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REVIEW ARTICLE

A Multiple and Multidimensional Linguistic Truth-Valued Reasoning Method and its Application in Multimedia Teaching Evaluation

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Abstract

With the expansion of the epidemic, online multimedia teaching has become a common trend. The reasoning model of multimedia teaching evaluation is a useful tool to infer the result of teaching effects and predict the tendency. However, the ambiguity in the linguistic-valued evaluation leads to reasoning problems always in the context with uncertainty. To make the reasoning model better deal with multiple and multidimensional reasoning problems in uncertainty environment, while considering both positive evidence and negative evidence at the same time, this paper mainly focuses on a linguistic truth-valued intuitionistic fuzzy layered aggregation (LTV-IFLA) reasoning method. First, based on the layered linguistic truth-valued intuitionistic fuzzy lattice (LTV-IFL), we realize aggregating the linguistic truth-valued information through the layered average aggregation (LAA) operator presented by this paper. Furthermore, a layered weighted average aggregation (LWAA) operator is proposed to consider setting different weights to achieve personalization of the reasoning results. Finally, a multiple multidimensional reasoning model which simulates the reasoning of human language is presented to illustrate the method's rationality and validity.

Keywords Lattice implication algebra · Linguistic truth-valued intuitionistic fuzzy reasoning · Multiple multidimensional reasoning · Layered aggregation operator

Abbreviations

LTV-IFLA	Linguistic truth-valued intuitionistic fuzzy layered aggregation
LTV-IFL	Layered linguistic truth-valued intuitionistic fuzzy lattice
LAA	Layered average aggregation
LWAA	Layered weighted average aggregation
IFSs	Intuitionistic fuzzy sets
LTV-LIA	Linguistic truth-valued lattice implication algebra
HFLTS	Hesitant fuzzy linguistic term sets
LTV-IFP	Linguistic truth-valued intuitionistic fuzzy pair
IFLA	Intuitionistic fuzzy layered aggregation

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1 Introduction

Multimedia can be described as "the combination of various digital media types, such as text, images, sound and video, into an integrated multisensory interactive application or presentation to convey a message or information to an audience" [1] (Fig. 1). As multimedia technology has the advantages of integrating multiple types of information into an interactive interface and enabling users to communicate with computers in real time through multiple senses, increasingly scholars are committed to multimedia research [2–6]. Among them, multimedia education technology [7] is the application of multimedia resources and materials

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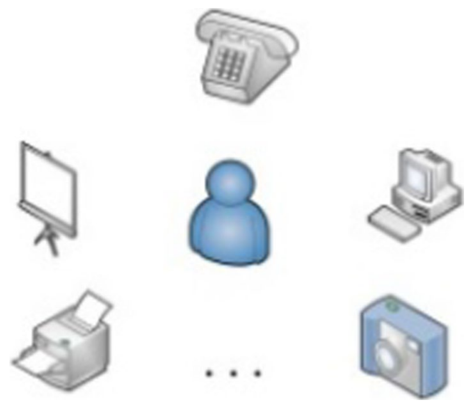

Fig. 1 Common multimedia devices

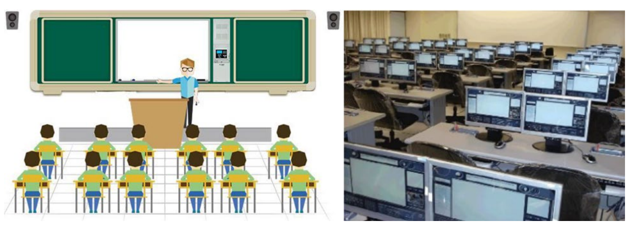

Fig. 2 Picture of multimedia teaching

to education, which provides a variety of possibilities for students to intervene in the teaching and learning process (Fig. 2). In addition, some researches combined multimedia technology with subjects such as Chinese teaching [8], physics [9], arts [10], physical education [11] and other disciplines to form a new interactive classroom teaching system, which improves teaching efficiency.

Teaching is the lifeline of a school, and the quality of teaching has an important impact on the cultivation and development of students' quality. Yong Nie [12] proposed an advanced tracking quality assessment method based on data mining to analyze various learning information that is beneficial to students, so as to obtain classification rules that can affect the learning effect toward students. In uncertainty linguistic environment, Liu et al. [13] proposed a kind of fuzzy linguistic concept lattice combining with fuzzy linguistic information, which can make the linguistic evaluation information more compact and the decision results more reasonable. The use of multimedia teaching classrooms has become a trend. How to evaluate multimedia classrooms is complicated. It is necessary to fully consider the needs of students and teachers and pay attention to the developmental function of evaluation. However, the results of evaluation are often inaccurate and vague, so how to better deal with the evaluation information in an uncertainty environment is a problem that should be handled.

Fuzzy sets theory [14] proposed by Zadeh helps us deal with the uncertainty and ambiguity that cannot be expressed by precise number when dealing with mathematical problems. However, because sometimes we need to consider positive evidence and negative evidence at the same time, fuzzy sets theory cannot express all information completely which makes it be subject to more and more restrictions and challenges. For solving this problem, Atanassov has put forward the concept of intuitionistic fuzzy sets (IFSs) [15, 16]. It can not only show the sustaining information from two aspects of non-membership degree and the membership degree, but also express neutral information effectively. Due to its better description for fuzzy variables, IFSs has been widely studied and put into various fields. Based on some transformation techniques, Chen et al. [17] proposed a new similarity measure between Atanassov's IFSs. Aggregating intuitionistic fuzzy information can be applied to multi criteria decision-making problems in the real world. Therefore, some researchers have proposed several aggregation operators on IFSs [18, 19].

According to the common sense, people in daily life use natural language for comparing, reasoning, evaluating rather than numerical language. The object itself is associated with incomparability and fuzziness. To deal with and characterize its properties and the uncertainty involved in its processing as well, in 1993, the lattice implication algebra was proposed [20]. By deep researching and expanding the lattice implication algebra, Zou et al. [21, 32] proposed a kind of LTV-IFL based on the point view of IFSs and linguistic truth-valued lattice implication algebra (LTV-LIA) which can express the information from both positive evidence and negative evidence in the form of linguistic-valued. Besides, there are other ways to deal with the linguistic-valued problems. An extension of fuzzy sets called hesitant fuzzy linguistic term sets (HFLTS) has been introduced by Rodríguez et al. [22]. By this method, the uncertainty caused by hesitation can be modeled, and a way to generate comparative linguistic expressions richer than single linguistic terms and close to the human beings' cognitive model can be provided. It quickly attracted many researchers to propose the new decision-making models using hesitant linguistic information [23–27].

Evaluation system can be better predicted and coordinated using reasoning strategy. It uses the information in the evaluation system according to the current input data and certain reasoning strategy, to solve the current problem, explain the external input facts and data, deduce the conclusion and guide the users. Because the evaluation system works by simulating human experts, the reasoning process should be similar to that of the experts when designing a reasoning model. Therefore, the intuitionistic fuzzy reasoning method that tends to human thinking has been continuously innovated and improved. As fuzzy sets theory cannot describe the data comprehensively, which has greatly limited the objectivity of fuzzy time series in uncertainty data forecasting, Wang et al. [28] proposed an intuitionistic fuzzy time series forecasting model and established forecast rules based on intuitionistic fuzzy approximate reasoning. Fuzzy Petri nets are also an important modeling tool for knowledge representation and reasoning, which have been extensively used in a lot of fields [29, 30]. Zou et al. [31]

proposed an approach to divide the LTV-IFL into layers to simplify the reasoning process. Two types of implication operators and a linguistic truth-valued intuitionistic fuzzy algebra reasoning algorithm have been proposed and applied to the human factors engineering. Based on the above analysis, this research is devoted to proposing an LTV-IFLA reasoning model to deal with the evaluation standards of multimedia teaching, which make the evaluation results have guiding significance to improve the teaching quality.

