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Cheap Talk with an Exit Option: A Model of Exit and Voice*

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Abstract

The paper presents a formal model of the exit and voice framework proposed by Hirschman [12]. More specifically, we modify Crawford and Sobel's [6] cheap talk model such that the sender of a cheap talk message has an exit option. We demonstrate that the presence of the exit option may increase the informativeness of cheap talk and improve welfare if the exit option is relatively attractive to the sender and relatively unattractive to the receiver. Moreover, it is verified that perfect information transmission can be approximated in the limit. The results suggest that the exit reinforces the voice in that the credibility of exit increases the informativeness of the voice.

Keywords: Exit, Voice, Cheap Talk, Informativeness, Credibility

Journal of Economic Literature Classification Numbers: D23, D82, J5, L22

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1 Introduction

Since the publication of the book *Exit, Voice, and Loyalty* by Hirschman [12], the exit-voice perspective has been widely adopted by studies in the field of political science, and it has also been extended to various studies on relationships and organizations, such as employer-employee (or union) relationships, buyer-seller relationships, hierarchies, public services, political parties, families, and adolescent development (See Hirschman [12] and [13]). Broadly speaking, the exit and the voice are alternative means of dealing with problems that arise within an ongoing relationship or organization. For example, consider an employer-employee relationship.[1] Suppose an employee finds himself/herself in undesirable situations regarding the conditions of employment, compensation packages, and rules at the workplace. In this situation, the employee usually has two options. One is to quit the job; this is the exit option. The other is to express his dissatisfaction directly to the employer; this is the voice option. Hirschman insists that the voice as well as the exit option is important for the sustainability of relationships and organizations—a concept that has been hitherto neglected in economics.

With regard to the workings of the exit and voice options in a real economy, how the exit interacts with the voice is a point of considerable interest, which is the main point of Hirschman's discussion. From one perspective, the exit works as a complement to the voice. Indeed, in Palgrave's dictionary [13], Hirschman briefly points out, "[t]he availability and threat of exit on the part of important customer or group of members may powerfully reinforce their voice."[2] However, it is not very clear why and how the exit can reinforce the voice. The present paper aims to clarify this by analyzing a formal model of exit and voice.

In this paper, the exit is regarded as a decision to terminate an ongoing relationship, whereas the voice is interpreted as an activity involving sending a costless message that serves for the improvement of the relationship. In other words, we identify the voice with "cheap talk" for transmitting useful information.[3] Among others, the model by Crawford

[1] See Freeman [9].

[2] In his 1970 book [12], Hirschman seems to emphasize on a substitute aspect between exit and voice. However, in 1987 Palgrave's dictionary [13], he turned to insist that a complementarity aspect of exit and voice is also important.

[3] As we see later, Banerjee and Somanathan [3] also identify voice as an activity of sending a cheap talk message.

and Sobel [6] (hereinafter referred to as CS) is the most successful one describing cheap talk with private information. We employ the CS model as the basis of the environment that we consider in the present paper and extend it to the situation in which an exit option is available.

The CS model has two players. One player possesses private information about the current state of the relationship, which is randomly drawn. In order to transmit the information, he sends a costless message to his partner, and the latter responds with a decision affecting both the players' payoffs. In CS, the latter is called the Receiver (R) and the former is called the Sender (S). In the present paper, we assume that S has an exit option after he observes R's decision. When S exercises the exit option, both players obtain their exit payoffs independent of the action chosen by R.

The key feature of our results is the difference between the players' payoff when S chooses to stay and an optimal action is chosen (maximum stay payoff) and one when she chooses to exit (exit payoff). Consider the case where R's difference is large and S's difference is small but positive. In this case, R's optimal action may induce S's exit as it differs from S's and cannot attain S's maximum stay payoff. Since R's large difference gives R a strong incentive to prevent S from exercising the exit option, R chooses an action close to the S's optimal action, instead of her own. Expecting R's response, S has a strong incentive to transmit more accurate information via cheap talk as more accurate information directly enhances his payoff. It then follows that the presence of S's exit option increases the informativeness of cheap talk, which in turn may increase not only S's payoff but also that of R. Moreover, we show that as S's difference approaches 0, the information transmission via cheap talk in the most informative equilibrium becomes almost perfect. In other words, the exit reinforces the voice in that the presence of the exit increases the informativeness of the voice. This is the main finding of this paper.

CS also shows that on the most efficient equilibrium, the more congruent both players' preferences are, the more informative is the cheap talk. In other words, when the exit option is not available, the informativeness of cheap talk is determined mainly by the degree of incongruence between the players' preferences. However, when the exit option is available, another determinant of the informativeness comes into play: *the credibility of exit*. A smaller difference between S's maximum stay payoff and exit payoff makes his choice of the exit option more credible, which in turn enables a more informative cheap talk transmission on the equilibrium. Thus, we show that informative cheap talk transmission can be carried out even if S's preference is not exactly similar to that of R. Our main claim—the exit reinforces the voice in that the credibility of exit increases the informativeness of the voice—is consistent with Hirschman's [13].

To the author's knowledge, the exit-voice perspective has seldom been analyzed in any formal model in economics despite the vast citations.[4] Banerjee and Somanathan's study [3] is one exception in that it presents a game-theoretical model of voice. Like us, they consider the voice as an activity of sending a cheap talk message. However, their model differs from ours in some respects. First, they do not consider the exit option, and therefore, they do not investigate the interplay between the exit and voice, which the present paper focuses on. However, they consider the collective aspect of voice formation, which is abstracted out from our model. In this regard, the present paper can be considered as a complement to their paper.

