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OPTIMAL COMPENSATION RULES FOR INTEGRATED SERVICES

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Abstract

Research on service compensation is rare. In this article we examine different compensation rules for integrated services (IS) which are produced jointly by a service provider and his client. Examples are consulting, advertising or management training. We distinguish three different compensation rules and compare them with joint profit maximization where both, service provider and client act as one organizational unit. The ompensation rules are (1) the input based compensation (IBC) characterized by a compensation that is based on work hours or work days, (2) the sales based compensation (SBC) with the compensation based on client's sales and (3) the profit based compensation (PBC) with compensation based on client's profits. We can show that under reasonable, realistic conditions the IBC (surprisingly) leads to better results for the service provider and for the client as compared to the PBC and the SBC.

Keywords

Services, Compensation rules, Pricing of services, Joint production

1 Introduction

Consultants are compensated on the basis of man-days they work on the client's project regardless of the profit the clientmakes from the project. Ad agencies are often compensated on the basis of the media costs for a campaign and not on the basis of the campaign's success. Training formanagers and workers is usually compensated based on the number of seminar days the training service company offers. Even if there exist additional cost factors, e.g. physical input costs such as printing material, or sometimes the compensation may be paid as a lump sum the usual way of compensating services is based on input related figures: The man-days in the consultant example, the media cost in the ad agency case and the seminar days in the training case. All these quantities are inputs for the service the service provider offers. Therefore we call a compensation based on such inputs an input based compensation (IBC). However, there may be different "unit prices". The unit price for one man-day offered by company A may be different to the compensation of one man-day offered by company B expressing different qualities or values of the service. The IBC is often used because the output of the service is not only determined by the work of the service provider but also by the work of the service client. If an ad agency gets an insufficient briefing the campaign may not be as successful as it could be with an excellent briefing. If there is no briefing at all the ad agencywill not be able to work on a campaign for that company since the ad agencies' employees do neither have any detailed information on current market position nor on future targets.

However, the management literature suggests that compensating for services should be based on the value for the client rather than on the service provider's cost (Lovelock et al. 1999). The client's value achieved due to the service can be interpreted as the additional turnover, which can easily be used as a basis for compensation. One could think of a fraction α of the additional sales value the client achieves determined by the service after costs. We call this compensation sales based compensation (SBC). A second way to think about client's value achieved could be the profit generated by the service. We will call it profit based compensation (PBC).

Another way to compensate for services is recommended in the marketing literature: "devise payfor-performance structures for contractible subsets but not for other sets." (Carson et al. 1999, p. 129). Taking the performance as a basis for compensation we could think about the whole profit generated by the joint service. Service provider and client could maximize the whole profit of the service i.e. a company and an ad agency could try to maximize the profit generated by an ad campaign and then share the profit. We will call this benchmark joint profit maximization (JPM).

We see a contradiction between the compensation rules in use compared to the compensation rule recommended in the managerial literature. As Noble and Gruca (1999) observe there is no systematic examination on the compensation of services. Berry and Yadav even state, that from a practical point of view "the pricing of services in the United States is a mess" (Berry and Yadav 1996, p. 41). Here, we try to make a first step in this direction.

King (1968) analyzes the compensation of ad agencies but does not take into account clients' input to the service. Compensation is typically discussed as an intra-organizational topic relating to sales agents (e.g. Coughlan and Sen 1989; Kissan and Thevaranjan 1998; Krafft 1999; Reichheld and Rogers 2005).

This article looks at different compensation rules for services as a phenomenon between organizations. We examine services that are produced jointly by service provider and client like consulting, advertising or management training.

We examine services with two fundamental characteristics: (1) The service requires input from service provider and client. Based on this first assumption we can be more specific about the nature of substitutability: (2) Service provider's input 356 H. Löbler et al. and client's input can be substituted by each other to some degree as specified below.

Although the input of service provider and client may be substituted to some extent by each other, both parties have still some core competence that cannot be substituted. Consulting firms are typically able to work with certain concepts or techniques based on the human capital that the client companies may not know or may not be able to apply. Client companies on the other hand provide specific company information that is a precondition for the consulting firm's work. Consulting firms do usually

not have this information before they cooperate with their clients. In the advertising agency example ad agencies but not the clients know how to design a marketing strategy whereas clients but not ad agencies know future plans and current market positions of the clients' firms. Finally, training firms but not students know how to teach specific skills and specific knowledge. Simultaneously, students but not training firms have to provide the time input to allow a learning process. The essence is that both parties involved in the service production are unable to achieve a positive service output without the other party's input. Obviously, there exist other services, e.g. car repair, where the client's input i.e. time can be neglected.

The second service characteristic implies that an increase in service provider's input may be substituted by the client's input. Consulting firms e.g. can decrease the number of hours they work in a client's company if the client's employees provide certain inputs, e.g. preparatory work, collecting information within the client firm or conducting interviews with employees. Advertising agencies may be able to decrease their input if the client provides more information about the market the client is competing in. Training firms may offer programs with smaller number of seminar days if the students to be trained increase the number of hours they work by themselves. In all these examples we observe substitutability between the service provider's and the client's inputs.

We define a service where both of these features apply as integrated service (IS) and a service that can be provided by the service provider without substantial client support as a non-integrated service (NIS). An IS is always provided by both parties, the service provider and the service client, either in a sequential processing, in a simultaneous processing or both combined. E.g. consultants need to discuss the work with their clients and in doing so the clients as well as the consultants substantially determine the outcome of the project. The consulting process includes simultaneous work on the project. After such discussions the clients may have to prepare some figures of their company which the consultants need for their further work. Here both parties are working together in a sequential processing. Compared to a NIS where the garage does not need any work of her client. In our article we focus on the compensation of IS.

The plan of the paper is as follows: In Section 2 we describe the model. First we focus on modeling integrated services and explaining basic assumptions and then we describe the modeling of different compensation rules. In Section 3 we derive profit maximizing inputs for different compensation rules: In Section 3.1 inputs for the profit based compensation rule (PBC), in Section 3.2 inputs for the sales based compensation rule (SBC) and in Section 3.3 the inputs for input based compensation rule (IBC). In Section 3.4 we develop a frame for comparing the compensation rules from different perspectives. In Section 4 we compare the different rules: First we compare PBC with IBC and then we compare SBC with IBC. Finally, we discuss the results and their limitations in the last section.