In this paper, by realizing the aggregation of multi-experts and multi-attributes, the evaluation information in the complex environment can be better processed. The remainder of this paper is organized as follows: Sect. 2 briefly reviews the knowledge of lattice implication algebra and their extension researches. Section 3 proposes two types of aggregation operators and an associated reasoning model is proposed. An illustrative example on multimedia teaching evaluation is given to show the applicability of the method. We draw some conclusions for this paper in the last section.

2 Preliminaries

This section introduces several definitions briefly which will help understanding this paper. The lattice implication algebra proposed by Xu is used for depicting the information with fuzziness and uncertainty.

Definition 1 [20] Let $(L, \vee, \wedge, O, I)$ be a bounded lattice with an order-reversing involution " ′", where O and I are the smallest and the greatest elements of L, respectively, and $\rightarrow: L \times L \rightarrow L$ is a mapping. $(L, \vee, \wedge, ', \rightarrow, O, I)$ is called a lattice implication algebra if the following conditions hold for any $x, y, z \in L$:

$$\begin{aligned}&(I_1)\, y \rightarrow (x \rightarrow z) = x \rightarrow (y \rightarrow z);\\&(I_2)\, x \rightarrow x = I;\\&(I_3)\, y\prime \rightarrow x\prime = x \rightarrow y;\\&(I_4)\, y \rightarrow x = x \rightarrow y = I implies x = y;\\&(I_5)\, (y \rightarrow x) \rightarrow x = (x \rightarrow y) \rightarrow y;\\&(I_6)\, (x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z);\\&(I_7)\, (x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z).\end{aligned} \tag{1}$$

Definition 2 [32] Based on $2n$-element LTV-LIA $L_{V(n\times 2)}$, $LI_{2n} = (LI_{2n}, \cup, \cap)$ is defined as a $2n$-element LTV-IFL (Fig. 3), where $((h_1,t),(h_1,f))$ and $((h_n,t),(h_n,f))$ are the minimum and maximum elements of LI_{2n}, respectively.

For any $((h_i,t),(h_j,f)) \in LI_{2n}$, we call $((h_i,t),(h_j,f))$ is a linguistic truth-valued intuitionistic fuzzy pair (LTV-IFP).

Definition 3 [32] For any $((h_i,t),(h_j,f))((h_k,t),(h_l,f)) \in LI_{2n}$, $i,j,k,l \in \{1,2,...,n\}$, "$\rightarrow$", "$\cup$", "$\cap$" and " ′" are defined as follows:

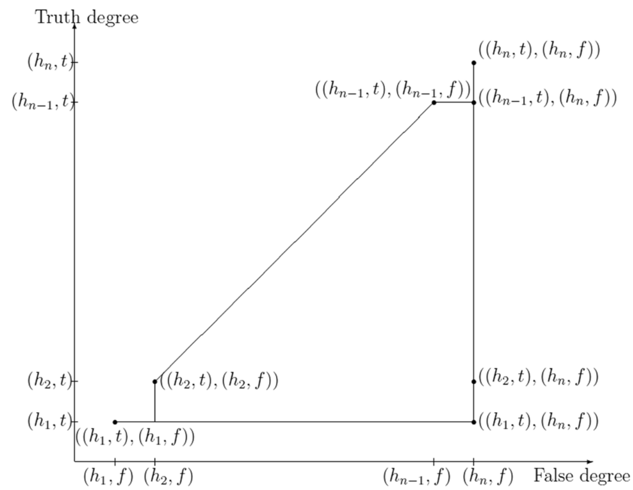

Fig. 3 Hasse diagram of LTV-IFL LI_{2n}

1. $((h_i,t),(h_j,f)) \cap ((h_k,t),(h_l,f)) = ((h_{min(i,k)},t),(h_{min(j,l)},f))$;
2. $((h_i,t),(h_j,f)) \cup ((h_k,t),(h_l,f)) = ((h_{max(i,k)},t),(h_{max(j,l)},f))$;
3. $((h_i,t),(h_j,f))\prime = ((h_{n-j+1},t),(h_{n-i+1},f))$;
4. $((h_i,t),(h_j,f)) \rightarrow ((h_k,t),(h_l,f)) = ((h_{min(n,n-i+k,n-j+l)},t),(h_{min(n,n-i+l)},f))$.

Definition 4 [32] A and B are linguistic truth-valued intuitionistic fuzzy matrixes. Suppose $a_{kl} \in A$, $b_{lp} \in B$, and "$\circ_l$" is the linguistic truth-valued intuitionistic fuzzy composite relation, then.

$$A \circ_l B = \begin{bmatrix} \overset{s}{\underset{j=1}{\vee}} (a_{1j} \wedge b_{j1}) & \cdots & \overset{s}{\underset{j=1}{\vee}} (a_{1j} \wedge b_{jp}) \\ \vdots & & \vdots \\ \overset{s}{\underset{j=1}{\vee}} (a_{kj} \wedge b_{j1}) & \cdots & \overset{s}{\underset{j=1}{\vee}} (a_{kj} \wedge b_{jp}) \end{bmatrix}. \tag{2}$$

To process the linguistic truth-valued information on the LTV-IFL, in [36], the elements on the intuitionistic fuzzy lattice can be layered. The fuzzy layered aggregation operators and their inverse operators help us calculating the information on the LTV-IFL.

Definition 5 [31] In LTV-IFL $LI_{2n} = (VI_{2n}, \cup, \cap)$, for any $((h_i,t),(h_j,f)) \in LI_{2n}$, $\varphi_{2n}: LI_{2n} \rightarrow [0,1]$ is called an intuitionistic fuzzy layered aggregation (IFLA) operator of LTV-IFA, where

$$\varphi_{2n}((h_i,t),(h_j,f)) = \left.\frac{i+j-2}{2n-2}\right|_i, i,j \in \{1,2,\ldots,n\}. \tag{3}$$

In IFLA operator, $\left.\frac{i+j-2}{2n-2}\right|_i$ means an IFLA number of LTV-IFA, where exists $2n-1$ layers, denoted by

$$\pounds(LI_{2n}) = 2n-1, \tag{4}$$

and $((h_i,t),(h_j,f))$ is on the $Layer\ i+j-2$, denoted by

$$Layer\big((h_i,t),(h_j,f)\big)=i+j-2, i+j-2\in\{0,1,\dots,2n-2\}, \tag{5}$$

and $|_i$ expresses the position of this layer.

Definition 6 [31] Let $\frac{k}{m}\Big|_l$ be an IFLA number, where $\frac{k}{m}\Big|_l\in[0|_1,1|_n]$, with $l\in\{1,2,\dots,n\}$. Then $\varphi_{2n}^{-1}:[0|_1,1|_n]\to LI_{2n}$ is called an IFLA inverse operator of LTV-IFA if

$$\varphi_{2n}^{-1}\Big(\frac{k}{m}\Big|_l\Big)=((h_l,t),(h_{k+2-l},f)), \tag{6}$$

where $\big((h_l,t),(h_{k+2-l,f})\big)\in LI_{m+2}$ is the corresponding LTV-IFP of the IFLA number $\frac{k}{m}\Big|_l$.

This paper takes the 6-element LTV-IFL as the specific research object which is shown in Fig. 4, probing further into other aggregation operators' calculation formulas and properties under the condition of layered.

