Computational and Applied Mathematics (2023) 42:176
https://doi.org/10.1007/s40314-023-02266-1

# TOPSIS approach for MCGDM based on intuitionistic fuzzy rough Dombi aggregation operations

**Azmat Hussain[1,2] · Tahir Mahmood[1] · Florentin Smarandache[3] · Shahzaib Ashraf[4]**

Received: 11 November 2022 / Revised: 5 February 2023 / Accepted: 6 March 2023 /
Published online: 10 May 2023


## Abstract

Atanassov presented the dominant notion of intuitionistic fuzzy sets which brought revolution in different fields of science since their inception. The operations of t-norm and t-conorm introduced by Dombi were known as Dombi operations and Dombi operational parameter possesses natural flexibility with the resilience of variability. The advantage of Dombi operational parameter is very important to express the experts' attitude in decision-making. This study aims to propose intuitionistic fuzzy rough TOPSIS method based on Dombi operations. For this, first we propose some new operational laws based on Dombi operations to aggregate averaging and geometric aggregation operators under the hybrid study of intuitionistic fuzzy sets and rough sets. On the proposed concept, we present intuitionistic fuzzy rough Dombi weighted averaging, intuitionistic fuzzy rough Dombi ordered weighted averaging, and intuitionistic fuzzy rough Dombi hybrid averaging operators. Moreover, on the developed concept, we present intuitionistic fuzzy rough Dombi weighted geometric, intuitionistic fuzzy rough Dombi ordered weighted geometric, and intuitionistic fuzzy rough Dombi hybrid geometric operators. The basic related properties of the developed operators are presented in detailed. Then the algorithm for MCGDM based on TOPSIS method for intuitionistic fuzzy

Communicated by Junsheng Qiao.

✉ Azmat Hussain
azmat.phdma66@gmail.com

Tahir Mahmood
tahirbakhat@iiu.edu.pk

Florentin Smarandache
fsmarandache@gmail.com

Shahzaib Ashraf
shahzaib.ashraf@kfueit.edu.pk

1 Department of Mathematics and Statistics, Faculty of Basic and Applied Sciences, International Islamic University Islamabad, Islamabad, Pakistan

2 Iclamic Girls School and College Parachinar, Parachinar, Pakistan

3 Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA

4 Institute of Mathematics, Khwaja Fareed University of Engineering and Information Technology, Rahim Yar Khan 64200, Pakistan

rough Dombi averaging and geometric operators is presented. By applying accumulated geometric operator, the intuitionistic fuzzy rough numbers are converted into the intuitionistic fuzzy numbers. The massive outbreak of the pandemic COVID-19 promoted the challenging scenario for the world organizations including scientists, laboratories, and researchers to conduct special clinical treatment strategies to prevent the people from COVID-19 pandemic. Additionally, an illustrative example is proposed to solve MCGDM problem to diagnose the most severe patient of COVID-19 by applying TOPSIS. Finally, a comparative analysis of the developed model is presented with some existing methods which show the applicability and superiority of the developed model.

**Keywords** IFS · Rough sets · Dombi operations · Averaging and geometric aggregation operators · TOPSIS · MCGDM

**Mathematics Subject Classification** 03E72 · 90B50 · 82B24

## 1 Introduction

The multi-criteria group decision-making (MCGDM) is the most significant and prominent methodology, in which a team of professional specialist evaluate alternatives for the selection of best optimal object based on multiple criteria. Group decision-making (DM) has the ability and capability to improve the assessment process by evaluating multiple conflicting criteria based on the performance of alternatives from independent aspects. In DM, it is hard to avoid the uncertainty due to the imprecise judgement by the professional specialist. The process of DM has engaged the attention of scholars in diverse directions around the world and gained the fruitful results by applying different approaches (Ishizaka and Nemery 2013; Xu 2004). To cope with vague and uncertain data, Zadeh (1965) originated the prominent concept of fuzzy sets (FS) and this concept has strong description of ambiguous information in MCGDM problems. After the inception of FS, researchers carried out different methods by applying the concept of FS in diverse directions (Biswas and Modak 2013; Debnath et al. 2018). Atanassov (1986) initiated the dominant notion of intuitionistic fuzzy set [IFS], having the property which is incorporated by the membership degree (MeD) and nonmembership degree (NonMeD) such that their sum belongs to [0, 1], which enables better description of the imperfect and imprecise date in DM problems. Thao and Nguyen (2018) put forwarded the concept of correlation coefficient and proposed for the same concept to determine the variance and covariance in sense of IFS. Chen et al. (2016) presented new fuzzy DM methods based on evidential reasoning strategy. Chen and Chun (2016) put forwarded the technique for TOPSIS method similarity measure based on intuitionistic fuzzy date. In DM, one of the most serious issue is to aggregate the preferences reports presented by the several professional experts to get a unique result. In this situation, aggregation operators (AO) play significant role to aggregate the collective information presented from the different sources. Xu (2007), Yager and Xu (2006) developed the dominant concepts of IFWA and IFWG aggregation operators and discussed their fundamental properties. Garg (2016) built up some improvement in averaging operators and proposed a series intuitionistic fuzzy interactive weighted averaging operators. Li (2010) originated idea of the generalized OWAO to aggregate the decision maker's assessment by applying intuitionistic fuzzy information and solved MADM problems on the proposed concept. Wei (2010) investigated the concept of IFOWGA operators and interval-valued IFOWGA operators and presented an illustrateFd example on the proposed model. The concept of Einstein operators was proposed by Wang and Liu (2012) by applying intuitionistic fuzzy information. Huang (2014) originated the idea of some new Hamacher operators by applying the idea of IFS and then applied the developed concept in DM. Hwang and Yoon (1981) initiated the dominant and top most method technique for order preference by similarity to ideal solution (TOPSIS). This model measures the shortest and farthest distance from PIS and NIS. Garg and Kumar (2018a) initiated the idea of exponential distance measure by applying the technique of TOPSIS method under interval-valued IFS and solve its application in DM. The concept of new distance measure was proposed by Shen et al. (2018) and by applying TOPSIS technique under intuitionistic fuzzy environment and studied its desirable properties. Zeng and Xiao (2016) originated TOPSIS technique based on averaging distance and initiated its desirable characteristics. Zeng et al. (2019) developed a new score function and used VIKOR and TOPSIS for ranking intuitionistic fuzzy numbers. Zulqarnain et al. (2021) proposed the model for TOPSIS approach using interval-valued intuitionistic fuzzy soft set based on correlation coefficient to aggregate the expert's decision by applying soft aggregation operators. By applying the idea of cosine function, Ye (2016) discussed the concept of two similarity measure. Garg and Kumar (2018b) proposed similarity measure using set pair analysis theory. Using the concept of direct operation, Song et al. (2017) put forward the notion of similarity measure under intuitionistic fuzzy environment. The geometrical interpretation of entropy measure under IFS was proposed by Szmidt and Kacprzyk (2001). A novel approach of entropy and similarity measure was proposed by Meng and Chen (2016) which is based on fuzzy measure. Lin and Ren (2014) proposed a new approach for entropy measure based on the weight determination. Garg (2018) made some improvement in cosine similarity measure. Yager (2013) addressed the shortcoming in IFS and originated the concept of Pythagorean fuzzy sets (PFS) which became a hot research area for scholars. The notions of averaging and geometric operators were proposed by Yager (2014). Peng et al. (2015) put forward some result in PFS. Hussain et al. (2020a, 2019) introduced the algebraic structure of PFS in semigroup and further presented its combined studies with soft and rough sets. Zhang (2014) proposed TOPSIS for PSF and described its application in DM. In spite of these, the concept of q-rung orthopair fuzzy sets (qROFS) was delivered by Yager (2016). Ali (2019) initiated the ideas of orbits and L-fuzzy sets in qROFS. Hussain et al. (2019a, 2020b; b) and Wang et al. (2020) proposed the combined study of qROFS with rough and soft sets. Shahzaib et al. (2021) investigated Einstein averaging and geometric operations for qROF rough sets through EDAS method. Feng et al. (2020a) proposed the idea of problem of ranking intuitionistic fuzzy values and presented Minkowski score functions of intuitionistic fuzzy values which generalized the score function for intuitionistic fuzzy value. Feng et al. (2020b) developed the idea of new extension of the PROMETHEE, by taking advantage of intuitionistic fuzzy soft sets. Huang et al. (2019) developed the dominance-based rough sets under intuitionistic fuzzy environment. Feng et al. (2018) improved the existing concepts and related results to generalized intuitionistic fuzzy sets. Li et al. (2023) proposed the notion of spherical fuzzy normalized projection for the dilemma of community-based epidemic prevention and control assessment method selection.

In 1982, Dombi (1982) investigated Dombi t-norm and Dombi t-conorm based on Dombi operational parameter. The concept of IFWA and IFWG operators based on Dombi operations was proposed by Seikh et al. (2021). The idea of Bonferroni mean operations was proposed by Lui et al. (2018) to aggregate the multi-attributes based on intuitionistic fuzzy aggregation operators and proposed its application in DM. Later, Chen and Ye (2017) made an effort to propose the Dombi operation in neurtrosophic information and constructed its application in DM. Wei and Wei (2018) initiated the hybrid study of Dombi operation with prioritized aggregation operators. Jana et al. (2018) put forward the idea of arithmetic and geometric operations based on bipolar fuzzy Dombi operations.

Pawlak (1982) initiated the prominent concept of rough set (RS) and this novel concept generalized the crisp set theory. The developed notion of RS theory handles the uncertainty and vagueness in more accurate way than classical set theory. From the inception, RS theory has been presented in different directions and proposed its applications in both practical and theoretical aspect as well. Dubois and Prade (1990) put forward the idea of fuzzy RS based on fuzzy relation. Cornelis et al. (2003) developed the combined structure of RS and IFS to get the dominant concept of intuitionistic fuzzy rough set (IFRS). The constrictive and axiomatic study of rough set was presented by Zhou and Wu (2008) by utilizing intuitionistic fuzzy rough aggregation operators. Zhou and Wu (2011) developed the idea of rough IFS and intuitionistic fuzzy rough soft (IFRS) by applying crisp and fuzzy relation. Bustince and Burillo (1996) developed the notion of intuitionistic fuzzy relation. By applying the generalized intuitionistic fuzzy relation, Zhang et al. (2012) proposed IFRS instead of intuitionistic fuzzy relation. Moreover, the combine study of RS, soft set, and IFS was investigated by Zhang et al. (2014) to obtaine the novel concepts of soft rough IFS and intuitionistic fuzzy soft RS based on crisp and fuzzy approximation spaces. By applying the intuitionistic fuzzy soft relation, Zhang et al. (2012) developed the concept of generalized intuitionistic fuzzy soft rough set. Chinram et al. (2021) presented the concept of intuitionistic fuzzy rough aggregation operators to aggregate the multi-assessment of experts to get a unique optimal option based on IFRWA, IFRWG, IFROWA, IFROWG, IFRHA, and IFRHG operators and by applying EDAS method to illustrate the DM application. Later on, Yahya et al. (2021) developed the intuitionistic fuzzy rough frank aggregation operators and discussed its basic properties. From the above analysis and discussion, it is clear that Dombi operations have natural resilience and flexibility to demonstrate the datum and questionable real-life issues more effectively. Furthermore, the behavior of general operational parameter $\beta$ in Dombi operations has more importance to express the decision maker's attitude. Different values are used for the operational parameter $\beta$ to judge the ranking results of the professional experts by applying the developed approach. From the best of our knowledge and above analysis up-till now, no application of Dombi operators with the hybrid study if IFS and rough sets by applying intuitionistic fuzzy averaging and geometric aggregation operators is reported in intuitionistic fuzzy environment. The performance of the developed intuitionistic fuzzy rough averaging and geometric operators is illustrated through MCGDM. Therefore, this motivates the current research to investigate averaging and geometric operators such as IFRWA, IFROWA, IFRHA, IFRWG, IFROWG, and IFRHG aggregation operators and investigated their desirable properties with details.

The remaining portion of this work is managed as follows: In Sect. 2, of the manuscript, some basic concepts are given which will be helpful for onward sections. Section 3, consists of Dombi operations and proposed some new operational laws based on Dombi operations to aggregate averaging operators and geometric operators. In Sect. 4, we investigated the notion of IFRDWA, IFRDOWA, and IFRDHA operators. Moreover, in Sect. 5, we developed the concept of IFRDWG, IFRDOWG, and IFRDHA operators. The fundamental related characteristics of the developed operators are presented in detailed. Section 6 discusses a step algorithm of TOPSIS method that was developed for MCGDM based on for intuitionistic fuzzy rough Dombi averaging and geometric operators. In Sect. 7, an illustrative example is proposed to solve MCGDM problem to diagnose the most severe patient of COVID-19 by applying TOPSIS technique. Finally, a comparative analysis of the developed model is presented with different previous models in literature which presents that the investigated concepts are more resilience and flexible than the developed models.

## 2 Preliminaries

This section includes the review of some elementary definitions, operations and their score values, which associate the existing literature with the developed concepts.

**Definition 1** (*Atanassov* 1986). Consider K to be a universal set, and IFS $\mathfrak{G}$on the set K is given as;

$$\mathfrak{G} = \{\langle \mathfrak{g}, \hbar_{\mathfrak{G}}(\mathfrak{g}), \delta_{\mathfrak{G}}(\mathfrak{g})\rangle | \mathfrak{g} \in K\},$$

where $\hbar_{\mathfrak{G}}, \delta_{\mathfrak{G}} : K \to [0, 1]$,represent the MeD and NonMeD of an object $\mathfrak{g} \in K$, to the set $\mathfrak{G}$ with $0 \leq \hbar_{\mathfrak{G}}(\mathfrak{g}) + \delta_{\mathfrak{G}}(\mathfrak{g}) \leq 1$. Moreover, $\pi_{\mathfrak{G}}(\mathfrak{g}) = 1 - (\hbar_{\mathfrak{G}}(\mathfrak{g}) + \delta_{\mathfrak{G}}(\mathfrak{g}))$denotes the hesitancy degree of an alternative$\mathfrak{g} \in K$. For simplicity,$\mathfrak{G} = \langle \mathfrak{g}, \hbar_{\mathfrak{G}}(\mathfrak{g}), \delta_{\mathfrak{G}}(\mathfrak{g})\rangle$ is denoted by $\mathfrak{G} = (\hbar_{\mathfrak{G}}, \delta_{\mathfrak{G}})$ and is called an intuitionistic fuzzy number (IFN) for$\mathfrak{g} \in K$.