2 The model

In this section, we first explain how we model integrated services. Then, we describe the three different compensation rules we compare as well as the decisions the service provider and the client have to make.

2.1 Modeling integrated services

Modeling the joint service production we use a very general production function, which assures that there is only an outcome if both, the service agent and the client, have positive inputs. The outcome Q is a function of the two inputs. We denote the service provider's input I_S (the agent) and the service client's input I_S . Q is the value of the service output produced by the two inputs. The input can e.g. be thought of man-days in a consulting environment or seminar days in the training environment.

$$Q(I_{\rm s}, I_{\rm c}) = bI_{\rm s}^{u}I_{\rm c}^{v}$$

The variables u and v are the elasticities of production. We assume that $0 \le u \le 1$ and $0 \le v \le 1$ as well as $0 \le u + v \le 1$. The joint production of the one service has a diminishing return in both arguments separately as well as jointly. We assume that $Q = Q(I_{s_r}, I_c)$ is twice differentiable in each argument.

It is assumed that both parties have full knowledge about all parameters of the model. The production function describes one project. Service provider and client have already agreed upon working together in that project but they have not yet negotiated about compensation and inputs.

Clearly, these requirements qualify the inputs as incomplete substitutes. These assumptions are plausible for a wide class of services, e.g. consulting and training services.

To maximize profits we have to assume a cost function for service provider as well as client. The cost function is for the service provider:

$$C_{\rm S}(I_{\rm S}) = c_{\rm S} I_{\rm S}^{\sigma}$$
 with $\sigma > 1$

and for the client:

$$C_{\rm C}(I_{\rm C}) = c_{\rm C} I_{\rm C}^{\tau}$$
 with $\tau > 1$

The cost function is convex. Part of the analysis is also valid for linear cost functions.

We assume that the service provider and client have already agreed that they work jointly on that project. However, they have not yet determined the compensation rule and their profit maximizing input which of course depends on the compensation rule.

To concentrate on the different compensation rules we exclude competition from our analysis in the sense that service provider and client have already agreed to cooperate but still have to negotiate the compensation. This seems reasonable because in the case of integrated services one service provider cannot be substituted easily by another service provider from the perspective of the client and vice versa after they have agreed to work together. E.g. consulting firms are specialized on different topics, they rely on different sets of human capital and, consequently, their services are not directly comparable. In a market context, this is reflected by different reputations for different areas of expertise of consulting firms and by relationships that very often last long.

2.2 Compensation rules and service provider's and client's decision

In this section, we describe the three different compensation rules mentioned above: profit-based compensation (PBC), sales-based compensation (SBC) and input-based compensation (IBC). We also examine joint profit maximization which does not serve as compensation rule but as a benchmark to compare the three compensation rules. The joint profit maximum is the highest profit service provider and client can achieve.

2.2.1 JPM—joint profit maximization

$$\max_{I_S, I_C} \Pi = Q(I_S, I_C) - C_C(I_C) - C_S(I_S)$$
(1)
maximizing total profit with respect to I_C and I_S .

The joint profit maximization serves as a benchmark for the comparison with the compensation rules.

The first compensation rule is the profit based compensation (PBC). Here the client offers a fraction α of her profit to the service provider, so the service provider gets the fraction of the client's profit minus her (the service provider's) cost. Each of them maximizes her profit with respect to her input. So the decision rules are:

For the service provider:

$$\max_{I_S} \Pi_S = \alpha (Q(I_S, I_C) - C_C(I_C)) - C_S(I_S)$$

$$= \alpha (bI_S^u I_C^v - c_C I_C^\tau) - c_S I_S^\sigma \text{ service provider's profit.}$$
(2)

For the client:

$$\max_{I_C} \Pi_C = (1 - \alpha) (Q(I_S, I_C) - C_C(I_C))$$

$$= (1 - \alpha) (bI_S^u I_C^v - c_C I_C^\tau) \text{ client's profit.}$$
(3)

2.2.3 SBC—sales based compensation

With this compensation rule the service provider gets a fraction α of the client's sales. Again, each of them maximizes her profit with respect to her input. For the service provider:

$$\max_{I_S} \Pi_S = \alpha Q(I_S, I_C) - C_S(I_S) = \alpha b I_S^u I_C^v - c_S I_S^\sigma \text{ service provider's profit.}$$
(4)

For the client:

$$\max_{I_C} \Pi_c = (1 - \alpha) Q(I_S, I_C) - C_C(I_C) = (1 - \alpha) b I_s^u I_c^v - c_c I_c^\tau \text{ client's profit.}$$
(5)

2.2.4 IBC—input based compensation

Now we assume that the service provider is compensated based on her input which is a quantity multiplied by a price p for an input unit which has to be negotiated. For the service provider:

$$\max_{I_S} \Pi_S = pI_S - C_S(I_S) = pI_S - c_S I_S^{\sigma}$$
(6)

For the client:

$$\max_{I_C} \Pi_C = Q(I_S, I_C) - pI_S - C_C(I_C) = bI_S^u I_C^v - pI_S - c_C I_C^\tau$$
(7)

Under these compensation rules the client has to give at least part of her profit to the service provider otherwise she would not get any input from him.

In Section 3, we analyze the profit maximizing inputs in terms of reaction functions. These reaction functions describe the input of one party (service provider or client) in terms of the input of the other party (service provider or client). In Section 4 we compare the profitability of the three compensation rules.

3 Inputs using different compensation rules

In all cases the service provider maximizes his profit with respect to his input whereas the client maximizes her profit according to her input. The client also has to decide about her input to maximize her profit. Appendix B gives a summary of all explicit solutions derived in this section.

3.1 Inputs under PBC

We now analyze PBC. The input reaction lines follow by maximizing Eqs. 2 and 3:

$$I_s^{optpbc} = \left(\frac{\alpha b u I_C^{\nu}}{\sigma c_s}\right)^{\frac{1}{\sigma - u}} \tag{8}$$

$$I_c^{optpbc} = \left(\frac{b\nu I_S^u}{\tau c_c}\right)^{\frac{1}{\tau-\nu}} \tag{9}$$

1

The service provider's reaction line depends on α whereas the client's does not. Both inputs are dependent on each other's input.

