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Delta-Modulator Based Quantised State Feedback Controller for T-S Fuzzy Networked Systems

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Abstract This paper proposes a Delta-Modulator (Δ-M) based quantised state feedback controller for Takagi-Sugeno (T-S) fuzzy networked systems. The Δ-M (a single bit quantiser) essentially belongs to one class of sliding mode quantisers (SMQs) and offers various advantages which include lower design complexity, lower cost and less noisy. For a prescribed quantisation error, the gains of the state-feedback controller and the quantiser are derived (both in continuous and discrete time domains) using linear matrix inequalities (LMIs) which ensures the stability of the overall system. The performance of the quantised control system is illustrated considering a practical communication network based on ZigBee protocol. The results of the simulation demonstrate that the proposed Δ-M based quantised controller could effectively achieve desired performance under various imperfections of the practical communication network.

1 Introduction

Nonlinear control based on T-S fuzzy model has been proven great success over the past few decades [1–4]. The fuzzy systems are proven to be universal approximator of nonlinear systems [1, 2, 5–12] which can represent a wide class of nonlinear systems by a set of local linear models interpolated by membership functions. During the last two decades, due to the astounding success of communication network and internet, a new class of control system, called networked control system (NCS), has emerged and many controllers are designed for this framework [13–16]. Although, NCS offers many benefits such as reducing system wiring, ease of system diagnosis and maintenance

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and increase in system agility, the communication network in the control loop gives rise to several new issues due to intermittent packet losses, delays and quantisation [17, 18].

In the past few decades, researchers have developed several effective control design methods and have proved their stability, both for linear and nonlinear systems [19–25]. Some of the existing control methods such as sliding mode control [26], event-triggered control [27], H_∞ control [28], formation tracking control [29], model predictive control [30], distributed control [31] and so on have been tailored for Networked control to mitigate the effects of various network imperfections and make the NCS more robust. These have been applied to many engineering applications such as fault detection, identifying cyberattacks, robot applications and many more.

One of the major issue with NCS is the bit-rate constraint (i.e. bandwidth utilisation) of the communication network. This problem can be alleviated through quantisation which is a process of mapping a large set of input values to a small set of output values where the continuous-time signals are represented by quantised signals. Therefore, the role of quantisers in the NCS is critical from the perspective of performance and stability. In the past, different types of quantisers have been proposed by researchers which include nearest neighbour quantisers [32], logarithmic quantisers [33], neural network quantisers [34] and so on. All these quantisers are essentially high-bit quantisers [35–38]. It is worth noting that the effectiveness of the quantisers in bandwidth utilisation increases with the decrease of quantisation levels. However, the accuracy of the generated control action decreases with decrease in the number of quantisation levels (quantisation error increases). Although by using a higher number of bits could reduce the quantisation error, this increases the delay in the control action. Because all the bits need to be received in order to reconstruct the input to the controller [39].

The single-bit quantiser is a possible solution to alleviate the problems associated with high-bit quantisers and achieve better network bandwidth utilisation. During the past decade, various researchers have used the single-bit quantisers based on either Δ-Modulator (Δ-M), Delta-Sigma Modulator ($\Delta\Sigma$-M), Hybrid-Delta Modulator (Δ_H-M) [18, 40, 41], to develop single-bit controllers. The output of these quantisers (modulators) are called bit-streams and the associated controllers are popularly known as bit-stream controllers [42]. The stability of bit-stream controllers have been established and the guidelines to tune the controllers' parameters have been reported in [43–47].

In these methods, instead of using a microprocessor to implement the control functions, the controllers are implemented in hardware using bit-streams inside programmable logic devices, such as field-programmable gate arrays (FPGAs). This technique differs from the traditional digital implementation where the continuous-time signal is represented by a single-bit signal. Moreover, since all control elements are implemented in parallel, the addition of extra functionality to a given design will have less impact on the timing of the system. This is in contrast to what happens in micro-controller based systems where

control functions are sequentially executed and this may exceed the available execution time with the addition of more functionalities.

Although the success of single-bit quantisers has been demonstrated in areas such as control, mobile communication, and biomedical applications [42, 48–53], their applications to nonlinear systems are not reported in the literature. The objective of the present study is therefore to investigate if Δ-M based single-bit quantised controllers can be designed for nonlinear systems which are represented by T-S fuzzy model.

In the present study, Δ-M based quantised state feedback controller is designed for T-S fuzzy class of nonlinear systems; both in continuous and discrete-time domains. The control actions of these controllers are transmitted as single-bit signal through the communication channel. Due to this, these controllers offer significant advantages which include consumption of less hardware resources and communication bandwidth. The main contributions of this paper are as follows:

(i.) Established the stability conditions for both continuous and discrete Δ-M based single-bit quantised feedback nonlinear systems.
(ii.) Determined the feedback gain and the quantiser gain which ensures that the quantisation error is bounded.
(iii.) Validated the theoretical findings using a real ZigBee protocol based communication system.

The rest of the paper is organised as follows. Section-2 and Section-3 discuss, continuous time Δ-M based control system and discrete time Δ-M based control system respectively and the stability conditions are derived using LMIs which ensures the stability of the closed loop system. The effectiveness of the proposed control strategy and the theoretical findings are validated using two simulation examples in Section-4 followed by conclusions on Section-5.

Notation: In the sequel the Euclidean norm is used for vectors. W^T and W^{-1} respectively denote the transpose and the inverse of any matrix W. $W > 0$ ($\geq 0, < 0, \leq 0$) denote symmetric positive definite (positive semi-definite, negative, negative semi-definite) matrix W and I denote the $n \times n$ identity matrix. If matrix dimensions are not explicitly stated, are assumed to be compatible with algebraic operations. The symbol $\star$ is used to represent a term that is induced by the symmetry.

2 Continuous Fuzzy State Feedback Controller

This section describes the procedure for designing Δ-M based quantised state feedback controller for continuous-time (CT) T-S fuzzy systems.

2.1 System Description

Consider a nonlinear system which is represented by a continuous time T-S fuzzy model using the following fuzzy IF-THEN rules:

IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip},
THEN,

$$\dot{x}(t) = A_{i,c}x(t) + B_{i,c}u(t), \qquad i = 1, 2, \ldots, r \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ denote respectively the states of the system and the control input. The matrices $A_{i,c}$ and $B_{i,c}$ are of appropriate dimensions. The number of IF-THEN rules equal to r and the fuzzy sets are denoted by M_{ij}. It is assumed that $z(t) = [z_1(t), z_2(t), \ldots, z_p(t)]^T$ is known. Then the system equation is given by:

$$\dot{x}(t) = \sum_{i=1}^{r} \lambda_i(z(t))\{A_{i,c}x(t) + B_{i,c}u(t)\} \tag{2}$$

where,

$$\lambda_i(z(t)) = \frac{w_i(z(t))}{\sum\limits_{i=1}^{r} w_i(z(t))};\ w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)) \tag{3}$$

where $Mij(\cdot)$ are the membership functions of the fuzzy sets M_{ij}. It is assumed that,

$$w_i(z(t)) \geq 0, i = 1, 2, \ldots, r, \text{ and } \sum_{i=1}^{r} w_i(z(t)) > 0, \forall t.$$

Hence $\lambda_i(z(t))$ satisfy,

$$\lambda_i(z(t)) \geq 0, i = 1, 2, \ldots, r, \text{ and } \sum_{i=1}^{r} \lambda_i(z(t)) = 1, \forall t.$$

2.2 Continuous-Time Delta Modulator

The block diagram of a multi-input multi-output (MIMO) CT Δ-M is shown in Figure. 1.