Let $\mathfrak{G} = (\hbar_{\mathfrak{G}}, \delta_{\mathfrak{G}})$ and $\mathfrak{G}_1 = \left(\hbar_{\mathfrak{G}_1}, \delta_{\mathfrak{G}_1}\right)$be the intuitionistic fuzzy numbers, then some fundamental operations on them are defined as:

(i) $\mathfrak{G} \cup \mathfrak{G}_1 = \left(\max\left(\hbar_{\mathfrak{G}}(\mathfrak{g}), \hbar_{\mathfrak{G}_1}(\mathfrak{g})\right), \min\left((\delta_{\mathfrak{G}}(\mathfrak{g}), \delta_{\mathfrak{G}_1}(\mathfrak{g}))\right)\right);$
(ii) $\mathfrak{G} \cap \mathfrak{G}_1 = \left(\min\left(\hbar_{\mathfrak{G}}(\mathfrak{g}), \hbar_{\mathfrak{G}_1}(\mathfrak{g})\right), \max\left(\delta_{\mathfrak{G}}(\mathfrak{g}), \delta_{\mathfrak{G}_1}(\mathfrak{g})\right)\right);$
(iii) $\mathfrak{G} \oplus \mathfrak{G}_1 = \left(\hbar_{\mathfrak{G}} + \hbar_{\mathfrak{G}_1} - \hbar_{\mathfrak{G}}\hbar_{\mathfrak{G}_1}, \delta_{\mathfrak{G}}\delta_{\mathfrak{G}_1}\right);$
(iv) $\mathfrak{G} \otimes \mathfrak{G}_1 = \left(\hbar_{\mathfrak{G}}\hbar_{\mathfrak{G}_1}, \delta_{\mathfrak{G}} + \delta_{\mathfrak{G}_1} - \delta_{\mathfrak{G}}\delta_{\mathfrak{G}_1}\right);$
(v) $\mathfrak{G} \leq \mathfrak{G}_1 if \hbar_{\mathfrak{G}}(\mathfrak{g}) \leq \hbar_{\mathfrak{G}_1}(\mathfrak{g}), \delta_{\mathfrak{G}}(\mathfrak{g}) \geq \delta_{\mathfrak{G}_1}(\mathfrak{g})$ for all$\mathfrak{g} \in K$;
(vi) $\mathfrak{G}^c = (\delta_{\mathfrak{G}}, \hbar_{\mathfrak{G}})$ where $\mathfrak{G}^c$ represents the complement of $\mathfrak{G}$;
(vii) $\alpha\mathfrak{G} = \left(1 - (1 - \hbar_{\mathfrak{G}})^{\alpha}, \delta_{\mathfrak{G}}^{\alpha}\right)$ for $\alpha \geq 1$;
(viii) $\mathfrak{G}^{\alpha} = \left(\hbar_{\mathfrak{G}}^{\alpha}, 1 - (1 - \delta_{\mathfrak{G}})^{\alpha}\right)$ for $\alpha \geq 1$.

**Definition 2** (*Chen and Tan* 1994) Let $\mathfrak{G} = (\hbar_{\mathfrak{G}}, \delta_{\mathfrak{G}})$ be an IFN. Then the score function of $\mathfrak{G}$ is defined as:

$$\overline{\overline{S}}(\mathfrak{G}) = \hbar_{\mathfrak{G}} - \delta_{\mathfrak{G}} \text{for} \overline{\overline{S}}(\mathfrak{G}) \in [-1, 1].$$

Greater the score, better the IFN is.

**Definition 3** (*Hong and Choi* 2000) Let $\mathfrak{G} = (\hbar_{\mathfrak{G}}, \delta_{\mathfrak{G}})$ be an IFN. Then the accuracy function of $\mathfrak{G}$ is defined as:

$$\overline{\overline{A}}(\mathfrak{G}) = \hbar_{\mathfrak{G}} + \delta_{\mathfrak{G}} \text{for} \overline{\overline{A}}(\mathfrak{G}) \in [0, 1].$$

**Definition 4** (*Zhou and Wu* 2008) Assume a fixed set $K$ and crisp intuitionistic fuzzy relation $\psi \in \text{IFS}(K \times K)$. Then

(i) For all $\mathfrak{g} \in K$, the relation $\psi$ is reflexive, if $\hbar_{\psi}(\mathfrak{g}, \mathfrak{g}) = 1$ and $\delta_{\psi}(\mathfrak{g}, \mathfrak{g}) = 0$.
(ii) For all $(\mathfrak{g}, \mathfrak{c}) \in K \times K$, the relation $\psi$ is symmetric, if $\hbar_{\psi}(\mathfrak{g}, \mathfrak{c}) = \hbar_{\psi}(\mathfrak{c}, \mathfrak{g})$ and $\delta_{\psi}(\mathfrak{g}, \mathfrak{c}) = \delta_{\psi}(\mathfrak{c}, \mathfrak{g})$.
(iii) For all $(\mathfrak{g}, d) \in K \times K$, the relation $\psi$ is transitive, if $\hbar_{\psi}(\mathfrak{g}, d) \geq \bigvee_{\mathfrak{c} \in K} \left\{\hbar_{\psi}(\mathfrak{g}, \mathfrak{c}) \vee \hbar_{\psi}(\mathfrak{c}, d)\right\}$ and $\delta_{\psi}(\mathfrak{g}, d) \geq \bigwedge_{\mathfrak{c} \in K} \left\{\delta_{\psi}(\mathfrak{g}, \mathfrak{c}) \wedge \delta_{\psi}(\mathfrak{c}, d)\right\}$.

**Definition 5** (*Chinram et al.* 2021) Consider $K$ as a universal of discourse such that $\psi$ be intuitionistic fuzzy relation over $K$, i.e., $\psi \in \text{IFS}(K \times K)$. Then the order pair $(K, \psi)$ is known to be intuitionistic fuzzy approximation space. Now any normal decision object $\mathfrak{B} \subseteq \text{IFS}(K)$, the lower and upper approximation of $\mathfrak{B}$ w.r.t intuitionistic fuzzy approximation space $(K, \psi)$are represented by $\underline{\psi}(\mathfrak{B})$ and $\overline{\psi}(\mathfrak{B})$ which is defined as:

$$\underline{\psi}(\mathfrak{B}) = \left\{\langle \mathfrak{g}, \hbar_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g}), \delta_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g})\rangle | \mathfrak{g} \in K\right\},$$

$$\overline{\psi}(\mathfrak{B}) = \left\{ \langle \mathfrak{g}, \hbar_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g}), \delta_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g}) \rangle | \mathfrak{g} \in K \right\},$$

where

$$\hbar_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g}) = \bigwedge_{\mathfrak{c} \in K} \left\{ \hbar_{\psi}(\mathfrak{g}, \mathfrak{c}) \wedge \hbar_{\mathfrak{B}}(\mathfrak{c}) \right\}, \delta_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g}) = \bigvee_{\mathfrak{c} \in K} \left\{ \delta_{\psi}(\mathfrak{g}, \mathfrak{c}) \vee \delta_{\mathfrak{B}}(\mathfrak{c}) \right\}$$
$$\hbar_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g}) = \bigvee_{\mathfrak{c} \in K} \left\{ \hbar_{\psi}(\mathfrak{g}, \mathfrak{c}) \bigvee \hbar_{\mathfrak{B}}(\mathfrak{c}) \right\}, \delta_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g}) = \bigwedge_{\mathfrak{c} \in K} \left\{ \delta_{\psi}(\mathfrak{g}, \mathfrak{c}) \bigwedge \delta_{\mathfrak{B}}(\mathfrak{c}) \right\}$$

with $0 \leq \hbar_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g}) + \delta_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g}) \leq 1$ and $0 \leq \hbar_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g}) + \delta_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g}) \leq 1$. As $\underline{\psi}(\mathfrak{B})$ and $\overline{\psi}(\mathfrak{B})$ are IFS, $\underline{\psi}(\mathfrak{B}), \overline{\psi}(\mathfrak{B}) : \mathrm{IFS}(K) \to \mathrm{IFS}(K)$ are lower and upper approximation operators. Therefore, the pair $\psi(\mathfrak{B}) = \left(\underline{\psi}(\mathfrak{B}), \overline{\psi}(\mathfrak{B})\right) = \left\{ \left(\mathfrak{g}, \langle \hbar_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g}), \delta_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g}) \rangle, \langle \hbar_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g}), \delta_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g}) \rangle \right) | \mathfrak{g} \in K \right\}$ is called intuitionistic fuzzy rough set (IFRS). For simplicity, $\psi(\mathfrak{B}) = \left(\underline{\psi}(\mathfrak{B}), \overline{\psi}(\mathfrak{B})\right) = \left(\langle \underline{\hbar}, \underline{\delta} \rangle, \langle \overline{\hbar}, \overline{\delta} \rangle\right)$ denotes the intuitionistic fuzzy rough number (IFRN).

**Definition 6** (*Chinram et al.* 2021) Let $\psi(\mathfrak{B}) = \left(\underline{\psi}(\mathfrak{B}), \overline{\psi}(\mathfrak{B})\right) = \left(\langle \underline{\hbar}, \underline{\delta} \rangle, \langle \overline{\hbar}, \overline{\delta} \rangle\right)$, be an IFRN. Then the score function of $\psi(\mathfrak{B})$ is defined as:

$$\overline{\overline{S}}(\mathfrak{B}) = \frac{1}{4}\left(2 + \underline{\hbar} + \overline{\hbar} - \underline{\delta} - \overline{\delta}\right) \text{for} \overline{\overline{S}}(\mathfrak{B}) \in [0, 1].$$

Greater the score better the IFRN is.

**Definition 7** (*Yahya et al.* 2021) Let $\psi(\mathfrak{B}) = \left(\underline{\psi}(\mathfrak{B}), \overline{\psi}(\mathfrak{B})\right) = \left(\langle \underline{\hbar}, \underline{\delta} \rangle, \langle \overline{\hbar}, \overline{\delta} \rangle\right)$, be an IFRN. Then the accuracy function of $\psi(\mathfrak{B})$ is defined as:

$$\overline{\overline{A}}(\mathfrak{B}) = \frac{1}{4}\left(\underline{\hbar} + \overline{\hbar} + \underline{\delta} + \overline{\delta}\right) \text{for} \overline{\overline{A}}(\mathfrak{B}) \in [0, 1].$$

## 3 Dombi operations

Dombi presented the pioneer concept Dombi operations known as Dombi product and Dombi sum, which are the special form of t-norms and t-conorms given in the following definition.

**Definition 8** (*Dombi* 1982). Consider that $\xi$ and $\upsilon$ belong to real numbers with $\beta \geq 1$. Then Dombi operations are elaborated as:

$$T_D(\xi, \upsilon) = \frac{1}{1 + \left\{ \left(\frac{1-\xi}{\xi}\right)^{\beta} + \left(\frac{1-\upsilon}{\upsilon}\right)^{\beta} \right\}^{\frac{1}{\beta}}}$$
$$T_{D'}(\xi, \upsilon) = 1 - \frac{1}{1 + \left\{ \left(\frac{\xi}{1-\xi}\right)^{\beta} + \left(\frac{\upsilon}{1-\upsilon}\right)^{\beta} \right\}^{\frac{1}{\beta}}}$$

**Definition 9** Let $\psi(\mathfrak{B}_1) = \left(\underline{\psi}(\mathfrak{B}_1), \overline{\psi}(\mathfrak{B}_1)\right) = \left(\langle \underline{\hbar_1}, \underline{\delta_1} \rangle, \langle \overline{\hbar_1}, \overline{\delta_1} \rangle\right)$ and $\psi(\mathfrak{B}_2) = \left(\underline{\psi}(\mathfrak{B}_2), \overline{\psi}(\mathfrak{B}_2)\right) = \left(\langle \underline{\hbar_2}, \underline{\delta_2} \rangle, \langle \overline{\hbar_2}, \overline{\delta_2} \rangle\right)$ be two IFRNs and $\alpha > 0$. Then some basic operations based on Dombi t-norms and t-conorms operations are given as:

(i)
$$\psi(\mathfrak{B}_1)\oplus\psi(\mathfrak{B}_2)=\left\{\begin{array}{l}\left(1-\dfrac{1}{1+\left\{\left(\frac{\underline{\hbar}_1}{1-\underline{\hbar}_1}\right)^{\beta}+\left(\frac{\underline{\hbar}_2}{1-\underline{\hbar}_2}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\dfrac{1}{1+\left\{\left(\frac{1-\underline{\delta}_1}{\underline{\delta}_1}\right)^{\beta}+\left(\frac{1-\underline{\delta}_2}{\underline{\delta}_2}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\\ \left(1-\dfrac{1}{1+\left\{\left(\frac{\overline{\hbar}_1}{1-\overline{\hbar}_1}\right)^{\beta}+\left(\frac{\overline{\hbar}_2}{1-\overline{\hbar}_2}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\dfrac{1}{1+\left\{\left(\frac{1-\overline{\delta}_1}{\overline{\delta}_1}\right)^{\beta}+\left(\frac{1-\overline{\delta}_2}{\overline{\delta}_2}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\end{array}\right\},$$

(ii)
$$\psi(\mathfrak{B}_1)\otimes\psi(\mathfrak{B}_2)=\left\{\begin{array}{l}\left(\dfrac{1}{1+\left\{\left(\frac{1-\underline{\hbar}_1}{\underline{\hbar}_1}\right)^{\beta}+\left(\frac{1-\underline{\hbar}_2}{\underline{\hbar}_2}\right)^{\beta}\right\}^{\frac{1}{\beta}}},1-\dfrac{1}{1+\left\{\left(\frac{\underline{\delta}_1}{1-\underline{\delta}_1}\right)^{\beta}+\left(\frac{\underline{\delta}_2}{1-\underline{\delta}_2}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\\ \left(\dfrac{1}{1+\left\{\left(\frac{1-\overline{\hbar}_1}{\overline{\hbar}_1}\right)^{\beta}+\left(\frac{1-\overline{\hbar}_2}{\overline{\hbar}_2}\right)^{\beta}\right\}^{\frac{1}{\beta}}},1-\dfrac{1}{1+\left\{\left(\frac{\overline{\delta}_1}{1-\overline{\delta}_1}\right)^{\beta}+\left(\frac{\overline{\delta}_2}{1-\overline{\delta}_2}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\end{array}\right\},$$

(iii)
$$\alpha\psi(\mathfrak{B}_1)=\left\{\left(1-\frac{1}{1+\left\{\alpha\left(\frac{\underline{\hbar}_1}{1-\underline{\hbar}_1}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\alpha\left(\frac{1-\underline{\delta}_1}{\underline{\delta}_1}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\left(1-\frac{1}{1+\left\{\alpha\left(\frac{\overline{\hbar}_1}{1-\overline{\hbar}_1}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\alpha\left(\frac{1-\overline{\delta}_1}{\overline{\delta}_1}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\},$$

(iv)
$$(\psi(\mathfrak{B}_1))^{\alpha}=\left\{\left(\frac{1}{1+\left\{\alpha\left(\frac{1-\underline{\hbar}_1}{\underline{\hbar}_1}\right)^{\beta}\right\}^{\frac{1}{\beta}}},1-\frac{1}{1+\left\{\alpha\left(\frac{\underline{\delta}_1}{1-\underline{\delta}_1}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\left(\frac{1}{1+\left\{\alpha\left(\frac{1-\overline{\hbar}_1}{\overline{\hbar}_1}\right)^{\beta}\right\}^{\frac{1}{\beta}}},1-\frac{1}{1+\left\{\alpha\left(\frac{\overline{\delta}_1}{1-\overline{\delta}_1}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}.$$

**Example 1** Let $\psi(\mathfrak{B}_1)=\left(\underline{\psi}(\mathfrak{B}_1),\overline{\psi}(\mathfrak{B}_1)\right)=(\langle 0.7,0.3\rangle,\langle 0.6,0.4\rangle)$ and $\psi(\mathfrak{B}_2)=\left(\underline{\psi}(\mathfrak{B}_2),\overline{\psi}(\mathfrak{B}_2)\right)=(\langle 0.8,0.1\rangle,\langle 0.5,0.2\rangle)$ be two IFRNs with $\beta=2$ and $\alpha>0$. Then some basic operations based on Dombi t-norms and t-conorms operations are given as:

$$\begin{aligned}\psi(\mathfrak{B}_1)\oplus\psi(\mathfrak{B}_2)&=\left\{\begin{array}{l}\left(1-\dfrac{1}{1+\left\{\left(\frac{0.7}{1-0.7}\right)^{2}+\left(\frac{0.8}{1-0.8}\right)^{2}\right\}^{\frac{1}{2}}},\dfrac{1}{1+\left\{\left(\frac{1-0.3}{0.3}\right)^{2}+\left(\frac{1-0.1}{0.1}\right)^{2}\right\}^{\frac{1}{2}}}\right),\\ \left(1-\dfrac{1}{1+\left\{\left(\frac{0.6}{1-0.6}\right)^{2}+\left(\frac{0.5}{1-0.5}\right)^{2}\right\}^{\frac{1}{2}}},\dfrac{1}{1+\left\{\left(\frac{1-0.4}{0.4}\right)^{2}+\left(\frac{1-0.2}{0.2}\right)^{2}\right\}^{\frac{1}{2}}}\right)\end{array}\right\}\\ &=\left\{\begin{array}{l}\left(1-\dfrac{1}{1+\{5.444+16\}^{\frac{1}{2}}},\dfrac{1}{1+\{5.444+81\}^{\frac{1}{2}}}\right),\\ \left(1-\dfrac{1}{1+\{2.25+1\}^{\frac{1}{2}}},\dfrac{1}{1+\{2.25+16\}^{\frac{1}{2}}}\right)\end{array}\right\}\\ &=\{(1-0.1776,0.09711),(1-0.3568,0.1897)\}\\ &=\{(0.8224,0.09711),(0.6432,0.1897)\}\end{aligned}$$

$$\psi(\mathfrak{B}_1)\otimes\psi(\mathfrak{B}_2)=\left\{\left(\frac{1}{1+\left\{\left(\frac{1-0.7}{0.7}\right)^{2}+\left(\frac{1-0.8}{0.8}\right)^{2}\right\}^{\frac{1}{2}}},1-\frac{1}{1+\left\{\left(\frac{0.3}{1-0.3}\right)^{2}+\left(\frac{0.1}{1-0.1}\right)^{2}\right\}^{\frac{1}{2}}}\right),\right.$$

$$\left.\left(\frac{1}{1+\left\{\left(\frac{1-0.6}{0.6}\right)^2+\left(\frac{1-0.5}{0.5}\right)^2\right\}^{\frac{1}{2}}}, 1-\frac{1}{1+\left\{\left(\frac{0.4}{1-0.4}\right)^2+\left(\frac{0.2}{1-0.2}\right)^2\right\}^{\frac{1}{2}}}\right)\right\}$$

$$= \left\{\left(\frac{1}{1+\{0.1837+0.0625\}^{\frac{1}{2}}}, 1-\frac{1}{1+\{0.5124+0.1235\}^{\frac{1}{2}}}\right),\right.$$

$$\left.\left(\frac{1}{1+\{0.4444+1\}^{\frac{1}{2}}}, 1-\frac{1}{1+\{1.1111+0.3215\}^{\frac{1}{2}}}\right)\right\}$$

$$= \{(0.6684, 1-0.5564), (0.4542, 1-0.4552)\}$$

$$= \{(0.6684, 0.4436), (0.4542, 0.4558)\}$$

Similarly, we can prove the parts (iii) and (iv).