Apart from the exit-voice perspective, the CS model *per se* has attracted considerable attention and has been extended to various directions. However, the exit option's effect on cheap talk has rarely been analyzed.[5] An exception, Matthews [16], deals with a cheap talk game with a congress and a president (the receiver and sender, respectively) with veto power, which is similar to the exit option in our model. In particular, the timing of the events in his model is approximately the same as that in ours. However, there is a large difference with respect to what private information pertains to. In Matthews, private information concerns the sender's preference, while in our model, it pertains to the current state of the relationship. One may consider such a difference to be small, but it leads to very different outcomes: in Matthews, the informativeness of cheap talk is constrained with a strict upper bound, independent of the exit payoff. However, in our model, we show that an equilibrium can be close to that with perfect information transmission to any degree. In other words, Matthews does not emphasize that the existence of the exit increases the informativeness of cheap talk, which is the main claim of the present paper.

Chiba and Leong [5] also consider the exit option's effect on cheap talk. However, in

[4]For efforts in the field of political science, see the survey by Dowding et al. [8]. For examaple, Gehlbach [10] presents a formal model of exit and voice. In his model, however, there is no asymmetric information, and therefore the voice does not play the role of an information transmitter.

[5]There is a relatively large amount of the literature on the effect of the exit option in the environments where the receiver can commit to delegation or message-contingent mechanism, while she cannot in this paper. See Gilligan and Krehbiel [11], Krishna and Morgan [14], Dessein [7], Marino [15], Mylovanov [17].

their model, it is *not the sender, but the receiver* who has an exit option. Although they also show that the presence of the exit option can facilitate the information transmission, the logic is completely different from ours. In their model, the change of the sender's incentive that occurs when the exit option is really exercised brings about the improvement of communication. On the other hand, in our model, the credibility of exit is important; the presence of the exit option *per se* serves for the improvement of communication even when it is *not* exercised on the equilibrium path.

Shimizu [18] is a companion paper which analyzes the effect of sender's exit option on information transmission in a model with a discrete state space. I demonstrate that it is possible in a finite states setting that the informative communication requires rather a moderate size of incongruence than a small size of that.

The rest of the paper is organized as follows. In Section 2, we present a formal model of exit and voice. In Section 3, we present the main claim of this paper. Finally, in Section 4, we summarize the results.

2 Setup

There are two players, namely, the sender (S, male) and the receiver (R, female). At the beginning of the game, the current state of the relationship between S and R, $t \in T$ is randomly chosen according to a probability distribution $F(t)$. A realized state is observed by S but not by R. On the basis of this observation, S chooses a message $m \in M$ to be sent to R. This message is cheap talk in that it is payoff-irrelevant. After R receives S's message, R chooses an action $a \in A$ relevant to both players' payoffs.

Up to this point, all elements are the same as in CS. Now, we introduce the concept of exit. After observing R's action, S chooses whether to stay or exit. If S chooses to stay, S's and R's payoffs are given by $y^S(t,a)$ and $y^R(t,a)$, respectively. If S chooses to exit, S's and R's payoffs are $U^S(t)$ and $U^R(t)$, respectively.

In this paper, we employ a more specific model.[6] We assume $T = M = [0,1]$ and $A = \mathbb{R}$.[7] In this model, $F(t)$ is a uniform distribution function on $[0,1]$ and S's and R's

[6] In an online appendix [19], we show that our main results apply to more general class of models.

[7] The assumption $T = M$ guarantees that S can send a direct message regarding a state.

stay payoffs are respectively expressed as

$$y^S(t,a) = Y^S - (t+b-a)^2,$$
$$y^R(t,a) = Y^R - (t-a)^2,$$

for some $b > 0$. Here, b is called *a bias* that represents a degree of incongruence between S's and R's optimal actions.[8] A uniform-quadratic environment was originally analyzed in Section 4 of CS. Furthermore, we assume that both players' exit payoffs are independent of t. We define the difference between i's maximum stay payoff and exit payoff by $D^i = Y^i - U^i$ for $i = S, R$.

We consider a perfect Bayesian equilibrium as an equilibrium concept. We also focus only on the class of equilibria with pure strategies. A pure strategy perfect Bayesian equilibrium is defined by $(\mu, p, \alpha, \epsilon)$, where

- $\mu : T \to M$: S's message strategy,
- $p : M \times T \to [0,1]$: R's posterior belief density function over T on the observation of m,
- $\alpha : M \to A$: R's action choice strategy, and
- $\epsilon : T \times A \to \{0,1\}$: S's exit strategy. To be more precise, $\epsilon = 1$ refers to exit and $\epsilon = 0$ refers to stay.

The equilibrium conditions are as follows:

- $\mu(t)$ must be an optimal message for type t of S given R's strategy and S's exit strategy, i.e.,

$$\mu(t) \in \arg\max_{m \in M} \left\{ \epsilon(t, \alpha(m)) U^S + (1 - \epsilon(t, \alpha(m))) y^S(t, \alpha(m)) \right\}, \quad \forall t \in T.$$

- R's posterior belief must be updated by adhering as much as possible to the Bayesian approach, i.e.,

$$\int_{t' \in T} \mathbb{I}\{\mu(t') = m\} dt' > 0 \Rightarrow p(m,t) = \frac{\mathbb{I}\{\mu(t) = m\}}{\int_{t' \in T} \mathbb{I}\{\mu(t') = m\} dt'},$$

where $\mathbb{I}$ is the identity function.

[8] For interpretations of biases in the real world, see the discussion in Dessein [7], among others.

