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RESEARCH ARTICLE

Exponential Synchronization of Inertial Complex-Valued Fuzzy Cellular Neural Networks with Time-Varying Delays via Periodically Intermittent Control

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Abstract
This paper mainly studies the exponential synchronization issue for the inertial complex-valued fuzzy cellular neural networks (ICVFCNNs) with time-varying delays via periodically intermittent control. To achieve exponential synchronization, we use a non-reduced order and non-separation approach, which is a supplement and innovation to the previous method. Based on directly constructing Lyapunov functional and a novel periodically intermittent control scheme, sufficient conditions for achieving the exponential synchronization of the ICVFCNNs are established. Finally, an example is given to illustrate the validity of the obtained results.

Keywords Inertial · Complex value · Fuzzy · Exponential synchronization · Periodically intermittent control

Abbreviations

ICVFCNNs	Inertial complex-valued fuzzy cellular neural networks
FCNNs	Fuzzy cellular neural networks
CVNNs	Complex-valued neural networks
RVNNs	Real-valued neural networks

1 Introduction

Since the fuzzy differential equations have played an active role in modeling various uncertain phenomena arising in applied sciences, they have attracted more attention [1–5]. As the cellular neural network implementation, fuzzy cellular neural networks (FCNNs) were initially proposed by Yang et al. in 1996. Since fuzzy neural networks are a combination of fuzzy logic and neural networks, they have the following advantages: more efficient storage of knowledge and processing of uncertain information, faster operation, better convergence, and stability. Therefore, they are widely used in mathematics, pattern recognition, computer science, artificial intelligence, optimal control, and so on [6–9]. Meanwhile, with the study of neural network theory and dynamical behavior, scholars have proposed a new neural network, namely an inertial neural network, which is generally described by second-order differential equations. The inertial neural network is represented its inertial properties by introducing inductance in the neural current [10, 11]. It is shown that the introduction of inertial terms in neural networks can not only substantially improve the disorderly search performance of neural networks but also is one of the essential methods to let the designed neural networks generate chaos and bifurcation. Therefore, it has important practical significance and theoretical value for the study of the dynamic behavior of inertial fuzzy cellular neural networks [12–14].

In recent years, the synchronization problem of neural networks has been widely studied since synchronization plays a vital role in many practical applications. There are many methods to study the synchronization problem, among which exponential synchronization has the advantages of fast synchronization rate and simple implementation, thus gaining the favor of researchers. Currently, many control techniques are introduced to synchronize neural networks, such as impulsive control, adaptive control, pinning control, and intermittent control. In particular, intermittent control is a discontinuous control strategy that can effectively reduce control costs because it is active only during the control

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interval. Therefore, the synchronization problem of neural networks with intermittent control is of great importance in practical applications and has been widely studied with fruitful results [15–19]. For example, the finite-time synchronization in [15] for delayed quaternion-valued neural networks was introduced via periodically intermittent control. The problem of quasi-synchronization in [16] for fractional-order heterogeneous dynamical networks was proposed via aperiodic intermittent pinning control.

At present, when studying inertial neural networks, the reduced-order approach was widely used, i.e., the second-order model was transformed into a first-order model by appropriate variable substitution [19–26]. For example, the finite-time and fixed-time synchronization in [22] for a class of inertial neural networks with multi-proportional delays. New criteria on periodicity and stabilization [24] for discontinuous uncertain inertial Cohen-Grossberg neural networks with proportional delays. The synchronization in [25] for coupled memristive inertial delayed neural networks with impulse and intermittent control. The problem of exponential synchronization in [26] for inertial neural networks with mixed time-varying delays via periodically intermittent control. The disadvantage of this method is that as the order decreases, the inertia term disappears, and its importance is not reflected in the reduced-order model. At the same time, it increases the system's dimensionality, which causes the complexity of the theoretical analysis. However, in existing literature, very few papers concentrate on the synchronization of inertial neural networks applying the non-reduced order method [27–29], which inspired us to study the inertial neural networks based on the new idea.

Meanwhile, complex-valued neural networks (CVNNs) are widely used because they have more advantages than real-valued neural networks (RVNNs) in computational power and processing speed. Generally, a frequently used approach is to split the CVNNs into two RVNNs and then discuss them separately, see [30–37]. Obviously, this approach increases the dimensionality of the model and the computational difficulty. To overcome the above shortcomings, the non-separation method was proposed in [38, 39] based on the theory of complex functions and the construction of suitable Lyapunov functions. To the best of our knowledge, exponential synchronization of the delayed ICVFCNNs under periodically intermittent control is not yet completely studied, which motivated our research.

Inspired by the previous works, the main objective of this paper is to eatablish some novel exponential synchronization criteria for the delayed ICVFCNNs. The main innovative contents are listed as follows:

(1) The ICVFCNNs introduced in this paper takes into account factors such as the inertia term, time-varying delays, fuzzy logic, and periodically intermittent control, this makes the model considered more versatile and practical applications.
(2) Based on the theory of complex functions and analysis techniques, this paper investigates exponential synchronization of the ICVFCNNs under periodically intermittent control using a non-reduced order and non-separation approach, which is more direct and more uncomplicated.
(3) The results of this paper are entirely new and supplement to the known results, and our method can be used to investigate the case of exponential synchronization in other types of neural networks with time delays.

The framework of this paper is organized as follows. In Sect. 2, our problems are formulated. The exponential synchronization is established in Sect. 3. Some illustrative numerical simulations are presented in Sect. 4. Section 5 draws a conclusion.

Notations : Let $\Theta = \{1, 2, \cdots, n\}$, $\mathbb{R}$ and $\mathbb{C}^n$ denote the set of real numbers and the set of n-dimensional complex-value vectors, respectively. Let $u \in \mathbb{C}$, define $\mid u \mid = \sqrt{u\overline{u}}$, where $\overline{u}$ is the conjugate of u. For each $u = (u_1, u_2, \cdots, u_n)^T \in \mathbb{C}^n$, $\|u\| = \sqrt{\sum_{l=1}^{n} \mid u_l \mid^2}$.

