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A preference foundation for Fehr and Schmidt's model of inequity aversion

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Abstract Fehr and Schmidt (FS) introduced an influential social utility function for individuals in interpersonal contexts that captures self-centered inequity aversion. The value of this social utility function lies in its exceptionally good balance between parsimony and fit. This paper provides a preference foundation for exactly the model of FS with preference conditions that exactly capture the exceptionally good balance of FS. Remarkably, FS is a special case of Schmeidler's rank-dependent utility for decision under uncertainty.

1 Introduction

It has been recognized in economics that individuals often care not only about their own payoff but also about the payoffs of others. Not only social planners, but also the members of society themselves care about the fairness and inequality of payoff distributions. Fehr and Schmidt (1999, FS from now on) introduced a social utility function that captures concerns about fairness in the sense of inequity aversion. Individuals dislike both being worse off than others, and others being worse off than themselves. Nevertheless, the latter is not very extreme, so that when all others receive nothing an individual does still want to maximize the own payoff. This paper gives a preference foundation of (exactly) FS.

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Virtually every major utility theory in economics has received its own preference foundation. One famous example is Koopmans (1960), who provided a preference foundation for discounted utility in intertemporal choice. Other examples include Artzner et al. (1999) for coherent measures of risk, Gilboa and Schmeidler (1989) for ambiguity and multiple priors, Harsanyi (1955) for utilitarianism, Savage (1954) for expected utility and Bayesian statistics, Schmeidler (1989) for nonexpected utility, Tversky and Kahneman (1992) for (cumulative) prospect theory, and von Neumann and Morgenstern (1944) for expected utility. Preference foundations give behavioral conditions that are necessary and sufficient for a model to hold. These conditions state the empirical meaning of a model in terms of observables. In empirical applications the model holds if and only if the behavioral conditions hold and it fails if and only if at least one of the behavioral conditions fails. Normatively, the model can be justified if one agrees that behavior should follow the behavioral conditions; it can be criticized otherwise. Thus, a preference foundation allows for the verification of the empirical validity and the normative appropriateness of a model.

Some recent studies provided preference foundations for models more general than FS (Neilson 2006; Sandbu 2008). From these preference foundations it does not follow how we can verify whether preferences satisfy exactly the model of FS. Although these papers provide necessary conditions for FS to hold, they do not provide sufficient conditions and therefore do not exactly identify FS's empirical content. In particular, they do not achieve the exceptionally good balance between parsimony and fit of FS's model.

Neilson (2006) considers general nonlinear utility of payoff differences between one individual and the others. He allows for, but does not identify a key feature of FS: that disadvantageous inequity is treated differently from advantageous inequity. Neilson's key condition is self-referent separability, which is similar to standard separability, but which treats one component, the payoff of the decision maker, differently. This self-referent separability is not sufficient to characterize FS, because it neither differentiates between advantageous and disadvantageous inequity, nor does it require additivity. It provides a useful starting point for obtaining generalizations of FS that reckon, for instance, with nonlinear utility.

Sandbu (2008) comes close to obtaining a preference foundation of FS. He distinguish between advantageous and disadvantageous inequity, by imposing a separability condition on 'rank-maintaining' distributions only. His homotheticity condition implies power utility. FS is the special case of Sandbu where the power of utility equals one. Sandbu does not give a preference condition that guarantees a power of one. Thus, his conditions are not sufficient to characterize exactly FS.

Instead of adding some extra conditions to Sandbu's foundation in order to provide a preference foundation of exactly FS, this paper aims at minimizing the total number of preference conditions and at keeping them as simple as possible. Thus, we start anew and focus entirely on FS. This is done to best reflect the empirical content of FS.