**Theorem 1** *Let* $\psi(\mathfrak{B}_1) = \left(\underline{\psi}(\mathfrak{B}_1), \overline{\psi}(\mathfrak{B}_1)\right)$ *and* $\psi(\mathfrak{B}_2) = \left(\underline{\psi}(\mathfrak{B}_2), \overline{\psi}(\mathfrak{B}_2)\right)$ *be two IFRNs and* $\alpha_1, \alpha_2 > 0$. *Then the following results are hold:*

(i) $\psi(\mathfrak{B}_1) \oplus \psi(\mathfrak{B}_2) = \psi(\mathfrak{B}_2) \oplus \psi(\mathfrak{B}_1)$,
(ii) $\psi(\mathfrak{B}_1) \otimes \psi(\mathfrak{B}_2) = \psi(\mathfrak{B}_2) \otimes \psi(\mathfrak{B}_1)$,
(iii) $\alpha_1(\psi(\mathfrak{B}_1) \oplus \psi(\mathfrak{B}_2)) = \alpha_1\psi(\mathfrak{B}_1) \oplus \alpha_1\psi(\mathfrak{B}_2)$,
(iv) $(\alpha_1 + \alpha_2)\psi(\mathfrak{B}_1) = \alpha_1\psi(\mathfrak{B}_1) \oplus \alpha_2\psi(\mathfrak{B}_1)$,
(v) $(\psi(\mathfrak{B}_1) \otimes \psi(\mathfrak{B}_2))^{\alpha_1} = (\psi(\mathfrak{B}_1))^{\alpha_1} \otimes (\psi(\mathfrak{B}_2))^{\alpha_1}$,
(vi) $(\psi(\mathfrak{B}_1))^{\alpha_1} \otimes (\psi(\mathfrak{B}_1))^{\alpha_2} = (\psi(\mathfrak{B}_1))^{(\alpha_1+\alpha_2)}$.

## 4 Average aggregation operators

The concept of aggregation operators has a significant role in DM to aggregate the multiple input information of different specialists into a single value. Here, we will address the concept of IFRDWA, IFRDOWA, and IFRDHA aggregation operators and present the important properties of these operators.

### 4.1 Intuitionistic fuzzy rough Dombi weighted averaging operators

**Definition 10** Assume that $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ be the family of IFRNs. Let $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ be the weight vector (WV) such that $\sum_{i=1}^{n}\varepsilon_i = 1$ and $\varepsilon_i \in [0, 1]$. Then the IFRDWA operator is a mapping $(\psi(\mathfrak{B}))^n \to \psi(\mathfrak{B})$, which is given as:

$$\mathrm{IFRDWA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \psi(\mathfrak{B}_3), \ldots, \psi(\mathfrak{B}_n)) = \left(\oplus_{i=1}^{n}\varepsilon_i\underline{\psi}(\mathfrak{B}_i), \oplus_{i=1}^{n}\varepsilon_i\overline{\psi}(\mathfrak{B}_i)\right).$$

**Theorem 2** *Let* $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ *be the family of IFRNs and* $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ *be the WV such that* $\sum_{i=1}^{n}\varepsilon_i = 1$*and* $\varepsilon_i \in [0, 1]$. *Then using IFRDWA operator, the aggregated result is described as:*

$$\mathrm{IFRDWA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \psi(\mathfrak{B}_3), \ldots, \psi(\mathfrak{B}_n)) = \left(\oplus_{i=1}^{n}\varepsilon_i\underline{\psi}(\mathfrak{B}_i), \oplus_{i=1}^{n}\varepsilon_i\overline{\psi}(\mathfrak{B}_i)\right)$$

$$
= \left\{ \left( 1 - \frac{1}{1 + \left\{ \sum_{i=1}^{n} \varepsilon_i \left( \frac{\underline{\hbar}_i}{1-\underline{\hbar}_i} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{ \sum_{i=1}^{n} \varepsilon_i \left( \frac{1-\underline{\delta}_i}{\underline{\delta}_i} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right), \left( 1 - \frac{1}{1 + \left\{ \sum_{i=1}^{n} \varepsilon_i \left( \frac{\overline{\hbar}_i}{1-\overline{\hbar}_i} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{ \sum_{i=1}^{n} \varepsilon_i \left( \frac{1-\overline{\delta}_i}{\overline{\delta}_i} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right) \right\}.
$$

***Proof*** By applying induction method to prove the required result.

Let $n = 2$, and now using the Dombi operational laws, we get

$$
\mathrm{IFRDWA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2)) = \left( \oplus_{i=1}^{2} \varepsilon_i \underline{\psi}(\mathfrak{B}_i), \oplus_{i=1}^{2} \varepsilon_i \overline{\psi}(\mathfrak{B}_i) \right)
$$

$$
= \left\{ \left( 1 - \frac{1}{1 + \left\{ \varepsilon_1 \left( \frac{\underline{\hbar}_1}{1-\underline{\hbar}_1} \right)^{\beta} + \varepsilon_2 \left( \frac{\underline{\hbar}_2}{1-\underline{\hbar}_2} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{ \varepsilon_1 \left( \frac{1-\underline{\delta}_1}{\underline{\delta}_1} \right)^{\beta} + \varepsilon_2 \left( \frac{1-\underline{\delta}_2}{\underline{\delta}_2} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right), \left( 1 - \frac{1}{1 + \left\{ \varepsilon_1 \left( \frac{\overline{\hbar}_1}{1-\overline{\hbar}_1} \right)^{\beta} + \varepsilon_2 \left( \frac{\overline{\hbar}_2}{1-\overline{\hbar}_2} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{ \varepsilon_1 \left( \frac{1-\overline{\delta}_1}{\overline{\delta}_1} \right)^{\beta} + \varepsilon_2 \left( \frac{1-\overline{\delta}_2}{\overline{\delta}_2} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right) \right\}
$$

$$
= \left\{ \left( 1 - \frac{1}{1 + \left\{ \sum_{i=1}^{2} \varepsilon_i \left( \frac{\underline{\hbar}_i}{1-\underline{\hbar}_i} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{ \sum_{i=1}^{2} \varepsilon_i \left( \frac{1-\underline{\delta}_i}{\underline{\delta}_i} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right), \left( 1 - \frac{1}{1 + \left\{ \sum_{i=1}^{2} \varepsilon_i \left( \frac{\overline{\hbar}_i}{1-\overline{\hbar}_i} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{ \sum_{i=1}^{2} \varepsilon_i \left( \frac{1-\overline{\delta}_i}{\overline{\delta}_i} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right) \right\}.
$$

The result is true for $n = 2$.

Assume that the required result holds for $n = k$, so we have

$$
\mathrm{IFRDWA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_k)) = \left( \oplus_{i=1}^{k} \varepsilon_i \underline{\psi}(\mathfrak{B}_i), \oplus_{i=1}^{k} \varepsilon_i \bar{\psi}(\mathfrak{B}_i) \right)
$$

$$
= \left\{ \left( 1 - \frac{1}{1 + \left\{ \sum_{i=1}^{k} \varepsilon_i \left( \frac{\underline{k}_i}{1-\underline{k}_i} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{ \sum_{i=1}^{k} \varepsilon_i \left( \frac{1-\underline{\delta}_i}{\underline{\delta}_i} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right), \right.
$$

$$\left(1-\frac{1}{1+\left\{\sum_{i=1}^{k}\varepsilon_i\left(\frac{\overline{k_i}}{1-\overline{k_i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{k}\varepsilon_i\left(\frac{1-\overline{\delta_i}}{\overline{\delta_i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\Bigg\}$$

Further, to prove $n=k+1$, we have

$$\begin{aligned}
&\text{IFRDWA}\{(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_k)),\psi(\mathfrak{B}_{k+1})\}\\
&=\left(\oplus_{i=1}^{k}\varepsilon_i\underline{\psi}(\mathfrak{B}_i),\oplus_{i=1}^{k}\varepsilon_i\overline{\psi}(\mathfrak{B}_i)\right)\oplus\left(\varepsilon_{k+1}\underline{\psi}(\mathfrak{B}_{k+1}),\varepsilon_{k+1}\overline{\psi}(\mathfrak{B}_{k+1})\right)\\
&=\left\{\left(1-\frac{1}{1+\left\{\sum_{i=1}^{k}\varepsilon_i\left(\frac{\underline{\hbar_i}}{1-\underline{\hbar_i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{k}\varepsilon_i\left(\frac{1-\underline{\delta_i}}{\underline{\delta_i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\right.\\
&\qquad\left.\left(1-\frac{1}{1+\left\{\sum_{i=1}^{k}\varepsilon_i\left(\frac{\overline{\hbar_i}}{1-\overline{\hbar_i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{k}\varepsilon_i\left(\frac{1-\overline{\delta_i}}{\overline{\delta_i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}\\
&\quad\oplus\left\{\left(1-\frac{1}{1+\left\{\varepsilon_{k+1}\left(\frac{\underline{\hbar_{k+1}}}{1-\underline{\hbar_{k+1}}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\varepsilon_{k+1}\left(\frac{1-\underline{\delta_{k+1}}}{\underline{\delta_{k+1}}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\right.\\
&\qquad\left.\left(1-\frac{1}{1+\left\{\varepsilon_{k+1}\left(\frac{\overline{\hbar_{k+1}}}{1-\overline{\hbar_{k+1}}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\varepsilon_{k+1}\left(\frac{1-\overline{\delta_{\varepsilon_{k+1}}}}{\overline{\delta_{k+1}}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}\\
&=\left\{\left(1-\frac{1}{1+\left\{\sum_{i=1}^{k+1}\varepsilon_i\left(\frac{\underline{\hbar_i}}{1-\underline{\hbar_i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{k+1}\varepsilon_i\left(\frac{1-\underline{\delta_i}}{\underline{\delta_i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\right.\\
&\qquad\left.\left(1-\frac{1}{1+\left\{\sum_{i=1}^{k+1}\varepsilon_i\left(\frac{\overline{\hbar_i}}{1-\overline{\hbar_i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{k+1}\varepsilon_i\left(\frac{1-\overline{\delta_i}}{\overline{\delta_i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}
\end{aligned}$$

Hence, the condition is true for $n \geq k+1$. Therefore, by induction principle, the result holds $\forall\, n \geq 1$.

As $\underline{\psi}(\mathfrak{B})$ and $\overline{\psi}(\mathfrak{B})$ are IFRNs, this implies $\oplus_{i=1}^{n}\varepsilon_i\underline{\psi}(\mathfrak{B}_i)$ and $\oplus_{i=1}^{n}\varepsilon_i\overline{\psi}(\mathfrak{B}_i)$ is also IFRNs. Therefore, from the above analysis, IFRDWA$(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n))$ also represents an IFRN based on IFR approximation space $(K,\psi)$.

**Theorem 3** *Let* $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ *be the family of IFRNs and* $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ *be the WV such that* $\sum_{i=1}^{n}\varepsilon_i = 1$ *and* $\varepsilon_i \in [0, 1]$. *Then some elementary properties are satisfied for IFRDWA operator.*

(i) *Idempotency* Let $\psi(\mathfrak{B}_i) = E(\mathfrak{B})\ \forall i = 1, 2, \ldots, n$ with $E(\mathfrak{B}) = \left(\underline{E}(\mathfrak{B}), \overline{E}(\mathfrak{B})\right) = \left(\langle\underline{\hbar}, \underline{\delta}\rangle, \langle\overline{\hbar}, \overline{\delta}\rangle\right)$. Then
$\text{IFRDWA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) = E(\mathfrak{B})$.

(ii) *Boundedness* Let $(\psi(\mathfrak{B}_i))^- = \left(\min_i \underline{\psi}(\mathfrak{B}_i), \min_i \overline{\psi}(\mathfrak{B}_i)\right)$ and $(\psi(\mathfrak{B}_i))^+ = \left(\max_i \underline{\psi}(\mathfrak{B}_i), \max_i \overline{\psi}(\mathfrak{B}_i)\right)$. Then
$(\psi(\mathfrak{B}_i))^- \leq IFRDWA(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \leq (\psi(\mathfrak{B}_i))^+$.

(iii) *Monotonicity* Consider the another family $\psi\left(\mathfrak{B}_i^{'}\right) = \left(\underline{\psi}\left(\mathfrak{B}_i^{'}\right), \overline{\psi}\left(\mathfrak{B}_i^{'}\right)\right)$ of IFRNs, such that $\underline{\psi}\left(\mathfrak{B}_i^{'}\right) \leq \underline{\psi}(\mathfrak{B}_i)$ and $\overline{\psi}\left(\mathfrak{B}_i^{'}\right) \leq \overline{\psi}(\mathfrak{B}_i)$. Then
$\text{IFRDWA}\left(\psi\left(\mathfrak{B}_1^{'}\right), \psi\left(\mathfrak{B}_2^{'}\right), \ldots, \psi\left(\mathfrak{B}_n^{'}\right)\right) \leq \text{IFRDWA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n))$.

(iv) *Shift invariance* Assume that $E\left(\mathfrak{B}^{'}\right) = \left(\underline{E}\left(\mathfrak{B}^{'}\right), \overline{E}\left(\mathfrak{B}^{'}\right)\right) = \left(\langle\underline{\hbar}^{'}, \underline{\delta}^{'}\rangle, \langle\overline{\hbar}^{'}, \overline{\delta}^{'}\rangle\right)$ be another IFRN. Then
$\text{IFRDWA}\left(\psi(\mathfrak{B}_1) \oplus E\left(\mathfrak{B}^{'}\right), \psi(\mathfrak{B}_2) \oplus E\left(\mathfrak{B}^{'}\right), \ldots, \psi(\mathfrak{B}_n) \oplus E\left(\mathfrak{B}^{'}\right)\right) = \text{IFRDWA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \oplus E\left(\mathfrak{B}^{'}\right)$.