In equilibrium where both reaction lines intersect, we get the explicit solution for the client's input:

$$I_C^{PBC} = \begin{bmatrix} \frac{1}{1 - \frac{u}{\sigma} - \frac{v}{\tau}} \left(\frac{u\alpha}{\sigma c_S}\right) \frac{u}{\sigma \left(1 - \frac{u}{\sigma} - \frac{v}{\tau}\right)} \left(\frac{v}{\tau c_C}\right) \frac{1 - \frac{u}{\sigma}}{1 - \frac{u}{\sigma} - \frac{v}{\tau}} \end{bmatrix}^{\frac{1}{\tau}}$$

and for the service provider's input:

$$I_{S}^{PBC} = \left[b^{\frac{1}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u\alpha}{\sigma c_{S}} \right)^{\frac{1-\frac{v}{\tau}}{\tau}-\frac{v}{\tau}} \left(\frac{v}{\tau c_{C}} \right)^{\frac{1}{\tau}\left(1-\frac{u}{\sigma}-\frac{v}{\tau}\right)} \right]^{\frac{1}{\sigma}}$$

Both inputs depend on α and therefore it is interesting to look for optimal α s from the different perspectives of the service provider and the client.

We find an optimal α for the service provider of α =1 which cannot be established by the service provider, because it implies zero profit for the client.

From the client's perspective the optimal share for the service provider is

$$\alpha = \frac{u}{\sigma \left(1 - \frac{\nu}{\tau}\right)} \tag{10}$$

which is between 0 and 1 so that both parties get a share of the profit.

Since this solution satisfies the client but not the service provider one could think about another solution that implies an α between $\alpha = \frac{u}{\sigma(1-\frac{\nu}{t})}$ and $\alpha=1$. A good candidate for such an α could be the aggregated profit maximizing α . We find that the aggregated profit maximizing α is 1 which also implies zero profit for the client. Since the aggregated profit maximizing α as well as the service provider's profit maximizing α equal 1 the client is left without profit. Therefore, we do not consider these cases any further.

Profits under PBC are

$$\Pi_{S}^{PBC} = c_{S} \frac{1 - u/\sigma - \nu/\tau}{u/\sigma} \left(b \left(\frac{u\alpha}{\sigma c_{S}} \right)^{1} - \frac{\nu}{\tau} \left(\frac{\nu}{\tau c_{C}} \right)^{\frac{\nu}{\tau}} \right)^{\frac{1}{1 - \frac{u}{\sigma} - \frac{\nu}{\tau}}}$$
$$\Pi_{S}^{PBC} = (1 - \alpha) c_{C} \frac{1 - \nu/\tau}{\nu/\tau} \left(b \left(\frac{u\alpha}{\sigma c_{S}} \right)^{\frac{u}{\sigma}} \left(\frac{\nu}{\tau_{c}} \right)^{1} - \frac{u}{\sigma} \right)^{\frac{1}{1 - \frac{u}{\sigma} - \frac{\nu}{\tau}}}$$

3.2 Inputs under SBC

We now analyze the case of SBC extensively and calculate profits under reasonable assumptions in order to be able to compare these profits to those of the other compensation rules.

In the SBC the client's input is lower but also depends on the service provider's input:

$$I_c^{optsbc} = \left(\frac{(1-\alpha)b\nu I_S^u}{\tau c_c}\right)^{\frac{1}{\tau - \nu}} \text{optsbc} = \text{optimum, sales based compensation}$$
(11)

Having the inputs of the client and knowing that they depend on the service provider's input it is crucial to know what the service provider's input is under different compensation rules. The optimal input of the service provider in that case is:

$$I_s^{optsbc} = \left(\frac{\alpha b u I_C^{\nu}}{\alpha c_s}\right)^{\frac{1}{\sigma-u}} \text{optsbc} = \text{optimum, sales based compensation}$$
(12)

The intersection of both input reaction lines forms a typical Nash equilibrium since if the service provider chooses any other input $I \leq I_S^{optsb}$ while the client holds on to I_C^{optsb} , the profit of S diminishes and vice versa.

The inputs in equilibrium are

$$I_{S}^{SBC} = \begin{bmatrix} \frac{1}{b^{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u\alpha}{\sigma c_{S}}\right)^{\frac{1-\frac{v}{\tau}}{\sigma}-\frac{v}{\tau}} \left(\frac{v(1-\alpha)}{\tau c_{C}}\right)^{\frac{v}{\tau}\left(1-\frac{u}{\sigma}-\frac{v}{\tau}\right)} \end{bmatrix}_{\tau}^{\frac{1}{\sigma}}$$
$$I_{C}^{SBC} = \begin{bmatrix} \frac{1}{b^{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u\alpha}{\sigma c_{S}}\right)^{\frac{v}{\sigma}\left(1-\frac{u}{\sigma}-\frac{v}{\tau}\right)} \left(\frac{v(1-\alpha)}{\tau c_{C}}\right)^{\frac{1-\frac{u}{\sigma}-\frac{v}{\tau}}{\sigma}-\frac{v}{\tau}} \end{bmatrix}_{\tau}^{\frac{1}{\sigma}}$$

The profits in this equilibrium are

$$\prod_{S}^{SBC} = c_{S} \frac{1 - u/\sigma}{u/\sigma} \left(b \left(\frac{u\alpha}{\sigma c_{S}} \right)^{1 - \frac{\nu}{\tau}} \left(\frac{\nu(1 - \alpha)}{\tau c_{C}} \right)^{\frac{\nu}{\tau}} \right)^{\frac{1}{1 - \frac{u}{\sigma} - \frac{\nu}{\tau}}}$$
(13)

$$\prod_{C}^{SBC} = c_{C} \frac{1 - \nu/\tau}{\nu/\tau} \left(b \left(\frac{u\alpha}{\sigma c_{S}} \right)^{\frac{u}{\sigma}} \left(\frac{\nu(1 - \alpha)}{\tau c_{C}} \right)^{1 - \frac{u}{\alpha}} \right)^{\frac{1}{1 - \frac{u}{\sigma} - \frac{v}{\tau}}}$$
(14)