The key preference condition for obtaining FS, covalent additivity, suggests a generalization of FS that is different from Neilson's and Sandbu's models. We show that, remarkably, FS is a special case of the rank-dependent model of Gilboa (1987) and

Schmeidler (1986, 1989), a relation that had not been observed in the literature before.[1] The rank-dependent model was introduced to explain behavior for Knightean uncertainty that deviates from expected utility, and it spurred a whole stream of literature on ambiguity. Tversky and Kahneman (1992) used it to correct a theoretical mistake in the most influential theory of risk and uncertainty today: prospect theory (Kahneman and Tversky 1979). For this new version of prospect theory it was possible to establish a preference foundation (Wakker and Tversky 1993), proving its theoretical soundness. Such a preference foundation had been missing for the original prospect theory of 1979. Our main axiom, covalent additivity, is a refinement of Schmeidler's comonotonic additivity. We show in particular that FS is a special case of a nonmonotonic version of Schmeidler (1986) model. This nonmonotonic version was axiomatized in general by De Waegenaere and Wakker (2001). Thus, the latter axiomatization is more general than FS and does not capture the particular balance of FS. It is remarkable that Schmeidler's idea provides the basis of a very influential model not only in decision under uncertainty, but also in interpersonal contexts.

2 The model

Assume $n+1$ individuals $0, \dots, n$. We model preferences $\succcurlyeq$ of individual 0 over *distributions* $x = (x_0, \dots, x_n)$ yielding *payoff* $x_i \in \mathbb{R}$ for all i. The notation $\succ$, $\preccurlyeq$, $\prec$, and $\sim$ is as usual. *Weak ordering* holds if $\succcurlyeq$ is *complete* ($x \succcurlyeq y$ or $y \succcurlyeq x$ for all distributions x, y) and transitive.

Preferences $\succcurlyeq$ can be represented by a social utility function U when $x \succcurlyeq y$ if and only if $U(x) \geq U(y)$. *FS* holds if preferences $\succcurlyeq$ can be represented by

$$U(x) = x_0 - \alpha \sum_{i=1}^{n} \max\{x_i - x_0, 0\} - \beta \sum_{i=1}^{n} \max\{x_0 - x_i, 0\} \qquad (1)$$

with $\alpha, \beta \geq 0$. For every unit by which another individual's payoff exceeds individual 0's payoff, individual 0's utility is decreased by α. For every unit by which another individual's payoff is lower than individual 0's payoff, individual 0's utility is decreased by β. Thus, α and β give the prices of being worse off or better off than others. We will next discuss the properties that preferences satisfy when FS holds. We first introduce some more notation.

For every distribution x and every individual $i = 1, \dots, n$ we define the *deviation* $d_i(x) = x_i - x_0$, i.e. the deviation of the payoff of individual i from the payoff of individual 0. A distribution x is *constant* if $x_i = x_0$ for all i, i.e. if $d_i(x) = 0$ for all i. For all individuals $i = 0, \dots, n$ and all distributions x the distribution $\mu_i x$ denotes the distribution x with the payoff of individual i replaced by μ. Every payoff $\mu \in \mathbb{R}$ is identified with a constant distribution yielding payoff μ for every individual. That is, instead of $(\mu, \dots, \mu)$ we will often write μ to denote a constant distribution. From the context it will be clear whether μ denotes a single payoff or a constant distribution.

[1] Similarly, Ben Porath and Gilboa (1994) showed that the Gini-index in inequality measurement is a special case of rank-dependent utility.

Constant monotonicity holds if $\mu \succcurlyeq \nu$ if and only if $\mu \geq \nu$. Constant monotonicity requires that, in the absence of inequality, higher payoffs are preferred to lower ones. If for a distribution x and a payoff μ we have $x \sim \mu$, then μ is a *constant equivalent* of x. Under constant monotonicity and transitivity every distribution can have at most one constant equivalent. *Constant equivalence* holds if every distribution has a constant equivalent. However unequal a distribution, under constant equivalence there is always a constant distribution that individual 0 finds equivalent. It can easily be verified that preferences satisfy constant monotonicity and constant equivalence if FS holds.

Two distributions x, y are *covalent* if for every individual $i = 1, \ldots, n$ the deviation has the same sign in both distributions: $d_i(x)d_i(y) \geq 0$. Thus, if two distributions x and y are covalent, then it cannot be the case that an individual i gets strictly more than individual 0 in x and strictly less than individual 0 in y.