2 Problem description

In this paper, the model of the ICVFCNNs with time-varying delays is presented as follows:

$$\begin{aligned} \ddot{x}_l(t) = &- a_l\dot{x}_l(t) - b_l x_l(t) + \sum_{j=1}^{n} c_{lj} f_j(x_j(t)) \\ &+ \sum_{j=1}^{n} d_{lj} f_j(x_j(t-\tau(t))) \\ &+ \sum_{j=1}^{n} e_{lj} u_j(t) + \bigwedge_{j=1}^{n} h_{lj} g_j(x_j(t-\delta(t)))ds \\ &+ \bigvee_{j=1}^{n} k_{lj} g_j(x_j(t-\delta(t))) \\ &+ \bigwedge_{j=1}^{n} s_{lj} u_j(t) + \bigvee_{j=1}^{n} q_{lj} u_j(t) + I_l(t), \ \ l \in \Theta, \end{aligned} \tag{1}$$

where $x_l(t) \in \mathbb{C}$ is the neural state variables of the lth neuron at time t, the second-order derivative is intituled as an inertial term of (1); $u_j(t) \in \mathbb{C}$ is the input of the jth neuron at time t; a_l and b_l are positive constants; c_{lj}, $d_{lj} \in \mathbb{R}$ are elements for feedback templates; $e_{lj} \in \mathbb{R}$ denotes elements for feed-forward template; h_{lj}, k_{lj}, s_{lj}, $q_{lj} \in \mathbb{R}$ are the elements of fuzzy feedback MIN template, fuzzy feedback MAX template, fuzzy feed forward MIN template, and fuzzy feed forward MAX template, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operations, respectively; $f_j(\cdot)$ and $g_j(\cdot) : \mathbb{C} \to \mathbb{C}$ are the activation functions of the jth neuron; $\tau(t), \delta(t) \in \mathbb{R}$ are the time delay, which satisfies $0 < \tau(t) < \hat{\tau}$, $\dot{\tau}(t) < \tau_1 < 1$, $0 < \delta(t) < \hat{\delta}$, $\dot{\delta}(t) < \delta_1 < 1$; $I_l(t) \in \mathbb{C}$ is the bias of the lth neuron at time t, $l, j \in \Theta$.

The initial condition of (1) is defined as

$$x_l(\eta) = \varphi_l(\eta), \dot{x}_l(\eta) = \hat{\varphi}_l(\eta), \ \ \eta \in [-\varpi, 0], \tag{2}$$

where $\varpi = \max\{\hat{\tau}, \hat{\delta}\}$, $\varphi_l(\cdot)$ and $\hat{\varphi}_l(\cdot)$ are bounded continuous functions, $l \in \Theta$.

Remark 1 Compared to the models proposed in [22, 26, 27, 30, 32, 34, 36], the model of system (1) is more general. For example, when fuzzy logic and inertial term are ignored, system (1) is degenerated into the first-order model in [22, 26, 27], and system (1) is reduced to the inertial complex-valued neural model in [36] if fuzzy logic are not considered.

The corresponding response system is proposed by the following equation:

$$\begin{aligned}
\ddot{y}_l(t) = & -a_l\dot{y}_l(t) - b_l y_l(t) + \sum_{j=1}^{n} c_{lj} f_j(y_j(t)) \\
& + \sum_{j=1}^{n} d_{lj} f_j(y_j(t-\tau(t))) \\
& + \sum_{j=1}^{n} e_{lj} u_j(t) + \bigwedge_{j=1}^{n} h_{lj} g_j(y_j(t-\delta(t)))ds \\
& + \bigvee_{j=1}^{n} k_{lj} g_j(y_j(t-\delta(t))) \\
& + \bigwedge_{j=1}^{n} s_{lj} u_j(t) + \bigvee_{j=1}^{n} q_{lj} u_j(t) + I_l(t) + M_l(t),
\end{aligned} \tag{3}$$

where $y_l(t) \in \mathbb{C}$ is the neural state variables of the lth neuron, $M_l(t)$ denotes a controller that will be designed, the other notations are the same as system (1), $l \in \Theta$.

The initial condition of 3 is defined as

$$y_l(\eta) = \psi_l(\eta), \dot{y}_l(\eta) = \hat{\psi}_l(\eta), \ \ \eta \in [-\varpi, 0], \tag{4}$$

where $\psi_l(\eta)$ and $\hat{\psi}_l(\eta)$ are bounded continuous functions, $l \in \Theta$.

To implement the exponentially synchronization of the ICVFCNNs (1) and (3), we designed the controllers as follows:

$$M_l(t) = \begin{cases} -\rho_l(\dot{z}_l(t) + z_l(t)), & nT \le t < (n+\nu)T, \\ 0, & (n+\nu)T \le t < (n+1)T, \end{cases} \tag{5}$$

where $\rho_l > 0$ denote control gains, $T > 0$ is called the control periodic, ν is called the control rate and $0 < \nu < 1$, $l \in \Theta$.