In this paper, we suppose $C=\{c_1,c_2\}$ is the meta linguistic value set and linguistic truth-valued intuitionistic fuzzy sub-pairs for LTV-IFPs, which can be close to the expression of human language. In the case of no confusion, it is still recorded as LI_{2n}.

3 Reasoning Model Based on the Layered 6-Element LTV-IFL

The refinement of information in the process of expression and calculation can make it better understood. In this section, we study an aggregation reasoning evaluation method based on the layered 6-element LTV-IFL.

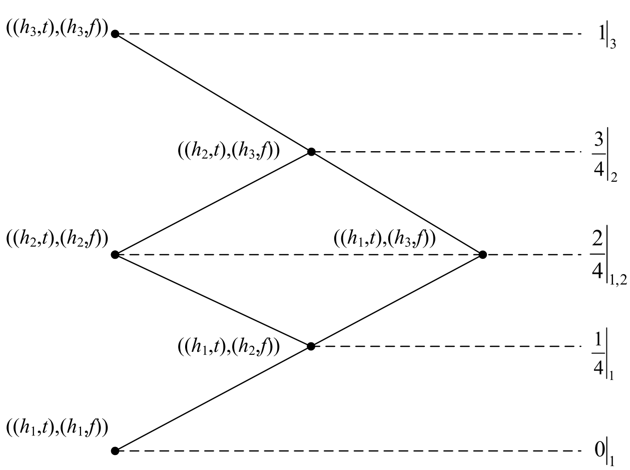

Fig. 4 Layered 6-element LTV-IFL

3.1 LAA Operator

The average aggregation algorithm is a very effective calculation method in the decision-making and evaluation process. In this part, we present a LAA operator and prove some properties of it as follows:

Definition 7 Suppose $\big((h_{i_k},c_1),(h_{j_k},c_2)\big)\in LI_6$, with $k=1,2,...,p$, then $\Upsilon_6:LI_6\to[0,1]$ is called a LAA operator based on the 6-element LTV-IFL, and

$$\begin{aligned}&\Upsilon_6\big(\big((h_{i_1},c_1),(h_{j_1},c_2)\big),\big((h_{i_2},c_1),(h_{j_2},c_2)\big),...,\big((h_{i_k},c_1),(h_{j_k},c_2)\big)\big)\\&=\frac{i_1+j_1+i_2+j_2+\cdots+i_k+j_k-2\times p}{4p}\Bigg|_{\frac{i_1+i_2+\cdots+i_k}{p}}\\&=\frac{\sum_{k=1}^{p}(i_k+j_k)-2p}{4p}\Bigg|_{\frac{\sum_{k=1}^{p}i_k}{p}}.\end{aligned} \tag{7}$$

Through the LAA operator we can aggregate intuitionistic fuzzy linguistic truth-valued information into intuitionistic fuzzy linguistic truth-valued layered aggregation number. In accordance with the practical meaning and the intuitionistic fuzzy layered theory, we map the intuitionistic fuzzy linguistic truth-valued layered aggregation number into 6-element LTV-IFL.

Definition 8 Suppose the layered number interval of the LAA operator is $[a,a+1]$, then a is defined as

$$a=Int\left(\frac{\sum_{k=1}^{p}(i_k+j_k)-2p}{p}\right). \tag{8}$$

Definition 9 Suppose $\lambda=a+\frac{1}{2}$ is the threshold valued, for any $\big((h_{i_k},c_1),(h_{j_k},c_2)\big)\in LI_6$, with $k=1,2,...,p$, its LAA number of 6-element LTV-IFL has two conditions as follows:

1. If $\frac{\sum_{k=1}^{p}(i_k+j_k)-2p}{4p}\geq a+\frac{1}{2}$, the layered number of LAA operator is $a+1$, and its LAA number of 6-element LTV-IFL is $\frac{a+1}{4}\Big|_s$, where $s=round\Big(\frac{\sum_{k=1}^{p}i_k}{p}\Big)$.
2. If $\frac{\sum_{k=1}^{p}(i_k+j_k)-2p}{4p}< a+\frac{1}{2}$, the layered number of LAA operator is a, and its LAA number of 6-element LTV-IFL is $\frac{a}{4}\Big|_s$, where $s=round\Big(\frac{\sum_{k=1}^{p}i_k}{p}\Big)$.

Theorem 1 *Suppose the LTV-IFPs* $\big((h_{i_k},c_1),(h_{j_k},c_2)\big)\in LI_6$,$k=1,2,...,p$, *then the properties of the LAA operator of the 6-element LTV-IFL are as follows*:

(1) *Idempotency*:

If there exists $\left((h_{i_2},c_1),(h_{j_2},c_2)\right)=\cdots=\left((h_{i_k},c_1),(h_{j_k},c_2)\right)=\left((h_{i_1},c_1),(h_{j_1},c_2)\right)$, *then* $\Upsilon_6\left(\left((h_{i_1},c_1),(h_{j_1},c_2)\right),\left((h_{i_2},c_1),(h_{j_2},c_2)\right),\dots,\left((h_{i_k},c_1),(h_{j_k},c_2)\right)\right)=\left((h_{i_1},c_1),(h_{j_1},c_2)\right)$.

(2) *Monotonicity*:

If $\left((h_{i_l},c_1),(h_{j_l},c_2)\right)\le\left((h_{i_k},c_1),(h_{j_k},c_2)\right)$, *then*

$$\Upsilon_6\left(\left((h_{i_1},c_1),(h_{j_1},c_2)\right),\left((h_{i_2},c_1),(h_{j_2},c_2)\right),\dots,\left((h_{i_l},c_1),(h_{j_l},c_2)\right)\right)$$
$$\le\Upsilon_6\left(\left((h_{i_1},c_1),(h_{j_1},c_2)\right),\left((h_{i_2},c_1),(h_{j_2},c_2)\right),\dots,\left((h_{i_k},c_1),(h_{j_k},c_2)\right)\right).$$

(3) *Boundedness*:

$$\begin{array}{l}\left((h_1,c_1),(h_1,c_2)\right)\le\Upsilon_6\left(\left((h_{i_1},c_1),(h_{j_1},c_2)\right),\dots,\right.\\ \left.\left((h_{i_k},c_1),(h_{j_k},c_2)\right)\right)\le\left((h_3,c_1),(h_3,c_2)\right)\end{array}.$$

(4) *Commutativity*:

If there exists $\left((h_{i_l},c_1),(h_{j_l},c_2)\right)$, *which is the any permutation of* $\left((h_{i_k},c_1),(h_{j_k},c_2)\right)$, *then*

$$\Upsilon_6\left(\left((h_{i_1},c_1),(h_{j_1},c_2)\right),\left((h_{i_2},c_1),(h_{j_2},c_2)\right),\dots,\left((h_{i_l},c_1),(h_{j_l},c_2)\right)\right)$$
$$=\Upsilon_6\left(\left((h_{i_1},c_1),(h_{j_1},c_2)\right),\left((h_{i_2},c_1),(h_{j_2},c_2)\right),\dots,\left((h_{i_k},c_1),(h_{j_k},c_2)\right)\right)$$

Proof. (1) When $\left((h_{i_2},c_1),(h_{j_2},c_2)\right)=\cdots=\left((h_{i_k},c_1),(h_{j_k},c_2)\right)=\left((h_{i_1},c_1),(h_{j_1},c_2)\right)$, it means $i_1=i_2=\ldots=i_k$, $j_1=j_2=\ldots=j_k$, so we get:

$$\Upsilon_6(A)=\left.\frac{i_1+j_1+i_2+j_2+\cdots+i_k+j_k-2\times p}{4p}\right|_{\frac{i_1+i_2+\cdots+i_k}{p}}$$
$$=\left.\frac{(i_1+j_1-2)\times p}{4p}\right|_{\frac{i_1\times p}{p}}=\left.\frac{(i_1+j_1-2)}{4}\right|_{i_1}=\left((h_{i_1},c_1),(h_{j_1},c_2)\right).$$