- $\alpha(m)$ must be an optimal action for R given R's posterior belief and S's exit strategy, i.e.,

$$\alpha(m) \in \arg\max_{a \in A} \int_{t \in T} \left\{ \epsilon(t,a) U^R + (1 - \epsilon(t,a)) y^R(t,a) \right\} p(m,t) dt, \quad \forall m \in M.$$

- $\epsilon(t,a)$ must be an optimal exit choice for type t of S given a realized action a, i.e.,

$$\forall t \in T, \forall a \in A, \begin{cases} y^S(t,a) < U^S & \Rightarrow \epsilon(t,a) = 1, \\ y^S(t,a) > U^S & \Rightarrow \epsilon(t,a) = 0. \end{cases}$$

We state that an action a is *induced on the equilibrium path* if there exists $t \in T$ such that $a = \alpha \circ \mu(t)$ and $\epsilon(t,a) = 0$. For ease of exposition, given an interval $\tau \subseteq T$ where $\inf \tau = \underline{t}$ and $\sup \tau = \bar{t}$, the posterior density function based on the observation that $t \in \tau$ is denote by f_τ; that is

$$f_\tau(t) = \begin{cases} \frac{1}{\bar{t} - \underline{t}} & \text{if } t \in \tau, \\ 0 & \text{otherwise.} \end{cases}$$

3 Results

3.1 Preliminary Results: Environment without an Exit Option

We first revisit CS's results in an environment without an exit option. If the exit option is not available, perfect information transmission via cheap talk does not occur. This is mainly because S has an incentive to send a upwardly biased message to R.

To be more precise, CS shows that in any equilibrium, there are finite intervals partitioning T and S informs R about which interval a true state is lying in via cheap talk. The necessary and sufficient condition for the existence of the equilibrium with N intervals is

$$b < \left\langle \frac{1}{2N(N-1)} \right\rangle, \tag{1}$$

where $\langle \cdot \rangle$ is the operator such that

$$\left\langle \frac{x}{y} \right\rangle = \begin{cases} \frac{x}{y}, & \text{if } y \neq 0, \\ \infty, & \text{if } y = 0, \ x \neq 0. \end{cases}$$

In other words, regarding N as the informativeness of the cheap talk, the informativeness is determined by the bias b. The smaller b is, the more intervals the equilibrium has.

Henceforth, we only focus on the most informative equilibrium or the equilibrium with the most intervals.[9] Indeed, CS shows that the equilibrium with the most intervals is Pareto superior to any other equilibrium with fewer intervals.

3.2 Environment with an Exit Option: Case of Large Bias

Hereafter, we consider the environment in which an exit option is available for S. In this situation, when D^R is so large that R has a strong incentive to avoid S's exit, the introduction of S's exit option drastically changes the nature of equilibria. To see this, consider the case in which $b \geq \frac{1}{2}$. (1) implies that if the exit option is not available, there exists only a babbling equilibrium. Even if the exit option is available, when S's difference D^S is sufficiently large, the existence of the exit option has no effect on equilibrium behavior since it is anticipated that S would never choose the exit option.

However, when S's exit becomes more credible, there can be equilibria in which some positive amount of information is transmitted via cheap talk. Consider the extreme case in which $D^S = 0$. In this case, S has no incentive to exercise the exit option if and only if R chooses S's optimal action $t + b$ at every state. Since it is anticipated that R would do the best to avoid S's exit, S has an incentive to perfectly informs R of the true state. In other words, perfect information transmission can be done via cheap talk.

To a less extreme degree, consider the case in which $D^S > 0$. Even in this case, there exists some informative equilibria as long as D^R is sufficiently large and D^S is sufficiently small. Indeed, we can construct such an equilibrium as follows: first, we recursively define a decreasing sequence $\{s_n\}_{n=0}^N$ and an associated action sequence $\{a_n\}_{n=1}^N$. We set $s_0 = 1$. For $n \geq 0$,

1. if $s_n = 0$, we stop the recursive process and denote n by N;

2. if $s_n > 0$, then we pick the larger solution a' of the equation $(s_n + b - a')^2 = D^S$ and name it as a_{n+1}. Furthermore, we pick up the smaller solution s' of the equation

[9] Che et al. [4] present a condition that selects the most informative equilibrium in uniform-quadratic models.

$(s' + b - a_{n+1})^2 = D^S$, and we define $s_{n+1} = \max\{s', 0\}$.[10]

Next, we specify equilibrium strategies such that

- the state space $[0, 1]$ is partitioned into $[s_N, s_{N-1}], (s_{N-1}, s_{N-1}], \ldots, (s_1, s_0]$,
- S informs R of which interval the realized state is lying on via cheap talk messages,
- when R is informed that the state is lying on the nth interval from the right, she chooses an action a_n, and
- when S is indifferent between exit and stay, he stays.

For each interval except $[s_N, s_{N-1}]$, the associated action is the only one that can deter S's exit when the state is lying on that interval. Therefore, R has no incentive to deviate from the equilibrium strategy when D^R is sufficiently large.[11]

For the last interval from the right, $[s_N, s_{N-1}]$, the associated action a_N is the one that makes S indifferent between exit and stay at the right end of the interval, s_{N-1}. This means that if R would choose a smaller action than a_N, S would exit around the right end. Since R has no incentive to choose a larger action than a_N, R has no incentive to choose an action other than a_N when she is informed that the state is lying on the last interval.

Furthermore, when D^S is sufficiently small,[12] it is verified that $N \geq 2$. In other words, some positive amount of information can be transmitted via cheap talk. While it is never exercised on the equilibrium path, the existence of the exit option can induce more informative communication.