Definition *Covalent additivity* holds if for all x, y, z that are pairwise covalent

$$x \succcurlyeq y \iff x + z \succcurlyeq y + z.$$

Consider two distributions x, y where for every individual i the payoff-ranking of individual i w.r.t individual 0 is the same in both distributions. Consider adding a certain amount to the payoff of individual j in both distributions such that the ranking of individual j w.r.t. individual 0 is not changed. Under covalent additivity this addition should not affect preference between the two distributions. By repeating such an addition for several individuals j, covalent additivity implies that adding a particular distribution to distributions x and y, without affecting the ranking of any individual j w.r.t. individual 0, should not affect preferences between the two distributions. Covalent additivity entails that for every individual the marginal utility from an increase in his payoff is constant as long as the ranking of his payoff w.r.t individual 0's payoff is unaffected. In choice under uncertainty and intertemporal choice marginal utilities are commonly diminishing. Neilson (2006) and Sandbu (2008) allow for diminishing utility. Covalent additivity and its implied constant marginal utility under preserved covalence is what makes FS a simple and thereby appealing model. FS's social utility can be calculated directly from payoffs without first requiring a specification of some nonlinear utility function. Yet, its predictions are consistent with the main empirical findings in many games and other applications. Covalent additivity is a strong assumption, though. One could imagine that abandoning the linearity of FS and considering nonlinear generalizations, might result in models that can predict behavior even better. Yet, abandoning linearity will come at the cost of more complicated models that will be harder to estimate. Future research should show whether the costs of more complicated estimations are worth the benefits of better predictions.

To show that FS implies covalent additivity, we first observe that the utility of the sum of two covalent distributions equals the sum of the utilities of the two distributions if FS holds. This property of the utility function is similar to Schmeidler's (1986) comonotonic additivity.

Observation 2.1 *If FS holds, then $U(x + y) = U(x) + U(y)$ for all covalent x, y.*

Proof We have $x_i + y_i \geq x_0 + y_0$ if $x_i \geq x_0$ and $y_i \geq y_0$. Similarly, $x_i + y_i \leq x_0 + y_0$ if $x_i \leq x_0$ and $y_i \leq y_0$. Thus, for two covalent distributions x, y we have $x_i + y_i \geq x_0 + y_0$ if $x_i > x_0$, and $x_i + y_i \leq x_0 + y_0$ if $x_i < x_0$. It follows that

$$\begin{aligned} U(x+y) = x_0 + y_0 - \alpha \sum_{i=1}^{n} \max\{x_i + y_i - x_0 - y_0, 0\} \\ - \beta \sum_{i=1}^{n} \max\{x_0 + y_0 - x_i - y_i, 0\} \\ &= x_0 - \alpha \sum_{i=1}^{n} \max\{x_i - x_0, 0\} - \beta \sum_{i=1}^{n} \max\{x_0 - x_i, 0\} \\ &\quad + y_0 - \alpha \sum_{i=1}^{n} \max\{y_i - y_0, 0\} - \beta \sum_{i=1}^{n} \max\{y_0 - y_i, 0\} \\ &= U(x) + U(y). \end{aligned}$$

□

The following observation now follows easily.

Observation 2.2 *If FS holds, then covalent additivity holds.*

As will be shown later in Observation 2.4, covalent additivity is the most important characteristic of FS. It will be shown that weak ordering, constant monotonicity, constant equivalence, and covalent additivity, together with a weak monotonicity condition to be defined later, imply that preferences can be represented by

$$x_0 - \sum_{i=1}^{n} \alpha_i \max\{x_i - x_0, 0\} - \sum_{i=1}^{n} \beta_i \max\{x_0 - x_i, 0\}, \tag{2}$$

with possibly negative coefficients α_i and β_i. The next few conditions will be needed in addition to the previous ones in order to obtain FS.

Definition *Disadvantage aversion* holds if $\mu_i 0 \preccurlyeq 0$ for all payoffs $\mu \geq 0$ and all $i = 1, \ldots, n$.

Disadvantage aversion means that individual 0 does not like another individual receiving a positive payoff μ when all other individuals, including himself, receive nothing. If FS holds, then preferences satisfy disadvantage aversion, because $\alpha \geq 0$: for payoff $\mu \geq 0$ and for $i = 1, \ldots, n$ we have $U(\mu_i 0) = -\alpha\mu \leq 0 = U(0)$.

Definition *Advantage aversion* holds if $\mu_i 0 \preccurlyeq 0$ for all payoffs $\mu \leq 0$ and all $i = 1, \ldots, n$.