(2) As $\left((h_{i_l},c_1),(h_{j_l},c_2)\right)\le\left((h_{i_k},c_1),(h_{j_k},c_2)\right)$, which means $i_l\le i_k$, $j_l\le j_k$,

we can get $i_1+j_1+i_2+j_2+\cdots+i_l+j_l-2\times p\le i_1+j_1+i_2+j_2+\cdots+i_k+j_k-2\times p$,
$\left.\frac{i_1+j_1+i_2+j_2+\cdots+i_l+j_l-2\times p}{4p}\right|_{\frac{i_1+i_2+\cdots+i_l}{p}}\le\left.\frac{i_1+j_1+i_2+j_2+\cdots+i_k+j_k-2\times p}{4p}\right|_{\frac{i_1+i_2+\cdots+i_k}{p}}$, then $\Upsilon_6\left(\left((h_{i_1},c_1),(h_{j_1},c_2)\right),\left((h_{i_2},c_1),(h_{j_2},c_2)\right),\dots,\left((h_{i_l},c_1),(h_{j_l},c_2)\right)\right)\le\Upsilon_6\left(\left((h_{i_1},c_1),(h_{j_1},c_2)\right),\left((h_{i_2},c_1),(h_{j_2},c_2)\right),\dots,\left((h_{i_k},c_1),(h_{j_k},c_2)\right)\right)$.

(3) The minimum value of the 6-element LTV-IFL is $\left((h_1,c_1),(h_1,c_2)\right)$, and the maximum value is $\left((h_3,c_1),(h_3,c_2)\right)$. When $i_k=j_k=1$, $p=1$, the result of LAA operator is $\left.\frac{0}{4}\right|_1$, and when $i_k=j_k=3$, the formula of LAA operator is processed as $\left.\frac{6p-2p}{4p}\right|_3$. As $p\ge1$, the result is $\left.\frac{4}{4}\right|_3$. Using the IFLA inverse operator we can get:
$\left((h_1,c_1),(h_1,c_2)\right)\le\Upsilon_6\left(\left((h_{i_1},c_1),(h_{j_1},c_2)\right),\dots,\left((h_{i_k},c_1),(h_{j_k},c_2)\right)\right)\le\left((h_3,c_1),(h_3,c_2)\right)$.

(4) When $\left((h_{i_l},c_1),(h_{j_l},c_2)\right)$ is the any permutation of $\left((h_{i_k},c_1),(h_{j_k},c_2)\right)$, for any i_k and j_k, there exists $i_l=i_k$, $j_l=j_k$. Therefore, we can easily get:
$\left.\frac{i_1+j_1+i_2+j_2+\cdots+i_l+j_l-2\times p}{4p}\right|_{\frac{i_1+i_2+\cdots+i_l}{p}}=\left.\frac{i_1+j_1+i_2+j_2+\cdots+i_k+j_k-2\times p}{4p}\right|_{\frac{i_1+i_2+\cdots+i_k}{p}}$,
which means $\Upsilon_6(B)=\Upsilon_6(A)$.

Remark 1 We use the IFLA operator to translate the LAA number into the form of the LTV-IFPs.

Example 1 Suppose $\left((h_1,c_1),(h_2,c_2)\right),\left((h_2,c_1),(h_2,c_2)\right),\left((h_1,c_1),(h_3,c_2)\right),\left((h_3,c_1),(h_3,c_2)\right)\in LI_6$. With the Definition 6 we can get the layered number interval is [2, 3], and according to the Definition 7, $\frac{9}{16}<\frac{5}{2}$, so the layered number is 2, and the corresponding LAA number is $\left.\frac{2}{4}\right|_2$. Using the IFLA inverse operator, we can get the LTV-IFP is $\left((h_2,c_1),(h_2,c_2)\right)$.

3.2 LWAA Operator

While we are doing decision making or evaluating, we always set the weights which will make results more accurate. Let S be the linguistic truth-valued intuitionistic fuzzy set, G be the experts set and W be the weight vector, then we define the LWAA operator on 6-element LTV-IFL as follows:

Definition 10 For any $\left((h_{i_k},c_1),(h_{j_k},c_2)\right)\in LI_6$, with $k=1,2,\dots,p$, let weight vector be $W=\left(\omega_1,\omega_2,\dots,\omega_p\right)$, where $0\le\omega_k\le1$ and $\sum_{k=1}^{p}\omega_k=1$, then $\ddot{\Upsilon}_6:LI_6\to[0,1]$ is called a LWAA operator based on the 6-element LTV-IFL, and

$$\ddot{\Upsilon}_6\left(\left((h_{i_1},c_1),(h_{j_1},c_2)\right),\left((h_{i_2},c_1),(h_{j_2},c_2)\right),\dots,\left((h_{i_k},c_1),(h_{j_k},c_2)\right)\right)$$
$$=\left.\frac{(i_1+j_1-2)\omega_1+(i_2+j_2-2)\omega_2+\cdots+(i_k+j_k-2)\omega_k}{4}\right|_{i_1\omega_1+i_2\omega_2+\cdots+i_k\omega_k}. \tag{9}$$

Theorem 2 *Suppose* $\left(\left(h_{i_k}, c_1\right), \left(h_{j_k}, c_2\right)\right) \in LI_6, k = 1, 2, ..., p$, *then the properties of the LWAA operator of the* 6*-element LTV-IFL are as follows*:

(1) *Idempotency*:

If there exists $\left(\left(h_{i_2}, c_1\right), \left(h_{j_2}, c_2\right)\right) = \cdots = \left(\left(h_{i_k}, c_1\right),\right.$ $\left(\left(h_{i_2}, c_1\right), \left(h_{j_2}, c_2\right)\right) = \cdots = \left(\left(h_{i_k}, c_1\right),\right.$, *then* $\ddot{\Upsilon}_6\left(\left(\left(h_{i_1}, c_1\right), \left(h_{j_1}, c_2\right)\right), \left(\left(h_{i_2}, c_1\right), \left(h_{j_2}, c_2\right)\right), ..., \left(\left(h_{i_k}, c_1\right), \left(h_{j_k}, c_2\right)\right)\right)$ $= \left(\left(h_{i_1}, c_1\right), \left(h_{j_1}, c_2\right)\right)$.