3.3 Characterization of No-Exit Equilibria

In this subsection, we focus on the equilibrium in which the exit option is never exercised on the equilibrium path. We assume that $D^S > 0$. We call it *no-exit equilibrium (NEE)*.

[10]We can derive more explicit forms as follows:

$$N\text{: a natural number satisfying } \frac{1}{2N} \leq \sqrt{D^S} < \frac{1}{2(N-1)},$$

$$s_n = \begin{cases} 1 - 2n\sqrt{D^S} & n = 0, \ldots, N-1, \\ 0 & n = N, \end{cases}$$

$$a_n = 1 + b - (2n-1)\sqrt{D^S} \quad n = 1, \ldots, N.$$

[11]More precisely, it is the case when $\sqrt{D^R} \geq \sqrt{D^S} + b$.

[12]More precisely, it is the case when $D^S < \frac{1}{4}$.

The following result asserts that any NEE is characterized by a partition of the state space consisting of finite number of intervals.[13]

Lemma 1 In any equilibrium, there are only finite actions induced on the equilibrium path.

All proofs are relegated to Appendix. Lemma 1 implies that any NEE $(\mu, p, \alpha, \epsilon)$ is characterized by a partition $\{\tau_n\}_{n=1,\dots,N}$ of $[0, 1]$ such that

- N is finite;
- τ_n is an interval for $n = 1, \dots, N$; and
- there exist $\{t_n\}_{n=0,\dots,N}$, $\{m_n\}_{n=1,\dots,N}$, $\{a_n\}_{n=1,\dots,N}$ such that
  - * $\inf \tau_n = t_{n-1}$ and $\sup \tau_n = t_n$ for $n = 1, \dots, N$,
  - * $0 = t_0 < t_1 < \dots < t_N = 1$,[14]
  - * $\mu(t) = m_n$ for any $t \in \tau_n$, $m_n \neq m_{n'}$ for $n \neq n'$, and therefore, $p(m_n, t) = f_{\tau_n}(t)$ for $n = 1, \dots n$,
  - * $\alpha(m_n) = a_n$ for $n = 1, \dots, N$, and
  - * $\epsilon(t, a_n) = 0$ for $t \in \tau_n$ and $n = 1, \dots, N$.

Below, we derive the equilibrium condition for NEE with N intervals. The following result shows that any interval of NEE is classified into three categories.

Lemma 2 Fix an NEE and an interval $\hat{\tau}$ from the equilibrium partition $\{\tau_n\}_{n=1,\dots,N}$. Let $\inf \hat{\tau} = \underline{t}$, $\sup \hat{\tau} = \bar{t}$, and $\hat{a} = \alpha \circ \mu(t)$ for $t \in \hat{\tau}$. Then, $\bar{t} - \underline{t} \leq 2\sqrt{D^S}$. Moreover, $\hat{\tau}$ belongs to any one of the following categories:

Interval $\mathcal{N}$: $\underline{t}$, $\bar{t}$, and $\hat{a}$ satisfy

- $\bar{t} - \underline{t} < 2\sqrt{D^S} - 2b$,
- $\hat{a} = \frac{\underline{t}+\bar{t}}{2}$,

[13] Generally, there can be an equilibrium with infinite number of intervals. This is because no length constraint is necessary for intervals in which S is induced to choose the exit option at any state (Interval $\mathcal{E}$ in an online appendix [19]).

[14] We denote a increasing (decreasing) sequence of thresholds by $\{t_n\}$ ($\{s_n\}$, respectively).

- $y^S(\underline{t}, \hat{a}) > U^S$, and
- $y^S(\bar{t}, \hat{a}) > U^S$.

Interval $\mathcal{A}$: $\underline{t}$, $\bar{t}$, and $\hat{a}$ satisfy

- $2\sqrt{D^S} > \bar{t} - \underline{t} \geq 2\sqrt{D^S} - 2b$,
- $\hat{a} = \bar{t} - \sqrt{D^S} + b$,
- $y^S(\underline{t}, \hat{a}) > U^S$, and
- $y^S(\bar{t}, \hat{a}) = U^S$.

Interval $\mathcal{F}$: $\underline{t}$, $\bar{t}$, and $\hat{a}$ satisfy

- $\bar{t} - \underline{t} = 2\sqrt{D^S}$,
- $\hat{a} = \bar{t} - \sqrt{D^S} + b$, and
- $y^S(\underline{t}, \hat{a}) = y^S(\bar{t}, \hat{a}) = U^S$.

Furthermore, in any interval, the receiver has an incentive to choose $\hat{a}$ if $\sqrt{D^R} \geq \sqrt{D^S} + b$.

Interval $\mathcal{N}$ is a non-accommodating interval in the sense that R can choose her optimal action without inducing S's exit. Interval $\mathcal{A}$ is an accommodating interval in that the constraint for no exit is binding at the right end of the interval and R has to choose an action more favorable for S than R's optimal action. Interval $\mathcal{F}$ is a fully accommodating interval in that the constraint for no exit is binding at both ends of the interval and any other action than $\hat{a}$ necessarily induces S to exercise the exit option at some states. This lemma also asserts that any NEE interval cannot be longer than $2\sqrt{D^S}$, for otherwise S would exercise the exit option at some states no matter which action R chooses.