Advantage aversion means that individual 0 does not like another individual receiving a negative payoff μ when all other individuals, including himself, receive nothing. If FS holds, then preferences satisfy advantage aversion, because $\beta \geq 0$: for payoff $\mu \leq 0$ and for $i = 1, \ldots, n$ we have $U(\mu_i 0) = \beta\mu \leq 0 = U(0)$.

Definition Preferences $\succcurlyeq$ satisfy *inequity aversion* if they satisfy disadvantage aversion and advantage aversion.

Definition Preferences $\succcurlyeq$ satisfy *anonymity* if $c_i 0 \sim c_j 0$ for all c and all individuals $i, j \in \{1, \ldots, n\}$.

Anonymity means that individual 0 does not favor one individual over the other. FS implies anonymity, because the coefficients α and β are independent of the individuals, i.e. they are independent of i.

The following theorem characterizes FS. As the proof is instructive, we provide it in the main text.

Theorem 2.3 *The following two statements are equivalent.*

(i) *FS holds* (Eq. 1).
(ii) *Preferences $\succcurlyeq$ satisfy*
   (a) *Weak ordering;*
   (b) *Constant monotonicity;*
   (c) *Constant equivalence;*
   (d) *Covalent additivity;*
   (e) *Inequity aversion;*
   (f) *Anonymity.*

In the following observation, we need a (minor) monotonicity condition. *Inequity monotonicity* holds if for every $i = 1, \ldots, n$ it is impossible to find $\mu, \nu \in \mathbb{R}$ with (1) $\mu\nu \geq 0$ and (2) $\mu_i 0 \prec 0$ and $\nu_i 0 \succ 0$. Inequity monotonicity means that if individual 0 dislikes being worse (better) off than others, then he dislikes this no matter how much worse (better) off he is.

Observation 2.4 *The following two statements are equivalent.*

(i′) *Preferences can be represented by*

$$x_0 - \sum_{i=1}^{n} \alpha_i \max\{x_i - x_0, 0\} - \sum_{i=1}^{n} \beta_i \max\{x_0 - x_i, 0\}$$

(ii′) *Preferences $\succcurlyeq$ satisfy*
   (a) *Weak ordering;*
   (b) *Inequity monotonicity;*
   (c) *Constant monotonicity;*
   (d) *Constant equivalence;*
   (e) *Covalent additivity;*

Proof of Theorem 2.3 and Observation 2.4 It was already shown that (i) implies (ii). We will now show that (ii) implies (i). Assume (ii). Before starting the proof, note first that inequity aversion implies inequity monotonicity.

Let $e(x)$ denote the constant equivalent of distribution x. From constant monotonicity we know that e represents preferences. For all $b, c \in \mathbb{R}_+$ and all $i = 1, \ldots, n$

we have, by covalent additivity and pairwise covalence of b_i0, c_i0, and $e(c_i0)$, the indifference $b_i0 + c_i0 \sim b_i0 + e(c_i0)$. Similarly, $b_i0 + e(c_i0) \sim e(b_i0) + e(c_i0)$. It follows that

$$\begin{aligned} e((b+c)_i0) &\sim (b+c)_i0 \\ &= b_i0 + c_i0 \\ &\sim b_i0 + e(c_i0) \\ &\sim e(b_i0) + e(c_i0). \end{aligned}$$

Thus,

$$e((b+c)_i0) = e(b_i0) + e(c_i0)$$

for $i = 1, \dots, n$. By inequity monotonicity $e(b_i0)$ is either non-negative for all $b \in \mathbb{R}_+$ or non-positive for all $b \in \mathbb{R}_+$. Thus, we can apply Theorem 1 of Section 2.1.1 in Aczél (1966). It follows that there is an $\alpha_i \in \mathbb{R}$ such that

$$e(b_i0) = -\alpha_i b$$

for all $b \in \mathbb{R}_+$ and all $i = 1, \dots, n$. By a similar argument we have a $\beta_i \in \mathbb{R}$ such that

$$e(b_i0) = \beta_i b$$

for all $b \in \mathbb{R}_-$ and all $i = 1, \dots, n$.