(v) *Homogeneity* For a real number $\alpha > 0$,
$\text{IFRDWA}(\alpha\psi(\mathfrak{B}_1), \alpha\psi(\mathfrak{B}_2), \ldots, \alpha\psi(\mathfrak{B}_n)) = \alpha\text{IFRDWA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n))$.

***Proof***

(i) *Idempotency* Since $\psi(\mathfrak{B}_i) = E(\mathfrak{B})\ \forall i = 1, 2, \ldots, n$ where $E(\mathfrak{B}) = \left(\underline{E}(\mathfrak{B}), \overline{E}(\mathfrak{B})\right) = \left(\langle\underline{\hbar}, \underline{\delta}\rangle, \langle\overline{\hbar}, \overline{\delta}\rangle\right)$, then by applying Theorem 2, we have

$$\text{IFRDWA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \psi(\mathfrak{B}_3), \ldots, \psi(\mathfrak{B}_n)) = \left(\oplus_{i=1}^{n}\varepsilon_i\underline{\psi}(\mathfrak{B}_i), \oplus_{i=1}^{n}\varepsilon_i\overline{\psi}(\mathfrak{B}_i)\right)$$

$$= \left\{\left(1 - \frac{1}{1 + \left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\underline{\hbar}_i}{1-\underline{\hbar}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\delta}_i}{\underline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\right.$$
$$\left.\left(1 - \frac{1}{1 + \left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\overline{\hbar}_i}{1-\overline{\hbar}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\overline{\delta}_i}{\overline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}$$

$$= \left\{\left(1 - \frac{1}{1 + \left\{\left(\frac{\underline{\hbar}}{1-\underline{\hbar}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{\left(\frac{1-\underline{\delta}}{\underline{\delta}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\right.$$

$$\left.\left.\left(1-\frac{1}{1+\left\{\left(\frac{\overline{\hbar}}{1-\overline{\hbar}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\left(\frac{1-\overline{\delta}}{\overline{\delta}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right)\right]$$
$$=\left(\underline{E}(\mathfrak{B}),\overline{E}(\mathfrak{B})\right)$$
$$=E(\mathfrak{B})$$

(ii) *Boundedness* As $(\psi(\mathfrak{B}_i))^-=\left(\left(\underline{\psi}(\mathfrak{B}_i)\right)^-,\left(\overline{\psi}(\mathfrak{B}_i)\right)^-\right)=\left[\left(\min_i\{\underline{\hbar_i}\},\max_i\{\underline{\delta_i}\}\right),\left(\min_i\left\{\overline{\hbar_i}\right\},\max_i\{\overline{\delta_i}\}\right)\right]$ and $(\psi(\mathfrak{B}_i))^+=\left(\left(\underline{\psi}(\mathfrak{B}_i)\right)^+,\left(\overline{\psi}(\mathfrak{B}_i)\right)^+\right)=\left[\left(\max_i\{\underline{\hbar_i}\},\min_i\{\underline{\delta_i}\}\right),\left(\max_i\left\{\overline{\hbar_i}\right\},\min_i\{\overline{\delta_i}\}\right)\right]$ and $\psi(\mathfrak{B}_i)=\left[\left(\underline{\hbar_i},\underline{\delta_i}\right),\left(\overline{\hbar_i},\overline{\delta_i}\right)\right]$. To verify that

$$(\psi(\mathfrak{B}_i))^-\leq IFRDWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n))\leq(\psi(\mathfrak{B}_i))^+$$

Since for each $i=1,2,\ldots,n$, we have

$$\min_i\underline{\hbar_i}\leq\underline{\hbar_i}\leq\max_i\underline{\hbar_i}\Rightarrow\frac{\min_i\underline{\hbar_i}}{1-\min_i\underline{\hbar_i}}\leq\frac{\underline{\hbar_i}}{1-\underline{\hbar_i}}\leq\frac{\max_i\underline{\hbar_i}}{1-\max_i\underline{\hbar_i}}$$
$$\Rightarrow1+\frac{\min_i\underline{\hbar_i}}{1-\min_i\underline{\hbar_i}}\leq1+\frac{\underline{\hbar_i}}{1-\underline{\hbar_i}}\leq1+\frac{\max_i\underline{\hbar_i}}{1-\max_i\underline{\hbar_i}}\Rightarrow\frac{1}{1+\frac{\max_i\underline{\hbar_i}}{1-\max_i\underline{\hbar_i}}}\leq\frac{1}{1+\frac{\underline{\hbar_i}}{1-\underline{\hbar_i}}}\leq\frac{1}{1+\frac{\min_i\underline{\hbar_i}}{1-\min_i\underline{\hbar_i}}}$$
$$\Rightarrow1-\frac{1}{1+\left(\sum_{i=1}^n\varepsilon_i\left(\frac{\min_i\underline{\hbar_i}}{1-\min_i\underline{\hbar_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}\leq1-\frac{1}{1+\left(\sum_{i=1}^n\varepsilon_i\left(\frac{\underline{\hbar_i}}{1-\underline{\hbar_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}$$
$$\leq1-\frac{1}{1+\left(\sum_{i=1}^n\varepsilon_i\left(\frac{\max_i\underline{\hbar_i}}{1-\max_i\underline{\hbar_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}$$
$$\Rightarrow1-\frac{1}{1+\frac{\min_i\underline{\hbar_i}}{1-\min_i\underline{\hbar_i}}}\leq1-\frac{1}{1+\left(\sum_{i=1}^n\varepsilon_i\left(\frac{\underline{\hbar_i}}{1-\underline{\hbar_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}\leq1-\frac{1}{1+\frac{\max_i\underline{\hbar_i}}{1-\max_i\underline{\hbar_i}}}$$
$$\Rightarrow\min_i\underline{\hbar_i}\leq1-\frac{1}{1+\left(\sum_{i=1}^n\varepsilon_i\left(\frac{\underline{\hbar_i}}{1-\underline{\hbar_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}\leq\max_i\underline{\hbar_i}$$

Next consider for every $i=1,2,\ldots,n$, consider that

$$\max_i\{\underline{\delta_i}\}\geq\underline{\delta_i}\geq\min_i\{\underline{\delta_i}\}\Rightarrow1-\min_i\{\underline{\delta_i}\}\geq1-\underline{\delta_i}\geq1-\max_i\{\underline{\delta_i}\}$$
$$\Rightarrow1+\frac{1-\min_i\{\underline{\delta_i}\}}{\min_i\{\underline{\delta_i}\}}\geq1+\frac{1-\underline{\delta_i}}{\underline{\delta_i}}\geq1+\frac{1-\max_i\{\underline{\delta_i}\}}{\max_i\{\underline{\delta_i}\}}$$

$$\Rightarrow 1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\min_i\{\underline{\delta_i}\}}{\min_i\{\underline{\delta_i}\}}\right)^{\beta}\right)^{\frac{1}{\beta}} \geq 1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\delta_i}}{\underline{\delta_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}$$

$$\geq 1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\max_i\{\underline{\delta_i}\}}{\max_i\{\underline{\delta_i}\}}\right)^{\beta}\right)^{\frac{1}{\beta}}$$

$$\Rightarrow \frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\max_i\{\underline{\delta_i}\}}{\max_i\{\underline{\delta_i}\}}\right)^{\beta}\right)^{\frac{1}{\beta}}} \geq \frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\delta_i}}{\underline{\delta_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}$$

$$\geq \frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\min_i\{\underline{\delta_i}\}}{\min_i\{\underline{\delta_i}\}}\right)^{\beta}\right)^{\frac{1}{\beta}}}$$

$$\Rightarrow \frac{1}{1+\frac{1-\max_i\{\underline{\delta_i}\}}{\max_i\{\underline{\delta_i}\}}} \geq \frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\delta_i}}{\underline{\delta_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}} \geq \frac{1}{1+\frac{1-\min_i\{\underline{\delta_i}\}}{\min_i\{\underline{\delta_i}\}}}$$

$$\Rightarrow \max_i\{\underline{\delta_i}\} \geq \frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\delta_i}}{\underline{\delta_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}} \geq \min_i\{\underline{\delta_i}\}$$

In the same way, we can prove that

$$\Rightarrow \min_i \overline{\hbar_i} \leq 1-\frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{\overline{\hbar_i}}{1-\overline{\hbar_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}} \leq \max_i \overline{\hbar_i}$$

and

$$\max_i\{\overline{\delta_i}\} \geq \frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\overline{\delta_i}}{\overline{\delta_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}} \geq \min_i\{\overline{\delta_i}\}$$

Thus, from the above analysis, we have

$$(\psi(\mathfrak{B}_i))^{-} \leq \text{IFRDWA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \leq (\psi(\mathfrak{B}_i))^{+}.$$

The proofs of (iii), (iv), and (v) can be followed from (i) and (ii).

## 4.2 Intuitionistic fuzzy rough Dombi ordered weighted averaging operators

**Definition 11** Assume that $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ be the family of IFRNs. Let $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n}\varepsilon_i = 1$ and $\varepsilon_i \in [0, 1]$. Then the aggregated result for IFRDOWA operator is a mapping $(\psi(\mathfrak{B}))^n \rightarrow \psi(\mathfrak{B})$, which is given as:

$$\text{IFRDOWA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \psi(\mathfrak{B}_3), \ldots, \psi(\mathfrak{B}_n)) = \left(\oplus_{i=1}^{n}\varepsilon_i\underline{\psi}(\mathfrak{B}_{\sigma i}), \oplus_{i=1}^{n}\varepsilon_i\overline{\psi}(\mathfrak{B}_{\sigma i})\right).$$

**Theorem 4** *Let* $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ *be the family of IFRNs and* $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ *be the WV such that* $\sum_{i=1}^{n}\varepsilon_i = 1$ *and* $\varepsilon_i \in [0, 1]$. *Then using IFRDOWA operator, the aggregated result is described as:*

$$IFRDOWA(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \psi(\mathfrak{B}_3), \ldots, \psi(\mathfrak{B}_n)) = \left(\oplus_{i=1}^{n}\varepsilon_i\underline{\psi}(\mathfrak{B}_{\sigma i}), \oplus_{i=1}^{n}\varepsilon_i\overline{\psi}(\mathfrak{B}_{\sigma i})\right)$$

$$= \left\{\left(1 - \frac{1}{1 + \left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\underline{\hbar}_{\sigma i}}{1-\underline{\hbar}_{\sigma i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\delta}_{\sigma i}}{\underline{\delta}_{\sigma i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\right.$$

$$\left.\left(1 - \frac{1}{1 + \left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\overline{\hbar}_{\sigma i}}{1-\overline{\hbar}_{\sigma i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\overline{\delta}_{\sigma i}}{\overline{\delta}_{\sigma i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\},$$

*where the IFRN* $\psi(\mathfrak{B}_{\sigma i}) = \left(\underline{\psi}(\mathfrak{B}_{\sigma i}), \overline{\psi}(\mathfrak{B}_{\sigma i})\right)$ *represents the largest permutation of the collection* $\psi(\mathfrak{B}_i)$.

***Proof*** Proof is straightforward.

**Theorem 5** *Let* $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ *be the family of IFRNs and* $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ *be the WV such that* $\sum_{i=1}^{n}\varepsilon_i = 1$ *and* $\varepsilon_i \in [0, 1]$. *Then some rudimentary axioms are discussed for IFRDOWA operator.*

(i) *Idempotency.* Let $\psi(\mathfrak{B}_i) = E(\mathfrak{B})\ \forall i = 1, 2, \ldots, n$ such that $E(\mathfrak{B}) = \left(\underline{E}(\mathfrak{B}), \overline{E}(\mathfrak{B})\right) = \left(\langle\underline{\hbar}, \underline{\delta}\rangle, \langle\overline{\hbar}, \overline{\delta}\rangle\right)$. Then
IFRDOWA$(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) = E(\mathfrak{B})$.

(ii) *Boundedness* Let $(\psi(\mathfrak{B}_i))^- = \left(\min_i\underline{\psi}(\mathfrak{B}_i), \min_i\overline{\psi}(\mathfrak{B}_i)\right)$ and $(\psi(\mathfrak{B}_i))^+ = \left(\max_i\underline{\psi}(\mathfrak{B}_i), \max_i\overline{\psi}(\mathfrak{B}_i)\right)$. Then
$(\psi(\mathfrak{B}_i))^- \leq$ IFRDOWA$(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \leq (\psi(\mathfrak{B}_i))^+$.

(iii) *Monotonicity* Consider another family $\psi\left(\mathfrak{B}_i^{'}\right) = \left(\underline{\psi}\left(\mathfrak{B}_i^{'}\right), \overline{\psi}\left(\mathfrak{B}_i^{'}\right)\right)$ of IFRNs, such that $\underline{\psi}\left(\mathfrak{B}_i^{'}\right) \leq \underline{\psi}(\mathfrak{B}_i)$ and $\overline{\psi}\left(\mathfrak{B}_i^{'}\right) \leq \overline{\psi}(\mathfrak{B}_i)$. Then
IFRDOWA$\left(\psi\left(\mathfrak{B}_1^{'}\right), \psi\left(\mathfrak{B}_2^{'}\right), \ldots, \psi\left(\mathfrak{B}_n^{'}\right)\right) \leq$ IFRDOWA$(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n))$.

(iv) *Shift invariance* Assume that $E\left(\mathfrak{B}^{'}\right) = \left(\underline{E}\left(\mathfrak{B}^{'}\right), \overline{E}\left(\mathfrak{B}^{'}\right)\right) = \left(\langle\underline{\hbar}^{'}, \underline{\delta}^{'}\rangle, \langle\overline{\hbar}^{'}, \overline{\delta}^{'}\rangle\right)$ be another IFRN. Then
IFRDOWA$\left(\psi(\mathfrak{B}_1) \oplus E\left(\mathfrak{B}^{'}\right), \psi(\mathfrak{B}_2) \oplus E\left(\mathfrak{B}^{'}\right), \ldots, \psi(\mathfrak{B}_n) \oplus E\left(\mathfrak{B}^{'}\right)\right) =$
IFRDOWA$(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \oplus E\left(\mathfrak{B}^{'}\right)$.

(v) *Homogeneity* For a real number $\alpha > 0$,
IFRDOWA$(\alpha\psi(\mathfrak{B}_1), \alpha\psi(\mathfrak{B}_2), \ldots, \alpha\psi(\mathfrak{B}_n)) = \alpha$IFRDOWA$(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n))$.

***Proof*** Proofs are straightforward.

### 4.3 Intuitionistic fuzzy rough Dombi hybrid averaging operators

In this portion of the manuscript, to examine relation of the hybrid aggregation operators with IFRDWA and IFRDOWA operators which weight both the ordered position and the arguments value itself, IFRDHA generalized both the operations. This subsection consists of the study of IFRDHA operator and discuss its rudimentary properties.

