

Commenced Publication in 1973

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Uncertain Projective Geometry

Statistical Reasoning
for Polyhedral Object Reconstruction



Springer

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Inaugural-Dissertation to obtain the degree Dr.-Ing.
Agricultural Faculty
Rheinische Friedrich-Wilhelms-Universität, Bonn, Germany

Referent: Prof. Dr.-Ing. W. Förstner
Coreferent: Prof. Dr. rer.nat. H.-P. Helfrich
Date of Exam: Friday, 13. Dezember 2002

Library of Congress Control Number: 2004104982

CR Subject Classification (1998): I.5, I.4, G.3, I.3.5, I.2.10

ISSN 0302-9743

ISBN 3-540-22029-1 Springer-Verlag Berlin Heidelberg New York

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© Springer-Verlag Berlin Heidelberg 2004
Printed in Germany

Typesetting: Camera-ready by author, data conversion by Boller Mediendesign
Printed on acid-free paper SPIN: 10999820 06/3142 5 4 3 2 1 0

Foreword

The last decade of computer vision can be characterized by the development of tools for geometric reasoning. Algebraic projective geometry, with its multilinear relations and its embedding into Grassmann-Cayley algebra, became the basic representation of multiple view geometry, on the one hand leading to a deep insight into the algebraic structure of the geometric relations and on the other hand leading to efficient and versatile algorithms for the calibration and orientation of cameras, for image sequence analysis as well as for 3D-object reconstruction. Prior to the use of projective geometry in computer vision, some kinds of features or information had to be extracted from the images. The inherent uncertainty of these automatically extracted image features, in a first instance image points and image line segments, required tracing the effects up to the inherently uncertain result. However, in spite of the awareness of the problem and the attempts to integrate uncertainty into geometric computations, especially joint final maximum-likelihood estimation of camera and object parameters, geometric reasoning under uncertainty rarely was seen as a topic on its own.

Reasons for the lack of integration of uncertainty may have been the redundancy in the representation of Euclidean entities using homogeneous coordinates leading to singularities in the statistical distributions, the non-linearity of the relations requiring approximations, which give biased results, and the dependency of both the numerical accuracy and the statistical properties on the coordinate systems chosen; another reason may have been the lower reward for performing good engineering.

The author provides a transparent integration of algebraic projective geometry and spatial reasoning under uncertainty with applications in computer vision. Examples of those applications are: grouping of spatial features, reconstruction of polyhedral objects, or calibration of projective cameras. He clearly cuts the boundary between rigorous but infeasible and approximate but practicable statistical tools for three-dimensional computations. In addition to the theoretical foundations of the proposed integration and the clear rules for performing geometric reasoning under uncertainty, he provides a collection of detailed algorithms that are implemented using now publicly available software.

The book will be of value to all scientists interested in algebraic projective geometry for image analysis, in statistical representations of two- and three-dimensional objects and transformations, and in generic tools for testing and estimating within the context of geometric multiple-view analysis.

Bonn, February 2004

Wolfgang Förstner

Preface

I wish to acknowledge the following people who directly or indirectly contributed to this work. First, I would like to thank my family, especially my parents, who have given me sufficient freedom to pursue my own interests and at the same time have shown me unconditional love.

I was very fortunate to have met wonderful people during my time at the University of Bonn. I refer in particular to Oliver who contributed to my life in numerous ways.

I also thank all the people at the University and the Institute of Photogrammetry in Bonn for a wonderful time I had there both as a student assistant and as a research assistant. In particular I wish to mention Kirsten Wolff, Andre Braunmandl, Heidi Hollander, Felicitas Lang, Wolfgang Schickler, Hardo Müller, Boudewijn Moonen, Thomas H. Kolbe, Petko and Annett Faber, Mirko Appel and many others. I also want to thank Thomas Läbe for coordinating the Java implementation of the algorithms described in this work.

I am deeply grateful that I had the opportunity to work with my advisor, Prof. Dr. Wolfgang Förstner. He was not only the initiator of this work and provided continuous support throughout the duration of the research, but more importantly he was a *Doktorvater* in the truest sense of the word. I extend my gratitude to Prof. Dr. Helfrich for his willingness to serve as a second reviewer for this work.

Lastly, I wish to express my thanks to Anette for her support during the last 15 months. I am still wondering why we did not meet earlier in life.

Bonn, October 2002

Stephan Heuel

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