For all $a \in \mathbb{R}$, all distributions x, all constant distributions c, and all individuals i we have that a_i0, $(x_0)_i x$, and c are pairwise covalent. Therefore, by repeated application of covalent additivity we have

$$\begin{aligned} e(x) \sim x &= x_0 + \sum_{i=1}^{n} (d_i(x))_i 0 \\ &\sim x_0 + e((d_1(x))_1 0) + \sum_{i=2}^{n} (d_i(x))_i 0 \\ &\vdots \\ &\sim x_0 + \sum_{i=1}^{n} e((d_i(x))_i 0) \\ &= x_0 - \sum_{i=1}^{n} \alpha_i \max\{x_i - x_0, 0\} - \sum_{i=1}^{n} \beta_i \max\{x_0 - x_i, 0\}. \end{aligned}$$

It follows that

$$e(x) = x_0 - \sum_{i=1}^{n} \alpha_i \max\{x_i - x_0, 0\} - \sum_{i=1}^{n} \beta_i \max\{x_0 - x_i, 0\},$$

which proves Observation 2.4. Disadvantage aversion implies that $\alpha_i \geq 0$ for all i. Similarly, advantage aversion implies that $\beta_i \geq 0$ for all i. By anonymity we have $\alpha_j = \alpha_i \equiv \alpha$ and $\beta_j = \beta_i \equiv \beta$ for all i, j, which proves the result. The Appendix shows that the preference conditions in Theorem 2.3 are independent. □

It is common to restrict α and β such that $\beta < 1/n$ and $\alpha \geq \beta$. We will now give the behavioral conditions underlying these restrictions.

Definition *Self-support* holds if $\mu_0 0 \succ 0$ for all payoffs $\mu > 0$.[2]

Self-support means that even if individual 0 is advantage averse, he still prefers receiving a positive payoff to receiving nothing if all other individuals receive nothing. Thus, keeping the payoff of others fixed, individual 0 will always want to maximize his own payoff.

Definition Individual 0 *prefers advantages over disadvantages* if $(-\mu)_i 0 \succcurlyeq \mu_i 0$ for all $\mu > 0$ and all $i = 1, \ldots, n$.

Corollary 2.5 *If FS holds, then*

(i) *self support holds if and only if* $\beta < 1/n$.
(ii) *individual 0 prefers advantages over disadvantages if and only if* $\alpha \geq \beta$.

3 Discussion

This paper has provided a preference foundation for the influential utility function introduced by Fehr and Schmidt (1999). This utility function captures self-centered inequity aversion in the sense that individuals dislike both being worse off than others, and others being worse off than themselves. FS's model has an exceptionally good balance between parsimony and fit. FS captures preferences in a linear way, which makes it a model which is easy to use. Yet, despite the strong assumption of linearity, FS can explain a wide range of findings in many games and other applications. The linearity of FS's model is implied by covalent additivity, the most important preference condition in this paper.

Covalent additivity emphasizes the ranking of other individuals' payoffs to the decision maker's payoff; as long as this ranking is unaffected, either increasing or decreasing an individual's payoff by a common amount in two distributions should not affect preferences between these distributions. The emphasis on rank raises the question how FS is related to Schmeidler's (1986, 1989) rank-dependent model.

[2] Note that, given FS, if the condition holds for one $\mu > 0$ it holds for all $\mu > 0$.

Remarkably, FS is a special case of rank-dependent utility with a non-monotonic weighting function.

Rank-dependent utility captures the fact that individuals are concerned about their rank in an income distribution (Rablen 2008). Under rank-dependent utility the weight that an individual attaches to the payoff of another individual depends on the rank of this other individual's payoff in the distribution of payoffs. It can, for instance, be the case that an individual weights the payoff of another individual when the latter is poorest more than when he is richest. Under FS an individual weights the payoffs of other individuals who are better off differently from the payoffs of other individuals who are worse off. The payoffs of all others who are worse off or all others who are better off are weighted equally.

It can be verified that FS is the special case of RDU (Schmeidler 1986; De Waegenaere and Wakker 2001) with utility $u(x_i) = x_i$ and a weighting function given by $W(I) = 1 - (n - |I| + 1)\beta$ if $0 \in I$ and $W(I) = -|I|\alpha$ if $0 \notin I$, where $|I|$ gives the number of individuals in the set I.[3] Since $\alpha, \beta \geq 0$ the weighting function is non-monotonic. De Waegenaere and Wakker (2001) provided a preference foundation for RDU with a non-monotonic weighting function and linear utility. Thus, an alternative preference foundation of FS could be obtained by taking the conditions of De Waegenaere and Wakker (2001) as a point of departure and then adding the extra conditions needed to specify FS.