**Definition 12** Assume that $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ be the family of IFRNs such that $\rho = (\rho_1, \rho_2, \ldots, \rho_n)^T$ be the WV such that $\sum_{i=1}^{n}\rho_i = 1$ and $\rho_i \in [0,1]$. Let $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ be the associated WV such that $\sum_{i=1}^{n}\varepsilon_i = 1$ and $\varepsilon_i \in [0,1]$. Then the aggregated result for IFRDHA operator is a mapping $(\psi(\mathfrak{B}))^n \to \psi(\mathfrak{B})$, which is given as:

$$\mathrm{IFRDHA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \psi(\mathfrak{B}_3), \ldots, \psi(\mathfrak{B}_n)) = \left(\oplus_{i=1}^{n}\varepsilon_i\underline{\psi}(\mathfrak{B}_{\widetilde{\sigma}i}), \oplus_{i=1}^{n}\varepsilon_i\overline{\psi}(\mathfrak{B}_{\widetilde{\sigma}i})\right).$$

**Theorem 6** *Let $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$be the family of IFRNs such that $\rho = (\rho_1, \rho_2, \ldots, \rho_n)^T$be the WV such that $\sum_{i=1}^{n}\rho_i = 1$and $\in [0,1]$. Let $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$be the associated WV such that $\sum_{i=1}^{n}\varepsilon_i = 1$and $\varepsilon_i \in [0,1]$. Then using IFRDHA operator, the aggregated result is described as:*

$$\mathrm{IFRDHA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \psi(\mathfrak{B}_3), \ldots, \psi(\mathfrak{B}_n)) = \left(\oplus_{i=1}^{n}\varepsilon_i\underline{\psi}(\mathfrak{B}_{\widetilde{\sigma}i}), \oplus_{i=1}^{n}\varepsilon_i\overline{\psi}(\mathfrak{B}_{\widetilde{\sigma}i})\right)$$

$$= \left\{\left(1 - \frac{1}{1 + \left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\underline{\hbar}_{\widetilde{\sigma}i}}{1-\underline{\hbar}_{\widetilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\delta}_{\widetilde{\sigma}i}}{\underline{\delta}_{\widetilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\right.$$
$$\left.\left(1 - \frac{1}{1 + \left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\overline{\hbar}_{\widetilde{\sigma}i}}{1-\overline{\hbar}_{\widetilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\overline{\delta}_{\widetilde{\sigma}i}}{\overline{\delta}_{\widetilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\},$$

*where the IFRN $\psi(\mathfrak{B}_{\widetilde{\sigma}i}) = n\rho_i\psi(\mathfrak{B}_i) = n\rho_i\left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$represents the largest permutation of the collection $\psi(\mathfrak{B}_i)$.*

***Proof*** Proof is straightforward.

**Theorem 7** *Let $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$be the family of IFRNs and $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$be the WV such that $\sum_{i=1}^{n}\varepsilon_i = 1$and $\varepsilon_i \in [0,1]$. Then some rudimentary characteristics are discussed for IFRDHA operator.*

(i) *Idempotency* Let $\psi(\mathfrak{B}_i) = E(\mathfrak{B})$ $\forall i = 1, 2, \ldots, n$ such that $E(\mathfrak{B}) = \left(\underline{E}(\mathfrak{B}), \overline{E}(\mathfrak{B})\right) = \left(\langle\underline{\hbar}, \underline{\delta}\rangle, \langle\overline{\hbar}, \overline{\delta}\rangle\right)$. Then
$\mathrm{IFRDHA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) = E(\mathfrak{B})$.

(ii) *Boundedness* Let $(\psi(\mathfrak{B}_i))^- = \left(\min_i\underline{\psi}(\mathfrak{B}_i), \min_i\overline{\psi}(\mathfrak{B}_i)\right)$ and $(\psi(\mathfrak{B}_i))^+ = \left(\max_i\underline{\psi}(\mathfrak{B}_i), \max_i\overline{\psi}(\mathfrak{B}_i)\right)$. Then
$(\psi(\mathfrak{B}_i))^- \leq \mathrm{IFRDHA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \leq (\psi(\mathfrak{B}_i))^+$.

(iii) *Monotonicity* Consider another family $\psi\left(\mathfrak{B}_i^{'}\right) = \left(\underline{\psi}\left(\mathfrak{B}_i^{'}\right), \overline{\psi}\left(\mathfrak{B}_i^{'}\right)\right)$ of IFRNs, such that $\underline{\psi}\left(\mathfrak{B}_i^{'}\right) \leq \underline{\psi}(\mathfrak{B}_i)$ and $\overline{\psi}\left(\mathfrak{B}_i^{'}\right) \leq \overline{\psi}(\mathfrak{B}_i)$. Then $\text{IFRDHA}\left(\psi\left(\mathfrak{B}_1^{'}\right), \psi\left(\mathfrak{B}_2^{'}\right), \ldots, \psi\left(\mathfrak{B}_n^{'}\right)\right) \leq \text{IFRDHA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n))$.

(iv) *Shift invariance* Assume that $E\left(\mathfrak{B}^{'}\right) = \left(\underline{E}\left(\mathfrak{B}^{'}\right), \overline{E}\left(\mathfrak{B}^{'}\right)\right) = \left(\langle \underline{\hbar}^{'}, \underline{\delta}^{'} \rangle, \langle \overline{\hbar}^{'}, \overline{\delta}^{'} \rangle\right)$ be another IFRN. Then $\text{IFRDHA}\left(\psi(\mathfrak{B}_1) \oplus E\left(\mathfrak{B}^{'}\right), \psi(\mathfrak{B}_2) \oplus E\left(\mathfrak{B}^{'}\right), \ldots, \psi(\mathfrak{B}_n) \oplus E\left(\mathfrak{B}^{'}\right)\right) = \text{IFRHWA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \oplus E\left(\mathfrak{B}^{'}\right)$.

(v) *Homogeneity* For a real number $\alpha > 0$, $\text{IFRDHA}(\alpha\psi(\mathfrak{B}_1), \alpha\psi(\mathfrak{B}_2), \ldots, \alpha\psi(\mathfrak{B}_n)) = \alpha\text{IFRDHA}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n))$.

Proofs are straightforward.

## 5 Geometric aggregation operators

Here, we will originate the novel notion of IFRDWG, IFRDOWG, and IFRDHG aggregation operators and present the important properties of these operators.

### 5.1 Intuitionistic fuzzy rough Dombi weighted geometric operators

**Definition 13** Assume that $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ be the family of IFRNs. Let $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ be the WV such that $\sum_{i=1}^n \varepsilon_i = 1$ and $\varepsilon_i \in [0, 1]$. Then the aggregated result for IFRDWG operator is a mapping $(\psi(\mathfrak{B}))^n \to \psi(\mathfrak{B})$, which is given as:

$$\text{IFRDWG}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \psi(\mathfrak{B}_3), \ldots, \psi(\mathfrak{B}_n)) = \left(\otimes_{i=1}^n \left(\underline{\psi}(\mathfrak{B}_i)\right)^{\varepsilon_i}, \otimes_{i=1}^n \left(\overline{\psi}(\mathfrak{B}_i)\right)^{\varepsilon_i}\right).$$

**Theorem 8** *Let $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ be the family of IFRNs and $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ be the WV such that $\sum_{i=1}^n \varepsilon_i = 1$ and $\varepsilon_i \in [0, 1]$. Then using IFRDWG operator, the aggregated result is described as*:

$$\text{IFRDWG}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \psi(\mathfrak{B}_3), \ldots, \psi(\mathfrak{B}_n)) = \left(\otimes_{i=1}^n \left(\underline{\psi}(\mathfrak{B}_i)\right)^{\varepsilon_i}, \otimes_{i=1}^n \left(\overline{\psi}(\mathfrak{B}_i)\right)^{\varepsilon_i}\right)$$

$$= \left\{ \left( \frac{1}{1 + \left\{ \sum_{i=1}^n \varepsilon_i \left( \frac{1 - \underline{\hbar}_i}{\underline{\hbar}_i} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, 1 - \frac{1}{1 + \left\{ \sum_{i=1}^n \varepsilon_i \left( \frac{\underline{\delta}_i}{1 - \underline{\delta}_i} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right), \right.$$
$$\left. \left( \frac{1}{1 + \left\{ \sum_{i=1}^n \varepsilon_i \left( \frac{1 - \overline{\hbar}_i}{\overline{\hbar}_i} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, 1 - \frac{1}{1 + \left\{ \sum_{i=1}^n \varepsilon_i \left( \frac{\overline{\delta}_i}{1 - \overline{\delta}_i} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right) \right\}.$$

***Proof*** Proof followed from Theorem 2.

**Theorem 9** *Let* $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ *be the family of IFRNs and* $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ *be the WV such that* $\sum_{i=1}^{n}\varepsilon_i = 1$ *and* $\varepsilon_i \in [0, 1]$. *Then the elementary result for IFRDWG operator are given as.*

(i) *Idempotency. Let* $\psi(\mathfrak{B}_i) = E(\mathfrak{B})\ \forall i = 1, 2, \ldots, n$ *such that* $E(\mathfrak{B}) = \left(\underline{E}(\mathfrak{B}), \overline{E}(\mathfrak{B})\right) = \left(\langle \underline{\hbar}, \underline{\delta}\rangle, \langle \overline{\hbar}, \overline{\delta}\rangle\right)$. *Then*
$IFRDWG(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) = E(\mathfrak{B})$.

(ii) *Boundedness. Let* $(\psi(\mathfrak{B}_i))^- = \left(\min_i \underline{\psi}(\mathfrak{B}_i), \min_i \overline{\psi}(\mathfrak{B}_i)\right)$ *and* $(\psi(\mathfrak{B}_i))^+ = \left(\max_i \underline{\psi}(\mathfrak{B}_i), \max_i \overline{\psi}(\mathfrak{B}_i)\right)$. *Then*
$(\psi(\mathfrak{B}_i))^- \leq IFRDWG(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \leq (\psi(\mathfrak{B}_i))^+$.

(iii) *Monotonicity. Consider another family* $\psi\left(\mathfrak{B}_i^{'}\right) = \left(\underline{\psi}\left(\mathfrak{B}_i^{'}\right), \overline{\psi}\left(\mathfrak{B}_i^{'}\right)\right)$ *of IFRNs, such that* $\underline{\psi}\left(\mathfrak{B}_i^{'}\right) \leq \underline{\psi}(\mathfrak{B}_i)$ *and* $\overline{\psi}\left(\mathfrak{B}_i^{'}\right) \leq \overline{\psi}(\mathfrak{B}_i)$. *Then*
$IFRDWG\left(\psi\left(\mathfrak{B}_1^{'}\right), \psi\left(\mathfrak{B}_2^{'}\right), \ldots, \psi\left(\mathfrak{B}_n^{'}\right)\right) \leq IFRDWG(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n))$.

(iv) *Shift invariance. Assume that* $E\left(\mathfrak{B}^{'}\right) = \left(\underline{E}\left(\mathfrak{B}^{'}\right), \overline{E}\left(\mathfrak{B}^{'}\right)\right) = \left(\langle \underline{\hbar}^{'}, \underline{\delta}^{'}\rangle, \langle \overline{\hbar}^{'}, \overline{\delta}^{'}\rangle\right)$ *be another IFRN. Then*
$IFRDWG\left(\psi(\mathfrak{B}_1) \oplus E\left(\mathfrak{B}^{'}\right), \psi(\mathfrak{B}_2) \oplus E\left(\mathfrak{B}^{'}\right), \ldots, \psi(\mathfrak{B}_n) \oplus E\left(\mathfrak{B}^{'}\right)\right) = IFRDWG(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \oplus E\left(\mathfrak{B}^{'}\right)$.

(v) *Homogeneity. For a real number* $\alpha > 0$,
$IFRDWG(\alpha\psi(\mathfrak{B}_1), \alpha\psi(\mathfrak{B}_2), \ldots, \alpha\psi(\mathfrak{B}_n)) = \alpha IFRDWG(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n))$.

***Proof*** Proofs are easy and straightforward.

## 5.2 Intuitionistic fuzzy rough Dombi ordered weighted geometric operators

**Definition 14** Assume that $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ be the family of IFRNs. Let $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n}\varepsilon_i = 1$ and $\varepsilon_i \in [0, 1]$. Then the aggregated result for IFRDOWG operator is a mapping $(\psi(\mathfrak{B}))^n \to \psi(\mathfrak{B})$, which is given as:

$$\text{IFRDOWG}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \psi(\mathfrak{B}_3), \ldots, \psi(\mathfrak{B}_n)) = \left(\otimes_{i=1}^{n}\left(\underline{\psi}(\mathfrak{B}_{\sigma i})\right)^{\varepsilon_i}, \otimes_{i=1}^{n}\left(\overline{\psi}(\mathfrak{B}_{\sigma i})\right)^{\varepsilon_i}\right).$$

**Theorem 10** *Let* $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ *be the family of IFRNs and* $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ *be the WV such that* $\sum_{i=1}^{n}\varepsilon_i = 1$ *and* $\varepsilon_i \in [0, 1]$. *Then using IFRDOWG operator, the aggregated result is described as:*

$$IFRDOWG(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) = \left(\otimes_{i=1}^{n}\left(\underline{\psi}(\mathfrak{B}_{\sigma i})\right)^{\varepsilon_i}, \otimes_{i=1}^{n}\left(\overline{\psi}(\mathfrak{B}_{\sigma i})\right)^{\varepsilon_i}\right)$$

$$= \left\{\left(\frac{1}{1 + \left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\hbar}_{\sigma i}}{\underline{\hbar}_{\sigma i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}, 1 - \frac{1}{1 + \left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\underline{\delta}_{\sigma i}}{1-\underline{\delta}_{\sigma i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\right.$$

$$\left(\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\overline{\hbar}_{\sigma i}}{\overline{\hbar}_{\sigma i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}, 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\overline{\delta}_{\sigma i}}{1-\overline{\delta}_{\sigma i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\Bigg)\Bigg\},$$

*where the IFRN* $\psi(\mathfrak{B}_{\sigma i}) = \left(\underline{\psi}(\mathfrak{B}_{\sigma i}), \overline{\psi}(\mathfrak{B}_{\sigma i})\right)$ *represents the largest permutation of the collection* $\psi(\mathfrak{B}_i)$.

***Proof*** Proof is straightforward.

**Theorem 11** *Let* $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ *be the family of IFRNs and* $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ *be the WV such that* $\sum_{i=1}^{n}\varepsilon_i = 1$ *and* $\varepsilon_i \in [0, 1]$. *Then some basic results are satisfied for the collection* $\psi(\mathfrak{B}_i)$ *by applying IFRDOWA operator.*

(i) *Idempotency*. Let $\psi(\mathfrak{B}_i) = E(\mathfrak{B})\ \forall i = 1, 2, \ldots, n$ such that $E(\mathfrak{B}) = \left(\underline{E}(\mathfrak{B}), \overline{E}(\mathfrak{B})\right) = \left(\langle\underline{\hbar}, \underline{\delta}\rangle, \langle\overline{\hbar}, \overline{\delta}\rangle\right)$. Then
IFRDOWG$(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) = E(\mathfrak{B})$.

(ii) *Boundedness*. Let $(\psi(\mathfrak{B}_i))^- = \left(\min_i \underline{\psi}(\mathfrak{B}_i), \min_i \overline{\psi}(\mathfrak{B}_i)\right)$ and $(\psi(\mathfrak{B}_i))^+ = \left(\max_i \underline{\psi}(\mathfrak{B}_i), \max_i \overline{\psi}(\mathfrak{B}_i)\right)$. Then
$(\psi(\mathfrak{B}_i))^- \leq IFRDOWG(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \leq (\psi(\mathfrak{B}_i))^+$.

(iii) *Monotonicity*. Consider another family $\psi\left(\mathfrak{B}_i'\right) = \left(\underline{\psi}\left(\mathfrak{B}_i'\right), \overline{\psi}\left(\mathfrak{B}_i'\right)\right)$ of IFRNs, such that $\underline{\psi}\left(\mathfrak{B}_i'\right) \leq \underline{\psi}(\mathfrak{B}_i)$ and $\overline{\psi}\left(\mathfrak{B}_i'\right) \leq \overline{\psi}(\mathfrak{B}_i)$. Then
IFRDOWG$\left(\psi\left(\mathfrak{B}_1'\right), \psi\left(\mathfrak{B}_2'\right), \ldots, \psi\left(\mathfrak{B}_n'\right)\right) \leq$ IFRDOWG$(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n))$.

(iv) *Shift invariance*. Assume that $E\left(\mathfrak{B}'\right) = \left(\underline{E}\left(\mathfrak{B}'\right), \overline{E}\left(\mathfrak{B}'\right)\right) = \left(\langle\underline{\hbar}', \underline{\delta}'\rangle, \langle\overline{\hbar}', \overline{\delta}'\rangle\right)$ be another IFRN. Then
IFRDOWG$\left(\psi(\mathfrak{B}_1) \oplus E\left(\mathfrak{B}'\right), \psi(\mathfrak{B}_2) \oplus E\left(\mathfrak{B}'\right), \ldots, \psi(\mathfrak{B}_n) \oplus E\left(\mathfrak{B}'\right)\right) =$
IFRDOWG$(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \oplus E\left(\mathfrak{B}'\right)$.