Denote $z_l(t) = y_l(t) - x_l(t)$, then the error system can be written as

$$\begin{cases}
\ddot{z}_l(t) & = -a_l\dot{z}_l(t) - b_l z_l(t) + \sum_{j=1}^{n} c_{lj}\tilde{f}_j(z_j(t)) \\
& \quad + \sum_{j=1}^{n} d_{lj}\tilde{f}_j(z_j(t-\tau(t))) \\
& \quad + \bigwedge_{j=1}^{n} h_{lj}\tilde{g}_j(z_j(t-\delta(t)))ds + \bigvee_{j=1}^{n} k_{lj}\tilde{g}_j(z_j(t-\delta(t))) \\
& \quad - \rho_l(\dot{z}_l(t) + z_l(t)), \ nT \le t < (n+\nu)T, \\
\ddot{z}_l(t) & = -a_l\dot{z}_l(t) - b_l z_l(t) + \sum_{j=1}^{n} c_{lj}\tilde{f}_j(z_j(t)) \\
& \quad + \sum_{j=1}^{n} d_{lj}\tilde{f}_j(z_j(t-\tau(t))) \\
& \quad + \bigwedge_{j=1}^{n} h_{lj}\tilde{g}_j(z_j(t-\delta(t)))ds + \bigvee_{j=1}^{n} k_{lj}\tilde{g}_j(z_j(t-\delta(t))), \\
& \quad (n+\nu)T \le t < (n+1)T,
\end{cases}$$

where $\tilde{f}_j(z_j(t)) = f_j(y_j(t)) - f_j(x_j(t)$ and $\tilde{f}_j(z_j(t-\tau(t))) = f_j(y_j(t-\tau(t))) - f_j(x_j(t-\tau(t)))$, $\tilde{g}_j(z_j(t-\delta(t))) = g_j(y_j(t-\delta(t)))–g_j(x_j(t-\delta(t)))$, $l, j \in \Theta$.

In the following, the definition of exponentially synchronization and a useful lemma are given.

Definition 1 ICVFCNNs (1) and (3) are said to achieve exponentially synchronization under periodically intermittent control, if there exist a constant $\omega > 0$ and a real number $M > 0$ such that

$$\|z(t)\| \le Me^{-\omega t}, \ \ t \ge 0,$$

where $z(t) = (z_1(t), z_2(t), ..., z_n(t))^T$, $\|z(t)\| = \sqrt{\sum_{l=1}^{n} | z_l(t) |^2}$.

Lemma 1 [40] *Let $V(t)$ is differentiable and positive on $[0, +\infty)$ and its derivative satisfies*

$$\begin{cases} \dot{V}(t) \le 0, & nT \le t < (n+\nu)T, \\ \dot{V}(t) \le \xi V(t), & (n+\nu)T \le t < (n+1)T, \end{cases}$$

where $n \in N = (0, 1, 2, \cdots)$, $T > 0$, $0 < \nu < 1$ and $\xi > 0$, then

$$V(t) \le V(0)e^{\xi(1-\nu)t}, \ \ t \ge 0.$$

3 Exponential synchronization

In this subsection, the sufficient conditions of exponentially synchronization of the drive-response ICVFCNNs will be obtain by developing some new Lyapunov functionals instead of the common reduced order and separation technique. In order to obtain these synchronization criteria, assume that the following conditions hold:

(H_1) There exist positive constants F_l, G_l such that for all $u, v \in \mathbb{C}$,

$$
\begin{aligned}
&\mid f_l(u) - f_l(v) \mid \leq F_l \mid u - v \mid, \\
&\mid g_l(u) - g_l(v) \mid \leq G_l \mid u - v \mid, \ l \in \Theta.
\end{aligned}
$$

(H_2) for any $l \in \Theta$, there exist positive constants β_l, λ such that $\Gamma_l < 0$, $\Upsilon_l < 0$, $\Pi_l^2 < 4\Gamma_l \Upsilon_l$, where

$$
\begin{aligned}
\Gamma_l =& \lambda\alpha_l + \beta_l(\lambda - b_l - \rho_l) \\
&+ \sum_{j=1}^{n} \Big[\beta_j \mid c_{jl} \mid F_l \\
&+ \frac{1}{2}\beta_l \Big(\mid c_{lj} \mid F_j + \mid d_{lj} \mid F_j + (\mid h_{lj} \mid + \mid k_{lj} \mid) G_j \Big) \Big] \\
&+ \sum_{j=1}^{n} \beta_j \Big(\frac{\mid d_{jl} \mid F_l}{1 - \tau_1} e^{2\lambda\hat{\tau}} + \frac{(\mid h_{jl} \mid + \mid k_{jl} \mid) G_l}{1 - \delta_1} e^{2\lambda\hat{\delta}} \Big), \\
\Upsilon_l =& \beta_l \Big(\lambda - a_l - \rho_l + 1 \Big) \\
&+ \frac{1}{2} \sum_{j=1}^{n} \beta_l \Big(\mid c_{lj} \mid F_j + \mid d_{lj} \mid F_j + (\mid h_{lj} \mid + \mid k_{lj} \mid) G_j \Big), \\
\Pi_l =& \alpha_l + \beta_l(2\lambda + 1 - b_l - 2\rho_l - a_l).
\end{aligned}
$$

Theorem 1 *Let (H_1)-(H_2) hold, then the systems (1) and (3) can achieve exponentially synchronization under the feedback controller (5) if $\lambda - \hat{\rho}(1 - \nu) > 0$, where $\hat{\rho} = \max_{l \in \Theta}\{\rho_l\}$.*

Proof Consider the following Lyapunov function:

$$
\begin{aligned}
V(t) =& \frac{1}{2} \sum_{l=1}^{n} \alpha_l z_l(t)\overline{z_l(t)} e^{2\lambda t} \\
&+ \frac{1}{2} \sum_{l=1}^{n} e^{2\lambda t} \beta_l (\dot{z}_l(t) + z_l(t)) \overline{(\dot{z}_l(t) + z_l(t))} \\
&+ \sum_{l=1}^{n} \sum_{j=1}^{n} \beta_l \mid d_{lj} \mid \frac{F_j}{1 - \tau_1} e^{2\lambda\hat{\tau}} \\
&\int_{t-\tau(t)}^{t} z_j(s)\overline{z_j(s)} e^{2\lambda s} ds \\
&+ \sum_{l=1}^{n} \sum_{j=1}^{n} \beta_l (\mid h_{lj} \mid + \mid k_{lj} \mid) \frac{G_j}{1 - \delta_1} e^{2\lambda\hat{\delta}} \\
&\int_{t-\delta(t)}^{t} z_j(s)\overline{z_j(s)} e^{2\lambda s} ds.
\end{aligned}
$$