It is remarkable that such a famous model as FS turns out to be a special case of yet another famous model, without this having been observed in the literature before. Rank-dependent utility may provide a promising direction for obtaining generalizations of FS. Moreover, FS illustrates the importance of generalizing RDU models to allow for non-monotonic weighting functions.

Appendix

This Appendix will show that the preference conditions used in the proof of Theorem 2.3 are independent. We will show that for each preference condition, dropping it from the list makes it impossible to characterize exactly FS.

From the proof of the theorem it follows that conditions (a)–(e), without anonymity, would not yield exactly FS, as the coefficient α_i and β_i would be allowed to differ for different individuals i. Similarly, inequity aversion is also required to obtain FS, as without it the coefficients would be allowed to be negative.

Neilson's model

$$U(x) = u(x_0) + \sum_{i=1}^{n} v(x_i - x_0)$$

would satisfy all axioms (a)–(f) except for covalent additivity.

[3] Sandbu's model is a special case of RDU with the same weighting function, but nonlinear utility.

The model

$$U(x) = -x_0 - \alpha \sum_{i=1}^{n} \max\{x_i - x_0, 0\} - \beta \sum_{i=1}^{n} \max\{x_0 - x_i, 0\}$$

would satisfy all axioms (a)–(f) except for constant monotonicity.

Consider preferences that are as follows. Every constant distribution is preferred to every non-constant distribution. Non-constant distributions are compared using

$$U(x) = x_0 - \alpha \sum_{i=1}^{n} \max\{x_i - x_0, 0\} - \beta \sum_{i=1}^{n} \max\{x_0 - x_i, 0\}$$

and constant distributions are compared using

$$U(x) = x_0 - \alpha \sum_{i=1}^{n} \max\{x_i - x_0, 0\} - \beta \sum_{i=1}^{n} \max\{x_0 - x_i, 0\},$$

but a non-constant distribution cannot be compared to a constant distribution using the same functional. By definition every constant distribution is preferred to every non-constant distribution. It can easily be verified that these preferences satisfy weak ordering, constant monotonicity, inequity aversion, and anonymity. Clearly, constant equivalence is not satisfied. Covalent additivity is satisfied. The only way in which covalent additivity could be violated is if adding the distribution z to two non-constant distributions $x \succcurlyeq y$ would make $y + z$ constant. This would require y and z not to be covalent, which is not allowed by covalent additivity.

It remains to be shown that condition (a) is required to obtain FS. Completeness is implied by transitivity, constant equivalence, and constant monotonicity. By constant equivalence every distribution has an equivalent constant distribution. Constant monotonicity implies that all constant distributions can be compared. Transitivity implies that all distributions can be compared through their constant equivalents. Thus, all distributions can be compared and completeness is satisfied.

Consider a setting with two individuals, i.e. $n = 1$. Let preferences be representable by

$$U(x) = x_0 - \alpha \sum_{i=1}^{n} \max\{x_i - x_0, 0\} - \beta \sum_{i=1}^{n} \max\{x_0 - x_i, 0\}$$

with $\beta > 1$, except when comparing two distributions that are not covalent. When two distributions are not covalent, they can be compared as follows:

$$\begin{aligned} (x_0, x_1) \succ (y_0, y_1) \quad &\text{whenever } x_0 > x_1 \text{ and } y_0 < y_1, \text{ and} \\ (x_0, x_1) \prec (y_0, y_1) \quad &\text{whenever } x_0 < x_1 \text{ and } y_0 > y_1. \end{aligned}$$

This preference relation is complete, but not transitive. We can find $x, y > 0$ such that the constant equivalent of $(x, 0)$ is smaller than the one of $(0, y)$, which, under transitivity and constant monotonicity would imply that $(x, 0) \prec (0, y)$. Note that, except for transitivity, all preference conditions are satisfied, as these conditions only concern comparisons of covalent distributions.

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