The important fact is that on the boundary state of two adjoining intervals, S must be indifferent between sending messages corresponding to the intervals. This implies that possible equilibrium configurations of intervals are restricted. For example, an interval $\mathcal{F}$ cannot be directly connected to an interval $\mathcal{N}$, for otherwise S will have a strict incentive to choose the action corresponding to the interval $\mathcal{N}$ at any state sufficiently close to the boundary point. By exhausting all possibilities, we can derive the equilibrium condition for S. The following is the formal statement:

Lemma 3 Given any NEE with N intervals,

(i) for $N = 1$, the equilibrium condition for the sender is $\sqrt{D^S} \geq \frac{1}{2N}$, and

(ii) for $N \geq 2$, a configuration of intervals is any one of the following five patterns:

(I) $\mathcal{N}, \ldots, \mathcal{N}$,

(II) $\mathcal{N}, \ldots, \mathcal{N}, \mathcal{A}$,

(III) $\mathcal{N}, \ldots, \mathcal{N}, \mathcal{A}, \mathcal{F}, \ldots, \mathcal{F}$,[15]

(IV) $\mathcal{A}, \mathcal{F}, \ldots, \mathcal{F}$, or

(V) $\mathcal{F}, \ldots, \mathcal{F}$.

The equilibrium condition for the sender in each case is the following:

(I) $b < \frac{1}{2N(N-1)}$ and $\sqrt{D^S} > \frac{1}{2N} + Nb$.

(II) $\sqrt{D^S} < 1 - (2N^2 - 4N + 1)b$ and $\frac{1}{2N} + \frac{(N-1)^2}{N}b < \sqrt{D^S} \leq \frac{1}{2N} + Nb$.

(III) $\sqrt{D^S} < \frac{1-(2i^2-4i+1)b}{2N-2i+1}$ and $\frac{1}{2N} + \frac{(i-1)^2}{N}b < \sqrt{D^S} \leq \frac{1}{2N} + \frac{i^2}{N}b$ for some $i = 2, \ldots, N-1$.

(IV) $\sqrt{D^S} < \frac{1}{2(N-1)}$ and $\frac{1}{2N} < \sqrt{D^S} \leq \frac{1}{2N} + \frac{1}{N}b$.

(V) $\sqrt{D^S} = \frac{1}{2N}$.

These conditions are illustrated in Figure 1. Given R's response, the equilibrium conditions consists of the indifference condition for S and the length constraints on each type of intervals. The details are as follows:

(I): The 1st condition is the indifference condition. The 2nd condition is the length constraint on $\mathcal{N}$.

(II): The 1st condition is the indifference condition. The 2nd conditions is the length constraints on $\mathcal{A}$.[16]

(III): The 1st condition is a mixture of the indifference condition and the length constraint on $\mathcal{F}$. The 2nd condition is the length constraint on $\mathcal{A}$.[17]

[15] This case occurs only if $N \geq 3$.

[16] It is verified that the length constraints on $\mathcal{N}$ are satisfied whenever the other conditions are satisfied.

[17] Similarly, it is verified that the other conditions implies the length constraint on $\mathcal{N}$.

(IV): The 1st condition is a mixture of the indifference condition and the length constraint on $\mathcal{F}$. The 2nd condition is the length constraint on $\mathcal{A}$.

(V): The condition is the length constraint on $\mathcal{F}$.

<Figures 1 should be inserted>

Combined with these lemmas, we derive the equilibrium condition for NEE.

Theorem 1 Suppose $\sqrt{D^R} \geq \sqrt{D^S} + b$. Then, an NEE with N intervals exists if and only if both (1) and (2) hold:

(1) Any one of (1-1)–(1-3) holds:[18]

(1-1) $b < \left\langle \frac{1}{2N(N-1)} \right\rangle$,

(1-2) $\sqrt{D^S} < \frac{1-(2i^2-4i+1)b}{2N-2i+1}$ for some $i = 2, \ldots, N$, or

(1-3) $\sqrt{D^S} < \left\langle \frac{1}{2(N-1)} \right\rangle$.

(2) $\sqrt{D^S} \geq \frac{1}{2N}$.

This theorem asserts that an NEE with sufficiently large number of intervals exists if and only if b is sufficiently small and/or D^S is sufficiently small as long as D^R is sufficiently large. Identifying the equilibrium number of intervals with *the informativeness of cheap talk*, we can interpret this result as follows: in our model, there are two determinants of the informativeness of cheap talk. One is the smallness of b, which refers to the degree of incongruence between S's and R's preferences. This is extensively discussed by CS and in other literature. The other is the smallness of D^S, which is newly found. We interpret the smallness of D^S as a degree of *S's credibility of exit*. In other words, the smaller D^S is, the more credible S's threat of exit is and the more informative information cheap talk can convey. Henceforth, we call such an NEE *an NEE driven by the credibility of exit*.[19]

Furthermore, the previous theorem implies the following important facts:

Corollary 1 Suppose that $\sqrt{D^R} > b$. Then, as U^S approaches Y^S (equivalently, D^S approaches 0), there exists a sequence of equilibria in which $\alpha \circ \mu(t)$ converges pointwise to $t + b$.

[18]For the definition of the operator $\langle \cdot \rangle$, see Section 3.1.

[19]To be more precise, an NEE driven by the credibility of exit is defined as an NEE that does not include $\mathcal{N}$ as an equilibrium interval, or NEE with a configuration of intervals (IV) or (V) in Lemma 3.

This corollary implies that approximately perfect information transmission is possible via cheap talk in the limit. The corollary also implies that the equilibrium actions converge to the optimal action for S at any state. In other words, our result implies that even if a commitment to delegation is impossible, the credibility of the exit option can bring about a similar outcome.