(2) *Monotonicity*:

If $\left(\left(h_{i_l}, c_1\right), \left(h_{j_l}, c_2\right)\right) \le \left(\left(h_{i_k}, c_1\right), \left(h_{j_k}, c_2\right)\right)$, *then*

$$\ddot{\Upsilon}_6\left(\left(\left(h_{i_1}, c_1\right), \left(h_{j_1}, c_2\right)\right), \left(\left(h_{i_2}, c_1\right), \left(h_{j_2}, c_2\right)\right), ..., \left(\left(h_{i_l}, c_1\right), \left(h_{j_l}, c_2\right)\right)\right) \le \ddot{\Upsilon}_6\left(\left(\left(h_{i_1}, c_1\right), \left(h_{j_1}, c_2\right)\right), \left(\left(h_{i_2}, c_1\right), \left(h_{j_2}, c_2\right)\right), ..., \left(\left(h_{i_k}, c_1\right), \left(h_{j_k}, c_2\right)\right)\right).$$

(3) *Boundedness*:

$$\left(\left(h_1, c_1\right), \left(h_1, c_2\right)\right) \le \ddot{\Upsilon}_6\left(\left(\left(h_{i_1}, c_1\right), \left(h_{j_1}, c_2\right)\right), ..., \left(\left(h_{i_k}, c_1\right), \left(h_{j_k}, c_2\right)\right)\right) \le \left(\left(h_3, c_1\right), \left(h_3, c_2\right)\right).$$

Proof. (1) As $\left(\left(h_{i_2}, c_1\right), \left(h_{j_2}, c_2\right)\right) = \cdots = \left(\left(h_{i_k}, c_1\right), \left(h_{j_k}, c_2\right)\right) = \left(\left(h_{i_1}, c_1\right), \left(h_{j_1}, c_2\right)\right), \omega_1 + \omega_2 + \cdots + \omega_k = 1$, which means $i_1 = i_2 = \ldots = i_k$, $j_1 = j_2 = \ldots = j_k$,

then we can get

$$\ddot{\Upsilon}_6(A) = \frac{\left(i_1 + j_1 - 2\right)\omega_1 + \left(i_2 + j_2 - 2\right)\omega_2 + \cdots + \left(i_k + j_k - 2\right)\omega_k}{4}\Bigg|_{i_1\omega_1 + i_2\omega_2 + \cdots + i_k\omega_k}$$

$$= \frac{\left(i_1 + j_1 - 2\right) \times \left(\omega_1 + \omega_2 + \cdots + \omega_k\right)}{4}\Bigg|_{i_1 \times \left(\omega_1 + \omega_2 + \cdots + \omega_k\right)}$$

$$= \frac{i_1 + j_1 - 2}{4}\Bigg|_{i_1} = \left(\left(h_{i_1}, c_1\right), \left(h_{j_1}, c_2\right)\right).$$

(2) As $\left(\left(h_{i_l}, c_1\right), \left(h_{j_l}, c_2\right)\right) \le \left(\left(h_{i_k}, c_1\right), \left(h_{j_k}, c_2\right)\right)$, we can get $i_l \le i_k$, $j_l \le j_k$, $i_l + j_l \le i_k + j_k$. Therefore

$$\frac{\left(i_1 + j_1 - 2\right)\omega_1 + \left(i_2 + j_2 - 2\right)\omega_2 + \cdots + \left(i_l + j_l - 2\right)\omega_k}{4}\Bigg|_{i_1\omega_1 + i_2\omega_2 + \cdots + i_l\omega_k} \le \frac{\left(i_1 + j_1 - 2\right)\omega_1 + \left(i_2 + j_2 - 2\right)\omega_2 + \cdots + \left(i_k + j_k - 2\right)\omega_k}{4}\Bigg|_{i_1\omega_1 + i_2\omega_2 + \cdots + i_k\omega_k},$$

$$\ddot{\Upsilon}_6\left(\left(\left(h_{i_1}, c_1\right), \left(h_{j_1}, c_2\right)\right), \left(\left(h_{i_2}, c_1\right), \left(h_{j_2}, c_2\right)\right), ..., \left(\left(h_{i_l}, c_1\right), \left(h_{j_l}, c_2\right)\right)\right) \le \ddot{\Upsilon}_6\left(\left(\left(h_{i_1}, c_1\right), \left(h_{j_1}, c_2\right)\right), \left(\left(h_{i_2}, c_1\right), \left(h_{j_2}, c_2\right)\right), ..., \left(\left(h_{i_k}, c_1\right), \left(h_{j_k}, c_2\right)\right)\right).$$

(3) $\left(\left(h_1, c_1\right), \left(h_1, c_2\right)\right) \le \ddot{\Upsilon}_6(A) \le \left(\left(h_3, c_1\right), \left(h_3, c_2\right)\right)$ means

$$\frac{0}{4}\Big|_1 \le \frac{\left(i_1+j_1-2\right)\omega_1 + \left(i_2+j_2-2\right)\omega_2 + \cdots + \left(i_k+j_k-2\right)\omega_k}{4}\Bigg|_{i_1\omega_1 + i_2\omega_2 + \cdots + i_k\omega_k} \le \frac{4}{4}\Big|_3,$$

so we need to proof $0 \le \left(i_1 + j_1 - 2\right)\omega_1 + \left(i_2 + j_2 - 2\right)\omega_2 + \cdots + \left(i_k + j_k - 2\right)\omega_k \le 4, 1 \le i_1\omega_1 + i_2\omega_2 + \cdots + i_k\omega_k \le 3$.

$\left(i_1 + j_1 - 2\right)\omega_1 + \left(i_2 + j_2 - 2\right)\omega_2 + \cdots + \left(i_k + j_k - 2\right)\omega_k = \left(i_1 + j_1\right)\omega_1 + \left(i_2 + j_2\right)\omega_2 + \cdots + \left(i_k + j_k\right)\omega_k - 2\left(\omega_1 + \omega_2 + \cdots + \omega_k\right)$,

as $\omega_1 + \omega_2 + \cdots + \omega_k = 1, 1 \le i_1 \le 3$, and $1 \le j_1 \le 3$,

then we can get $2 \le i_1 + j_1 \le 6, 2\omega_1 \le \left(i_1 + j_1\right) \le 6\omega_1, ..., 2\omega_k \le \left(i_k + j_k\right) \le 6\omega_k$,

$2\left(\omega_1 + \omega_2 + \cdots + \omega_k\right) \le \left(i_1 + j_1\right) + \left(i_2 + j_2\right) ... + \left(i_k + j_k\right) \le 6\left(\omega_1 + \omega_2 + \cdots + \omega_k\right)$,

$0 \le \left(i_1 + j_1 - 2\right)\omega_1 + \left(i_2 + j_2 - 2\right)\omega_2 + \cdots + \left(i_k + j_k - 2\right)\omega_k \le 4$.

In a similar way, $1 \le i_1\omega_1 + i_2\omega_2 + \cdots + i_k\omega_k \le 3$, then we get $\left(\left(h_1, c_1\right), \left(h_1, c_2\right)\right) \le \ddot{\Upsilon}_6(A) \le \left(\left(h_3, c_1\right), \left(h_3, c_2\right)\right)$.

Definition 11 For any $\left(\left(h_{i_k}, c_1\right), \left(h_{j_k}, c_2\right)\right) \in LI_6$, with $k = 1, 2, ..., p$, let weight vector be $W = \left(\omega_1, \omega_2, ..., \omega_p\right)$, where $0 \le \omega_k \le 1$ and $\sum_{k=1}^{p} \omega_k = 1$, then its LWAA number of 6-element LTV-IFL is:

$$\frac{round\left(\left(i_1 + j_1 - 2\right)\omega_1 + \cdots + \left(i_k + j_k - 2\right)\omega_k\right)}{4}\Bigg|_{round\left(i_1\omega_1 + \cdots + i_k\omega_k\right)}. \tag{10}$$

Remark 2 We use the IFLA operator to translate the LWAA number into the form of the LTV-IFP.

Example 2 Suppose $\left(\left(h_2, c_1\right), \left(h_2, c_2\right)\right), \left(\left(h_2, c_1\right), \left(h_3, c_2\right)\right), \left(\left(h_1, c_1\right), \left(h_2, c_2\right)\right), \left(\left(h_1, c_1\right), \left(h_3, c_2\right)\right) \in LI_6$, and corresponding weights are $\omega_1 = 0.4, \omega_2 = 0.3, \omega_3 = 0.2, \omega_4 = 0.1$. From the Definition 9 we can get the LWAA number is $\frac{2}{4}\Big|_2$, and using the IFLA inverse operator, the LTV-IFP is $((h_2, c_1), (h_2, c_2))$.