In the next step of optimization we are now interested in those values of the service provider's share lpha which the service provider and the client want to secure when negotiating the compensation rule in the first place. For this purpose, Π_S^{SBC} and Π_C^{SBC} have to be maximized as functions of α . From Eq. 13 it is clear that Π_S^{SBC} is maximal if and only if

$$Z := \prod_{S}^{SBC} \frac{1-\frac{u}{\sigma}-\frac{v}{\tau}}{\left(\frac{u}{\sigma c_{S}(1-u/\sigma)}\right)^{1-\frac{u}{\sigma}-\frac{v}{\tau}} b^{-1} \left(\frac{\sigma c_{S}}{u}\right)^{1-\frac{v}{\tau}} \left(\frac{\tau c_{C}}{v}\right)^{\frac{v}{\tau}} = \alpha^{1-\frac{v}{\tau}} (1-\alpha)^{\frac{v}{\tau}}$$

is maximal. Since the first derivative $dZ/d\alpha$ vanishes if and only if $\alpha = 1 - v/\tau$ while the second derivative $d^2Z / d\alpha^2$ is negative throughout (0; 1), Z and thus Π_S^{SBC} takes its maximum for $\alpha = 1 - v/\tau$. Analogous considerations show that Π_c^{SBC} becomes maximal if $\alpha = u/\sigma$.

While the service provider is willing to give the client a share of $\frac{v}{t}Q$, corresponding to the elasticity of the production function divided by the elasticity of the cost function w.r.t. the client's input, the client grants the service provider $\frac{u}{\sigma}Q$, corresponding to the production elasticity divided by the elasticity of the cost function w.r.t. the service provider's input. Therefore, we have

$$\alpha Q + (1-\alpha)Q = \left(1 - \frac{\nu}{\tau}\right)Q + \left(1 - \frac{u}{\sigma}\right)Q = \left(2 - \frac{u}{\sigma} - \frac{\nu}{\tau}\right)Q > Q.$$

Both parties together want to have more than they have to distribute so they end up in a distribution conflict.

Another perspective to find an α is to maximize the aggregated profit $\Pi_{agg}^{SBC} = \Pi_{S}^{SBC} + \Pi_{C}^{SBC}$ with respect to α . Given the conflict from the individual perspectives of both parties, this α could be a good candidate for an agreement. The profit generated by this α is higher as compared with the case where one the party can establish her optimal α .

Because of Eqs. 13 and 14 one has

$$\prod_{agg}^{SBC} = \left(1 - \alpha \frac{u}{\sigma} - (1 - \alpha) \frac{v}{\tau}\right) \left(b\left(\frac{\alpha u}{\sigma c_S}\right)^{\frac{u}{\sigma}} \left(\frac{(1 - \alpha)v}{\tau c_C}\right)^{\frac{v}{\tau}}\right)^{\frac{1}{1 - \frac{u}{\sigma} - \frac{v}{\tau}}}$$

which becomes maximal if and only if $Z(\alpha) = (1 - \alpha \frac{u}{\sigma} - (1 - \alpha) \frac{v}{\tau}) \alpha^{\frac{u}{1 - \frac{u}{\sigma} - \frac{v}{\tau}}} \times (1 - \alpha)^{\frac{t}{1 - \frac{u}{\sigma} - \frac{v}{\tau}}}$ does.

By setting $dZ/d\alpha$ to zero, a quadratic equation for α is obtained if $\frac{u}{\sigma} \neq \frac{v}{\tau}$; of its two solutions, only

$$\alpha = \frac{1}{\frac{u}{\sigma} - \frac{v}{\tau}} \left(\frac{u}{\sigma} \left(1 - \frac{v}{\tau} \right) - \sqrt{\frac{u}{\sigma} \left(1 - \frac{u}{\sigma} \right) \frac{v}{\tau} \left(1 - \frac{v}{\tau} \right)} \right)$$
(15)

1

lies in (0; 1); for $\frac{u}{\sigma} = \frac{v}{\tau}$, a linear equation arises which is solved by $\alpha = 1/2$. To secure the presence of a maximum, one checks that $d^2Z/d\alpha^2 < 0$.

3.3 Profits under IBC

Next, we examine IBC rule. The client's reaction function is the same:

$$I_c^{optibc} = \left(\frac{b\nu I_S^u}{\tau c_c}\right)^{\frac{1}{\tau - \nu}}$$
(16)

Under IBC the service provider's optimal input does not depend on the client's input as under the other compensation rules:

$$I_{S}^{optibc} = \left(\frac{p}{c_{S}\sigma}\right)^{\overline{\sigma-1}} \tag{17}$$

While I_s is fully determined by the input unit price p, I_c is still dependent on I_s . Knowing the service provider's costs, the client can predict $I_s = I_s(p)$ which allows her to determine her input based on p,

$$I_C^{IBC} = \left[\left(\frac{b\nu}{\tau c_C} \right)^{\frac{1}{1 - \frac{\nu}{\tau}}} \left(\frac{p}{c_S \sigma} \right)^{\frac{u}{(\sigma - 1)\left(1 - \frac{\nu}{\tau}\right)}} \right]^{\frac{1}{\tau}}$$

With the inputs I_s and I_c , the profits of both parties become

$$\Pi_{S}^{IBC}(p) = \left(\frac{1}{c_{S}\sigma}\right)^{\frac{1}{\sigma-1}} \left(1 - \frac{1}{\sigma}\right)^{\frac{\sigma}{\sigma-1}},$$
$$\Pi_{C}^{IBC}(p) = \left(1 - \frac{\nu}{\tau}\right)^{\frac{1}{1-\frac{\nu}{\tau}}} \left(\frac{\nu}{\tau c_{C}}\right)^{\frac{\nu}{\tau}} \frac{\frac{\nu}{\tau}}{1-\frac{\nu}{\tau}} \left(\frac{p}{\sigma c_{S}}\right)^{\frac{(\sigma-1)(1-\frac{\nu}{\tau})}{\sigma-1}} - p\left(\frac{p}{\sigma c_{S}}\right)^{\frac{1}{\sigma-1}}$$

Here again, as in the PBC and SBC cases we look from the client's perspective to determine the price p, and thereby optimize profit Π_C^{IBC} w.r.t. this parameter.