For $nT \leq t < (n + \nu)T$, the derivative of $V(t)$ is estimated as follows:

$$
\begin{aligned}
\dot{V}(t) =& e^{2\lambda t} \sum_{l=1}^{n} \Big[\lambda\alpha_l z_l(t)\overline{z_l(t)} + \beta_l \lambda (\dot{z}_l(t) + z_l(t)) \overline{(\dot{z}_l(t) + z_l(t))} \\
&+ \frac{1}{2}\alpha_l \Big(\dot{z}_l(t)\overline{z_l(t)} + z_l(t)\overline{\dot{z}_l(t)} \Big) + \frac{1}{2}\beta_l (\ddot{z}_l(t) \\
&+ \dot{z}_l(t)) \overline{(\dot{z}_l(t) + z_l(t))} \\
&+ \frac{1}{2}\beta_l (\dot{z}_l(t) + z_l(t)) \overline{(\ddot{z}_l(t) + \dot{z}_l(t))} \Big] \\
&+ \sum_{l=1}^{n} \sum_{j=1}^{n} \beta_l \mid d_{lj} \mid \frac{F_j}{1 - \tau_1} e^{2\lambda\hat{\tau}} \\
&\times \Big(z_j(t)\overline{z_j(t)} e^{2\lambda t} - z_j(t - \tau(t)) \\
&\overline{z_j(t - \tau(t))} e^{2\lambda(t - \tau(t))} (1 - \tau'(t)) \Big) \\
&+ \sum_{l=1}^{n} \sum_{j=1}^{n} \beta_l (\mid h_{lj} \mid + \mid k_{lj} \mid) \frac{G_j}{1 - \delta_1} e^{2\lambda\hat{\delta}} \\
&\Big(z_j(t)\overline{z_j(t)} e^{2\lambda t} \\
&- z_j(t - \delta(t)) \overline{z_j(t - \delta(t))} e^{2\lambda(t - \delta(t))} (1 - \delta'(t)) \Big) \\
\leq& e^{2\lambda t} \sum_{l=1}^{n} \Big[\Big(\lambda\alpha_l + \beta_l(\lambda - b_l - \rho_l) \Big) z_l(t)\overline{z_l(t)} \\
&+ \beta_l \Big(\lambda - a_l - \rho_l + 1 \Big) \\
&\times \dot{z}_l(t)\overline{\dot{z}_l(t)} + \Big(\alpha_l + \beta_l(2\lambda + 1 - b_l - 2\rho_l - a_l) \Big) \\
&Re(\dot{z}_l(t)\overline{z_l(t)}) \Big] \\
&+ \sum_{l=1}^{n} \sum_{j=1}^{n} \beta_l \mid d_{lj} \mid \frac{F_j}{1 - \tau_1} e^{2\lambda\hat{\tau}} \\
&\Big(z_j(t)\overline{z_j(t)} e^{2\lambda t} - z_j(t - \tau(t)) \overline{z_j(t - \tau(t))} \\
&\times e^{2\lambda(t - \tau(t))} (1 - \tau'(t)) \Big) + \sum_{l=1}^{n} \sum_{j=1}^{n} \beta_l (\mid h_{lj} \mid \\
&+ \mid k_{lj} \mid) \frac{G_j}{1 - \delta_1} e^{2\lambda\hat{\delta}} \\
&\times \Big(z_j(t)\overline{z_j(t)} e^{2\lambda t} - z_j(t - \delta(t)) \\
&\overline{z_j(t - \delta(t))} e^{2\lambda(t - \delta(t))} (1 - \delta'(t)) \Big) \\
&+ e^{2\lambda t} \sum_{l=1}^{n} \sum_{j=1}^{n} \beta_l^2 \Big[\mid c_{lj} \mid Re(\overline{\dot{z}_l(t)} \tilde{f}_j(z_j(t))) \\
&+ \mid d_{lj} \mid Re(\overline{\dot{z}_l(t)} \tilde{f}_j(z_j(t - \tau(t)))) \\
&+ (\mid h_{lj} \mid + \mid k_{lj} \mid) Re(\overline{\dot{z}_l(t)} \tilde{g}_j(z_j(t - \delta(t)))) \Big] \\
&+ e^{2\lambda t} \sum_{l=1}^{n} \sum_{j=1}^{n} \beta_l \gamma_l \\
&\times \Big[\mid c_{lj} \mid Re(\overline{z_l(t)} \tilde{f}_j(z_j(t))) \\
&+ \mid d_{lj} \mid Re(\overline{z_l(t)} \tilde{f}_j(z_j(t - \tau(t)))) \\
&+ (\mid h_{lj} \mid + \mid k_{lj} \mid) Re(\overline{z_l(t)} \tilde{g}_j(z_j(t - \delta(t)))) \Big].
\end{aligned} \tag{6}
$$