3.3 Multiple Multidimensional Reasoning Model

During the process of designing the reasoning model, first, we confirm the meta linguistic and fuzzy information. Then we use the LWAA operator and the IFLA inverse operator to deal with the linguistic-valued fuzzy information to obtain the estimated evaluation from different experts to different objects with different attributes. Second, we establish the LTV-IFLA multiple multidimensional reasoning model. Finally, we get intuitionistic fuzzy reasoning results in the form of linguistic-valued information.

Definition 12 Suppose $a_k = ((h_{i_k}, c_1), (h_{j_k}, c_2)), k = 1, 2, ..., p$ is a LTV-IFP, "$\otimes_\vee$" is the composite relation of linguistic truth-valued intuitionistic fuzzy information, then

$$a_1 \otimes_\vee a_2 \otimes_\vee ... \otimes_\vee a_k = \left[((h_{i_1}, c_1), (h_{j_1}, c_2))\ ((h_{i_2}, c_1), (h_{j_2}, c_2))\ \cdots\ ((h_{i_k}, c_1), (h_{j_k}, c_2))\right]^T. \quad (11)$$

We establish the 6-element linguistic truth-valued intuitionistic fuzzy layered multiple multidimensional reasoning model as follows. Each rule in n rules contains m attributes. The intuitionistic fuzzy linguistic truth-valued information comes from 6-element LTV-IFL, which means A, B in this model are the linguistic truth-valued intuitionistic fuzzy information:

If x_1 is $A_{11} \otimes_\vee A_{12} \otimes_\vee ... \otimes_\vee A_{1m}$, then y is B_1 (Rules)

If x_2 is $A_{21} \otimes_\vee A_{22} \otimes_\vee ... \otimes_\vee A_{2m}$, then y is B_2

$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

If x_n is $A_{n1} \otimes_\vee A_{n2} \otimes_\vee ... \otimes_\vee A_{nm}$, then y is B_n

If x^* is $A^*_1 \otimes_\vee A^*_2 \otimes_\vee ... \otimes_\vee A^*_m$ (Facts)

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then y is B^* (Conclusion)

We take the intuitionistic fuzzy relation between A and B in intuitionistic fuzzy reasoning model as the composite opertion of the reasoning conclusions, which is denoted by IR.

Remark 3 Intuitionistic fuzzy relation IR can be any intuitionistic fuzzy relation which has existed or can be defined by ourselves.

For p experts evaluate n objects with m attributes, after confirming the meta linguistic value set, we use the LWAA operator to aggregate the linguistic truth-valued evaluation information and restore results to LTV-IFPs using the IFLA inverse operator. Then we work out each fuzzy relation $IR_1, IR_2, ..., IR_n$ in each rule of aggregated rules. Finally we use A^* to do the compound operation "$\circ_l$" with $IR_1, IR_2, ..., IR_n$ to obtain n reasoning results $B^*_1, B^*_2, ..., B^*_n$ and get the disjunction or the conjunction B^*. Above method can be detailed as following steps:

Step 1: Confirm meta linguistic value set $C = \{c_1, c_2\}$ and collect linguistic truth-valued intuitionistic fuzzy information of p experts to n objects from m attributes;

Step 2: Use the LWAA operator to calculate the linguistic truth-valued information, according to the layered number interval $[a, a+1]$ and the threshold value $\lambda = a + \frac{1}{2}$ to obtain the LWAA operator number;

Step 3: Use the IFLA inverse operator to restore numbers into the form of LTV-IFPs;

Step 4: According to the aggregated information to establish the model of the 6-element LTV-IFLA multiple multidimensional reasoning model;

Step 5: Work out each fuzzy relation $IR_1, IR_2, ..., IR_n$ in each rule of aggregated rules, $IR_i = A_i \rightarrow B_i, 1 \le i \le n$;

Step 6: Use A^* to do the compound operation "$\circ_l$" with $IR_1, IR_2, ..., IR_n$ to obtain n reasoning results $B^*_1, B^*_2, ..., B^*_n$, where $B^*_i = A^* \circ_l IR_i, 1 \le i \le n$;

Step 7: Do the disjunctive operation or the conjunctive operation of $B^*_1, B^*_2, ..., B^*_n$;

Step 8: Obtain the final result B^* and H.

We depict the main reasoning processing phases of our proposed 6-element LTV-IFLA multiple multidimensional reasoning model in Fig. 5.

3.4 Case Study

While fully affirming the role of multimedia teaching in realizing the optimization of the teaching process, it is obviously necessary to objectively consider the negative factors in the application of multimedia teaching and scientifically evaluate the quality and efficiency of multimedia teaching. It can better apply multimedia technology to the classroom teaching process, so that it can fully tap the advantages of multimedia teaching. We split the evaluation standards of multimedia teaching into multimedia courseware design level, multimedia audiovisual effects, and combination degree of course and courseware. We use the LWAA operator and combine the 6-element LTV-IFLA multiple multidimensional reasoning model to do the reasoning evaluation

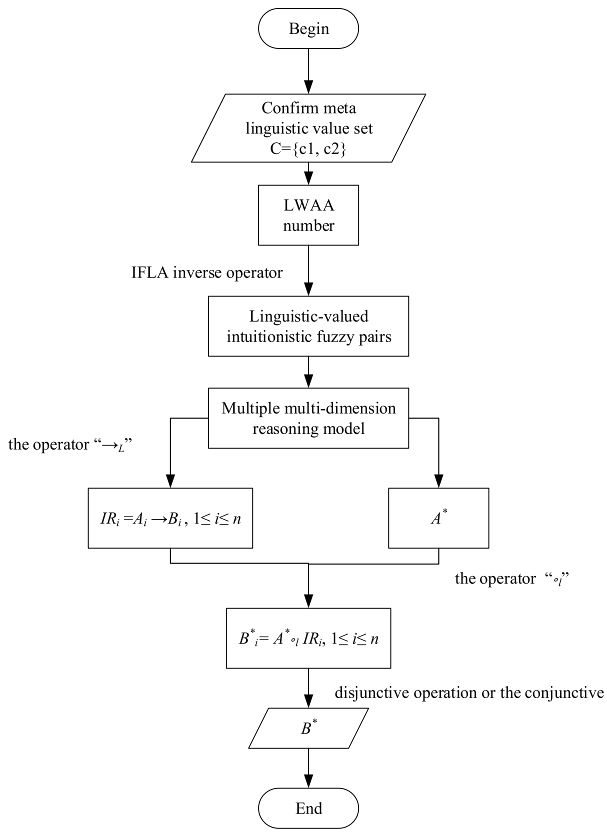

Fig. 5 Flow chart of the reasoning algorithm

of students' recognition of different multimedia teaching courses.

Here is a teaching method M_3 waiting to be reasoned about the degree of teaching effect, where meta linguistic value set $C = \{H(high), L(low)\}$. The experts have finished the linguistic truth-valued evaluation of three kinds of evaluation information (Class1 evaluation:$\omega_1 = 0.3$, Class2 evaluation:$\omega_2 = 0.4$ and Class3 evaluation:$\omega_3 = 0.3$) of M_3 and other two teaching methods M_1 and M_2 on three attributes. The information is shown in Table 1.

Using LWAA operator to calculate the linguistic truth-valued evaluation information in Table 1 can obtain the aggregation of evaluation information of teaching methods, which is shown in Table 2.

Restore these LWAA numbers to the form of LTV-IFPs using the IFLA operator which is shown in Table 3.