The condition $d\Pi_C^{IBC} \frac{}{dp} = 0$ implies that the appropriate choice is given by

$$p = \left(b^{\sigma-1}(c_S\sigma)^{1-u-\frac{\nu}{\tau}}\left(\frac{\nu}{\tau c_C}\right)^{(\sigma-1)\frac{\nu}{\tau}}\left(\frac{u}{\sigma}\right)^{(\sigma-1)\left(1-\frac{\nu}{\tau}\right)}\right)\frac{1}{\sigma\left(1-\frac{u}{\sigma}-\frac{\nu}{\tau}\right)}.$$
(18)

Taking the service provider's perspective we get an infinite optimal p which obviously does not make economic sense.

As in the PBC and SBC case we want to know the *p* that maximizes the aggregated profit $\Pi_{agg}^{SBC} = \Pi_{S}^{SBC} + \Pi_{C}^{SBC}$. The profit-maximizing price *p* reads

$$p = \left(b^{(\sigma-1)}(c_S\sigma)^{\sigma\left(1-\frac{u}{\sigma}-\frac{\nu}{\tau}\right)} \left(\frac{\nu}{\tau c_C}\right)^{(\sigma-1)\frac{\nu}{\tau}} \left(\frac{u}{\sigma c_S}\right)^{(\sigma-1)\left(1-\frac{\nu}{\tau}\right)}\right) \frac{1}{\sigma\left(1-\frac{u}{\sigma}-\frac{\nu}{\tau}\right)}$$
(19)

3.4 A perspective on the comparison of profits

In our model we use a two-stage optimization. It is based on the idea that both parties maximize their profit with respect to their inputs for a given α and p in the second stage. The following table summarizes the results with respect to the optimization of α respectively p from different perspectives using different compensation rules.

1

4). The SBC case makes sense from all three perspectives (case 2, 5 and 8). Moreover, the solution is symmetric for the client's and the service provider's α (case 2 and 8) and the maximization of aggregated profit leads to an α between the two individual solutions (case 5). Finally, in IBC we obtain an optimal *p* from the perspective of the customer (case 3) as well as a higher *p* from the perspective of aggregated profit maximization (case 6) but we do not get a meaningful solution from the service provider's perspective (case 9).

Perspectives	α in PBC	α in SBC	p in IBC
Client's profit maximization Aggregated profit maximization Service provider's profit maximization	$0 < \alpha < 1$ (1) $\alpha = 1$ (4) $\alpha = 1$ (7)	$\alpha = u/\sigma (2)$ α between C's α and S's $\alpha (5)$ $\alpha = 1 - v/\tau (8)$	p as above (3) p higher as C's p (assures joint profit max) (6) ∞ (9)

Table 1 Perspectives on the comparison of compensation rules

We now compare the three compensation rules.

4 A comparison of compensation rules

To compare the three compensation rules we keep one of the perspectives specified above and use only meaningful results of α and p. So, we first compare PBC and IBC from the client's perspective only (cases 1 vs. 3 in Table 1). Then we compare SBC and IBC from the client's perspective (cases 2 vs. 3 in Table 1) as well as from the perspective of aggregated profit maximization (cases 5 vs. 6 in Table 1). We do not compare PBC and SBC because it turns out that IBC dominates PBC. To do so we use the reaction lines of the involved parties. The intersection of their reaction lines determines the Nash equilibrium.

4.1 Comparing PBC and IBC

To compare PBC and IBC we distinguish two cases. First, we compare profits assuming that the client maximizes her α given the PBC rule and her p given the IBC rule. Secondly, we compare the sum of both parties' profits given the client's optimal α and p. We do not compare the PBC and IBC rules for aggregated profit maximizing α and p.

(1) Comparing client's profits in case 1 to case 3, i.e. given the client's optimal α and p. Comparing profits under PBC and IBC we first focus on client's profits given that the client can determine the α and p. The client's profit under PBC with the client's optimal α is

$$\Pi_{C}^{*C \ PBC} = b^{\overline{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u}{\sigma c_{S}}\right)^{\overline{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{v}{\tau c_{C}}\right)^{\overline{\frac{v}{\tau}}} \left(1-\frac{u}{\sigma}-\frac{v}{\tau}\right) \left(\frac{u}{\sigma}\right)^{\overline{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(1-\frac{u}{\sigma}-\frac{v}{\tau}\right)^{\overline{\frac{u}{\sigma}}} \left(1-\frac{v}{\sigma}-\frac{v}{\tau}\right)^{\overline{\frac{u}{\sigma}}} \left(1-\frac{v}{\sigma}-\frac{v}{\tau}\right)^{\overline{\frac{u}{\sigma}}} \left(1-\frac{v}{\sigma}-\frac{v}{\tau}\right)^{\overline{\frac{u}{\sigma}}} \left(1-\frac{v}{\tau}-\frac{v}{\tau}\right)^{\overline{\frac{u}{\sigma}}} \left(1-\frac{v}{\sigma}-\frac{v}{\tau}\right)^{\overline{\frac{u}{\sigma}}} \left(1-\frac{v}{\sigma}-\frac{v}$$

Comparing both profits leads to

$$\Rightarrow \frac{\prod_{C}^{*_{C PBC}}}{\prod_{C}^{*_{C PBC}}} = \left(\frac{u}{1-\frac{\nu}{\tau}}\right)^{\frac{u}{\sigma}} \frac{\frac{u}{\sigma}}{1-\frac{u}{\sigma}-\frac{\nu}{\tau}} < 1, \text{ which is true under the given assumptions because:} u < 1-\nu < 1-\frac{\nu}{\tau}.$$

(2) Comparing both parties' aggregated profits in case 1 to case 3, i.e. given the client's optimal α and p. The following figure shows the reaction functions under the two compensation rules.



Fig. 1 JPM and reaction functions for IBC and PBC

In Fig. 1 as well as the following figures the abscissa shows the service provider's input and the ordinate shows the customer's input. Figure 1 shows two scenarios for plausible example values (see Appendix A). The client's reaction function is the same for IBC and PBC rules and is indicated by R(C,IBC)=R(C,PBC). In addition, the inputs maximizing joint profit determine a point that lies on this reaction function and is indicated by JPM.