Corollary 2 Suppose that $\sqrt{D^R} > b$ and $b < \frac{1}{2\sqrt{3}}$. Then, if U^S is sufficiently close to Y^S (equivalently, D^S is sufficiently close to 0), there exists an equilibrium in the environment with the exit, in which S's and R's ex ante payoffs are both larger than those in any equilibrium in the environment without the exit.

This corollary implies that the presence of S's exit option increases the ex ante payoff of R as well as S. Therefore, giving S an exit option is Pareto-improving.

Remark 1 Corollaries 1 and 2 imply that, as U^S approaches Y^S, an equilibrium outcome nearly seems to be one that occurs when R would delegate her decision right to S, and it is desirable even for R as long as b is sufficiently small. We can also show that an almost delegation outcome can be carried out by the following simple contract. Let us consider the situation in which players' maximum stay payoffs are determined by the splitting of the joint surplus Y. Before a state is realized, R proposes a contract that specifies an allocation of Y between S's share Y^S and R's share Y^R. This contract does not depend upon the message. If $Y > U^S + U^R + b^2$, an allocation with $Y^S = U^S + \varepsilon$ and $Y^R = Y - Y^S$ for sufficiently small ε leads to an almost delegation outcome because it satisfies the premise of Theorem 1 for a large N.[20]

3.4 Characterization of General Equilibria

The condition $\sqrt{D^R} \geq \sqrt{D^S} + b$ is the sufficient condition for Theorem 1. We can show that it is also necessary for the existence of informative NEE driven by the credibility of exit.

[20] Dessein [7] shows that the manager (the receiver) can realize more efficient outcome by delegating her decision right to her subordinate (the sender) who has a specialized information about the current state. However, it also has been recognized that delegating the formal decision right to the subordinate might be very difficult because the manager often has a temptation to overrule the subordinate's decision (Baker et al. [2] and Alonso and Matouschek [1]). Interpreting our results from this point of view, it can be said that the credibility of exit is one way to successful delegation.

In an online appendix [19], we derive the conditions for equilibria including those other than NEEs. Generally, it can be the case that R intentionally induces S to exercise the exit option. Also, in the online appendix, we investigate equilibria when D^R is not so large and present an example in which an increase in R's exit payoff can lead to a *decrease* in her equilibrium payoff.

What happens in the example is as follows. When R's exit payoff is not so large, R attempts to deter S from exercising the exit and then there exists an informative NEE driven by the credibility of exit. Once R's exit payoff, however, exceeds some threshold level, R does no longer do her best to stop S's exit. This brings about a situation in which any informative NEE cannot be sustained and there is only a babbling equilibrium. In other words, an increase in R's exit payoff deteriorates the informativeness of equilibrium communication, which in turn reduces her equilibrium payoff. All in all, we conclude that R's exit aversion is necessary for the credibility of exit to serve for the informative communication.

4 Concluding Remarks

This paper investigates the interplay between exit and voice by analyzing a modified version of Crawford and Sobel's [6] model in which the sender has an exit option after the receiver makes a decision. The key feature of our results is the difference between players' maximum stay payoff and exit payoff. We find that in the case where the receiver's difference is large and the sender's difference is small but positive, the latter's exit is so credible that the former makes a decision that is desirable to the latter so as to prevent him from exercising the exit option; through this, accurate information can be transmitted via cheap talk on the equilibrium. In other words, it is shown that the informativeness of cheap talk is determined by not only the degree of incongruence between both players' preferences but also the credibility of the sender's exit, which is measured by the smallness of the sender's difference. Furthermore, the perfect information transmission via cheap talk can be approximated in the limit. To the author's knowledge, these results are unprecedented in the literature on cheap talk with private information.

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Appendix

Proof of Lemma 1

Suppose, to the contrary, that there exists an equilibrium with infinite actions induced on the equilibrium path. Then, there must exist actions a_1, a_2, and a_3 induced on the equilibrium path such that $a_1 < a_2 < a_3$ and $a_3 - a_1 < \min\{\sqrt{D^S}, b\}$. Taking S's incentive into consideration, the following must hold:

$$\{t|\alpha \circ \mu(t) = a_2\} \subseteq (\max\{a_1 - b, 0\}, a_3 - b) \neq \emptyset.$$

Since for any $t < a_3 - b$,

$$\sqrt{D^S} > a_3 - a_1 > t + b - a_1 > t + b - a_2,$$

$\epsilon(t, a_1) = \epsilon(t, a_2) = 0$ must hold for any t such that $\alpha \circ \mu(t) = a_2$. On the other hand, for any $t < a_3 - b$,

$$a_2 > a_1 > a_3 - b > t.$$

This means

$$-\int_{\{t|\alpha\circ\mu(t)=a_2\}} (t - a_1)^2 dt > -\int_{\{t|\alpha\circ\mu(t)=a_2\}} (t - a_2)^2 dt.$$

It follows that R would deviate to choosing a_1 after receiving a message inducing a_2. This is a contradiction. ∎

Proof of Lemma 2

First of all, if $t < \hat{a} - b - \sqrt{D^S}$ or $t > \hat{a} - b + \sqrt{D^S}$, then $\epsilon(t, \hat{a}) = 1$. This implies that the length of any NEE interval must be less than or equal to $2\sqrt{D^S}$. Henceforth, we focus on the case in which $\bar{t} - \underline{t} \leq 2\sqrt{D^S}$.

Let us denote R's expected equilibrium payoff on choosing a conditional on the assumption that $t \in \hat{\tau}$ by $\tilde{V}^R(a)$ and, for ease of exposition, define $W(a) = (\bar{t} - \underline{t})(\tilde{V}^R(a) - U^R)$.