If $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip}, then the dynamics of MIMO CT Δ-M is described as:

$$s(t) = x(t) - \hat{x}(t) \tag{4a}$$

$$\dot{\hat{x}}(t) = \Theta_{i,c}\,\mathrm{sgn}(s(t)), \quad i = 1, 2, \ldots, r. \tag{4b}$$

where $\hat{x}(t) \in \mathbb{R}^n$, $s(t) \in \mathbb{R}^n$ and $\Theta_{i,c} \in \mathbb{R}^{n\times n}$ denote the quantised signal, the switching signal (the quantisation error) and gain of the ith two-level quantiser respectively. Let us rewrite (4b) as:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} \lambda_i(z(t))\Theta_{i,c}\mathrm{sgn}(s(t)) \tag{5}$$

Further, $\mathrm{sgn}(s(t)) \in [\{-1, 1\}, \cdots, \{-1, 1\}]^T \in \mathbb{R}^n$, where,

$$\mathrm{sgn}(s_j(t)) = \begin{cases} +1, \text{ if } & s_j(t) \geq 0; \\ -1, \text{ if } & s_j(t) < 0; \end{cases} \quad j = (1, 2, \cdots, n).$$

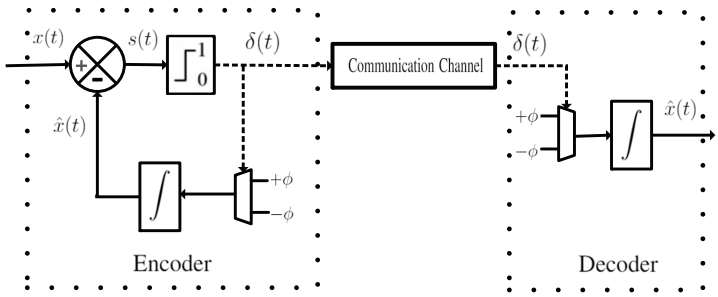


Fig. 1 Continuous-time Delta-Modulator (Δ-M)

2.3 Quantised State-Feedback Control System

Using (1) and (4), the quantised state feedback control system can be described as:

$$\dot{x}(t) = A_{i,c}\, x(t) + B_{i,c}\, \hat{u}(t) \tag{6a}$$

$$\dot{\hat{x}}(t) = \Theta_{i,c}\, \mathrm{sgn}(s(t)) \tag{6b}$$

$$\hat{u}(t) = K_{i,c}\, \hat{x}(t) \tag{6c}$$

$$s(t) = x(t) - \hat{x}(t) \tag{6d}$$

where $K_{i,c}$ is the state-feedback gain. For the sake of convenience, rewrite (6) as:

$$\begin{aligned}\dot{x}(t) = &\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(z(t))\lambda_j(z(t))\{A_{i,c} + B_{i,c}K_{j,c}\}\, x(t) \\ &- \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(z(t))\lambda_j(z(t))B_{i,c}K_{j,c}\, s(t)\end{aligned} \tag{7a}$$

$$\begin{aligned}\dot{s}(t) = &\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(z(t))\lambda_j(z(t))\{A_{i,c} + B_{i,c}K_{j,c}\}\, x(t) \\ &- \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(z(t))\lambda_j(z(t))B_{i,c}K_{j,c}\, s(t) \\ &- \sum_{i=1}^{r} \lambda_i(z(t))\Theta_{i,c}\, \mathrm{sgn}(s(t)).\end{aligned} \tag{7b}$$

Theorem 1 *For known $A_{i,c}$, $B_{i,c}$ $K_{i.c}$, $\Theta_{i,c}$, $i = 1, \ldots, r$ and the maximum quantisation error ϵ, the quantised fuzzy system (7) is exponentially stable if there exists a common symmetric positive-definite matrix $R_c > 0$ and matrices Λ_c for $i = 1, \ldots, r$ and Φ_c for $i < j \leq r$ such that,*

$$\Lambda_c = \begin{bmatrix} \lambda_{11,c} & \lambda_{12,c} \\ \lambda_{21,c} & \lambda_{22,c} \end{bmatrix} < 0, \quad \Phi_c = \begin{bmatrix} \phi_{11,c} & \phi_{12,c} \\ \phi_{21,c} & \phi_{22,c} \end{bmatrix} < 0 \tag{8}$$

where

$$\begin{aligned}
\lambda_{11,c} &= \{A_{i,c} + B_{i,c}K_{i,c}\}^T\{P_1^c + P_3^c\} \\
&\quad + \{P_1^c + P_3^c\}\{A_{i,c} + B_{i,c}K_{i,c}\} \\
\lambda_{12,c} &= -P_1^c B_{i,c}K_{i,c} + \{A_{i,c} + B_{i,c}K_{i,c}\}^T\{P_2^c + P_3^c\} \\
&\quad - P_3^c\{B_{i,c}K_{i,c} + \frac{\Theta_{i,c}}{\epsilon}\} \\
\lambda_{21,c} &= -K_{i,c}^T B_{i,c}^T P_1^c + \{P_2^c + P_3^c\}\{A_{i,c} + B_{i,c}K_{i,c}\} \\
&\quad - \{B_{i,c}K_{i,c} + \frac{\Theta_{i,c}}{\epsilon}\}^T P_3^c \\
\lambda_{22,c} &= -\{B_{i,c}K_{i,c} + \frac{\Theta_{i,c}}{\epsilon}\}^T P_2^c - K_{i,c}^T B_{i,c}^T P_3^c \\
&\quad - P_2^c\{B_{i,c}K_{i,c} + \frac{\Theta_{i,c}}{\epsilon}\} - P_3^c B_{i,c}K_{i,c} \\
\phi_{11,c} &= G_{ij,c}^T\{P_1^c + P_3^c\} + \{P_1^c + P_3^c\}G_{ij,c} \\
\phi_{12,c} &= -P_1^c H_{ij,c} + G_{ij,c}^T\{P_2^c + P_3^c\} - P_3^c I_{ij,c} \\
\phi_{21,c} &= -H_{ij,c}^T P_1^c + \{P_2^c + P_3^c\}G_{ij,c} - I_{ij,c}^T P_3^c \\
\phi_{22,c} &= -I_{ij,c}^T P_2^c - P_2^c I_{ij,c} - H_{ij,c}^T P_3^c - P_3^c H_{ij,c}
\end{aligned}$$

and

$$\begin{aligned}
G_{ij,c} &= \frac{\{A_{i,c} + B_{i,c}K_{j,c}\} + \{A_{i,c} + B_{j,c}K_{i,c}\}}{2} \\
H_{ij,c} &= \frac{\{B_{i,c}K_{j,c}\} + \{B_{j,c}K_{i,c}\}}{2} \\
I_{ij,c} &= \frac{\{B_{i,c}K_{j,c} + \frac{\Theta_{i,c}}{\epsilon}\} + \{B_{j,c}K_{i,c} + \frac{\Theta_{i,c}}{\epsilon}\}}{2}
\end{aligned}$$

Proof Consider a Lyapunov function as:

$$V(t) = \xi_c(t)^T R_c \xi_c(t) \tag{9}$$

where $\xi_c(t) = [x(t),\, s(t)]^T$ and

$$R_c = \begin{bmatrix} P_1^c & P_3^c \\ \star & P_2^c \end{bmatrix} > 0 \tag{10}$$

Partial differentiation of (9) gives:

$$\dot{V}(t) = \dot{\xi_c}^T(t) R_c \xi_c(t) + \xi_c^T(t) R_c \dot{\xi}_c(t) \tag{11}$$

where

$$\begin{aligned}\dot{x}(t)^T P_1^c x(t) = {} & x^T(t)\left\{\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(A_{i,c}+B_{i,c}K_{i,c})\right\}^T P_1^c x(t)\\ & + x^T(t)\left\{2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(G_{ij,c})\right\}^T P_1^c x(t)\\ & + s^T(t)\left\{-\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(B_{i,c}K_{i,c})\right\}^T P_1^c x(t)\\ & + s^T(t)\left\{-2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(H_{ij,c})\right\}^T P_1^c x(t) \qquad (12)\end{aligned}$$

$$\begin{aligned}x^T(t)P_1^c\dot{x}(t) = {} & x^T(t)P_1^c\left\{\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(A_{i,c}+B_{i,c}K_{i,c})\right\}x(t)\\ & + x^T(t)P_1^c\left\{2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(G_{ij,c})\right\}x(t)\\ & + x^T(t)P_1^c\left\{-\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(B_{i,c}K_{i,c})\right\}s(t)\\ & + x^T(t)P_1^c\left\{-2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(H_{ij,c})\right\}s(t) \qquad (13)\end{aligned}$$