As already mentioned the service provider's reaction function in the IBC case is a straight line independent from the client's input indicated by R(S,IBC). The service provider's reaction function in the PBC case is indicated by R(S,PBC).

The intersection of R(S,IBC) and R(C,IBC) is the input equilibrium using the IBC compensation rule and indicated by EQ(IBC). The intersection of R(S,PBC) and R(C,PBC) is the input equilibrium using the PBC compensation rule indicated by EQ(PBC). In the IBC equilibrium both inputs are higher compared to the PBC equilibrium. For both compensation rules both inputs are below the aggregate profit maximum JPM inputs, point JPM. Since the iso-profit lines are ellipsoids around JPM the two equilibrium points on the R(C,PBC) are easy to compare in terms of profits. The EQ(IBC) profit must be higher than the EQ(PBC) profit.

This result can easily be generalized:

Drawing from the results of Section 3 we find the following relationships between the inputs for the service provider and for the client respectively for PBC divided by IBC,

$$\frac{I_S^{PBC}}{I_S^{IBC}} = (\alpha\sigma)^{\frac{1-\frac{\nu}{\tau}}{1-\frac{u}{\sigma}-\frac{\nu}{\tau}}}, \qquad \frac{I_C^{PBC}}{I_C^{IBC}} = (\alpha\sigma)^{\frac{u}{\sigma(1-\frac{u}{\sigma}-\frac{\nu}{\tau})}}$$

As described above we use the client's optimal α which is $\propto = \frac{u}{\sigma(1-\frac{v}{t})}$. Then we get

$$\frac{I_S^{PBC}}{I_S^{IBC}} = \left(\frac{u}{1-\frac{v}{\tau}}\right)^{\frac{1-\frac{v}{\tau}}{1-\frac{u}{\sigma}-\frac{v}{\tau}}}, \qquad \frac{I_C^{PBC}}{I_C^{IBC}} = \left(\frac{u}{1-\frac{v}{\tau}}\right)^{\frac{u}{\sigma\left(1-\frac{u}{\sigma}-\frac{v}{\tau}\right)}}$$

Note that we *must* have $u + v < 1 \rightarrow u < 1 - v < 1 - \frac{v}{t} \rightarrow \frac{u}{1 - \frac{v}{t}} < 1$. This means that the numerator is always smaller than the denominator in both fractions. Since the exponents are between 0 and 1 for both fractions, inputs under IBC are always greater than inputs under PBC. Going back to the Fig. 1 with the reaction functions this implies that inputs under IBC are always closer to JPM than inputs under PBC. We conclude that the sum of profits under IBC must always be higher than the sum of profits under PBC. We did not discuss the service provider's profits because given the results it is easy for the client to pay a fixed amount to the service provider so that both profits are higher compared to the PBC case. Therefore, IBC is a superior compensation rule as compared with PBC in the case of integrated services.

Next, since it is clear now that IBC is better than PBC we compare IBC and SBC to find the profit maximizing compensation rule.

4.2 Comparing SBC and IBC

In this section to compare SBC and IBC we distinguish three cases. First, we compare profits assuming that the client maximizes her α given the SBC rule and her p given the IBC rule. Secondly, we compare the sum of both parties' profits given the client's optimal α and p. Thirdly, we compare aggregated profits assuming that the α and p is derived from maximizing the profits of both parties.

(1) Comparing client's profits in case 2 to case 3, i.e. given the client's optimal α and p. Comparing profits under SBC and IBC we first focus on client's profits given that the client can determine the α and p. The client's profit under SBC with the client's optimal α is

$$\prod_{C}^{*C SBC} = b^{\frac{1}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u}{\sigma c_{S}}\right)^{\frac{u}{\sigma}} \left(\frac{v}{\tau c_{C}}\right)^{\frac{v}{\tau}} \left(\frac{v}{\tau c_{C}}\right)^{\frac{v}{\tau}} \left(1-\frac{v}{\tau}\right) \left(\frac{u}{\sigma}\right)^{\frac{u}{\sigma}-\frac{v}{\tau}} \left(1-\frac{u}{\sigma}\right)^{\frac{1-\frac{u}{\sigma}}{\tau}-\frac{v}{\tau}}$$

The client's profit under IBC with the client's optimal price p is

$$\prod_{C}^{*CIBC} = b^{\frac{1}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u}{\sigma c_{S}}\right)^{\frac{u}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{v}{\tau c_{C}}\right)^{\frac{v}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \frac{\left(1-\frac{u}{\sigma}-\frac{v}{\tau}\right)}{\sigma^{\frac{u}{\sigma}}}$$

Comparing both profits leads to

$$\frac{\prod_{C}^{*CSBC}}{\prod_{C}^{*CIBC}} = \frac{1 - \frac{v}{\tau}}{1 - \frac{u}{\sigma} - \frac{v}{\tau}} \cdot u^{\frac{u}{\sigma} - \frac{v}{\tau}} \cdot \left(1 - \frac{u}{\sigma}\right)^{\frac{1 - \frac{u}{\sigma}}{1 - \frac{u}{\sigma} - \frac{v}{\tau}}}$$

Numerical analysis shows that this term is always below 1. Therefore, we conclude that the client's profit is always higher under the IBC rule as compared to the SBC rule given that the client chooses her optimal α and p, respectively.

(2) Comparing both parties' aggregated profits in case 2 to case 3, i.e. given the client's optimal α and p. Figure 2 shows the reaction functions under the three compensation rules and is based on the same principles as Fig. 1 above.

Figure 2 shows the reaction functions of the service provider R(S,SBC) and of the client R(C,SBC). We first note that the reaction function of the customer R(C,SBC) differs from the client's reaction function under any other compensation rule (remember that the other reaction function for the customer is identical for two regimes R(C,IBC), R(C,PBC) and intersects the point JPM). R(S,SBC) represents the service supplier's reaction function under SBC. The equilibrium under SBC is represented by EQ(SBC).

To compare the equilibrium profits under SBC and IBC we start by comparing the inputs. For the example we can easily see that the service supplier's as well as the customer's inputs are substantially lower as compared to IBC.