By means of the theory of complex functions and (H_1), then

$$
\begin{aligned}
&\sum_{l=1}^{n}\sum_{j=1}^{n}\beta_l \mid c_{lj}\mid Re(\overline{\dot{z}_l(t)}\tilde{f}_j(z_j(t)))\\
&\leq \sum_{l=1}^{n}\sum_{j=1}^{n}\beta_l \mid c_{lj}\mid F_j\overline{\dot{z}_l(t)}z_j(t) \qquad (7)\\
&\leq \frac{1}{2}\sum_{l=1}^{n}\sum_{j=1}^{n}\Big(\beta_l \mid c_{lj}\mid F_j\dot{z}_l(t)\overline{\dot{z}_l(t)}+\beta_j \mid c_{jl}\mid F_l z_l(t)\overline{z_l(t)}\Big),
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{l=1}^{n}\sum_{j=1}^{n}\beta_l \mid d_{lj}\mid Re(\overline{\dot{z}_l(t)}\tilde{f}_j(z_j(t-\tau(t))))\\
&\leq \frac{1}{2}\sum_{l=1}^{n}\sum_{j=1}^{n}\beta_l \mid d_{lj}\mid F_j\Big(\dot{z}_l(t)\overline{\dot{z}_l(t)}+z_j(t-\tau(t))\overline{z_j(t-\tau(t))}\Big), \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{l=1}^{n}\sum_{j=1}^{n}\beta_l(\mid h_{lj}\mid+\mid k_{lj}\mid)Re(\overline{\dot{z}_l(t)}\tilde{g}_j(z_j(t-\delta(t))))\\
&\leq \frac{1}{2}\sum_{l=1}^{n}\sum_{j=1}^{n}\beta_l(\mid h_{lj}\mid+\mid k_{lj}\mid) \qquad (9)\\
&G_j\Big(\dot{z}_l(t)\overline{\dot{z}_l(t)}+z_j(t-\delta(t))\overline{z_j(t-\delta(t))}\Big),
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{l=1}^{n}\sum_{j=1}^{n}\beta_l \mid c_{lj}\mid Re(\overline{z_l(t)}\tilde{f}_j(z_j(t)))\\
&\leq \frac{1}{2}\sum_{l=1}^{n}\sum_{j=1}^{n}\Big(\beta_l \mid c_{lj}\mid F_j z_l(t)\overline{z_l(t)}+\beta_j \mid c_{jl}\mid F_l z_l(t)\overline{z_l(t)}\Big), \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{l=1}^{n}\sum_{j=1}^{n}\beta_l \mid d_{lj}\mid Re(\overline{z_l(t)}\tilde{f}_j(z_j(t-\tau(t))))\\
&\leq \frac{1}{2}\sum_{l=1}^{n}\sum_{j=1}^{n}\beta_l \mid d_{lj}\mid F_j\Big(z_l(t)\overline{z_l(t)}+z_j(t-\tau(t))\overline{z_j(t-\tau(t))}\Big), \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{l=1}^{n}\sum_{q=1}^{n}\beta_l(\mid h_{lj}\mid+\mid k_{lj}\mid)Re(\overline{z_l(t)}\tilde{g}_j(z_j(t-\delta(t))))\\
&\leq \frac{1}{2}\sum_{l=1}^{n}\sum_{j=1}^{n}\beta_l(\mid h_{lj}\mid+\mid k_{lj}\mid) \qquad (12)\\
&G_j\Big(z_l(t)\overline{z_l(t)}+z_j(t-\delta(t))\overline{z_j(t-\delta(t))}\Big).
\end{aligned}
$$

Substituting (7–12) into (6), one has

$$
\begin{aligned}
\dot{V}(t)\leq{}& e^{2\lambda t}\sum_{l=1}^{n}\Bigg\{\lambda\alpha_l+\beta_l(\lambda-b_l-\rho_l)\\
&+\sum_{j=1}^{n}\Big[\beta_j\mid c_{jl}\mid F_l+\frac{1}{2}\beta_l\Big(\mid c_{lj}\mid F_j\\
&+\mid d_{lj}\mid F_j+(\mid h_{lj}\mid+\mid k_{lj}\mid)G_j\Big)\Big]\\
&+\sum_{j=1}^{n}\beta_j\Big(\frac{\mid d_{jl}\mid F_l}{1-\tau_1}e^{2\lambda\hat{\tau}}\\
&+\frac{(\mid h_{jl}\mid+\mid k_{jl}\mid)G_l}{1-\delta_1}e^{2\lambda\hat{\delta}}\Big)\Bigg\}z_l(t)\overline{z_l(t)}\\
&+e^{2\lambda t}\sum_{l=1}^{n}\Big[\beta_l\Big(\lambda-a_l-\rho_l+1\Big)\\
&+\frac{1}{2}\sum_{j=1}^{n}\beta_l\Big(\mid c_{lj}\mid F_j+\mid d_{lj}\mid F_j+(\mid h_{lj}\mid\\
&+\mid k_{lj}\mid)G_j\Big)\Big]\dot{z}_l(t)\overline{\dot{z}_l(t)}\\
&+e^{2\lambda t}\sum_{l=1}^{n}\Big(\alpha_l+\beta_l(2\lambda+1-b_l-2\rho_l-a_l)\Big)\\
&Re(\dot{z}_l(t)\overline{z_l(t)})\\
={}& e^{2\nu t}\sum_{l=1}^{n}\Big[\Gamma_l z_l(t)\overline{z_l(t)}\\
&+\Upsilon_l\dot{z}_l(t)\overline{\dot{z}_l(t)}+\frac{\Pi_l}{2}\Big(\dot{z}_l(t)\overline{z_l(t)}+\overline{\dot{z}_l(t)}z_l(t)\Big)\Big].
\end{aligned}
$$

Let $\triangle=\{l\in\Theta:\Gamma_l=0\}$ and from (H_2), we have $\Pi_l=0$ for $l\in\triangle$. Meanwhile, note that $\Gamma_l\leq 0$, $\Upsilon_l\leq 0$ and $\Pi_l^2\leq 4\Gamma_l\Upsilon_l$, then

$$
\begin{aligned}
\dot{V}(t)\leq{}& e^{2\lambda t}\sum_{l\in\Lambda\setminus\Delta}^{n}\Gamma_l\Big(\dot{z}_l(t)+\frac{\Pi_l}{2\Gamma_l}z_l(t)\Big)\overline{\Big(\dot{z}_l(t)+\frac{\Pi_l}{2\Gamma_l}z_l(t)\Big)}\\
&+e^{2\lambda t}\sum_{l\in\Lambda\setminus\Delta}^{n}\Big(\Upsilon_l-\frac{\Pi_l^2}{4\Gamma_l}\Big)z_l(t)\overline{z_l(t)}\\
\leq{}& 0.
\end{aligned}
$$