$$\begin{aligned}\dot{s}^T(t)P_2^c s(t) = {} & x^T(t)\left\{\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(A_{i,c}+B_{i,c}K_{i,c})\right\}^T P_2^c s(t)\\ & + x^T(t)\left\{2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(G_{ij,c})\right\}^T P_2^c s(t)\\ & + s^T(t)\left\{-\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(B_{i,c}K_{i,c}+\frac{\Theta_{i,c}}{\epsilon})\right\}^T P_2^c s(t)\\ & + s^T(t)\left\{-2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(I_{ij,c})\right\}^T P_2^c s(t) \qquad (14)\end{aligned}$$

$$
\begin{aligned}
s^T(t)P_2^c\dot{s}(t) = {} & s^T(t)P_2^c\left\{\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(A_{i,c}+B_{i,c}K_{i,c})\right\}x(t) \\
& + s^T(t)P_2^c\left\{-2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(G_{ij,c})\right\}x(t) \\
& + s^T(t)P_2^c\left\{-\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(B_{i,c}K_{i,c}+\frac{\Theta_{i,c}}{\epsilon})\right\}s(t) \\
& + s^T(t)P_2^c\left\{-2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(I_{ij,c})\right\}s(t) \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
\dot{s}(t)^T P_3^c x(t) = {} & x^T(t)\left\{\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(A_{i,c}+B_{i,c}K_{i,c})\right\}^T P_3^c x(t) \\
& + x^T(t)\left\{2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(G_{ij,c})\right\}^T P_3^c x(t) \\
& + s^T(t)\left\{-\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(B_{i,c}K_{i,c}+\frac{\Theta_{i,c}}{\epsilon})\right\}^T P_3^c x(t) \\
& + s^T(t)\left\{-2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(I_{ij,c})\right\}^T P_3^c x(t) \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
s^T(t)P_3^c\dot{x}(t) = {} & s^T(t)P_3^c\left\{\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(A_{i,c}+B_{i,c}K_{i,c})\right\}x(t) \\
& + s^T(t)P_3^c\left\{2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(G_{ij,c})\right\}x(t) \\
& + s^T(t)P_3^c\left\{-\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(B_{i,c}K_{i,c})\right\}s(t) \\
& + s^T(t)P_3^c\left\{-2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(H_{ij,c})\right\}s(t) \qquad (17)
\end{aligned}
$$

$$\begin{aligned}\dot{x}^T(t)P_3^c s(t) = x^T(t)&\left\{\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(A_{i,c}+B_{i,c}K_{i,c})\right\}^T P_3^c s(t)\\ &+ x^T(t)\left\{2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(G_{ij,c})\right\}^T P_3^c s(t)\\ &+ s^T(t)\left\{-\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(B_{i,c}K_{i,c})\right\}^T P_3^c s(t)\\ &+ s^T(t)\left\{-2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(H_{ij,c})\right\}^T P_3^c s(t) \qquad (18)\end{aligned}$$

$$\begin{aligned}x^T(t)P_3^c\dot{s}(t) = x^T(t)P_3^c&\left\{\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(A_{i,c}+B_{i,c}K_{i,c})\right\}x(t)\\ &+ x^T(t)P_3^c\left\{-2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(G_{ij,c})\right\}x(t)\\ &+ x^T(t)P_3^c\left\{-\sum_{i=1}^{r}\lambda_i(z(t))\lambda_i(z(t))(B_{i,c}K_{i,c}+\frac{\Theta_{i,c}}{\epsilon})\right\}s(t)\\ &+ x^T(t)P_3^c\left\{-2\sum_{i<j}\lambda_i(z(t))\lambda_j(z(t))(I_{ij,c})\right\}s(t) \qquad (19)\end{aligned}$$

Note that $\mathrm{sgn}(s(t)) = \frac{s(t)}{|s(t)|}$ and ϵ is the maximum quantisation error (i.e. $|s(t)| = \epsilon$). Simplifying (12)-(19) gives:

$$\dot{V}(t) = \xi_c^T(t)\Lambda_c\xi_c(t),\ i = 1,\ldots,r, \qquad (20)$$

and

$$\dot{V}(t) = \xi_c^T(t)\Phi_c\xi_c(t),\ i < j \leq r, \qquad (21)$$

which completes the proof of Theorem 1.

Theorem 2 *For given $A_{i,c}$, $B_{i,c}$, $i = 1,\ldots,r$ and the maximum quantisation error ϵ, the quantised fuzzy system (7) is exponentially stable if there exists a matrix $X_c > 0$ and unknown matrices $\alpha_{i,c}$, $\beta_{i,c}$, $i = 1,\ldots,r$ where the feedback gain $K_{i,c}$ and quantiser gain $\Theta_{i,c}$ in (7) is given by,*

$$K_{i,c} = \alpha_{i,c} * X_c^{-1},\ \Theta_{i,c} = \beta_{i,c} * X_c^{-1} \qquad (22)$$

Proof Substituting $P_1^c = P_2^c = P_c$ and $P_3^c = 0$, in (8) gives,

$$\Lambda_c = \begin{bmatrix}\lambda_{11,c} & \lambda_{12,c}\\ \lambda_{21,c} & \lambda_{22,c}\end{bmatrix} < 0,\ \ i = 1,\ldots,r \qquad (23)$$

where

$$\begin{aligned}
\lambda_{11,c} &= \{A_{i,c} + B_{i,c}K_{i,c}\}^T P_c + P_c\{A_{i,c} + B_{i,c}K_{i,c}\} \\
\lambda_{12,c} &= -P_c B_{i,c}K_{i,c} + \{A_{i,c} + B_{i,c}K_{i,c}\}^T P_c \\
\lambda_{21,c} &= -K_{i,c}^T B_{i,c}^T P_c + P_c\{A_{i,c} + B_{i,c}K_{i,c}\} \\
\lambda_{22,c} &= -\{B_{i,c}K_{i,c} + \frac{\Theta_{i,c}}{\epsilon}\}^T P_c - P_c\{B_{i,c}K_{i,c} + \frac{\Theta_{i,c}}{\epsilon}\}
\end{aligned}$$

and

$$\Phi_c = \begin{bmatrix} \phi_{11,c} & \phi_{12,c} \\ \phi_{21,c} & \phi_{22,c} \end{bmatrix} < 0, \quad i < j \leq r \tag{24}$$

where

$$\begin{aligned}
\phi_{11,c} &= G_{ij,c}^T P_c + P_c G_{ij,c} \\
\phi_{12,c} &= -P_c H_{ij,c} + G_{ij,c}^T P_c \\
\phi_{21,c} &= -H_{ij,c}^T P_c + P_c G_{ij,c} \\
\phi_{22,c} &= -I_{ij,c}^T P_c - P_c I_{ij,c}
\end{aligned}$$

Pre and post multiply (23) and (24) by $L_c = diag\{X_c, X_c\}$, where $X_c = P_c^{-1}$gives,

$$\Lambda_c = \begin{bmatrix} \lambda_{11,c} & \lambda_{12,c} \\ \lambda_{21,c} & \lambda_{22,c} \end{bmatrix} < 0, \quad i = 1, \ldots, r \tag{25}$$

where

$$\begin{aligned}
\lambda_{11,c} &= X_c A_{i,c}^T + \alpha_{i,c}^T B_{i,c}^T + A_{i,c}X_c + B_{i,c}\alpha_{i,c} \\
\lambda_{12,c} &= -B_{i,c}\alpha_{i,c} + X_c A_{i,c}^T + \alpha_{i,c}^T B_{i,c}^T \\
\lambda_{21,c} &= -\alpha_{i,c}^T B_{i,c}^T + A_{i,c}X_c + B_{i,c}\alpha_{i,c} \\
\lambda_{22,c} &= -\alpha_{i,c}^T B_{i,c}^T - \frac{\beta_{i,c}^T}{\epsilon} - B_{i,c}\alpha_{i,c} - \frac{\beta_{i,c}}{\epsilon}
\end{aligned}$$

and

$$\Phi_c = \begin{bmatrix} \phi_{11,c} & \phi_{12,c} \\ \phi_{21,c} & \phi_{22,c} \end{bmatrix} < 0, \quad i < j \leq r \tag{26}$$

where

$$\begin{aligned}
\phi_{11,c} &= 2X_cA_{i,c}^T + \alpha_{i,c}^TB_{j,c}^T + \alpha_{j,c}^TB_{i,c}^T + 2A_{i,c}X_c \\
&\quad + B_{i,c}\alpha_{j,c} + B_{j,c}\alpha_{i,c} \\
\phi_{12,c} &= -B_{i,c}\alpha_{j,c} - B_{j,c}\alpha_{i,c} + 2X_cA_{i,c}^T + \alpha_{i,c}^TB_{j,c}^T \\
&\quad + \alpha_{j,c}^TB_{i,c}^T \\
\phi_{21,c} &= -\alpha_{i,c}^TB_{j,c}^T - \alpha_{j,c}^TB_{i,c}^T + 2A_{i,c}X_c + B_{i,c}\alpha_{j,c} \\
&\quad + B_{j,c}\alpha_{i,c} \\
\phi_{22,c} &= -\alpha_{i,c}^TB_{j,c}^T - \alpha_{j,c}^TB_{i,c}^T - 2\frac{\beta_{i,c}^T}{\epsilon} \\
&\quad - B_{i,c}\alpha_{j,c} - B_{j,c}\alpha_{i,c} - 2\frac{\beta_{i,c}}{\epsilon}
\end{aligned}$$