This result can be extended by comparing the general formulas for the inputs of service provider and the customer derived in Section 3:

$$\frac{I_{S}^{SBC}}{I_{S}^{IBC}} = \begin{bmatrix} \left(\frac{\alpha}{\sigma}\right)^{\frac{1-\frac{\nu}{\tau}}{1-\frac{u}{\sigma}-\frac{\nu}{\tau}}} (1-\alpha)^{\frac{\nu}{\tau}\left(1-\frac{u}{\sigma}-\frac{\nu}{\tau}\right)} \end{bmatrix}^{\frac{1}{\sigma}}$$

$$\frac{I_{S}^{SBC}}{I_{S}^{IBC}} = \begin{bmatrix} \left(\frac{\alpha}{\sigma}\right)^{\frac{\nu}{\sigma\left(1-\frac{u}{\sigma}-\frac{\nu}{\tau}\right)}} (1-\alpha)^{\frac{1-\frac{u}{\sigma}}{1-\frac{u}{\sigma}-\frac{\nu}{\tau}}} \end{bmatrix}^{\frac{1}{\tau}}$$

Since α is between 0 and 1, the values α/σ and $(1-\alpha)$ are between 0 and 1. Consequently, the fraction of inputs under SBC divided by the inputs under IBC are smaller than 1 for the service provider and for the customer. It is clear now that the inputs under SBC are smaller than under IBC and therefore IBC implies higher aggregate profits as compared with SBC. We did not discuss the service provider's profits because given the results it is easy for the client to pay a fixed amount to the service provider so that both profits are higher compared to the SBC case. Consequently, IBC is always superior to SBC.

(3) Comparing both parties' aggregated profits in case 5 to case 6, i.e. given the optimal α and p from aggregated profit maximizing. Up to now we based our comparisons on the assumption that the client determines the α and p for the three compensation rules. Now, we compare the aggregate profits under SBC and IBC assuming that the α for SBC and the p for IBC maximizes the sum of the profits of both parties.

Figure 3 shows the reaction functions based on these assumptions for SBC and IBC. It is clear that IBC is always superior to SBC because the equilibrium point for IBC now is the same as the JPM point. The equilibrium point for SBC on the other hand never reaches the JPM.



Fig. 2 SBC and IBC reaction functions with client's optimal α and p

In general terms we have:

The sum of both parties' profits under the two compensation rules is

$$\Pi_{agg}^{SBC} = b^{\overline{1 - \frac{u}{\sigma} - \frac{v}{\tau}}} \left(\frac{\alpha u}{\sigma c_s}\right)^{\overline{1 - \frac{u}{\sigma} - \frac{v}{\tau}}} \left(\frac{(1 - \alpha)v}{\tau c_c}\right)^{\overline{1 - \frac{u}{\sigma} - \frac{v}{\tau}}} \quad 0 < \alpha < 1$$
$$\Pi_{agg}^{IBC} = b^{\overline{1 - \frac{u}{\sigma} - \frac{v}{\tau}}} \left(\frac{u}{\sigma c_s}\right)^{\overline{1 - \frac{u}{\sigma} - \frac{v}{\tau}}} \left(\frac{v}{\tau c_c}\right)^{\overline{1 - \frac{u}{\sigma} - \frac{v}{\tau}}} \left(1 - \frac{u}{\sigma} - \frac{v}{\tau}\right)$$

To compare both profits we use the ratio

$$\frac{\prod_{agg}^{SBC}}{\prod_{agg}^{IBC}} = \frac{1 - \alpha \frac{u}{\sigma} - (1 - \alpha) \frac{v}{\tau}}{1 - \frac{u}{\sigma} - \frac{v}{\tau}} \cdot \alpha^{\frac{u}{\sigma} - \frac{v}{\tau}} (1 - \alpha)^{\frac{v}{\tau}} \cdot \frac{v}{\tau}.$$

The term $\frac{\prod_{agg}^{SBC}}{\prod_{agg}^{IBC}}$ is smaller than 1 for each α between 0 and 1, therefore also for the optimal α .

 $\begin{array}{l} \text{Raising the expression} \quad \frac{1-\alpha_{\sigma}^{\underline{u}}-(1-\alpha)_{\tau}^{\underline{v}}}{1-\frac{u}{\sigma}-\frac{v}{\tau}}, \\ \alpha \end{array} \xrightarrow{\frac{u}{\sigma}} \frac{\frac{v}{\tau}}{1-\frac{u}{\sigma}-\frac{v}{\tau}} \\ \text{which is greater than 0 we get} \quad \left(\frac{1-\alpha_{\sigma}^{\underline{u}}-(1-\alpha)_{\tau}^{\underline{v}}}{1-\frac{u}{\sigma}-\frac{v}{\tau}}\right)^{1-\frac{u}{\sigma}-\frac{v}{\tau}} \\ \alpha_{\sigma}^{\underline{u}}(1-\alpha)^{\frac{v}{\tau}} \end{array} \xrightarrow{\frac{v}{\tau}} \\ \text{This is a weighted geometric mean of the three positive numbers } \\ \left(\frac{1-\alpha_{\sigma}^{\underline{u}}-(1-\alpha)_{\tau}^{\underline{v}}}{1-\frac{u}{\sigma}-\frac{v}{\tau}}\right), \\ \alpha, \\ (1-\alpha). \end{array}$



Fig. 3 SBC and IBC reaction functions with α and p maximizing both parties' aggregated profit

We now show that this geometric mean is smaller than 1. Using the weighted arithmetic mean of the same numbers with the same weights, we get:

$$\left(1 - \frac{u}{\sigma} - \frac{v}{\tau}\right) \cdot \left(\frac{1 - \alpha \frac{u}{\sigma} - (1 - \alpha)\frac{v}{\tau}}{1 - \frac{u}{\sigma} - \frac{v}{\tau}}\right) + \frac{u}{\sigma}\alpha + \frac{v}{\tau}(1 - \alpha) = 1.$$

From the arithmetic-geometric mean inequality it follows that $\frac{\prod_{agg}^{SBC}}{\prod_{agg}^{IBC}} < 1$.