(v) *Homogeneity*. For a real number $\alpha > 0$,
IFRDOWG$(\alpha\psi(\mathfrak{B}_1), \alpha\psi(\mathfrak{B}_2), \ldots, \alpha\psi(\mathfrak{B}_n)) = \alpha$IFRDOWG$(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n))$.

***Proof*** Proofs are easy and straightforward.

### 5.3 Intuitionistic fuzzy rough Dombi hybrid geometric operators

In this portion of the manuscript, to examine relation of the hybrid geometric operators with IFRDWG and IFRDOWG operators which weight both the ordered position and the arguments value itself, IFRDHG generalized both the operations. This subsection consists of the study of IFRDHG operator and discuss its rudimentary properties.

**Definition 15** Assume that $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ be the family of IFRNs such that $\rho = (\rho_1, \rho_2, \ldots, \rho_n)^T$ be the WV such that $\sum_{i=1}^{n}\rho_i = 1$ and $\rho_i \in [0, 1]$. Let

$\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ be the associated WV such that $\sum_{i=1}^{n} \varepsilon_i = 1$ and $\varepsilon_i \in [0, 1]$. Then the IFRDHG operator is a mapping $(\psi(\mathfrak{B}))^n \to \psi(\mathfrak{B})$, which is given as:

$$\mathrm{IFRDHG}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \psi(\mathfrak{B}_3), \ldots, \psi(\mathfrak{B}_n)) = \left(\otimes_{i=1}^{n}\left(\underline{\psi}(\mathfrak{B}_{\tilde{\sigma}i})\right)^{\varepsilon_i}, \otimes_{i=1}^{n}\left(\overline{\psi}(\mathfrak{B}_{\tilde{\sigma}i})\right)^{\varepsilon_i}\right).$$

**Theorem 12** *Let* $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$ *be the family of IFRNs such that* $\rho = (\rho_1, \rho_2, \ldots, \rho_n)^T$ *be the WV such that* $\sum_{i=1}^{n} \rho_i = 1$*and* $\in [0, 1]$. *Let* $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$ *be the associated WV such that* $\sum_{i=1}^{n} \varepsilon_i = 1$*and* $\varepsilon_i \in [0, 1]$. *Then using IFRDHG operator, the aggregated result is described as:*

$$IFRDHG(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \psi(\mathfrak{B}_3), \ldots, \psi(\mathfrak{B}_n)) = \left(\otimes_{i=1}^{n}\left(\underline{\psi}(\mathfrak{B}_{\tilde{\sigma}i})\right)^{\varepsilon_i}, \otimes_{i=1}^{n}\left(\overline{\psi}(\mathfrak{B}_{\tilde{\sigma}i})\right)^{\varepsilon_i}\right)$$

$$= \left\{\left(\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\hbar}_{\tilde{\sigma}i}}{\underline{\hbar}_{\tilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}, 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\underline{\delta}_{\tilde{\sigma}i}}{1-\underline{\delta}_{\tilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\right.$$

$$\left.\left(\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\overline{\hbar}_{\tilde{\sigma}i}}{\overline{\hbar}_{\tilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}, 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\overline{\delta}_{\tilde{\sigma}i}}{1-\overline{\delta}_{\tilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\},$$

*where the IFRN* $\psi(\mathfrak{B}_{\tilde{\sigma}i}) = n\rho_i\psi(\mathfrak{B}_i) = n\rho_i\left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$*represent the largest permutation of the collection* $\psi(\mathfrak{B}_i)$.

***Proof*** Proof is straightforward.

**Theorem 13** *Let* $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i), \overline{\psi}(\mathfrak{B}_i)\right)$*be the family of IFRNs and* $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)^T$*be the WV such that* $\sum_{i=1}^{n} \varepsilon_i = 1$*and* $\varepsilon_i \in [0, 1]$. *Then some basic results are satisfied for the collection* $\psi(\mathfrak{B}_i)$*by applying IFRDOWG operator.*

(i) *Idempotency*. Let $\psi(\mathfrak{B}_i) = E(\mathfrak{B})\ \forall i = 1, 2, \ldots, n$ such that $E(\mathfrak{B}) = \left(\underline{E}(\mathfrak{B}), \overline{E}(\mathfrak{B})\right) = \left(\langle\underline{\hbar}, \underline{\delta}\rangle, \langle\overline{\hbar}, \overline{\delta}\rangle\right)$. Then
$\mathrm{IFRDHG}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) = E(\mathfrak{B})$.

(ii) *Boundedness*. Let $(\psi(\mathfrak{B}_i))^- = \left(\min_i \underline{\psi}(\mathfrak{B}_i), \min_i \overline{\psi}(\mathfrak{B}_i)\right)$ and $(\psi(\mathfrak{B}_i))^+ = \left(\max_i \underline{\psi}(\mathfrak{B}_i), \max_i \overline{\psi}(\mathfrak{B}_i)\right)$. Then
$(\psi(\mathfrak{B}_i))^- \le \mathrm{IFRDHG}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \le (\psi(\mathfrak{B}_i))^+$.

(iii) *Monotonicity*. Consider another family $\psi\left(\mathfrak{B}_i^{'}\right) = \left(\underline{\psi}\left(\mathfrak{B}_i^{'}\right), \overline{\psi}\left(\mathfrak{B}_i^{'}\right)\right)$ of IFRNs, such that $\underline{\psi}\left(\mathfrak{B}_i^{'}\right) \le \underline{\psi}(\mathfrak{B}_i)$ and $\overline{\psi}\left(\mathfrak{B}_i^{'}\right) \le \overline{\psi}(\mathfrak{B}_i)$. Then
$\mathrm{IFRDHG}\left(\psi\left(\mathfrak{B}_1^{'}\right), \psi\left(\mathfrak{B}_2^{'}\right), \ldots, \psi\left(\mathfrak{B}_n^{'}\right)\right) \le \mathrm{IFRDHG}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n))$.

(iv) *Shift invariance*. Assume that $E\left(\mathfrak{B}^{'}\right) = \left(\underline{E}\left(\mathfrak{B}^{'}\right), \overline{E}\left(\mathfrak{B}^{'}\right)\right) = \left(\langle\underline{\hbar}^{'}, \underline{\delta}^{'}\rangle, \langle\overline{\hbar}^{'}, \overline{\delta}^{'}\rangle\right)$ be another IFRN. Then
$\mathrm{IFRDHG}\left(\psi(\mathfrak{B}_1) \oplus E\left(\mathfrak{B}^{'}\right), \psi(\mathfrak{B}_2) \oplus E\left(\mathfrak{B}^{'}\right), \ldots, \psi(\mathfrak{B}_n) \oplus E\left(\mathfrak{B}^{'}\right)\right) = \mathrm{IFRHWG}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \oplus E\left(\mathfrak{B}^{'}\right)$.

(v) *Homogeneity*. For a real number $\alpha > 0$,
$\mathrm{IFRDHG}(\alpha\psi(\mathfrak{B}_1), \alpha\psi(\mathfrak{B}_2), \ldots, \alpha\psi(\mathfrak{B}_n)) = \alpha\mathrm{IFRDHG}(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n))$.

***Proof*** Proofs are easy and straightforward.

## 6 A new approach for MCGDM hybrid with TOPSIS technique

In this portion of the manuscript, we will present the general structure of the TOPSIS and step-wise algorithm for TOPSIS technique based on MCGDM.

In real life, group DM is one of the most significant process, in which the professional experts of different genre present their input evaluations for every alternative against all criteria to get the most desirable solution. Assume that the set $K = \{\mathfrak{g}_1, \mathfrak{g}_2, \ldots, \mathfrak{g}_n\}$ of $n$ objects and let $\widetilde{C} = \{\widetilde{C}_1, \widetilde{C}_2, \ldots, \widetilde{C}_m\}$ be the set corresponding criteria with WV $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m)^T$ such that $\sum_{j=1}^{m}\varepsilon_j$ with $\varepsilon_j \in [0, 1]$. Let $G = \{G_1, G_2, \ldots, G_t\}$ be a set of professional specialist who assign their personal views for each alternatives with respect to corresponding criteria with WV $v = (v_1, v_2, \ldots, v_t)^T$ such that $\sum_{l=1}^{t} v_l$ with $v_l \in [0, 1]$. The decision experts present their evaluation in the form of IFRNs and collectively represented in the form of decision matrix $\mathcal{M} = [\psi(\mathfrak{B}_{ij})]_{m\times n}$, and then defined the accumulated geometric operator to transform the IFRNs into IFNs which is defined by:

**Definition 16** Let $\psi(\mathfrak{B}) = \left(\underline{\psi}(\mathfrak{B}), \overline{\psi}(\mathfrak{B})\right) = \left(\langle\underline{\hbar}, \underline{\delta}\rangle, \langle\overline{\hbar}, \overline{\delta}\rangle\right)$ be an intuitionistic fuzzy rough number. Then transform the intuitionistic fuzzy rough number into intuitionistic fuzzy number by applying accumulated geometric operator (AGO), which is defined as:

$$\mathfrak{G} = (\hbar_{\mathfrak{G}}, \delta_{\mathfrak{G}}) = \left(\underline{\psi}(\mathfrak{B}), \overline{\psi}(\mathfrak{B})\right)^{0.5} = \left(\left(\underline{\hbar}\overline{\hbar}\right)^{0.5}, \left(\underline{\delta}\overline{\delta}\right)^{0.5}\right)$$

The combined opinions of decision experts are expressed in the form of intuitionistic fuzzy rough decision matrix $\mathcal{M} = [\psi(\mathfrak{B}_{ij})]_{m\times n}$ which is transformed into an intuitionistic fuzzy decision matrix $\mathcal{M} = \left[\mathfrak{G}\left(\hbar_{\mathfrak{G}ij}, \delta_{\mathfrak{G}_{ij}}\right)\right]_{m\times n}$ by applying AGO.

Furthermore, the technique of TOPSIS method is applied to calculate the IF-PIS $\mathcal{P}^+$ and IF-NIS $\mathcal{P}^-$ of the transformed decision matrix via the score function which is defined as:

$$\begin{aligned}
\mathcal{P}^+ &= \left\{\langle\widetilde{C}_j, \max\left\{\overline{\overline{S}}\left(\widetilde{C}_j(\mathfrak{g}_i)\right)\right\}\rangle | i = 1, \ldots, n, j = 1, \ldots, m\right\} \\
&= \left\{\langle\widetilde{C}_1, (\hbar_1^+, \delta_1^+)\rangle, \langle\widetilde{C}_2, (\hbar_2^+, \delta_2^+)\rangle, \ldots, \langle\widetilde{C}_m, (\hbar_m^+, \delta_m^+)\rangle\right\} \\
\mathcal{P}^- &= \left\{\langle\widetilde{C}_j, \min\left\{\overline{\overline{S}}\left(\widetilde{C}_j(\mathfrak{g}_i)\right)\right\}\rangle | i = 1, \ldots, n, j = 1, \ldots, m\right\} \\
&= \left\{\langle\widetilde{C}_1, (\hbar_1^-, \delta_1^-)\rangle, \langle\widetilde{C}_2, (\hbar_2^-, \delta_2^-)\rangle, \ldots, \langle\widetilde{C}_m, (\hbar_m^-, \delta_m^-)\rangle\right\}
\end{aligned}$$

Calculate the shortest distance $D^+$ and farthest distance $D^-$ between the each object $\mathfrak{g}_i$ and the IF-PIS and IF-NIS

$$\begin{aligned}
D^+(\mathfrak{g}_i, \mathcal{P}^+) &= \sum_{j=1}^{n}\varepsilon_j d\left(\widetilde{C}_j(\mathfrak{g}_i), \widetilde{C}_j(\mathcal{P}^+)\right) \\
&= \frac{1}{2}\sum_{j=1}^{n}\varepsilon_j\left(\left|\hbar_{ij} - \hbar^+{}_j\right| + \left|\delta_{ij} - \delta^+{}_j\right| + \left|\pi_{ij} - \pi^+{}_j\right|\right)\text{for}\, p > 1 \\
D^-(\mathfrak{g}_i, \mathcal{P}^-) &= \sum_{j=1}^{n}\varepsilon_j d\left(\widetilde{C}_j(\mathfrak{g}_i), \widetilde{C}_j(\mathcal{P}^-)\right)
\end{aligned}$$

$$= \frac{1}{2}\sum_{j=1}^{n} \varepsilon_j \left( \left| \hbar_{ij} - \hbar^-{}_j \right| + \left| \delta_{ij} - \delta^-{}_j \right| + \left| \pi_{ij} - \pi^-{}_j \right| \right) \text{for} p > 1$$

Generally the objects having smaller the value of shortest distance $D^+(\mathfrak{g}_i, \mathcal{P}^+)$ is better the alternative $\mathfrak{g}_i$ and the object havinh bigger the value of farthest distance $D^-(\mathfrak{g}_i, \mathcal{P}^-)$ is better that alternative $\mathfrak{g}_i$.

$$D^+_{min}(\mathfrak{g}_i, \mathcal{P}^+) = \min_{1 \le i \le n} D^+(\mathfrak{g}_i, \mathcal{P}^+), \; D^-_{max}(\mathfrak{g}_i, \mathcal{P}^-) = \max_{1 \le i \le n} D^-(\mathfrak{g}_i, \mathcal{P}^-)$$

Finally, from the above analysis, the ranking of all alternatives was calculated according to the corresponding criteria and arranged them in a specific order to get the optimum value.

$$\xi(\mathfrak{g}_i) = \frac{D^-(\mathfrak{g}_i, \mathcal{P}^-)}{D^-_{max}(\mathfrak{g}_i, \mathcal{P}^-)} - \frac{D^+(\mathfrak{g}_i, \mathcal{P}^+)}{D^+_{min}(\mathfrak{g}_i, \mathcal{P}^+)}.$$

**Algorithm** From the above analysis, the step-wise decision algorithm for the developed approach consists of the following steps:

*Step 1* The decision experts present their evaluation in the form of IFRNs and collectively expressed in the form of IFR decision matrix given by:

$$\mathcal{M} = \left[ \psi(\mathfrak{B}_{ij}) \right]_{m \times n}.$$

*Step 2* Aggregate the expressed combine decision assessment of the professional experts by applying the developed approach to get a single decision matrix in the form of IFR decision matrix.

*Step 3* The collective aggregated evaluation information of decision experts in the form of IF rough decision matrix $\mathcal{M} = \left[ \psi(\mathfrak{B}_{ij}) \right]_{m \times n}$ is transformed into an intuitionistic fuzzy decision matrix $\mathcal{M} = \left[ \mathfrak{G}\left( \hbar_{\mathfrak{G}ij}, \delta_{\mathfrak{G}_{ij}} \right) \right]_{m \times n}$ by applying AGO.

*Step 4* Calculate the IF-PIS $\mathcal{P}^+$ and IF-NIS $\mathcal{P}^-$ of the transformed decision matrix via the score function.

*Step 5* Calculate the shortest distance $D^+$ and farthest distance $D^-$ between the alternative $\mathfrak{g}_i$ and the IF-PIS and IF-NIS.

*Step 6* Finally, by applying the ranking function $\xi(\mathfrak{g}_i)$, arrange the ranking information in a specific order to get the optimum object.

## 7 Illustrative example

The massive outbreak of the pandemic COVID-19 promoted the challenging scenario for the world organizations including scientists, laboratories, and researchers to conduct special clinical treatment strategies to prevent the people from COVID-19 pandemic. Globally, COVID-19 pandemic affected the human race and hit hard on them in terms of health and economy. The most severe symptoms, which need medical attention are, low level of oxygen in the body, pneumonia, sometime failure of vital organs such as kidneys, heart, and liver. Studies also reported loss of taste and smell. The common symptoms reported by CDC is mentioned somewhere in this article, but we here studied the symptoms with sever disease that are associated to most distinctive comorbidities SARS-CoV-2 infection. We will discuss the severity of the disease with symptoms through an illustrative example.