For $(n+\nu)T\leq t<(n+1)T$, similar to the preceding proof, we have

$$
\begin{aligned}
\dot{V}(t) \le & e^{2\lambda t}\sum_{l=1}^{n}\Bigg\{\lambda\alpha_l+\beta_l(\lambda-b_l)\\
&+\sum_{j=1}^{n}\Bigg[\beta_j\mid c_{jl}\mid F_l+\frac{1}{2}\beta_l\Big(\mid c_{lj}\mid F_j\\
&+\mid d_{lj}\mid F_j+(\mid h_{lj}\mid+\mid k_{lj}\mid)G_j\Big)\Bigg]\\
&+\sum_{j=1}^{n}\beta_j\Big(\frac{\mid d_{jl}\mid F_l}{1-\tau_1}e^{2\lambda\hat{\tau}}\\
&+\frac{(\mid h_{jl}\mid+\mid k_{jl}\mid)G_l}{1-\delta_1}e^{2\lambda\hat{\delta}}\Big)\Bigg\}z_l(t)\overline{z_l(t)}\\
&+e^{2\lambda t}\sum_{l=1}^{n}\Bigg[\beta_l\Big(\lambda-a_l+1\Big)\\
&+\frac{1}{2}\sum_{j=1}^{n}\beta_l\Big(\mid c_{lj}\mid F_j+\mid d_{lj}\mid F_j\\
&+(\mid h_{lj}\mid+\mid k_{lj}\mid)G_j\Big)\Bigg]\dot{z}_l(t)\overline{\dot{z}_l(t)}\\
&+e^{2\lambda t}\sum_{l=1}^{n}\Big(\alpha_l+\beta_l(2\lambda+1-b_l-a_l)\Big)Re(\dot{z}_l(t)\overline{z_l(t)})\\
\le & e^{2\lambda t}\sum_{l=1}^{n}\Bigg\{\lambda\alpha_l+\beta_l(\lambda-b_l-\rho_l)\\
&+\sum_{j=1}^{n}\Bigg[\beta_j\mid c_{jl}\mid F_l+\frac{1}{2}\beta_l\\
&\Big(\mid c_{lj}\mid F_j+\mid d_{lj}\mid F_j+(\mid h_{lj}\mid+\mid k_{lj}\mid)G_j\Big)\Bigg]\\
&+\sum_{j=1}^{n}\beta_j\Big(\frac{\mid d_{jl}\mid F_l}{1-\tau_1}e^{2\lambda\hat{\tau}}\\
&+\frac{(\mid h_{jl}\mid+\mid k_{jl}\mid)G_l}{1-\delta_1}e^{2\lambda\hat{\delta}}\Big)\Bigg\}z_l(t)\overline{z_l(t)}\\
&+e^{2\lambda t}\sum_{l=1}^{n}\beta_l\rho_l z_l(t)\overline{z_l(t)}\\
&+e^{2\lambda t}\sum_{l=1}^{n}\Bigg[\beta_l\Big(\lambda-a_l-\rho_l+1\Big)\\
&+\frac{1}{2}\sum_{j=1}^{n}\beta_l\Big(\mid c_{lj}\mid F_j\\
&+\mid d_{lj}\mid F_j+(\mid h_{lj}\mid+\mid k_{lj}\mid)G_j\Big)\Bigg]\dot{z}_l(t)\overline{\dot{z}_l(t)}\\
&+e^{2\lambda t}\sum_{l=1}^{n}\beta_l\rho_l\dot{z}_l(t)\overline{\dot{z}_l(t)}\\
&+e^{2\lambda t}\sum_{l=1}^{n}\Big(\alpha_l+\beta_l(2\lambda+1-b_l-2\rho_l-a_l)\Big)\\
&Re(\dot{z}_l(t)\overline{z_l(t)})\\
&+e^{2\lambda t}\sum_{l=1}^{n}2\beta_l\rho_l Re(\dot{z}_l(t)\overline{z_l(t)})\\
\le & e^{2\lambda t}\sum_{l=1}^{n}\beta_l\rho_l(z_l(t)\overline{z_l(t)}+\dot{z}_l(t)\overline{\dot{z}_l(t)}+2Re(\dot{z}_l(t)\overline{z_l(t)}))\\
= & e^{2\lambda t}\sum_{l=1}^{n}\beta_l\rho_l(\dot{z}_l(t)+z_l(t))\overline{(\dot{z}_l(t)+z_l(t))}\\
\le & 2\hat{\rho}V(t).
\end{aligned}
$$

For all $t \ge 0$, we have

$$V(t) \le V(0)e^{2\hat{\rho}(1-\nu)t}.$$

Hence,

$$\|z(t)\|^2=\sum_{l=1}^{n}z_l(t)\overline{z_l(t)}\le\frac{2}{\alpha}e^{-2\lambda t}V(t)\le\frac{2V(0)}{\alpha}e^{-2[\lambda-\hat{\rho}(1-\nu)]t},$$

where $\alpha=\min_{l\in\Lambda}\{\alpha_l\}$. Therefore,

$$\|z(t)\|\le\sqrt{\frac{2V(0)}{\alpha}}e^{-[\lambda-\hat{\rho}(1-\nu)]t},$$

which means that the exponential synchronization is realized. The proof is achieved. □

Remark 2 Without the traditional variable transformation method in the reports of [19, 25, 26] based on intermittent control scheme, some new Lyapunov functionals are constructed to directly analyze the synchronization problem of inertial neural networks. The new technique is more direct and concise compared to the previous reduced order technique.

Remark 3 Actually, various complex-valued neural networks have been studied to achieve synchronization by separating the complex-valued neural networks into two real-valued systems [30–37]. Different from these work, exponential synchronization is reached in this paper for the inertial complex-valued neural networks by the theory of complex-variable functions.