Note that in (25) and (26)

$$\alpha_{i,c} = K_{i,c} * X_c,\ \beta_{i,c} = \Theta_{i,c} * X_c \tag{27}$$

which completes the proof of Theorem 2.

Design summary of the continuous-time Δ-M based quantised state-feedback control system are described in algorithm 1.

Algorithm 1: Implementation of continuous-time quantised control system.

Input : T-S fuzzy model
Output: State-feedback gain K_c and Quantiser gain Θ_c

Check the LMI in Eq. (8),
`*/ Check stability conditions`
if *Eq.* (8)< 0, **then**
  $P_1^c = P_2^c = P_c$, $P_3^c = 0$
end
Pre and post-multiply Eq. (23) & Eq. (24) by $L_c = diag(X_c, X_c); X_c = P_c^{-1}$
`*/ Determination of` X_c, α_c `&` β_c
Solve LMIs in Eq. (25) and Eq. (26) to obtain X_c, α_c & β_c
`*/ Computation of State-feedback gain and Quantiser gain`
$K_c = \alpha_c X_c^{-1}$, $\Theta_c = \beta_c X_c^{-1}$

3 Discrete Fuzzy State Feedback Controller

This section gives the design procedure of Δ-M based quantised state feedback controller for discrete-time (DT) T-S fuzzy systems. Note that although the

design of this controller follows similar steps of continuous time T-S fuzzy systems (described in section-2), the design of feedback gain and quantisation gain as well as the stability conditions are different in discrete domain.

3.1 System Description

Consider a nonlinear system which is represented by a discrete time T-S fuzzy model with the following IF-THEN rules:

IF $z_1(k)$ is M_{i1} and ... and $z_p(k)$ is M_{ip},
THEN

$$x(k+1) = A_{i,d}x(k) + B_{i,d}u(k), \qquad i = 1, 2, \ldots, r \tag{28}$$

where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$ respectively denote the states of the system and the control input. The matrices $A_{i,d}$ and $B_{i,d}$ are of appropriate dimensions. There are r number of IF-THEN rules and the fuzzy sets are denoted by M_{ij}. It is assumed that $z(k) = [z_1(k), z_2(k), \ldots, z_p(k)]^T$ is a known. Then the system equation is given by:

$$x(k+1) = \sum_{i=1}^{r} \lambda_i(z(k))\{A_{i,d}x(k) + B_{i,d}u(k)\} \tag{29}$$

3.2 Discrete-Time Delta Modulator

Note that this controller uses discrete time Δ-M whose design procedure is different than the continuous time Δ-M. The block diagram of a MIMO discrete-time Δ-M is shown in Figure. 2.

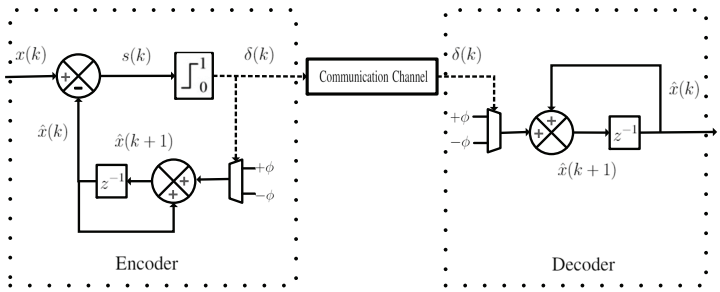


Fig. 2 Discrete-time Delta-Modulator (Δ-M)

If $z_1(k)$ is M_{i1} and ... and $z_p(k)$ is M_{ip}, then the dynamics of MIMO DT Δ-M is described as:

$$s(k) = x(k) - \hat{x}(k) \tag{30a}$$

$$\hat{x}(k+1) = \hat{x}(k) + \Theta_{i,d}\,\mathrm{sgn}(s(k)),\ i = 1, 2, \ldots, r, \tag{30b}$$

where $\Theta_{i,d}$ is the quantisation gain of the ith 2-level quantiser. Rewrite (30b) as:

$$\hat{x}(k+1) = \hat{x}(k) + \sum_{i=1}^{r} \lambda_i(z(k))\Theta_{i,d}\mathrm{sgn}(s(k)) \tag{31}$$

Further, $\mathrm{sgn}(s(k)) \in [\{-1,1\}, \cdots, \{-1,1\}]^T \in \mathbb{R}^n$ for all $s(k) \in \mathbb{R}^n$, where

$$\mathrm{sgn}(s_j(k)) = \begin{cases} +1, \text{ if } \ s_j(k) \ \geq \ 0; \\ -1, \text{ if } \ s_j(k) \ < \ 0; \end{cases} j = (1, 2, \cdots, n).$$

3.3 Quantised State-Feedback Control System

Using (28) and (30), the quantised state-feedback control system can be expressed as:

$$x(k+1) = A_{i,d}\, x(k) + B_{i,d}\, \hat{u}(k) \tag{32a}$$
$$\hat{x}(k+1) = \hat{x}(k) + \Theta_{i,d}\, \mathrm{sgn}(s(k)) \tag{32b}$$
$$\hat{u}(k) = K_{i,d}\, \hat{x}(k) \tag{32c}$$
$$s(k) = x(k) - \hat{x}(k) \tag{32d}$$

From (32):

$$\begin{aligned} x(k+1) = &\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(z(k))\lambda_j(z(k))\{A_{i,d} + B_{i,d}K_{j,d}\}x(k) \\ &- \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(z(k))\lambda_j(z(k))B_{i,d}K_{j,d}\, s(k) \end{aligned} \tag{33a}$$

$$\begin{aligned} s(k+1) = &\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(z(k))\lambda_j(z(k))\{A_{i,d} + B_{i,d}K_{j,d}\}x(k) \\ &- \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(z(k))\lambda_j(z(k))B_{i,d}K_{j,d}\, s(k) \\ &- x(k) + s(k) - \sum_{i=1}^{r} \lambda_i(z(k))\Theta_{i,d}\, \mathrm{sgn}(s(k)) \end{aligned} \tag{33b}$$