Assume that a team of experts doctors including $D_1$, $D_2$, and $D_3$ are called to diagnose the most severe illness of COVID-19 patient with WV $\varepsilon = (0.326, 0.352, 0.322)^T$ such that

$\sum_{j=1}^{m}\varepsilon_j$ with $\varepsilon_j \in [0, 1]$. The experts examined four patients $\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3$, and $\mathfrak{g}_4$. According to the recent study by the collaboration of different organizations, majority exhibited clinical criteria such as $\widetilde{C}_1$ = fever, $\widetilde{C}_2$ = dry cough, $\widetilde{C}_3$ = fatigue, $\widetilde{C}_4$ = diarrhea and $\widetilde{C}_5$ = shortness of breath, with WV $\vartheta = (0.215, 0.218, 0.212, 0.231, 0.124)^T$. Further, the decision maker presented their evaluation report in the form of IFRNs for each patient $\mathfrak{g}_i$ with respect to their corresponding criteria. Now, the step-wise algorithm was applied for the developed approach to diagnose the most severe ill patient by taking the operational parameter $\beta = 2$.

### 7.1 For IFRDWA/IFRDWG operator

*Step 1* The decision experts expressed their judgement in the form of IFRNs and collectively represented in the form of IFR decision matrix given in Tables 1, 2, 3.

*Step 2* Aggregate the collective decision information of the professional experts given in Tables 1, 2, 3, by applying the IFRDWA/IFRDWG operator to get a single decision matrix in the form of IFR decision matrix which is given in Table 4.

*Step 3* The collective aggregated evaluation information of decision experts in the form of IF rough decision matrix $\mathcal{M} = \left[\psi\left(\mathfrak{B}_{ij}\right)\right]_{m\times n}$ is transformed into an intuitionistic fuzzy decision matrix $\mathcal{M} = \left[\mathfrak{G}\left(\hbar_{\mathfrak{G}ij}, \delta_{\mathfrak{G}_{ij}}\right)\right]_{m\times n}$ by applying AGO, which is defined as:

$$\mathfrak{G} = (\hbar_{\mathfrak{G}}, \delta_{\mathfrak{G}}) = \left(\underline{\psi}(\mathfrak{B}), \overline{\psi}(\mathfrak{B})\right)^{0.5} = \left(\left(\underline{\hbar}\overline{\hbar}\right)^{0.5}, \left(\underline{\delta}\overline{\delta}\right)^{0.5}\right).$$

The collective evaluation information of decision experts in the form of intuitionistic fuzzy decision matrix by applying AGO is given in Table 5.

*Step 4* Determine the IF-PIS $\mathcal{P}^+$ and IF-NIS $\mathcal{P}^-$ of the transformed decision matrix given in Table 5, by applying the score function given in Definition 6.

$$\mathcal{P}^+ = \{(0.7304, 0.1443), (0.8434, 0.1437), (0.7915, 0.1425), (0.8474, 0.1159), (0.7853, 0.1296)\}$$

$$\mathcal{P}^- = \{(0.5772, 0.1723), (0.5486, 0.1881), (0.6631, 0.1620), (0.5131, 0.1441), (0.6955, 0.1607)\}$$

*Step 5* Calculate the shortest distance $D^+\left(\mathfrak{g}_i, \mathcal{P}^+\right)$ and farthest distance $D^-\left(\mathfrak{g}_i, \mathcal{P}^-\right)$ between the alternative $\mathfrak{g}_i$ and the IF-PIS and IF-NIS, which is given in Table 6.

*Step 6* Finally, by applying the ranking function $\xi(\mathfrak{g}_i)$, arrange the ranking information in a specific order to get the optimum object, which is illustrated in Table 7.

### 7.2 Comparative study for the effectiveness of the proposed approaches

The TOPSIS method is one of the most significant technique to cope MCDM problems, in which the target is to get the optimal object having highest score value known as PIS and the object with the least score value is known as NIF. To present the ability and resilience of proposed approach by applying IF rough aggregation operators based on Dombi t-norms and t-conorms hybrid with TOPSIS method, we made a comparison of the investigated concept with several previous models in literature such as IFWA operator by Xu (2007), IFWG operator by Xu and Yager (2006), IF TOPSIS method by Yinghui and Wenlu (2015), IFRWA operator by Yahya et al. (2021), IFRWA and IFRWG operators based on EDAS method by Chinram et al. (2021), IFDWA and IFDWG operators by Seikh and Mandal (2021). If we consider Tables 1, 2, 3, and 4, then the aggregation operators presented in Xu (2007), Xu and Yager (2006), Seikh and Mandal (2021) and Yinghui and Wenlu (2015) are not

**Table 1** IFR evaluation information $D_1$

| | $\widetilde{C}_1$ | $\widetilde{C}_2$ | $\widetilde{C}_3$ | $\widetilde{C}_4$ | $\widetilde{C}_5$ |
|---|---|---|---|---|---|
| $\mathfrak{g}_1$ | $((0.7, 0.1), (0.4, 0.2))$ | $((0.7, 0.2), (0.6, 0.4))$ | $((0.5, 0.2), (0.8, 0.2))$ | $((0.8, 0.1), (0.7, 0.2))$ | $((0.4, 0.2), (0.5, 0.3))$ |
| $\mathfrak{g}_2$ | $((0.6, 0.2), (0.9, 0.1))$ | $((0.5, 0.3), (0.9, 0.1))$ | $((0.9, 0.1), (0.6, 0.3))$ | $((0.4, 0.1), (0.5, 0.1))$ | $((0.8, 0.1), (0.6, 0.2))$ |
| $\mathfrak{g}_3$ | $((0.8, 0.2), (0.7, 0.2))$ | $((0.4, 0.1), (0.3, 0.2))$ | $((0.4, 0.2), (0.7, 0.1))$ | $((0.9, 0.1), (0.6, 0.2))$ | $((0.6, 0.3), (0.7, 0.2))$ |
| $\mathfrak{g}_4$ | $((0.4, 0.1), (0.6, 0.3))$ | $((0.8, 0.1), (0.7, 0.3))$ | $((0.9, 0.1), (0.6, 0.2))$ | $((0.5, 0.2), (0.8, 0.1))$ | $((0.7, 0.1), (0.9, 0.1))$ |

**Table 2** IFR evaluation information $D_2$

| | $\widetilde{C}_1$ | $\widetilde{C}_2$ | $\widetilde{C}_3$ | $\widetilde{C}_4$ | $\widetilde{C}_5$ |
|---|---|---|---|---|---|
| $\mathfrak{g}_1$ | ((0.4, 0.1), (0.3, 0.3)) | ((0.9, 0.1), (0.5, 0.2)) | ((0.8, 0.1), (0.7, 0.2)) | ((0.9, 0.1), (0.4, 0.1)) | ((0.6, 0.1), (0.8, 0.2)) |
| $\mathfrak{g}_2$ | ((0.5, 0.3), (0.8, 0.2)) | ((0.7, 0.1), (0.9, 0.1)) | ((0.7, 0.2), (0.5, 0.2)) | ((0.4, 0.3), (0.3, 0.2)) | ((0.5, 0.2), (0.9, 0.1)) |
| $\mathfrak{g}_3$ | ((0.7, 0.3), (0.6, 0.1)) | ((0.6, 0.2), (0.7, 0.3)) | ((0.8, 0.1), (0.6, 0.3)) | ((0.8, 0.2), (0.9, 0.1)) | ((0.4, 0.1), (0.3, 0.2)) |
| $\mathfrak{g}_4$ | ((0.2, 0.1), (0.5, 0.2)) | ((0.9, 0.1), (0.6, 0.2)) | ((0.3, 0.1), (0.4, 0.3)) | ((0.5, 0.4), (0.7, 0.3)) | ((0.6, 0.2), (0.8, 0.1)) |

**Table 3** IFR evaluation information $D_3$

| | $\widetilde{C}_1$ | $\widetilde{C}_2$ | $\widetilde{C}_3$ | $\widetilde{C}_4$ | $\widetilde{C}_5$ |
|---|---|---|---|---|---|
| $g_1$ | $((0.6, 0.3), (0.8, 0.1))$ | $((0.6, 0.2), (0.9, 0.1))$ | $((0.7, 0.3), (0.9, 0.1))$ | $((0.5, 0.3), (0.9, 0.1))$ | $((0.8, 0.1), (0.4, 0.3))$ |
| $g_2$ | $((0.7, 0.1), (0.4, 0.2))$ | $((0.8, 0.1), (0.7, 0.2))$ | $((0.4, 0.2), (0.7, 0.2))$ | $((0.3, 0.2), (0.8, 0.2))$ | $((0.7, 0.3), (0.5, 0.1))$ |
| $g_3$ | $((0.5, 0.3), (0.8, 0.2))$ | $((0.3, 0.2), (0.5, 0.3))$ | $((0.8, 0.1), (0.4, 0.3))$ | $((0.7, 0.2), (0.5, 0.3))$ | $((0.9, 0.1), (0.2, 0.3))$ |
| $g_4$ | $((0.7, 0.2), (0.6, 0.4))$ | $((0.2, 0.1), (0.4, 0.1))$ | $((0.6, 0.4), (0.5, 0.2))$ | $((0.6, 0.1), (0.9, 0.1))$ | $((0.7, 0.2), (0.8, 0.1))$ |

**Table 4** Aggregated results by applying IFRWA operator

| | $\widetilde{C}_1$ | $\widetilde{C}_2$ | $\widetilde{C}_3$ |
|---|---|---|---|
| $\mathfrak{g}_1$ | $((0.6197, 0.1173), (0.6984, 0.1479))$ | $((0.8478, 0.1382), (0.8390, 0.1493))$ | $((0.7352, 0.1438), (0.8521, 0.1413))$ |
| $\mathfrak{g}_2$ | $((0.6275, 0.1479), (0.8501, 0.1409))$ | $((0.7311, 0.1175), (0.8827, 0.1143))$ | $((0.8422, 0.1409), (0.6275, 0.2201))$ |
| $\mathfrak{g}_3$ | $((0.7319, 0.2512), (0.7353, 0.1382))$ | $((0.4995, 0.1409), (0.6026, 0.2512))$ | $((0.7678, 0.1145), (0.6221, 0.1542))$ |
| $\mathfrak{g}_4$ | $((0.5808, 0.1143), (0.5736, 0.2596))$ | $((0.8531, 0.1), (0.6221, 0.1473))$ | $((0.8391, 0.1182), (0.5240, 0.2220))$ |

| | $\widetilde{C}_4$ | $\widetilde{C}_5$ |
|---|---|---|
| $\mathfrak{g}_1$ | $((0.8537, 0.1173), (0.8411, 0.1145))$ | $((0.7116, 0.1145), (0.7118, 0.2484))$ |
| $\mathfrak{g}_2$ | $((0.3752, 0.1474), (0.7019, 0.1409))$ | $((0.7302, 0.1468), (0.8447, 0.1145))$ |
| $\mathfrak{g}_3$ | $((0.8532, 0.1409), (0.8447, 0.1438))$ | $((0.8385, 0.1175), (0.5769, 0.2198))$ |
| $\mathfrak{g}_4$ | $((0.5822, 0.1501), (0.8521, 0.1194))$ | $((0.6752, 0.1409), (0.8591, 0.1))$ |

**Table 5** Intuitionistic fuzzy decision matrix after the use of AGO

| | $\widetilde{C}_1$ | $\widetilde{C}_2$ | $\widetilde{C}_3$ | $\widetilde{C}_4$ | $\widetilde{C}_5$ |
|---|---|---|---|---|---|
| $\mathfrak{g}_1$ | (0.6579, 0.1317) | (0.8434, 0.1437) | (0.7915, 0.1425) | (0.8474, 0.1159) | (0.7117, 0.1686) |
| $\mathfrak{g}_2$ | (0.7304, 0.1443) | (0.8034, 0.1159) | (0.7270, 0.1761) | (0.5131, 0.1441) | (0.7853, 0.1296) |
| $\mathfrak{g}_3$ | (0.7336, 0.1863) | (0.5486, 0.1881) | (0.6911, 0.1329) | (0.8489, 0.1423) | (0.6955, 0.1607) |
| $\mathfrak{g}_4$ | (0.5772, 0.1723) | (0.7285, 0.1214) | (0.6631, 0.1620) | (0.6797, 0.1338) | (0.7616, 0.1187) |

**Table 6** Result obtained for IFRDWA operator by applying IFR TOPSIS method

| | $D^+(\mathfrak{g}_i, \mathcal{P}^+)$ | $D^-(\mathfrak{g}_i, \mathcal{P}^-)$ | $\xi(\mathfrak{g}_i)$ | Ranking |
|---|---|---|---|---|
| $\mathfrak{g}_1$ | 0.0275 | 0.1891 | 0 | 1 |
| $\mathfrak{g}_2$ | 0.1057 | 0.1162 | −3.2327 | 2 |
| $\mathfrak{g}_3$ | 0.1149 | 0.1204 | −3.5469 | 3 |
| $\mathfrak{g}_4$ | 0.1331 | 0.0859 | −4.3899 | 4 |

capable to aggregate the illustrative example presented in Sect. 7. However, the aggregation operators investigated by Chinram et al. (2021) work but these operators are the special cases of the investigated operators. Furthermore, the influence of operational parameter $\beta$ provides additional space to the decision makers to use their skill and expertise. Dombi operators have general capability and provide additional space in evaluation process to the decision makers. Some of the existing models such as Xu (2007); Xu and Yager (2006); Seikh and Mandal (2021); Chinram et al. (2021); Yinghui and Wenlu (2015) have lack of this operational parameter. The collectively aggregated ranking result of the existing and developed approaches is given in Table 8. The influence of operational parameter $\beta$ plays significant role in DM. Different values are used for the operational parameter $\beta$ to judge the ranking result of proposed approaches IFRDWA and IFRDWG operators. The raking results based on different values of operational parameter $\beta$ in the range of $2 \leq \beta \leq 10$, for both IFRWA and IFRWG operators, are shown in Table 9. From Table 9, it is clear that the ranking results and best optimal object is same that is $\mathfrak{g}_1$. From the analysis of existing models and proposed approaches, it is clear that the investigated approach provides extra flexibility and capability than the previous methods.