Remark 4 In [39], the authors used a non-reduced order approach to study global dissipativity of real-valued neutral-type inertial neural networks. Compared to the work, a class of more general systems, complex-valued inertial neural networks are considered in this paper.

4 Numerical illustration

Considering the ICVFCNNs with time-varying delays is given as follows:

$$
\begin{aligned}
\ddot{x}_l(t)=&-a_l\dot{x}_l(t)-b_lx_l(t)+\sum_{j=1}^{n}c_{lj}f_j(x_j(t))\\
&+\sum_{j=1}^{n}d_{lj}f_j(x_j(t-\tau(t)))\\
&+\sum_{j=1}^{n}e_{lj}u_j(t)+\bigwedge_{j=1}^{n}h_{lj}g_j(x_j(t-\delta(t)))ds\\
&+\bigvee_{j=1}^{n}k_{lj}g_j(x_j(t-\delta(t)))\\
&+\bigwedge_{j=1}^{n}s_{lj}u_j(t)+\bigvee_{j=1}^{n}q_{lj}u_j(t)+I_l(t),\ \ l\in\Theta,
\end{aligned}
\tag{13}
$$

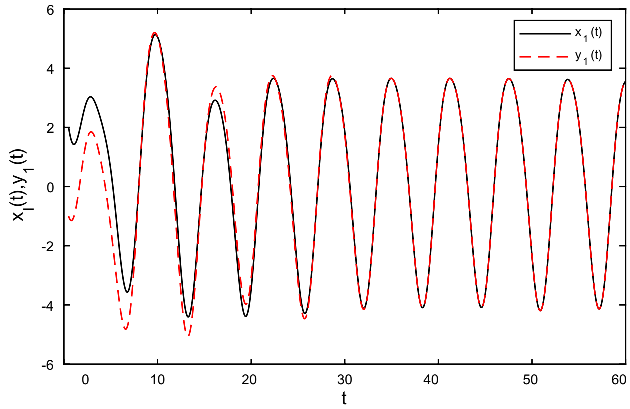

Fig. 1 Synchronization curves of x_1 and y_1

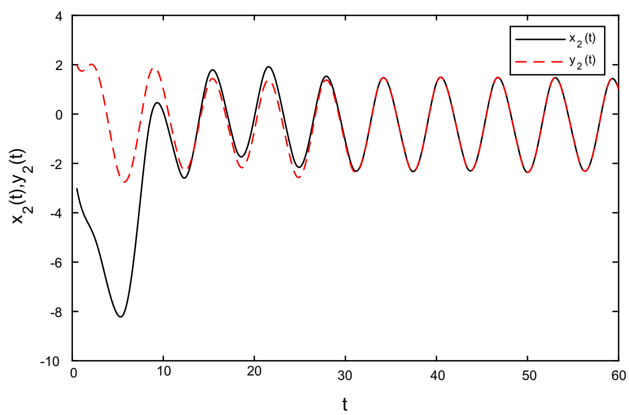

Fig. 2 Synchronization curves of x_2 and y_2

and the corresponding response system is proposed by the following equation:

$$\begin{aligned}\ddot{y}_l(t) =& -a_l\dot{y}_l(t) - b_l y_l(t) + \sum_{j=1}^{n} c_{lj} f_j(y_j(t)) \\ &+ \sum_{j=1}^{n} d_{lj} f_j(y_j(t-\tau(t))) \\ &+ \sum_{j=1}^{n} e_{lj} u_j(t) + \bigwedge_{j=1}^{n} h_{lj} g_j(y_j(t-\delta(t)))ds \\ &+ \bigvee_{j=1}^{n} k_{lj} g_j(y_j(t-\delta(t))) \\ &+ \bigwedge_{j=1}^{n} s_{lj} u_j(t) + \bigvee_{j=1}^{n} q_{lj} u_j(t) + I_l(t) + M_l(t),\ \ l \in \Theta,\end{aligned} \tag{14}$$

where $\Theta = \{1,2\}$, $\tau(t) = \delta(t) = 0.5(1+\sin 2t)$, $a_1 = 0.3$, $a_2 = 0.5$, $b_1 = 0.4$, $b_2 = 0.2$, $\rho_1 = 6$, $\rho_2 = 8$, $f_j(\cdot) = g_j(\cdot)$ $= \tanh(Re(\cdot)) + i\tanh(Im(\cdot))$, and

$$(c_{lj})_{2\times 2} = \begin{pmatrix} 0.3 & -0.1 \\ -0.2 & 0.1 \end{pmatrix}, (d_{lj})_{2\times 2} = \begin{pmatrix} 0.1 & -0.4 \\ -0.5 & -0.3 \end{pmatrix},$$
$$(e_{lj})_{2\times 2} = \begin{pmatrix} 1.0\ amp;0.5 \\ 0.4 \quad 0.2 \end{pmatrix}, (h_{lj})_{2\times 2} = \begin{pmatrix} 0.1 & -0.1 \\ -0.3 & 0.2 \end{pmatrix},$$
$$(k_{lj})_{2\times 2} = \begin{pmatrix} -0.6 & 0.1 \\ 0.2 & 0.3 \end{pmatrix}, (s_{lj})_{2\times 2} = \begin{pmatrix} 0.5\ amp;\ 0.5 \\ 0.2 \quad 0.2 \end{pmatrix},$$
$$(q_{lj})_{2\times 2} = \begin{pmatrix} 0.4\ amp;0.3 \\ 0.2 \quad 0.1 \end{pmatrix}, \begin{pmatrix} I_1(t) \\ I_2(t) \end{pmatrix} = \begin{pmatrix} 2\sin t + i\sin 2t \\ \cos t + i\sin t \end{pmatrix},$$
$$u_1(t) = \sin 2t + i\cos 2t, \quad u_2(t) = \cos t + i\sin t.$$

The initial value are give as $\varphi_1(\eta) = 2 + 3i$, $\varphi_2(\eta) = -3 + 0.5i$, $\hat{\varphi}_1(\eta) = \hat{\varphi}_2(\eta) = -2 + 2i$, $\psi_1(\eta) = -1 + i$, $\psi_2(\eta) = 2 - 2i$, $\hat{\psi}_1(\eta) = \hat{\psi}_2(\eta) = -1 + 2i$, $\eta \in [-1, 0]$. Under periodically intermittent control, Figs. 1 and 2 show that synchronization curves of x_l and y_l, $l = 1, 2$.