Theorem 3 *For known $A_{i,d}$, $B_{i,d}$ $K_{i.d}$, $\Theta_{i,d}$, $i = 1, \ldots, r$ and the maximum quantisation error ϵ, the quantised fuzzy system (33) is exponentially stable if there exists a common symmetric positive-definite matrix $R_d > 0$ and matrices Λ_d for $i = 1, \ldots, r$ and Φ_d for $i < j \leq r$ such that,*

$$\Lambda_d = \begin{bmatrix} \lambda_{11,d} & \lambda_{12,d} \\ \lambda_{21,d} & \lambda_{22,d} \end{bmatrix} < 0, \ \Phi_d = \begin{bmatrix} \phi_{11,d} & \phi_{12,d} \\ \phi_{21,d} & \phi_{22,d} \end{bmatrix} < 0 \tag{34}$$

where

$$
\begin{aligned}
\lambda_{11,d} = {} & \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_1^d \{A_{i,d} + B_{i,d}K_{i,d}\} - P_1^d \\
& - P_2^d\{A_{i,d} + B_{i,d}K_{i,d}\} - \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_2^d \\
& + \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_2^d \{A_{i,d} + B_{i,d}K_{i,d}\} + P_2^d \\
& - P_3^d\{A_{i,d} + B_{i,d}K_{i,d}\} - \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_3^d \\
& + 2\{A_{i,d} + B_{i,d}K_{i,d}\}^T P_3^d \{A_{i,d} + B_{i,d}K_{i,d}\} \\
\lambda_{12,d} = {} & -\{A_{i,d} + B_{i,d}K_{i,d}\}^T P_1^d \{B_{i,d}K_{i,d}\} - P_2^d \\
& + P_2^d\{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\} + \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_2^d \\
& - \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_2^d \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\} \\
& + P_3^d B_{i,d}K_{i,d} - \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_3^d \{B_{i,d}K_{i,d}\} \\
& + \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_3^d - P_3^d \\
& - \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_3^d \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\} \\
\lambda_{21,d} = {} & -\{B_{i,d}K_{i,d}\}^T P_1^d \{A_{i,d} + B_{i,d}K_{i,d}\} - P_2^d \\
& + P_2^d\{A_{i,d} + B_{i,d}K_{i,d}\} + \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}^T P_2^d \\
& - \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}^T P_2^d \{A_{i,d} + B_{i,d}K_{i,d}\} \\
& + K_{i,d}^T B_{i,d}^T P_3^d - \{B_{i,d}K_{i,d}\}^T P_3^d \{A_{i,d} + B_{i,d}K_{i,d}\} \\
& + P_3^d\{A_{i,d} + B_{i,d}K_{i,d}\} - P_3^d \\
& - \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}^T P_3^d \{A_{i,d} + B_{i,d}K_{i,d}\} \\
\lambda_{22,d} = {} & K_{i,d}^T B_{i,d}^T P_1^d B_{i,d}K_{i,d} - P_2^d\{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\} \\
& - \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}^T P_2^d \\
& + \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}^T P_2^d \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\} \\
& - P_3^T B_{i,d}K_{i,d} - K_{i,d}^T B_{i,d}^T P_3^d \\
& + \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}^T P_3^d \{B_{i,d}K_{i,d}\} \\
& + \{B_{i,d}K_{i,d}\}^T P_3^d \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}
\end{aligned}
$$

$$\begin{aligned}
\phi_{11,d} &= 4G_{ij,d}^T P_1^d G_{ij,d} - P_1^d - 2P_2^d G_{ij,d} + P_2^d \\
&- 2G_{ij,d}^T P_2^d + 4G_{ij,d}^T P_2^d G_{ij,d} - 2P_3^d G_{ij,d} \\
&+ 4G_{ij,d}^T P_3^d G_{ij,d} - 2G_{ij,d}^T P_3^d + 4G_{ij,d}^T P_3^d G_{ij,d} \\
\phi_{12,d} &= -4G_{ij,d}^T P_1^d H_{ij,d} - P_2^d + 2P_2^d I_{ij,d} + 2G_{ij,d}^T P_2^d \\
&- 4G_{ij,d}^T P_2^d I_{ij,d} + 2P_3^d H_{ij,d} - 4G_{ij,d}^T P_3^d H_{ij,d} \\
&+ 2G_{ij,d}^T P_3^d - 4G_{ij,d}^T P_3^d I_{ij,d} - P_3^d \\
\phi_{21,d} &= -4H_{ij,d}^T P_1^d G_{ij,d} - P_2^d + 2P_2^d G_{ij,d} + 2I_{ij,d}^T P_2^d \\
&- 4I_{ij,d}^T P_2^d G_{ij,d} + 2P_3^d G_{ij,d} - 4I_{ij,d}^T P_3^d G_{ij,d} \\
&- P_3^d + 2H_{ij,d}^T P_3^d - 4H_{ij,d}^T P_3^d G_{ij,d} \\
\phi_{22,d} &= 4H_{ij,d}^T P_1^d H_{ij,d} - 2P_2^d I_{ij,d} - 2I_{ij,d}^T P_2^d \\
&+ 4I_{ij,d}^T P_2^d I_{ij,d} - 2P_3^d H_{ij,d} - 2H_{ij,d}^T P_3^d \\
&+ 4I_{ij,d}^T P_3^d H_{ij,d} + 4H_{ij,d}^T P_3^d I_{ij,d}
\end{aligned}$$

and

$$\begin{aligned}
G_{ij,d} &= \frac{\{A_{i,d} + B_{i,d}K_{j,d}\} + \{A_{i,d} + B_{j,d}K_{i,d}\}}{2} \\
H_{ij,d} &= \frac{\{B_{i,d}K_{j,d}\} + \{B_{j,d}K_{i,d}\}}{2} \\
I_{ij,d} &= \frac{\{B_{i,d}K_{j,d} + \frac{\Theta_{i,d}}{\epsilon}\} + \{B_{j,d}K_{i,d} + \frac{\Theta_{i,c}}{\epsilon}\}}{2}
\end{aligned}$$

Proof Proof is similar to Theorem 1.

Theorem 4 *For given $A_{i,d}$, $B_{i,d}$, $i = 1, \ldots, r$ and the maximum quantisation error ϵ, the quantised fuzzy system (33) is exponentially stable if there exists a matrix $X_d > 0$ and unknown matrices $\alpha_{i,d}$, $\beta_{i,d}$, $i = 1, \ldots, r$ where the feedback gain $K_{i,d}$ and quantiser gain $\Theta_{i,d}$ in (7) is given by,*

$$K_{i,d} = \alpha_{i,d} * X_d^{-1},\ \Theta_{i,d} = \beta_{i,d} * X_d^{-1} \tag{35}$$

Proof Substituting $P_1^d = P_2^d = P_d$ and $P_3^d = 0$, in (34) gives,

$$\Lambda_d = \begin{bmatrix} \lambda_{11,d} & \lambda_{12,d} \\ \lambda_{21,d} & \lambda_{22,d} \end{bmatrix} < 0, \quad i = 1, \ldots, r \tag{36}$$

where

$$\begin{aligned}
\lambda_{11,d} &= 2\{A_{i,d} + B_{i,d}K_{i,d}\}^T P_d \{A_{i,d} + B_{i,d}K_{i,d}\} \\
&\quad - P_d\{A_{i,d} + B_{i,d}K_{i,d}\} - \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_d \\
\lambda_{12,d} &= -\{A_{i,d} + B_{i,d}K_{i,d}\}^T P_d \{B_{i,d}K_{i,d}\} - P_d \\
&\quad + P_d\{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\} + \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_d \\
&\quad - \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_d \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\} \\
\lambda_{21,d} &= -\{B_{i,d}K_{i,d}\}^T P_d \{A_{i,d} + B_{i,d}K_{i,d}\} - P_d \\
&\quad + P_d\{A_{i,d} + B_{i,d}K_{i,d}\} + \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}^T P_d \\
&\quad - \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}^T P_d \{A_{i,d} + B_{i,d}K_{i,d}\} \\
\lambda_{22,d} &= K_{i,d}^T B_{i,d}^T P_d B_{i,d} K_{i,d} - P_d\{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\} \\
&\quad - \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}^T P_d \\
&\quad + \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}^T P_d \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}
\end{aligned}$$

and

$$\Phi_d = \begin{bmatrix} \phi_{11,d} & \phi_{12,d} \\ \phi_{21,d} & \phi_{22,d} \end{bmatrix} < 0, \quad i < j \leq r \tag{37}$$

where

$$\begin{aligned}
\phi_{11,d} &= 8G_{ij,d}^T P_d G_{ij,d} - 2P_d G_{ij,d} - 2G_{ij,d}^T P_d \\
\phi_{12,d} &= -4G_{ij,d}^T P_d H_{ij,d} - P_d + 2P_d I_{ij,d} + 2G_{ij,d}^T P_d \\
&\quad - 4G_{ij,d}^T P_d I_{ij,d} \\
\phi_{21,d} &= -4H_{ij,d}^T P_d G_{ij,d} - P_d + 2P_d G_{ij,d} + 2I_{ij,d}^T P_d \\
&\quad - 4I_{ij,d}^T P_d G_{ij,d} \\
\phi_{22,d} &= 4H_{ij,d}^T P_d H_{ij,d} - 2P_d I_{ij,d} - 2I_{ij,d}^T P_d \\
&\quad + 4I_{ij,d}^T P_d I_{ij,d}
\end{aligned}$$

