4 Contrastive analysis

From Table 31, although the ranking of SF-CPT–CoCoSo method is somewhat different from the existing methods, the optimal alternative selection of all methods is consistent. The above comparative analysis firmly demonstrates the feasibility and rationality of our presented model. Among the existing methods, each method has its own advantages in decision

Table 31 The ranking for different methods

Methods	Rankings
SWAM (Gundogdu and Kahraman 2019b)	$St_2 > St_3 > St_5 > St_4 > St_1$
SWGM (Gundogdu and Kahraman 2019b)	$St_2 > St_5 > St_1 > St_4 > St_3$
SFNWAA (Ashraf et al. 2019b)	$St_2 > St_3 > St_5 > St_4 > St_1$
SFNWGA (Ashraf et al. 2019b)	$St_2 > St_5 > St_3 > St_4 > St_1$
SFWAI (Ju et al. 2021)	$St_2 > St_3 > St_5 > St_4 > St_1$
SFWGI (Ju et al. 2021)	$St_2 > St_5 > St_3 > St_4 > St_1$
SF-TOPSIS (Kutlu Gündoğdu and Kahraman 2021)	$St_2 > St_5 > St_3 > St_4 > St_1$
SF-CODAS (Gundogdu and Kahraman 2019c) (threshold parameter $\mu = 0.02$)	$St_2 > St_3 > St_5 > St_4 > St_1$
SF-WASPAS (Gundogdu and Kahraman 2019d) (threshold parameter $\mu = 0.5$)	$St_2 > St_3 > St_5 > St_4 > St_1$
SF-CoCoSo (Peng and Li 2022) (compromise coefficient $\mu = 0.5$)	$St_2 > St_5 > St_3 > St_4 > St_1$
The proposed SF-CPT–CoCoSo	$St_2 > St_5 > St_4 > St_3 > St_1$

making. SWAM, SWGM, SFNWAA and SFWGA operators emphasize the overall impact, whereas SFWAI as well as SFWGI operators focus on the individual effect. SF-TOPSIS and SF-CODAS approaches evaluate each alternative by measuring its distance from the ideal solutions. SF-WASPAS and SF-CoCoSo approaches integrate WSM and WPM in evaluating each set of attributes for different alternatives, and the SF-CoCoSo approach is superior to the SF-WASPAS approach because it implements aggregation strategies based on the attitude of compromise. However, the proposed method in this article not only takes advantage of the merits of the CoCoSo method, but also integrates CPT into the the decision-making process to fully simulate the psychological behavior characteristics of DMs for facing risks. Moreover, we utilize the proposed DIMs to effectively extend the D-CRITIC approach in getting the unknown attribute weight legitimately under SFSs. Hence, the presented model makes the evaluation results more valid and scientific for LSEVCS. Furthermore, the advantages of the different approaches are also displayed in more detail in Table 32.

5 Conclusions

In this paper, we use CPT to improve the traditional CoCoso method and build a novel risk-based MAGDM model for solving uncertainty issues under SFSs. Firstly, we propose some new spherical fuzzy DIMs based on Dice and Jaccard indexes and discuss some special cases of them. Secondly, considering DMs' psychological risk factors and different preferences of DMs to attribute set in decision making, we incorporate CPT into the CoCoSo method under SFSs to develop an SF-CPT–CoCoSo model to MAGDM. At the same time, in view of the advantage of the D-CRITIC method in using distance correlation to capture nonlinear relations between attributes, we utilize the proposed DIMs to extend D-CRITIC method for the first time to SFSs in obtaining unknown attribute weight information. Eventually, the established model is utilized for LSEVCS issue to demonstrate the applicability of the SF-CPT–CoCoSo model. Afterward, sensitivity analysis and comparative discussion further illustrate the stability, validity and superiority of the developed model. Therefore, the main

Table 32 Comparison of the characteristics of different methods

Methods	Detect information widely	Adjust parameter flexibly	Capture the mental behavior of DMs fully	Model nonlinear relationships among attributes
SWAM (Gundogdu and Kahraman 2019b)	√	×	×	×
SWGM (Gundogdu and Kahraman 2019b)	√	×	×	×
SFNWAA (Ashraf et al. 2019b)	√	×	×	×
SFNWGA (Ashraf et al. 2019b)	√	×	×	×
SFWAI (Ju et al. 2021)	√	×	×	×
SFWGI (Ju et al. 2021)	√	×	×	×
SF-TOPSIS (Kutlu Gündoğdu and Kahraman 2021)	√	×	×	×
SF-CODAS (Gundogdu and Kahraman 2019c)	√	√	×	×
SF-WASPAS (Gundogdu and Kahraman 2019d)	√	√	×	×
SF-CoCoSo (Peng and Li 2022)	√	√	×	×
The proposed SF-CPT–CoCoSo	√	√	√	√

achievements of this article can be summarized as follows: (1) many new spherical fuzzy Dice and Jaccard DIMs are proposed to provide more options for detecting the differences between SFSs or inputs; (2) a novel risk MAGDM model called SF-CPT–CoCoSo is developed to solve uncertain problems; (3) D-CRITIC method is extended for the first time to effectively obtain unknown attribute weights under SFSs; (4) the proposed model is applied for LSEVCS to demonstrate its applicability. Moreover, sensitivity analysis and further comparison demonstrate the stability, legality and superiority of the SF-CPT–CoCoSo model; (5) the established model affords DMs more choices in settling MAGDM issues and also offers some reference on the extension of the CPT–CoCoSo method in other decision-making environments. All in all, the established model adopts SFSs to deeply excavate uncertain evaluation information, combines CPT with CoCoSo method to evaluate alternatives under the premise of fully considering DMs' risk preferences and introduces D-CRITIC method for the first time into SFSs to reasonably get attribute weights. Hence, the established model makes the decision results more reasonable and scientific.

However, the established model also has some limitations, such as the relatively complex calculation process. In particular, although distance correlation in the D-CRITIC approach can efficaciously model the nonlinear relationship between attributes, its computational cost is high. Therefore, in the following research, on one hand, we will further explore a simple and efficient algorithm to compute distance correlation to improve D-CRITIC method, so that the D-CRITIC method can be widely extended in other fuzzy environments. On the other hand, we will consider the combination of the D-CRITIC method and some subjective weight methods [such as AHP (Saaty 1980a, b), BWM (Rezaei 2015), FOCUM (Pamucar et al. 2018) and KEMIRA (Krylovas et al. 2014)] to design some more sound attribute weight calculation algorithm to enhance the effectiveness of decision making. In addition, this article only discusses the proposed model in the case of attribute values as SFNs. But in the actual decision, many complex information forms may appear in the evaluation process due to the uncertainty of DMs and the diversity of attributes, such as spherical linguistic FSs (Ashraf et al. 2018), spherical soft FSs (Perveen et al. 2019), T-spherical hesitant FSs (Al-Quran 2021) and interval-valued T-SFSs (Garg et al. 2022). Therefore, in future studies, we will also focus on the further extension of the CPT–CoCoSo method in the aforementioned environment to develop more MAGDM models for dealing with various uncertain problems (Zhang et al. 2021; Liu et al. 2021; Wu et al. 2014; Lu et al. 2019; Wang et al. 2019b).

Declarations

Conflict of interest There is no conflict of interest for the authors.

Ethical standard This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent There is no individual participant included in the study.

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