Note that $\hat{\tau} = \tau_1 = \hat{\delta} = \delta_1 = 0.5, F_1 = F_2 = G_1 = G_2 = 1$, $\hat{\rho} = 8$. Choose $T = 3$, $\lambda = 0.6$, $\nu = 0.95$, $\alpha_1 = 1$, $\alpha_2 = 2$, $\beta_1 = 0.5$, $\beta_2 = 0.4$ and by calculation, we can get

$$\Gamma_1 = -5.11 < 0,\ \Gamma_2 = -2.14 < 0,\ \Upsilon_1 = -2.5 < 0,$$
$$\Upsilon_2 = -2.86 < 0,\ \Pi_1 = -4.25,\ \Pi_2 = -3.8.$$
$$\lambda - \hat{\rho}(1-\nu)0.6 - 0.8(1 - 0.95) = 0.2 > 0.$$

It is easy to verify that the conditions of Theorem 1 hold. Consequently, the ICVFCNNs 13 and 14 are exponentially synchronized under periodically intermittent control, which is demonstrated by Fig. 3.

In addition, let the time-varying delays $\tau(t) = 0.5\cos^2(2t)$ and $\delta(t) = 0.5\sin^2(2t)$, all the other parameters and the error initial conditions are the same as the above for systems (13) and (14), then it is easy to verify that the conditions of Theorem 1 hold. Under the intermittent control, Figure 4 shows time responses of the error variables $z_1(t)$, $z_2(t)$. From Figs. 3 and 4, the synchronization time in Fig. 3 is shorter than that in Fig. 4, which indicates that the synchronization of (13) and (14) is delay-dependent. So, Figs. 1, 2, 3, 4, testify the validity of the results for Theorem 1.

5 Conclusion

This paper has discussed the global exponential synchronization for ICVFCNNs with mixed time-varying delays via periodically intermittent control. Based on the theory of complex functions and the construction of suitable

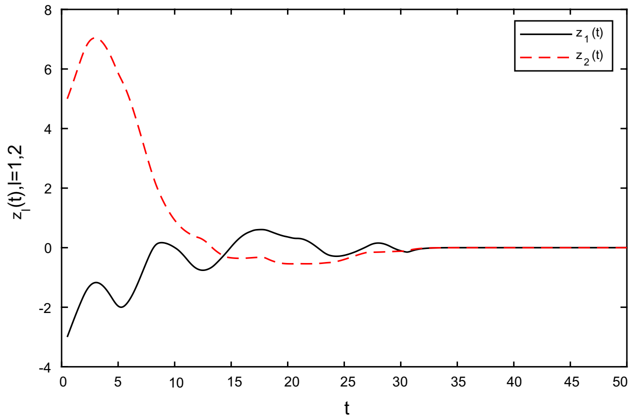

Fig. 3 The evolutions of $z_1(t)$ and $z_2(t)$ with $\tau(t) = \delta(t) = 0.5(1 + \sin 2t)$

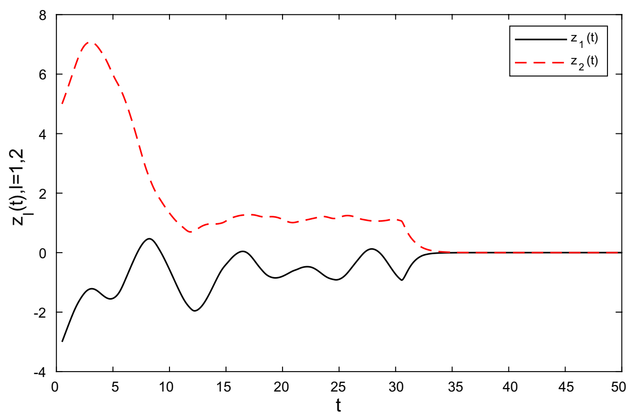

Fig. 4 The evolutions of $z_1(t)$ and $z_2(t)$ with $\tau(t) = 0.5\cos^2(2t)$, $\delta(t) = 0.5\sin^2(2t)$

Lyapunov functions, the non-reduced order and non-separation approach was introduced to investigate the synchronization problems of delayed ICVFCNNs. Compared to previous results, the method used in this article is more concise and practical, which is an entirely new attempt. Further, it can be utilized to study other dynamic models, e.g., fractional-order models [41, 42], impulsive model [43], stochastic model [44], etc.

In addition, because fixed-time synchronization does not depend on the system's initial conditions and is only related to the parameters of the system, thus reducing the requirements in practical applications. Currently, fixed-time synchronization for inertial complex-valued fuzzy cellular neural networks has been extensively discussed by using reduced-order transform. However, there seems few related results about the topic based on the direct method of Lyapunov functional instead of the reduced order technique. The interesting and challenging issues will be examined in our latest work.

Author Contributions WP: conceptualization, methodology, writing-original draft, writing-review and editing. LXC: funding acquisition, writing-original draft, writing-review and editing. ZTW: writing-review and editing.

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Availability of Data and Material The data that support the findings of this study are available on request from the first author (WP).

Declarations

Conflict of Interest The authors declare that they have no competing interests.

Ethical Approval and Consent to Participate Not applicable.

Consent for Publication Not applicable.

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