Using Schur compliment gives [38],

$$\Lambda_d = \begin{bmatrix} \lambda_{11,d} & \lambda_{12,d} & \lambda_{13,d} \\ \lambda_{12,d}^T & \lambda_{22,d} & \lambda_{23,d} \\ \star & \star & \lambda_{33,d} \end{bmatrix} < 0, \quad i = 1, \ldots, r \tag{38}$$

where

$$
\begin{aligned}
\lambda_{11,d} &= \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_d \{A_{i,d} + B_{i,d}K_{i,d}\} \\
&\quad - P_d\{A_{i,d} + B_{i,d}K_{i,d}\} - \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_d \\
\lambda_{12,d} &= P_d\{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\} + \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_d \\
&\quad - \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_d \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\} - P_d \\
\lambda_{13,d} &= \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_d \\
\lambda_{22,d} &= -P_d\{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\} - \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}^T P_d \\
&\quad + \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}^T P_d \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\} \\
\lambda_{23,d} &= -K_{i,d}^T B_{i,d}^T P_d \\
\lambda_{33,d} &= -P_d
\end{aligned}
$$

and

$$
\Phi_d = \begin{bmatrix} \phi_{11,d} & \phi_{12,d} & \phi_{13,d} \\ \phi_{12,d}^T & \phi_{22,d} & \phi_{23,d} \\ \star & \star & \phi_{33,d} \end{bmatrix} < 0, \quad i < j \leq r \tag{39}
$$

where

$$
\begin{aligned}
\phi_{11,d} &= 4G_{ij,d}^T P_d G_{ij,d} - 2P_d G_{ij,d} - 2G_{ij,d}^T P_d \\
\phi_{12,d} &= -P_d + 2P_{i,d} I_{ij,d} + 2G_{ij,d}^T P_d - 4G_{ij,d}^T P_d I_{ij,d} \\
\phi_{13,d} &= 2G_{ij,d}^T P_d^T \\
\phi_{22,d} &= -2P_d I_{ij,d} - 2I_{ij,d}^T P_d + 4I_{ij,d}^T P_d I_{ij,d} \\
\phi_{23,d} &= -2H_{ij,d}^T P_d^T \\
\phi_{33,d} &= -P_d
\end{aligned}
$$

Using Schur compliment again gives [38],

$$
\Lambda_d = \begin{bmatrix} \lambda_{11,d} & \lambda_{12,d} & \lambda_{13,d} & \lambda_{14,d} \\ \star & \lambda_{22,d} & \lambda_{23,d} & \lambda_{24,d} \\ \star & \star & \lambda_{33,d} & 0 \\ \star & \star & \star & \lambda_{44,d} \end{bmatrix} < 0, \quad i = 1, \ldots, r \tag{40}
$$

where

$$\begin{aligned}
\lambda_{11,d} &= -P_d\{A_{i,d} + B_{i,d}K_{i,d}\} - \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_d \\
\lambda_{12,d} &= P_d\{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\} + \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_d - P_d \\
\lambda_{13,d} &= \lambda_{14,d} = \{A_{i,d} + B_{i,d}K_{i,d}\}^T P_{i,d} \\
\lambda_{22,d} &= -P_d\{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\} - \{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}^T P_d \\
\lambda_{23,d} &= -K_{i,d}^T B_{i,d}^T P_d \\
\lambda_{24,d} &= -\{B_{i,d}K_{i,d} + \frac{\Theta_{i,d}}{\epsilon}\}^T P_d \\
\lambda_{33,d} &= \lambda_{44,d} = -P_d
\end{aligned}$$

and

$$\Phi_d = \begin{bmatrix} \phi_{11,d} & \phi_{12,d} & \phi_{13,d} & \phi_{14,d} \\ \star & \phi_{22,d} & \phi_{23,d} & \phi_{24,d} \\ \star & \star & \phi_{33,d} & 0 \\ \star & \star & \star & \phi_{44,d} \end{bmatrix} < 0, \quad i < j \leq r \tag{41}$$

where

$$\begin{aligned}
\phi_{11,d} &= -2P_d G_{ij,d} - 2G_{ij,d}^T P_d \\
\phi_{12,d} &= -P_d + 2P_d I_{ij,d} + 2G_{ij,d}^T P_d \\
\phi_{13,d} &= \phi_{14,d} = 2G_{ij,d}^T P_d^T \\
\phi_{22,d} &= -2P_d I_{ij,d} - 2I_{ij,d}^T P_d \\
\phi_{23,d} &= -2H_{ij,d}^T P_d^T \\
\phi_{24,d} &= -2I_{ij,d}^T P_d^T \\
\phi_{33,d} &= \phi_{44,d} = -P_d
\end{aligned}$$

Pre-multiplying and post-multiplying (40) and (41) by $L_d = diag\{X_d, X_d, X_d, X_d\}$ where $X_d = P_d^{-1}$ gives,

$$\Lambda_d = \begin{bmatrix} \lambda_{11,d} & \lambda_{12,d} & \lambda_{13,d} & \lambda_{14,d} \\ \star & \lambda_{22,d} & \lambda_{23,d} & \lambda_{24,d} \\ \star & \star & \lambda_{33,d} & 0 \\ \star & \star & \star & \lambda_{44,d} \end{bmatrix} < 0, \quad i = 1, \ldots, r \tag{42}$$

where

$$\lambda_{11,d} = -A_{i,d}X_d - B_{i,d}\alpha_{i,d} - X_dA_{i,d}^T - \alpha_{i,d}^TB_{i,d}^T$$
$$\lambda_{12,d} = B_{i,d}\alpha_{i,d} + \frac{\beta_{i,d}}{\epsilon} + X_dA_{i,d}^T + \alpha_{i,d}^TB_{i,d}^T - X_d$$
$$\lambda_{13,d} = \lambda_{14,d} = X_dA_{i,d}^T + \alpha_{i,d}^TB_{i,d}^T$$
$$\lambda_{22,d} = -B_{i,d}\alpha_{i,d} - \frac{\beta_{i,d}}{\epsilon} - \alpha_{i,d}^TB_{i,d}^T - \frac{\beta_{i,d}^T}{\epsilon}$$
$$\lambda_{23,d} = -\alpha_{i,d}^TB_{i,d}^T$$
$$\lambda_{24,d} = -\alpha_{i,d}^TB_{i,d}^T - \frac{\beta_{i,d}^T}{\epsilon}$$
$$\lambda_{33,d} = \lambda_{44,d} = -X_d$$

and

$$\Phi_d = \begin{bmatrix} \phi_{11,d} & \phi_{12,d} & \phi_{13,d} & \phi_{14,d} \\ \star & \phi_{22,d} & \phi_{23,d} & \phi_{24,d} \\ \star & \star & \phi_{33,d} & 0 \\ \star & \star & \star & \phi_{44,d} \end{bmatrix} < 0, \quad i < j \leq r \tag{43}$$

where

$$\begin{aligned}
\phi_{11,d} &= -2A_{i,d}X_d - B_{i,d}\alpha_{j,d} - B_{j,d}\alpha_{i,d} - 2X_dA_{i,d}^T \\
&\quad - \alpha_{j,d}^TB_{i,d}^T - \alpha_{i,d}^TB_{j,d}^T \\
\phi_{12,d} &= -X_d + 2\frac{\beta_{i,d}}{\epsilon} + B_{i,d}\alpha_{j,d} + B_{j,d}\alpha_{i,d} \\
&\quad + 2X_dA_{i,d}^T + \alpha_{j,d}^TB_{i,d}^T + \alpha_{i,d}^TB_{j,d}^T \\
\phi_{13,d} &= \phi_{14,d} = 2X_dA_{i,d}^T + \alpha_{j,d}^TB_{i,d}^T + \alpha_{i,d}^TB_{j,d}^T \\
\phi_{22,d} &= -2\frac{\beta_{i,d}}{\epsilon} - B_{i,d}\alpha_{j,d} - B_{j,d}\alpha_{i,d} \\
&\quad - 2\frac{\beta_{i,d}^T}{\epsilon} - \alpha_{j,d}^TB_{i,d}^T - \alpha_{i,d}^TB_{j,d}^T \\
\phi_{23,d} &= -\alpha_{j,d}^TB_{i,d}^T - \alpha_{i,d}^TB_{j,d}^T \\
\phi_{24,d} &= -2\frac{\beta_{i,d}^T}{\epsilon} - \alpha_{j,d}^TB_{i,d}^T - \alpha_{i,d}^TB_{j,d}^T \\
\phi_{33,d} &= \phi_{44,d} = -X_d
\end{aligned}$$

Note that in (42) and (43)

$$\alpha_{i,d} = K_{i,d} * X_d, \ \beta_{i,d} = \Theta_{i,d} * X_d \tag{44}$$

which completes the proof of Theorem 2.