## 8 Conclusion

The MCGDM is a pre-plan technique in which the group of professional decision makers presented their evaluation report to get the best and logical alternative among the several objects. Decision-making is a hard and complex process to access the conflicting characteristics of each alternative from different aspect. Hence, intellectual experts are engaged in this technique to improve the evaluation process to get more accurate and intelligent decision with experience and skills in addition to mental maturity. For this shortcoming, Atanassov (1986) investigated the prominent concept of IFS which is characterized by the membership and nonmembership grades. This study aims to propose the intuitionistic fuzzy rough set

**Table 7** Ranking order of function $\xi(\mathfrak{g}_i)$ for optimum object

| Proposed method | Score Values $\xi(\mathfrak{g}_i)$ | | | | Ranking |
|---|---|---|---|---|---|
| | $\mathfrak{g}_1$ | $\mathfrak{g}_2$ | $\mathfrak{g}_3$ | $\mathfrak{g}_4$ | |
| IFRDWA proposed | 0.0000, | − 3.2327, | − 3.5469, | − 4.3899 | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$ |
| IFRDWG proposed | 0.0000, | — 0.3131, | − 0.4616, | − 0.8582 | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_4 \geq \mathfrak{g}_3$ |

**Table 8** Comparative study of the proposed model with some existing approaches

| Methods | $\xi(\mathfrak{g}_i)$ | Ranking |
|---|---|---|
| IFWA (Xu and Yager 2006) | $0.6523,\ 0.5781,\ 0.5646,\ 0.5361$ | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$ |
| IF TOPSIS (Yinghui and Wenlu 2015) | $0.8041,\ 0.4434,\ 0.5694,\ 0.4631$ | $\mathfrak{g}_1 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4 \geq \mathfrak{g}_2$ |
| IFRFWA based on EDAS (Yahya et al. 2021) | $0.8966,\ 0.6654,\ 0.247,\ 0.3567$ | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_4 \geq \mathfrak{g}_3$ |
| IFRWA based on EDAS (Chinram et al. 2021) | $0.8584,\ 0.5703,\ 0.2734,\ 0.2234$ | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$ |
| IFDWA (Seikh and Mandal 2021) | $0.6747,\ 0.6081,\ 0.6113,\ 0.5530$ | $\mathfrak{g}_1 \geq \mathfrak{g}_3 \geq \mathfrak{g}_2 \geq \mathfrak{g}_4$ |
| IFWG (Xu and Yager 2006) | $0.6356,\ 0.5507,\ 0.5362,\ 0.5253$ | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$ |
| IFDWG (Seikh and Mandal 2021) | $0.6136,\ 0.5064,\ 0.5022,\ 0.5107$ | $\mathfrak{g}_1 \geq \mathfrak{g}_4 \geq \mathfrak{g}_3 \geq \mathfrak{g}_2$ |
| IFRWG based on EDAS (Chinram et al. 2021) | $0.7789,\ 0.6357,\ 0.3677,\ 0.2043$ | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$ |
| IFRDWA proposed | $0.0000,\ -3.2327,\ -3.5469,\ -4.3899$ | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$ |
| IFRDWG proposed | $0.0000,\ -0.1313,\ -0.4616,\ -0.8582$ | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$ |

based on Dombi norms and t-norms, and then by applying TOPSIS approach to aggregate averaging and geometric aggregation operators. Further, on the presented concept, we investigated IFRDWA, IFRDOWA, and IFRDHA operators. Moreover, on the proposed model, we initiated IFRDWG, IFRDOWG, and IFRDHG operators. The important properties of the proposed approach are presented in detailed. The algorithm for MCGDM hybrid with TOPSIS method is presented for intuitionistic fuzzy rough Dombi averaging and geometric aggregation operators. The novel concept of accumulated geometric operator is applied on proposed model to convert the intuitionistic fuzzy rough numbers to intuitionistic fuzzy numbers. The massive outbreak of the pandemic COVID-19 promoted the challenging scenario for the world organizations including scientists, laboratories, and researchers to conduct special clinical treatment strategies to prevent the people from COVID-19 pandemic. In addition, an illustrative example is proposed to solve MCGDM problem to diagnose the most severe patient of COVID-19 by applying TOPSIS. Finally, a comparative analysis of the developed model is presented with some existing approaches which shows the applicability and preeminence of the investigated model. As the prominent concepts of intuitionistic fuzzy sets and rough sets have broad applications, so by keeping in view of the future, we will extend the proposed concept to hesitant fuzzy sets, neutrosophic fuzzy sets, bipolar fuzzy sets, soft sets and will study different aggregation operators by applying the presented model.

**Table 9** Ranking result based on different parameter $\beta$, for IFRWA and IFRWG operators

| $\beta$ | The IFRDWA operator | | The IFRDWG operator | |
|---|---|---|---|---|
| | Score value for $\xi(\mathfrak{g}_i)$ | Ranking result | Score value for $\xi(\mathfrak{g}_i)$ | Ranking result |
| $\beta = 2$ | 0, −3.2327, −3.5469, −4.3899 | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$ | 0, −0.3131, −0.4616, −0.8582 | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$ |
| $\beta = 3$ | 0, −4.4821, −4.6355, −5.7402 | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$ | 0, −0.3943, −0.4598, −0.8727 | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$ |
| $\beta = 5$ | 0, −4.7495, −4.6601, −6.2513 | $\mathfrak{g}_1 \geq \mathfrak{g}_3 \geq \mathfrak{g}_2 \geq \mathfrak{g}_4$ | 0, −0.3291, −0.4541, −0.7768 | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$ |
| $\beta = 8$ | 0, −5.1842, −4.9268, −6.7728 | $\mathfrak{g}_1 \geq \mathfrak{g}_3 \geq \mathfrak{g}_2 \geq \mathfrak{g}_4$ | 0, −0.3397, −0.4569, −0.7674 | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$ |
| $\beta = 10$ | 0, −5.3057, −4.9896, −6.9064 | $\mathfrak{g}_1 \geq \mathfrak{g}_3 \geq \mathfrak{g}_2 \geq \mathfrak{g}_4$ | 0, −0.3446, −0.4592, −0.7674 | $\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$ |

**Acknowledgements** The authors are highly thankful to Editor-in-Chief, Associate Editor and the anonymous referees for their valuable comments and suggestions for improving the quality of paper.

**Data availability** The data used in this article are artificial and hypothetical, and anyone can use these data before prior permission by just citing this article.

## Declarations

**Conflict of interest** The authors declare that they have no conflicts of interest.

**Ethical approval** This article does not contain any studies with human participants or animals performed by any of the authors.

## References

Ali MI (2019) Another view on q-rung orthopair fuzzy sets. Int J Intell Syst 33:2139–2153

Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Set Syst 20:87–96

Biswas A, Modak N (2013) A fuzzy goal programming technique for multi objective chance constrained programming with normally distributed fuzzy random variables and fuzzy numbers. Int J Math Oper Res 5:551–570

Bustince H, Burillo P (1996) Structures on intuitionistic fuzzy relations. Fuzzy Sets Syst 78(3):293–303

Chen SM, Chun TL (2016) Multicriteria decision making based on the TOPSIS method and similarity measures between intuitionistic fuzzy values. Inf Sci 367:279–295

Chen SM, Tan JM (1994) Handling multicriteria fuzzy decision making problem based on vague set theory. Fuzzy Sets Syst 67(2):163–172

Chen J, Ye J (2017) Some single-valued neutrosophic Dombi weighted aggregation operators for multi attribute decision making. Symmetry 9(82):1–11

Chen SM, Cheng SH, Chiou CH (2016) Fuzzy multiattribute group decision making based on intuitionistic fuzzy sets and evidential reasoning methodology. Inf Fusion. 27:215–227

Chinram R, Hussain A, Mahmood T, Ali MI (2021) EDAS method for multi criteria decision group decision making based on intuitionistic fuzzy rough aggregation operators. IEEE Access 9:10199–10216

Cornelis C, De Cock M, Kerre EE (2003) Intuitionistic fuzzy rough sets: at the crossroads of imperfect knowledge. Expert Syst 20(5):260–269

Debnath J, Majumder D, Biswas A (2018) Air quality assessment using interval type-2 weighted fuzzy inference system. Ecol Inf 46:133–146

Dombi J (1982) A general class of fuzzy operators, the De Morgan class of fuzzy operators and fuzziness measures induced by fuzzy operators. Fuzzy Sets Syst 8(2):149–163

Dubois D, Prade H (1990) Rough fuzzy sets and fuzzy rough sets. Int J Gen Syst 17(2–3):191–209

Feng F, Fujita H, Ali MI, Yager RR, Liu X (2018) Another view on generalized intuitionistic fuzzy soft sets and related multiattribute decision making methods. IEEE Trans Fuzzy Syst 27(3):474–488

Feng F, Zheng Y, Alcantud JCR, Wang Q (2020a) Minkowski weighted score functions of intuitionistic fuzzy values. Mathematics 8(7):1143

Feng F, Xu Z, Fujita H, Liang M (2020b) Enhancing PROMETHEE method with intuitionistic fuzzy soft sets. Int J Intell Syst 35(7):1071–1104

Garg H (2016) Some series of intuitionistic fuzzy interactive averaging aggregation operators. Springer plus 5(1):999

Garg H (2018) An improved cosine similarity measure for intuitionistic fuzzy sets and their applications to decision-making process. Hacettepe J Math Stat 47(6):1585–1601

Garg H, Kumar K (2018a) A novel exponential distance and its based TOPSIS method for interval-valued intuitionistic fuzzy sets using connection number of SPA theory. Artif Intell Rev 53:1–30

Garg H, Kumar K (2018b) An advanced study on the similarity measures of intuitionistic fuzzy sets based on the set pair analysis theory and their application in decision making. Soft Comput 22(15):4959–4970

Hong DH, Choi CH (2000) Multi-criteria fuzzy decision-making problem based on vague set theory. Fuzzy Sets Syst 114(1):103–113

Huang JY (2014) Intuitionistic fuzzy Hamacher aggregation operators and their application to multiple attribute decision making. J Intell Fuzzy Syst 27(1):505–513

Huang B, Li H, Feng G, Zhou X (2019) Dominance-based rough sets in multi-scale intuitionistic fuzzy decision tables. Appl Math Comput 348:487–512
Hussain A, Mahmood T, Ali MI (2019) Rough Pythagorean fuzzy ideals in semigroups. Comput Appl Math 38(2):67
Hussain A, Ali MI, Mahmood T (2020a) Pythagorean fuzzy soft rough sets and their applications in decision-making. J Tabah Univ Sci 14(1):101–113
Hussain A, Ali MI, Mahmood T, Munir M (2020b) Q-Rung orthopair fuzzy soft average aggregation operators and their application in multicriteria decision making. Int J Intell Syst 35(4):571–599
Hussain A, Ali MI, Mahmood T (2019) Covering based q-rung orthopair fuzzy rough set model hybrid with TOPSIS for multi-attribute decision making. J Intell Fuzzy Syst 1–13
Hussain A, Ali MI, Mahmood T (2019) Hesitant q-rung orthopair fuzzy aggregation operators with their applications in multi-criteria decision making. Iran J Fuzzy Syst. 1903–5074
Hwang CL, Yoon K (1981) Multiple attribute decision making: methods and applications. Springer, New York
Ishizaka A, Nemery P (2013) Multi-criteria decision analysis: methods and software. Wiley, West Sussex
Jana C, Pal M, Wang JQ (2018) Bipolar fuzzy Dombi aggregation operators and its application in multiple-attribute decision making process. J Ambient Intell Hum Comp 10(9):3533–3549
Li DF (2010) Multi-attribute decision-making method based on generalized OWA operator with intuitionistic fuzzy sets. Expert Syst Appl 37(12):8673–8678
Li Z, Liu A, Miao J, Yang Y (2023) A three-phase method for spherical fuzzy environment and application to community epidemic prevention management. Expert Syst Appl 211:118601
Liu M, Ren H (2014) A new intuitionistic fuzzy entropy and application in multi-attribute decision making. Information 5(4):587–601
Liu P, Liu J, Chen SM (2018) Some intuitionistic fuzzy Dombi Bonferonni operators and their application to multi-attribute group decision-making. J Oper Res Soc 69(1):1–24
Meng F, Chen X (2016) Entropy and similarity measure of Atanassov intuitionistic fuzzy sets and their application to pattern recognition based on fuzzy measures. Pattern Anal Appl 19:11–20
Pawlak Z (1982) Rough sets. Int J Comput Inf Sci 11(5):341–356
Peng X, Yang Y (2015) Some results for Pythagorean fuzzy sets. Int J Intell Syst 30:1133–1160
Seikh MR, Mandal U (2021) Intuitionistic fuzzy Dombi aggregation operators and their application to multiple attribute decision-making. Gran Comp 6(3):473–488
Shahzaib A, Rehman N, Hussain A, AlSalman H, Gumaei AH (2021) Rung orthopair fuzzy rough einstein aggregation information-based EDAS method: applications in robotic Agrifarming. Comput Intell Neurosci 2021:1–21
Shen F, Ma X, Li Z, Xu ZS, Cai D (2018) An extended intuitionistic fuzzy TOPSIS method based on a new distance measure with an application to credit risk evaluation. Inf Sci 428:105–119
Song Y, Wang X, Quan W, Huang W (2017) A new approach to construct similarity measure for intuitionistic fuzzy sets. Soft Comput 23(6):1985–1998
Szmidt E, Kacprzyk J (2001) Entropy for intuitionistic fuzzy sets. Fuzzy Sets Syst 118(3):467–477
Thao X, Nguyen X (2018) A new correlation co-efficient of intuitionistic fuzzy sets and its application. J Intell Fuzzy Syst. 35(2):1959–1968
Wang W, Liu X (2012) Intuitionistic fuzzy information aggregation using Einstein operations. IEEE Trans Fuzzy Syst 20(5):923–938
Wang Y, Hussain A, Mahmood T, Ali MI, Wu H, Jin Y (2020) Decision making based on q-rung orthopair fuzzy soft rough sets. Math Prob Eng 2020:1–21
Wei G (2010) Some induced geometric aggregation operators with intuitionistic fuzzy information and their application to group decision making. Appl Soft Comput 10(2):423–431
Wei G, Wei Y (2018) Some single-valued neutrosophic Dombi prioritized weighted aggregation operators in multiple attributes decision-making. J Intell Fuzzy Syst 35(2):2001–2013
Xu ZS (2004) Uncertain multi-attribute decision making. Tsinghua University Press, Beijing
Xu ZS (2007) Intuitionistic fuzzy aggregation operators. IEEE Trans Fuzzy Syst 15:1179–1187
Xu ZS, Yager RR (2006) Some geometric aggregation operators based on intuitionistic fuzzy sets. Int J Gen Syst 35:417–433
Yager RR (2013) Pythagorean fuzzy subsets. In: Proceedings of the joint IFSA world congress and NAFIPS annual meeting edmonton Canada, pp 57–61
Yager RR (2014) Pythagorean membership grades in multicriteria decision-making. IEEE Trans Fuzzy Syst 22:958–965
Yager RR (2016) Generalized orthopair fuzzy sets. IEEE Trans Fuzzy Syst 25:1222–1230
Yahya M, Naeem M, Abdullah S, Qiyas M, Aamir M (2021) A Novel approach on the intuitionistic fuzzy rough frank aggregation operator-based EDAS method for multicriteria group decision-making. Complexity 2021:1–24

Ye J (2016) Similarity measures of intuitionistic fuzzy sets based on cosine function for the decision making of mechanical design schemes. J Intell Fuzzy Syst 30:151–158
Yinghui W, Wenlu L (2015) The application of intuitionistic fuzzy set TOPSIS method in employee performance appraisal. Int J u-and e-Serv Sci Technol 8(3):329–344
Zadeh LA (1965) Fuzzy sets. Inf Control 8:338–353
Zeng S, Xiao Y (2016) TOPSIS method for intuitionistic fuzzy multiple criteria decision making and its application to investment selection. Kybernetes 45(2):282–296
Zeng S, Chen SM, Kuo LW (2019) Multiattribute decision making based on novel score function of intuitionistic fuzzy values and modified VIKOR method. Inf Sci 488:76–92
Zhang Z (2012) Generalized intuitionistic fuzzy rough sets based on intuitionistic fuzzy coverings. Inf Sci 198:186–206
Zhang X, Xu Z (2014) Extension of TOPSIS to multiple criteria decision-making with pythagorean fuzzy sets. Int J Intell Syst 29:1061–1078
Zhang X, Zhou B, Li P (2012) A general frame for intuitionistic fuzzy rough sets. Inf Sci 216:34–49
Zhang H, Shu L, Liao S (2014) Intuitionistic fuzzy soft rough set and its application in decision making. Abstr Appl Anal 2014:287314
Zhou L, Wu WZ (2008) On generalized intuitionistic fuzzy rough approximation operators. Inform Sci 178:2448–2465
Zhou L, Wu WZ (2011) Characterization of rough set approximations in Atanassov intuitionistic fuzzy set theory. Comput Math Appl 62(1):282–296
Zulqarnain RM, Xin XL, Saqlain M, Khan WA (2021) TOPSIS method based on the correlation coefficient of interval-valued intuitionistic fuzzy soft sets and aggregation operators with their application in decision-making. J Math 2021:1–16