According to the reasoning model, we can get the evaluation information and final evaluation results obtained by the two teaching methods in three aspects: multimedia courseware design level, multimedia audiovisual effects, and combination degree of course and courseware. Aiming at the evaluation information of the existing teaching method M_3, the evaluation results can be obtained by applying this model. The reasoning calculation process is as follows:

Table 1 Collection of linguistic truth-valued evaluation information

Teaching methods	Index and results	Class1	Class2	Class3
$M1$	Multimedia courseware design level	$((h1, H), (h3, L))$	$((h2, H), (h3, L))$	$((h1, H), (h2, L))$
	Multimedia audiovisual effects	$((h3, H), (h3, L))$	$((h2, H), (h3, L))$	$((h3, H), (h3, L))$
	Combination degree of course and courseware	$((h2, H), (h3, L))$	$((h3, H), (h3, L))$	$((h2, H), (h2, L))$
	Result	$((h2, H), (h3, L))$		
$M2$	Multimedia courseware design level	$((h1, H), (h1, L))$	$((h1, H), (h2, L))$	$((h2, H), (h2, L))$
	Multimedia audiovisual effects	$((h1, H), (h2, L))$	$((h1, H), (h3, L))$	$((h2, H), (h3, L))$
	Combination degree of course and courseware	$((h1, H), (h2, L))$	$((h2, H), (h2, L))$	$((h2, H), (h3, L))$
	Result	$((h1, H), (h2, L))$		
$M3$	Multimedia courseware design level	$((h2, H), (h3, L))$	$((h3, H), (h3, L))$	$((h2, H), (h2, L))$
	Multimedia audiovisual effects	$((h1, H), (h3, L))$	$((h2, H), (h2, L))$	$((h1, H), (h2, L))$
	Combination degree of course and courseware	$((h2, H), (h3, L))$	$((h2, H), (h3, L))$	$((h2, H), (h2, L))$

Table 2 Aggregated evaluation information

Teaching methods	Multimedia courseware design level	Multimedia audiovisual effects	Combination degree of course and courseware	Results
$M1$	$\frac{2}{4}\Big\|_1$	$\frac{4}{4}\Big\|_3$	$\frac{3}{4}\Big\|_2$	$((h2, H), (h3, L))$
$M2$	$\frac{1}{4}\Big\|_1$	$\frac{2}{4}\Big\|_1$	$\frac{2}{4}\Big\|_2$	$((h1, H), (h2, L))$
$M3$	$\frac{3}{4}\Big\|_2$	$\frac{2}{4}\Big\|_1$	$\frac{3}{4}\Big\|_2$	

Table 3 Aggregated evaluation information in form of the LTV-IFPs

Teaching methods	Multimedia courseware design level	Multimedia audiovisual effects	Combination degree of course and courseware	Results
$M1$	$((h1, H), (h3, L))$	$((h3, H), (h3, L))$	$((h2, H), (h3, L))$	$((h2, H), (h3, L))$
$M2$	$((h1, H), (h2, L))$	$((h1, H), (h3, L))$	$((h2, H), (h2, L))$	$((h1, H), (h2, L))$
$M3$	$((h2, H), (h3, L))$	$((h1, H), (h3, L))$	$((h2, H), (h3, L))$	

$$IR_1 = \left(A_{11} \otimes_\vee A_{12} \otimes_\vee A_{13}\right) \to B_1 = \begin{bmatrix} \left(\left(h_1,H\right),\left(h_3,L\right)\right) \\ \left(\left(h_3,H\right),\left(h_3,L\right)\right) \\ \left(\left(h_2,H\right),\left(h_3,L\right)\right) \end{bmatrix} \to \left[\left(\left(h_2,H\right),\left(h_3,L\right)\right)\right] = \begin{bmatrix} \left(\left(h_3,H\right),\left(h_3,L\right)\right) \\ \left(\left(h_2,H\right),\left(h_3,L\right)\right) \\ \left(\left(h_3,H\right),\left(h_3,L\right)\right) \end{bmatrix},$$

$$IR_2 = \left(A_{21} \otimes_\vee A_{22} \otimes_\vee A_{23}\right) \to B_2 = \begin{bmatrix} \left(\left(h_1,H\right),\left(h_2,L\right)\right) \\ \left(\left(h_1,H\right),\left(h_3,L\right)\right) \\ \left(\left(h_2,H\right),\left(h_2,L\right)\right) \end{bmatrix} \to \left[\left(\left(h_1,H\right),\left(h_2,L\right)\right)\right] = \begin{bmatrix} \left(\left(h_3,H\right),\left(h_3,L\right)\right) \\ \left(\left(h_2,H\right),\left(h_3,L\right)\right) \\ \left(\left(h_2,H\right),\left(h_3,L\right)\right) \end{bmatrix},$$

$$B_1^* = A^* \circ_l IR_1 = (A^*)^T \circ_l IR_1$$

$$= \left[\ \left(\left(h_1,H\right),\left(h_3,L\right)\right)\ \left(\left(h_3,H\right),\left(h_3,L\right)\right)\ \ \left(\left(h_2,H\right),\left(h_3,L\right)\right)\ \right] \circ_l \begin{bmatrix} \left(\left(h_3,H\right),\left(h_3,L\right)\right) \\ \left(\left(h_2,H\right),\left(h_3,L\right)\right) \\ \left(\left(h_3,H\right),\left(h_3,L\right)\right) \end{bmatrix},$$

$$= \left[\left(\left(h_2,H\right),\left(h_3,L\right)\right)\right]$$

$$B_2^* = A^* \circ_l IR_2 = (A^*)^T \circ_l IR_2$$

$$= \left[\ \left(\left(h_1,H\right),\left(h_2,L\right)\right)\ \left(\left(h_1,H\right),\left(h_3,L\right)\right)\ \ \left(\left(h_2,H\right),\left(h_2,L\right)\right)\ \right] \circ_l \begin{bmatrix} \left(\left(h_3,H\right),\left(h_3,L\right)\right) \\ \left(\left(h_2,H\right),\left(h_3,L\right)\right) \\ \left(\left(h_2,H\right),\left(h_3,L\right)\right) \end{bmatrix},$$

$$= \left[\left(\left(h_2,H\right),\left(h_3,L\right)\right)\right]$$

$$B^* = B_1^* \vee B_2^* = \left(\left(h_2,H\right),\left(h_3,L\right)\right).$$

The result of M_3 is $((h_2, H), (h_3, L))$, which means that the teaching effect of the teaching method M_3 comprehensively is good. According to Table 3, we can see that M_3 is a little weaker at multimedia audiovisual effects compared with the other two aspects, which can be improved to get better teaching effect.

As the complexity of information increases, this reasoning model can well handle the aggregation of information and reasoning of multiple multidimensional teaching evaluation under the environment described by language. Compared with the evaluation of education under the intuitionistic fuzzy environment proposed in the literature [11], this paper better handles the linguistic-valued information that cannot be represented by the numerical value, which reduces the loss of information. It more intuitively and vividly shows the reasoning effect of multiple multidimensional education evaluation information.

4 Conclusions

With the increasing popularity of online education, it is very important to grasp the situation of students and their evaluation of teaching in time to improve the teaching quality. The multiple multidimensional reasoning model of multimedia teaching evaluation proposed in this paper can better combine multiple classes of students to evaluate from different perspective of both positive and negative aspects at the same time, so as to make the reasoning results more comprehensive. According to the layered operators of the LTV-IFL, the LWAA operator which we proposed is used to aggregate and transform the intuitionistic fuzzy linguistic truth-valued information. According to the established layered reasoning model, the algorithm we provided can obtain the result which conform to human thinking habits and better deal with complex information under the uncertainty environment.