Design summary of the discrete-time Δ-M based quantised state-feedback control system are given in algorithm 2.

Algorithm 2: Implementation of discrete-time quantised control system

Input : T-S fuzzy model
Output: State-feedback gain K_d and Quantiser gain Θ_d

Check the LMI in Eq. (34),

`*/ Check stability conditions`

if *Eq.* (34)< 0, **then**

  $P_1^d = P_2^d = P_d$, $P_3^d = 0$

end

`*/ Convert the Bi-linear Matrix inequality into LMI using Schur Compliment`

Use Schur compliment on Eq. (36) and Eq. (37) twice

Pre and post-multiply Eq. (40) & Eq. (41) by
$L_d = diag(X_d, X_d, X_d, X_d); X_d = P_d^{-1}$

`*/ Determination of` X_d, α_d `&` β_d

Solve LMIs in Eq. (42) and Eq. (43) to obtain X_d, α_d & β_d

`*/ Computation of State-feedback gain and Quantiser gain`

$K_d = \alpha_d X_d^{-1}$, $\Theta_d = \beta_d X_d^{-1}$

4 Simulation Results

The effectiveness of the proposed controller design and implementation framework in a practical networked environment is demonstrated considering two examples. In this study, the communication network is implemented using ZigBee protocol.

4.1 Networked Control System

The block diagram of the networked control system is shown in Figure. 3 where the network is simulated using ZigBee protocol. This study follows the common practice, reported in literature, where the wireless communication channel is implemented only on one side of the networked control system to transmit the control signal from the controller node to the system. During the simulations, it is found out that the transmission delay τ of this network equals to 0.02 seconds.

The format of the transmitted data packet using ZigBee protocol is given in Table-1. Note that the length of the information in the transmitted data packet is assumed to be μ number of bytes, where μ equals to the total number of system states. For each control action, the length of each data packet (Υ):

$$\Upsilon = \mu + 7$$

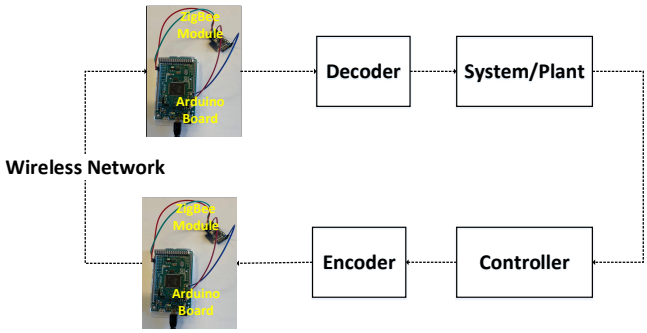


Fig. 3 Networked control using ZigBee protocol based communication network.

Table 1 Data packet format.

Name	Length(bytes)
Header	2
Source Port	1
Destination Port	1
Receiver Address	2
Data	μ
Packet End	1

The average transmission time T is given by:

$$T = \frac{8 \times \Upsilon}{\eta} \tag{45}$$

where η is the average rate of radio transmission of the ZigBee module. The average energy consumed by each transmission E is calculated as:

$$E = \upsilon \times \iota \times T \tag{46}$$

where υ and ι denote the operating voltage and current respectively. The values of different parameters of the ZigBee module used here are:$\eta = 250$ K bits/s; $\upsilon = 3.3$V; $\iota = 0.3$A.

4.2 Example 1

This example illustrates the effectiveness of the proposed controller in continuous time domain which is equipped with a continuous time Δ-M. Consider the nonlinear system described by [54, 55]:

$$\dot{x}_1(t) = -0.1x_1(t) - 0.5x_1^3(t) + 0.1x_2(t) \tag{47a}$$
$$\dot{x}_2(t) = -x_1(t) - 10x_2(t) + u(t) \tag{47b}$$

The nonlinear system (47) can be represented by the following fuzzy model:
IF $x_1(t)$ is M_i;

THEN $\dot{x}(t) = A_i x(t) + bu(t), \quad i = 1, 2;$ where

$$A_1 = \begin{bmatrix} -0.1 & 0.1 \\ -1 & -10 \end{bmatrix}, A_2 = \begin{bmatrix} -4.6 & 0.1 \\ -1 & -10 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The gains of the state-feedback controller and the quantiser are designed following the procedures described in the section-2. In this simulation, the initial conditions x_0 of the plant is considered to be $[0.01, -0.01]^T$ and the maximum quantisation error ϵ is taken to be 0.1. The quantisation gains Θ_1, Θ_2 and the feedback gains K_1, K_2 are given by:

$$\Theta_1 = \begin{bmatrix} 0.0660 & 0.2871 \\ 0.3186 & -0.5346 \end{bmatrix}, \Theta_2 = \begin{bmatrix} 0.0938 & 0.4696 \\ 0.3052 & -0.2321 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} -2.6637 & 9.0867 \end{bmatrix}, K_2 = \begin{bmatrix} -2.0574 & 7.4270 \end{bmatrix}$$

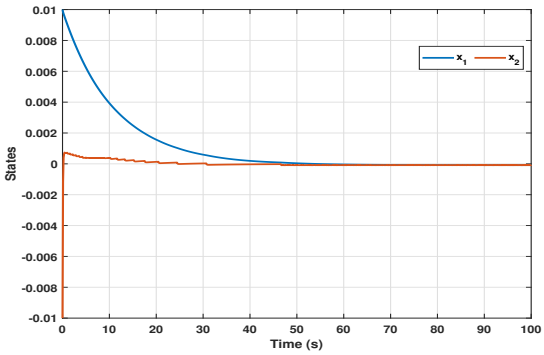
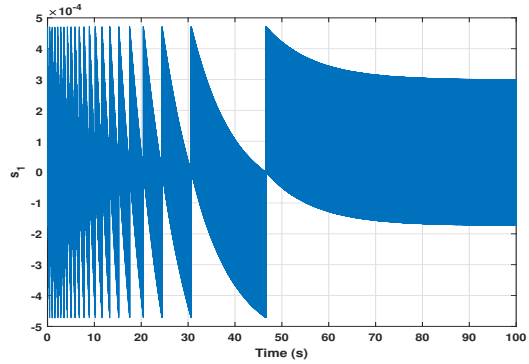
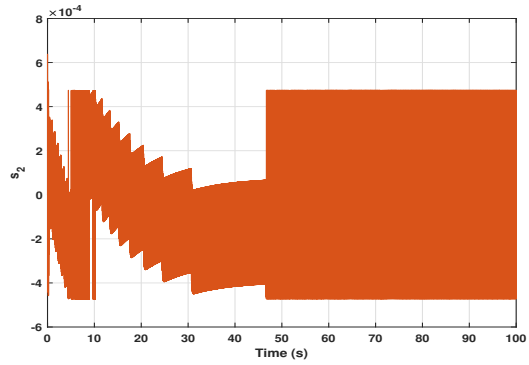


Fig. 4 Dynamics of the states for the example 1.

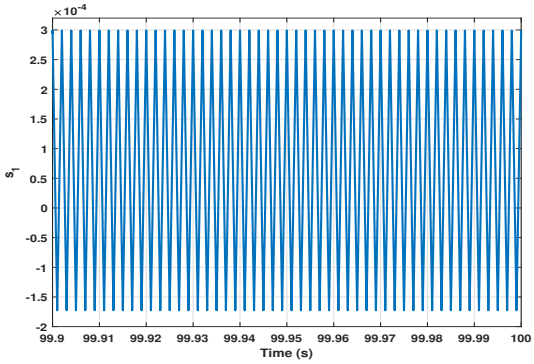
The dynamics of the states and the switching function are shown in Figure. 4 and Figure. 5 respectively. From Figure. 4, it is observed that all the states converge to zero within finite time. Further,the switching function starts inside region $\Omega < \epsilon$, as can be seen in Figure. 5(a) and 5(b), and stays there indefinitely within that region (see Figure. 5(c) and 5(d)). The results of practical simulation confirms with the theoretical findings. It is worth noting that although the results are shown for $\epsilon = 0.1$, the performance of the proposed controller is not significantly affected by other choices of ϵ. However, its value should not be considered to be very low; as this will result in high chattering.