Since the sum of profits in the IBC case is higher compared to the SBC case we can assure with a fixed amount of money paid from one party to the other that the profit of each individual party is higher in the IBC case.

We conclude that IBC is superior to SBC and PBC under all meaningful assumptions.

5 Discussion and further research

To our knowledge compensation rules for services have not been analyzed in a theoretical setting. We examined a special kind of services, namely integrated services where service provider's and client's input is necessary to get an output. This is a very common kind of service which can be found in consulting, advertising and training. The management literature claims that the compensation of these kind of services as well as others should be based on service value created for the client. The client's value can be measured by additional sales or additional profits. So we analyzed a sales-based

compensation as well as a profit-based compensation and compared it to the commonly used input compensation. For the analysis we developed a model with a very general production and cost function and assumed no uncertainty and perfect information.

As the literature recommends we expected the value-based compensation to be superior to the input-based compensation. Surprisingly, we found that the inputbased compensation is superior to both forms of value-based compensation.

In the service management literature authors have argued based on plausibility that valued based pricing, i.e. sales or profit based compensation schemes are superior to input based compensation schemes (Lovelock et al. 1999). However, we are not aware of any modeling approach that supports this conclusion. Based on our model we find exactly the opposite: Input-based compensation is superior to value-based compensation. We are able to extend that result even to the conclusion that input-based compensation is superior to profit-based compensation. Our result may be even stronger in the case of uncertainty which is left for further research. Uncertainty typically means that value cannot be observed, especially the link between a particular input and its effect on the generated value. Input can be observed. So, in an uncertain environment input-based compensation may be the only way to compensate. Our results underpin the practitioners' behavior from a theoretical perspective. Our result is also compatible with case-wise empirical observations. In many cases in service industries such as consulting, advertising and training service providers are compensated based on the number of hours or number or work days they deliver within a particular project. Contrary to the suggestion of the management literature these industry habits make sense in the light of our model.

The economic reason behind our result is the client's ability to govern the service provider's input using the input-based compensation. Since the service provider's input depends mainly on the price he exactly gives the input which is appropriate for the given price. Therefore both parties do not end up in a conflict. On the other hand using the value-based compensation none of both parties can govern the other party's input and therefore both parties do not care about the input but focus on the share of profit they get. Hence, they end up in a conflict. From this perspective practitioners do quite well in using input based compensation.

Variable	Notation for variable	Value
Elasticities of production	u and v	0.45
Variable and marginal costs	$c_{ m S}$ and $c_{ m C}$	0.30
Cost elasticity service provider	α	1.30
Cost elasticity client	au	1.10

Appendix A: Values for reaction function examples

Appendix B: Explicit solutions for the inputs

$$\begin{split} I_{S}^{PBC} &= \left[b^{\frac{1}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u\alpha}{\sigma c_{S}} \right)^{\frac{1-\frac{v}{\tau}}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{v}{\tau c_{C}} \right)^{\frac{v}{\tau(1-\frac{u}{\sigma}-\frac{v}{\tau})}} \right]^{\frac{1}{\sigma}} \\ I_{C}^{PBC} &= \left[b^{\frac{1}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u\alpha}{\sigma c_{S}} \right)^{\frac{u}{\sigma(1-\frac{u}{\sigma}-\frac{v}{\tau})}} \left(\frac{v}{\tau c_{C}} \right)^{\frac{1-\frac{u}{\sigma}}{\tau}} \right]^{\frac{1}{\tau}} \\ I_{S}^{IBC} &= \left[b^{\frac{1}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u}{\sigma^{2} c_{S}} \right)^{\frac{1-\frac{v}{\tau}}{\tau}} \left(\frac{v}{\tau c_{C}} \right)^{\frac{v}{\tau(1-\frac{u}{\sigma}-\frac{v}{\tau})}} \right]^{\frac{1}{\sigma}} \\ I_{C}^{IBC} &= \left[b^{\frac{1}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u}{\sigma^{2} c_{S}} \right)^{\frac{u}{\sigma(1-\frac{u}{\sigma}-\frac{v}{\tau})}} \left(\frac{v}{\tau c_{C}} \right)^{\frac{1-\frac{u}{\sigma}}{\tau}} \right]^{\frac{1}{\tau}} \\ I_{S}^{SBC} &= \left[b^{\frac{1}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u\alpha}{\sigma c_{S}} \right)^{\frac{1-\frac{v}{\tau}}{\tau}} \left(\frac{v(1-\alpha)}{\tau c_{C}} \right)^{\frac{v}{\tau(1-\frac{u}{\sigma}-\frac{v}{\tau})}} \right]^{\frac{1}{\sigma}} \\ I_{C}^{SBC} &= \left[b^{\frac{1}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u\alpha}{\sigma c_{S}} \right)^{\frac{u}{\sigma(1-\frac{u}{\sigma}-\frac{v}{\tau})}} \left(\frac{v(1-\alpha)}{\tau c_{C}} \right)^{\frac{1-\frac{u}{\sigma}}{\tau}} \right]^{\frac{1}{\sigma}} \\ I_{S}^{SBC} &= \left[b^{\frac{1}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u\alpha}{\sigma c_{S}} \right)^{\frac{1-\frac{v}{\tau}}{\tau}} \left(\frac{v(1-\alpha)}{\tau c_{C}} \right)^{\frac{1-\frac{u}{\sigma}}{\tau}} \right]^{\frac{1}{\sigma}} \\ I_{S}^{JPM} &= \left[b^{\frac{1}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u}{\sigma c_{S}} \right)^{\frac{1-\frac{v}{\tau}}{\tau}} \left(\frac{v}{\tau c_{C}} \right)^{\frac{1-\frac{u}{\tau}}{\tau}} \right]^{\frac{1}{\sigma}} \\ I_{C}^{JPM} &= \left[b^{\frac{1}{1-\frac{u}{\sigma}-\frac{v}{\tau}}} \left(\frac{u}{\sigma c_{S}} \right)^{\frac{u}{\sigma(1-\frac{u}{\sigma}-\frac{v}{\tau})}} \left(\frac{v}{\tau c_{C}} \right)^{\frac{1-\frac{u}{\tau}}{\tau}} \right]^{\frac{1}{\tau}} \end{aligned}$$

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