In this paper, we assign the weights by ourselves, in an artificial way, which makes it not accurate. It could be better if the weights are calculated in a natural way. As we calculate the LTV-IFPs with the subscripts, there will still be information loss. Therefore, assigning weights more reasonable and establishing a system to evaluate with linguistic truth-valued more accurate is that our next research will focus on.

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Authors' Contributions YL drafted the manuscript, studied layered linguistic truth-valued intuitionistic fuzzy lattice and realized its application. NL proved the theorems and properties in the manuscript. HL consulted relevant literatures and research status at home and abroad. HZ designed the experiment and collected relevant parameters. XL helped to reviewed the manuscript. LZ conceived of the study, and helped to draft the manuscript. All authors read and approved the final manuscript.

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Declarations

Conflict of interest Authors have no conflict of interest to declare.

Ethics Approval and Consent to Participate Not applicable.

Consent for Publication I would like to declare on behalf of my co-authors that the work described was original research that has not been published previously, and not under consideration for publication elsewhere, in whole or in part. All the authors listed have approved the manuscript that is enclosed.

Availability of Data and Materials Data sharing is not applicable to this article as no data sets were generated or analyzed during the current study.

References

1. Spector, J.M., Merrill, M.D., Elen, J., Bishop, M.J.: Handbook of Research on Educational Communications and Technology. Springer, Berlin (2013)
2. Molina, A.I., Navarro, O., Ortega, M., Lacruz, M.: Evaluating multimedia learning materials in primary education using eye tracking. Comput. Standards Interfaces. 59(AUG), 45–60 (2018)
3. Mutlu-Bayraktar, D., Cosgun, V., Altan, T.: Cognitive load in multimedia learning environments: a systematic review[J]. Comput. Educ. **141**, 103618 (2019)
4. Heo, M., Toomey, N.: Learning with multimedia: the effects of gender, type of multimedia learning resources, and spatial ability[J]. Comput. Educ. **146**, 103747 (2020)
5. Bhargava, A., Bansal, A.: Novel coronavirus (COVID-19) diagnosis using computer vision and artificial intelligence techniques: a review. Multimed. Tools Appl. 1–16 (2021)
6. Le, Y., Liu, J., Deng, C., Dai, D.: Heart rate variability reflects the effects of emotional design principle on mental effort in multimedia learning. Comput. Hum. Behav. **89**, 40–47 (2018)
7. Li, N., Chen, X., Subramani, S., Kadry, S. N.: Improved fuzzy “ssisted multimedia” ssistive technology for engineering education. Comput. Appl. Eng. Educ. (8) (2020)
8. Wu, Y., Su, C., Wei, H.: A study on the application of computer multimedia in Chinese teaching of higher vocational colleges. J. Phys: Conf. Ser. **1533**(2), 022059 (2020)
9. Li, W., Su, L.: Analysis of Effective Application of Multimedia Technology in College Physics Teaching[C].
10. Xie, Z.: Symmetry for multimedia-aided art teaching based on the form of animation teaching organization and social network. Symmetry. **12**(4), 671 (2020)
11. Liu, S.: Research on the teaching quality evaluation of physical education with intuitionistic fuzzy TOPSIS method. J. Intell. Fuzzy Syst. **40**(5), 1–10 (2021)
12. Nie, Y.: On-line classroom visual tracking and quality evaluation by an advanced feature mining technique - ScienceDirect. Signal Process Image Commun. **84**, 115817 (2020)
13. Liu, P., Cui, H., Cao, Y., Hou, X., Zou, L.: A method of multimedia teaching evaluation based on fuzzy linguistic concept lattice. Multimed. Tools Appl. **78**(21), 30975–31001 (2019)
14. Zadeh, L.A.: Fuzzy sets. J. Inf. Control. **8**(3), 338–353 (1965)
15. Atanassov, K.: Lntuionistic fuzzy sets. Fuzzy Sets Syst. **20**(1), 87–96 (1986)
16. Atanassov, K.: More on intuionistic fuzzy sets. Fuzzy Sets Syst. **33**(1), 37–45 (1989)
17. Chen, S.M., Chang, C.H.: A novel similarity measure between Atanassov’s intuitionistic fuzzy sets based on transformation techniques with applications to pattern recognition. Inf. Sci. **291**, 96–114 (2015)
18. Zou, X.Y., Chen, S.M., Fan, K.Y.: Multiple attribute decision making using improved intuitionistic fuzzy weighted geometric operators of intuitionistic fuzzy values. Inf. Sci. **535**, 242–253 (2020)
19. Kumar, K., Chen, S.M.: Multiattribute decision making based on the improved intuitionistic fuzzy Einstein weighted averaging operator of intuitionistic fuzzy values. Inf. Sci. **568**, 369–383 (2021)
20. Xu, Y., Ruan, D., Qin, K. Y., Liu, J.: Lattice-valued logic. Stud. Fuzzy Soft Comput. 132 (2003)
21. Zou, L., Liu, X., Pei, Z.: Implication operator on the set of ∨-irreducible element in linguistic truth-valued intuitionistic fuzzy lattice. Int. J. Mach. Learn. Cybern. **4**, 365–372 (2013)
22. Rodríguez, R.M., Martínez, L., Herrera, F.: Hesitant fuzzy linguistic term sets for decision making. IEEE Trans. Fuzzy Syst. **20**(1), 109–119 (2011)
23. Beg, I., Rashid, T.: TOPSIS for hesitant fuzzy linguistic term sets. Int. J. Intell. Syst. **28**(12), 1162–1171 (2013)
24. Hao, J., Chiclana, F.: Attitude quantifier based possibility distribution generation method for hesitant fuzzy linguistic group decision making. Inf. Sci. 341–360 (2020)

25. Li, P., Wei, C.: A case-based reasoning decision-making model for hesitant fuzzy linguistic information. Int. J. Fuzzy Syst. **20**(7), 2175–2186 (2018)
26. Krishankumar, R., Ravichandran, K.S., Shyam, V., Sneha, S.V., Kar, S., Garg, H.: Multi-attribute group decision-making using double hierarchy hesitant fuzzy linguistic preference information. Neural Comput. Appl. **32**(17), 14031–14045 (2020)
27. Lin, M., Chen, Z., Chen, R., Fujita, H.: Evaluation of startup companies using multicriteria decision making based on hesitant fuzzy linguistic information envelopment analysis models. Int. J. Intell. Syst. **36**(5), 2292–2322 (2021)
28. Wang, Y. N., Lei, Y., Fan, X., Wang, Y.: Intuitionistic fuzzy time series forecasting model based on intuitionistic fuzzy reasoning. Math. Probl. Eng. 1–12 (2016)
29. Yue, W., Liu, X., Li, S., Gui, W., Xie, Y.: Knowledge representation and reasoning with industrial application using interval-valued intuitionistic fuzzy Petri nets and extended TOPSIS. Int. J. Mach. Learn. Cybern. **12**(4), 987–1013 (2021)
30. Xu, X.G., Xiong, Y., Xu, D.H., Liu, H.C.: Bipolar fuzzy Petri nets for knowledge representation and acquisition considering non-cooperative behaviors. Int. J. Mach. Learn. Cybern. **11**(10), 2297–2311 (2020)
31. Zou, L., Wen, X., Wang, Y.: Linguistic truth-valued intuitionistic fuzzy reasoning with applications in human factors engineering. Information Sciences. 201–216 (2016)
32. Zou, L., Shi, P., Pei, Z., Xu, Y.: On an algebra of linguistic truth-valued intuitionistic lattice-valued logic. J. Intell. Fuzzy Syst. **24**(3), 447–456 (2013)