Then, we obtain the following expression:

$$W(a) =$$

$$\begin{cases} 0 & \text{if } a \in A_1 = (-\infty, \underline{t} + b - \sqrt{D^S}], \\ (a - b + \sqrt{D^S} - \underline{t})D^R - \int_{\underline{t}}^{a-b+\sqrt{D^S}} (t-a)^2 dt & \text{if } a \in A_2 = (\underline{t} + b - \sqrt{D^S}, \bar{t} + b - \sqrt{D^S}), \\ (\bar{t} - \underline{t})D^R - \int_{\underline{t}}^{\bar{t}} (t-a)^2 dt & \text{if } a \in A_3 = [\bar{t} + b - \sqrt{D^S}, \underline{t} + b + \sqrt{D^S}], \\ (\bar{t} - a + b + \sqrt{D^S})D^R - \int_{a-b-\sqrt{D^S}}^{\bar{t}} (t-a)^2 dt & \text{if } a \in A_4 = (\underline{t} + b + \sqrt{D^S}, \bar{t} + b + \sqrt{D^S}), \\ 0 & \text{if } a \in A_5 = [\bar{t} + b + \sqrt{D^S}, \infty). \end{cases}$$

In any NEE, the optimal action must lie on A_3, for otherwise some type of S would choose an exit option. The unique local maximizer on A_3, denoted by a^*, is

$$a^* = \begin{cases} \frac{\underline{t}+\bar{t}}{2} & \text{if } \bar{t} - \underline{t} < 2\sqrt{D^S} - 2b, \\ \bar{t} + b - \sqrt{D^S} & \text{if } \bar{t} - \underline{t} \geq 2\sqrt{D^S} - 2b. \end{cases}$$

Note that $a^* = \hat{a}$ in any case. When $\bar{t} - \underline{t} < 2\sqrt{D^S} - 2b$, it is verified that $y^S(\underline{t}, a^*) > 0$ and $y^S(\bar{t}, a^*) > 0$ hold. In the case of $\bar{t} - \underline{t} \geq 2\sqrt{D^S} - 2b$ it is verified $y^S(\bar{t}, a^*) = 0$ and

$$y^S(\underline{t}, a^*) \begin{cases} > U^S & \text{if } \bar{t} - \underline{t} < 2\sqrt{D^S}, \\ = U^S & \text{if } \bar{t} - \underline{t} = 2\sqrt{D^S}, \end{cases}$$

hold.

In both cases, it is verified that $\sqrt{D^R} \geq \sqrt{D^S} + b$ implies that $W(a^*) \geq 0$ and W has no local maximum on A_2 and A_4. It follows that a^* is a global optimal action. ∎

Proof of Lemma 3

Condition (i) is immediately proved from Lemma 2. Throughout the proof, we consider $N \geq 2$. On the boundary point of two adjoining intervals, S must be indifferent between sending actions corresponding to the intervals. By Lemma 2, they must be any one of the following cases:

- $\mathcal{N}\mathcal{N}$,
- $\mathcal{N}\mathcal{A}$,
- $\mathcal{A}\mathcal{F}$, or
- $\mathcal{F}\mathcal{F}$.

This implies that possible configurations of intervals of NEE is restricted to (I)–(V).

Consider (I). In this type of equilibrium, by the analysis in CS (see Section 3.1),

$$t_n = nt_1 + 2n(n-1)b, \quad n = 0, \ldots, N,$$

where

$$t_1 = \frac{1 - 2N(N-1)b}{N}.$$

The equilibrium condition is

$$\begin{aligned}
&t_1 - t_0 > 0, \\
&t_N - t_{N-1} < 2\sqrt{D^S} - 2b.
\end{aligned}$$

Then, we obtain

$$\begin{aligned}
&b < \left\langle \frac{1}{2N(N-1)} \right\rangle, \\
&\sqrt{D^S} > \frac{1}{2N} + Nb.
\end{aligned}$$

Consider (II). In this type of equilibrium,

$$\begin{aligned}
t_n &= \begin{cases} nt_1 + 2n(n-1)b, & n = 0, \ldots, N-1, \\ 1, & n = N, \end{cases} \\
a_n &= \begin{cases} \frac{t_{n-1}+t_n}{2}, & n = 1, \ldots, N-1, \\ 1 - \sqrt{D^S} + b, & n = N. \end{cases}
\end{aligned}$$

The equilibrium condition is

$$\begin{aligned}
&y^S(t_{N-1}, a_{N-1}) = y^S(t_{N-1}, a_N), \\
&t_1 - t_0 > 0, \\
&t_{N-1} - t_{N-2} < 2\sqrt{D^S} - 2b, \\
&t_N - t_{N-1} > 0, \\
&2\sqrt{D^S} > t_N - t_{N-1} \geq 2\sqrt{D^S} - 2b.
\end{aligned}$$

Then, we obtain

$$\begin{aligned}
&\sqrt{D^S} < 1 - (2N^2 - 4N + 1)b, \\
&\frac{1}{2N} + \frac{(N-1)^2}{N} b < \sqrt{D^S} \leq \frac{1}{2N} + Nb,
\end{aligned}$$

where

$$t_1 = \frac{2 - 2\sqrt{D^S} - 2(2N^2 - 4N + 1)b}{2N - 1}.$$

Consider (III). Given any $i = 2, \ldots, N-1$, consider the following configuration:

$$\underbrace{\mathcal{N}, \ldots, \mathcal{N}}_{i-1 \text{ times}}, \mathcal{A}, \underbrace{\mathcal{F}, \ldots, \mathcal{F}}_{N-i \text{ times}}.$$