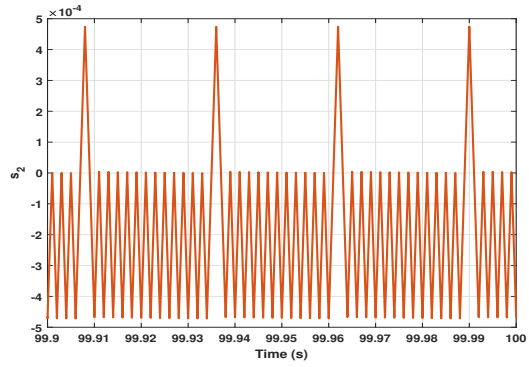
(a)



(b)



(c)



(d)

Fig. 5 Dynamics of the switching function for the example 1: (a) s_1; (b) s_2; (c) Zig-zag behaviour of s_1; (d) Zig-zag behaviour of s_2;

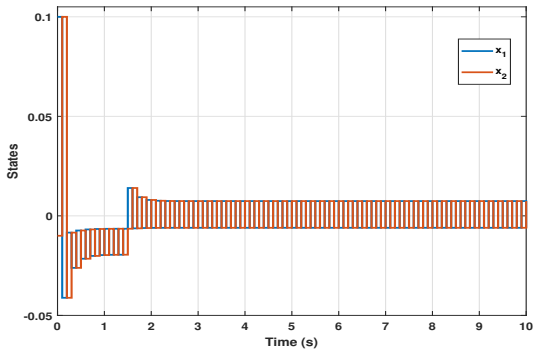


Fig. 6 Dynamics of the states for the example 2.

4.3 Example 2

This example demonstrates the effectiveness of the discrete-time quantised state feedback controller design which uses a discrete-time Δ-M described in section-3. Consider a Hẽnon system described by [4]:

$$x_1(k+1) = -\{cx_1(k) + (1-c)x_1(k)\}^2 + 0.3x_2(k) + u(k) \tag{48a}$$
$$x_2(k+1) = cx_1(k) + (1-c)x_1(k) \tag{48b}$$

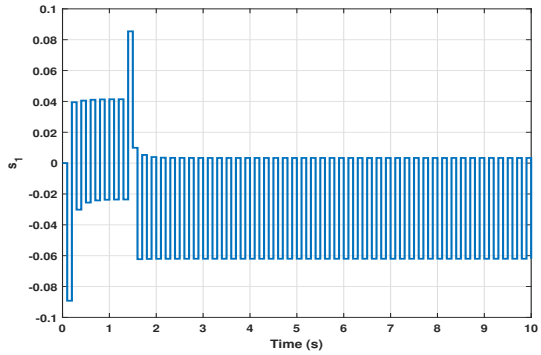
where the constant $c \in [0,1]$ is the retarded coefficient. Let $\theta(k) = cx_1(k) + (1-c)x_1(k)$. Assume that $\theta(k) \in [-m, m], m > 0$. Then the nonlinear system in (48) can be represented by the following T-S fuzzy model:

***Plant Rule 1*:**
IF $\theta(k)$ is $-m$,
THEN $x(k+1) = A_1x(k) + B_1u(k)$
***Plant Rule 2*:**
IF $\theta(k)$ is m,
THEN $x(k+1) = A_2x(k) + B_2u(k)$
where

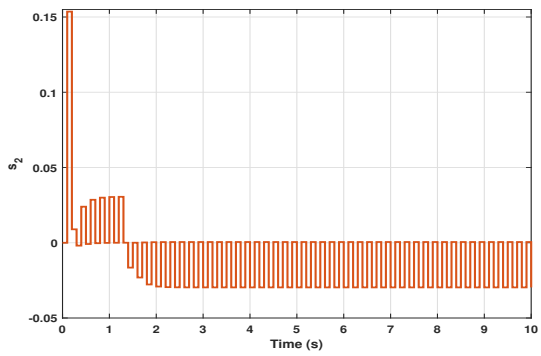
$$A_1 = \begin{bmatrix} c \times m & 0.3 \\ c & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -c \times m & 0.3 \\ c & 0 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The gains of the state-feedback controller and the quantiser are designed following the procedures described in section-3 for the parameter values $c = 0.1$ and $m = 2$. In this simulation, the initial conditions x_0 of the plant is considered to be $[0.1, -0.01]^T$, sampling time $h = 0.1$ and the maximum quantisation error ϵ is taken to be 0.2. The quantisation gains Θ_1, Θ_2 and the feedback gains K_1, K_2 are given by:

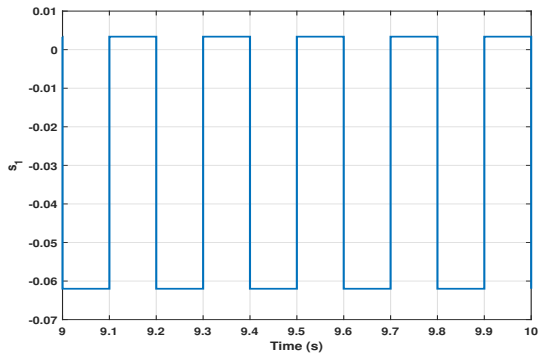
$$\Theta_1 = \begin{bmatrix} 0.0930 & 0.0897 \\ -0.0012 & 0.1370 \end{bmatrix}, \Theta_2 = \begin{bmatrix} 0.1005 & 0.1002 \\ -0.0177 & 0.1962 \end{bmatrix}$$



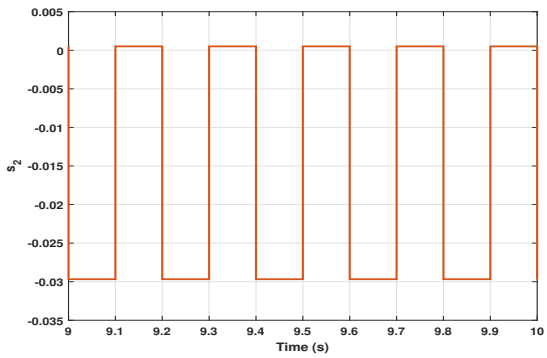
(a)



(b)



(c)



(d)

Fig. 7 Dynamics of the switching function for the example 2
: (a) s_1; (b) s_2; (c) Zig-zag behaviour of s_1; (d) Zig-zag behaviour of s_2;

$$K_1 = \begin{bmatrix} 0.0892 & -0.5920 \end{bmatrix}, K_2 = \begin{bmatrix} 0.3602 & -0.5604 \end{bmatrix}$$

The dynamics of the states and the switching function are shown in Figure. 6 and Figure. 7 respectively. From Figure. 6, it is observed that all the states converge towards zero within finite time. Further,the switching function starts inside region $\Omega < \epsilon$, as can be seen in Figure. 7(a) and 7(b), and stays there indefinitely within that region (see Figure. 7(c) and 7(d)). The period of the switching function s_1 and s_2 is 2 ((see Figure. 7(c) and 7(d)). The results of practical simulation confirms with the theoretical findings.

Note that, although the examples considered in this study are simple systems, the procedure of the proposed controller design is generic and can be applied to all the nonlinear systems which are represented by T-S fuzzy models. The smaller number of IF-Then rules is not a limitation of this controller.

5 Conclusions

Quantised single-bit state feedback controller is designed for nonlinear T-S fuzzy networked systems where Δ-M is used as the quantiser. The stability conditions Δ-M is derived using LMIs for both in continuous and discrete time domains. The effectiveness of the control strategy is shown considering a real ZigBee protocol based communication network with inherent imperfections such as bit-rate constraints, packet losses, transmission delays and so on using two simulated examples. The simulation results confirm the theoretical findings.

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