In this type of equilibrium,

$$t_n = \begin{cases} nt_1 + 2n(n-1)b, & n = 0, \ldots, i-1, \\ 1 - 2(N-n)\sqrt{D^S}, & n = i, \ldots, N, \end{cases}$$

$$a_n = \begin{cases} \frac{t_{n-1}+t_n}{2}, & n = 1, \ldots, i-1, \\ t_n - \sqrt{D^S} + b, & n = i, \ldots, N. \end{cases}$$

The equilibrium condition is

$$\begin{aligned} & y^S(t_{i-1}, a_{i-1}) = y^S(t_{i-1}, a_i), \\ & t_1 - t_0 > 0, \\ & t_{i-1} - t_{i-2} < 2\sqrt{D^S} - 2b, \\ & t_i - t_{i-1} > 0, \\ & 2\sqrt{D^S} > t_i - t_{i-1} \geq 2\sqrt{D^S} - 2b. \end{aligned}$$

Then, we obtain

$$\begin{aligned} & \sqrt{D^S} < \frac{1 - (2i^2 - 4i + 1)b}{2N - 2i + 1}, \\ & \frac{1}{2N} + \frac{(i-1)^2}{N} b < \sqrt{D^S} \leq \frac{1}{2N} + \frac{i^2}{N} b, \end{aligned}$$

where

$$t_1 = \frac{2 - 2(2N - 2i + 1)\sqrt{D^S} - 2(2i^2 - 4i + 1)b}{2i - 1}.$$

Consider (IV). In this type of equilibrium,

$$t_n = \begin{cases} 0, & n = 0, \\ 1 - 2(N-n)\sqrt{D^S}, & n = 1, \ldots, N. \end{cases}$$

The equilibrium condition is

$$
\begin{aligned}
&t_1 - t_0 > 0, \\
&2\sqrt{D^S} > t_1 - t_0 \geq 2\sqrt{D^S} - 2b.
\end{aligned}
$$

Then, we obtain

$$
\begin{aligned}
\sqrt{D^S} &< \frac{1}{2(N-1)}, \\
\frac{1}{2N} < \sqrt{D^S} &\leq \frac{1}{2N} + \frac{1}{N}b.
\end{aligned}
$$

The derivation of the equilibrium condition for (V) is immediate. Since in this type of equilibrium,

$$t_n = 2\sqrt{D^S}n, \quad n = 0, \dots, N,$$

it must hold that $t_N = 1$, or equivalently,

$$\sqrt{D^S} = \frac{1}{2N}.$$

∎

Proof of Corollary 1

It is obtained directly from Theorem 1 and on the basis of the fact that each interval has a length of $2\sqrt{D^S}$ or less (Lemma 2). ∎

Proof of Corollary 2

By Corollary 1, it is obvious that the sequence of S's ex ante equilibrium payoffs V^S converges to Y^S as $D^S \to 0$. Similarly, as for R's ex ante equilibrium payoff V^R,

$$V^R - V^S = Y^R - Y^S + b\int_0^1 (2t + b - 2\alpha \circ \mu(t))^2 dt \to Y^R - Y^S - b^2$$

as $D^S \to 0$. Then, V^R converges to $Y^R - b^2$. On the other hand, according to CS, S's and R's largest equilibrium ex ante payoffs in the environment without the exit are as follows:

$$
\begin{aligned}
\hat{V}^S &= Y^S - \frac{4N^2(N^2+2)b^2+1}{12N^2}, \\
\hat{V}^R &= Y^R - \frac{4N^2(N^2-1)b^2+1}{12N^2},
\end{aligned}
$$

respectively, where N is the largest natural number satisfying (1). By a direct calculation, if $b < \frac{1}{2\sqrt{3}}$, then

$$\hat{V}^S < Y^S,$$
$$\hat{V}^R < Y^R - b^2.$$

This completes the proof. ∎

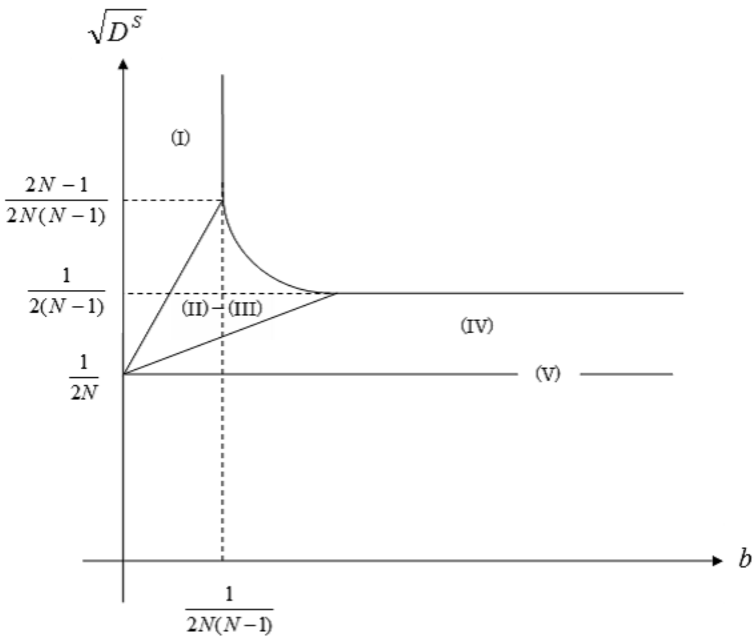

Figure 1: Equilibrium condition of NEE with N intervals ($N \geq 2$)