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> Multidimensional Welfare Comparisons of EU Member States Before, During, and After the Financial Crisis:
> A Dominance Approach

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# Multidimensional Welfare Comparisons of EU Member 

## States Before, During, and After the Financial Crisis:

## A Dominance Approach

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#### Abstract

How did the financial crisis affect population welfare in EU member states in key dimensions such as income, health, and education? We seek to answer this question by way of welfare comparisons between countries and within countries over time, using EU-SILC data. Our study is novel in using a multidimensional first order dominance comparison approach on the basis of multi-level ordinal data. We find that the countries most often dominated are southern and eastern European member states, and the dominant countries are mostly northern and western European member states. However, for most country comparisons, there is no dominance relationship. Moreover, only a few member states have experienced a temporal dominance improvement in welfare, while no member states have experienced a temporal dominance deterioration during the financial crisis.


[^1]Keywords: first order dominance, multidimensional well-being, multi-level indicators, EU-SILC

JEL classification: I31, I32, O52.

## 1 Introduction

The recent financial and economic crisis has had a major impact on EU member states. The effects on key macroeconomic indicators at the country level such as GDP growth, public debt, inflation, etc. have been widely analyzed (e.g., European Commission, 2009). It has also been shown that the financial and economic crisis has affected income at the individual level (e.g., De Beer, 2012). However, it has long been recognized that welfare is a multidimensional phenomenon, which is not adequately measured by income (e.g., Sen, 1970, 1976, Arrow, 1971, Kolm, 1977), and much less is known regarding the impact of the crisis on welfare when taking a multidimensional view. Two questions are examined in this paper: How has the crisis changed the relative multidimensional welfare of EU member states? Has multidimensional welfare improved or deteriorated for each state during the financial crisis?

Previous multidimensional welfare comparisons of European countries have used methods that rely on a priori assumptions about the relative importance of different dimensions of welfare and a weighting scheme reflecting these. Examples of such methods are the Human Development Index (HDI), which focuses on three dimensions of welfare, namely a long and healthy life, being knowledgeable, and having a decent standard of living (UNDP, 1990, 2014), and the Multidimensional Poverty Index (MPI) (e.g. Alkire and Apablaza, 2016). The approach of assigning weights to each dimension, and thereby defining a single composite measure of welfare, is convenient for applications since it enables the analyst to make a complete ranking of the populations being compared. However, a challenge when applying such an approach is that there is no natural or generally agreed methodology on how to set the weights, and in practice the weights are often set equally in each dimension. As Ravallion (2011) points out, the producer of the index is essentially free to
set the unusually large number of "moving parts" that make up the index. ${ }^{1}$ Even if there is wide agreement that one dimension should be given a higher weight than another, it is rarely clear how this should be translated into actual weights. In addition, it is likely that there are significant differences in EU member states' preferences for different dimensions of welfare. One specific weighting scheme may thus not be appropriate to describe generic preferences in all European countries.

The challenges described above have fostered a focus on dominance methods that are robust to different weighting schemes in multidimensional welfare comparisons. These methods enable comparisons across different weighting schemes or, put differently, broad classes of underlying social welfare functions (e.g., Atkinson and Bourguignon, 1982, 1987, Bourguignon, 1989, Atkinson, 1992, Bourguignon and Chakravarty, 2003, Duclos et al., 2006, 2007, Gravel et al., 2009, Gravel and Mukhopadhyay, 2010, Duclos and Échevin, 2011, Muller and Trannoy, 2011, Gravel and Moyes, 2012). However, while the above mentioned studies develop methods that offer great flexibility in terms of weighting of dimensions, these methods all assume that the indicators are suitably scaled (or one out of two as in Gravel and Moyes, 2012), i.e., the methods are not suited for the analysis of ordinal data. Specifically, a common feature of the above-mentioned studies is the application of conditions that are typically formulated in terms of specified signs on the second (or higher) order partial or cross-derivatives of the underlying social welfare function.

In this paper we make multidimensional population welfare comparisons without relying on a priori chosen weights, nor on methods that require assumptions about the substitutability/complimentarity between dimensions. The natural concept for such comparisons is first order (stochastic) dominance (FOD), also known as the usual stochastic order in probability theory (e.g., Shaked and Shanthikumar, 2007). One finite distribution first order dominates another finite distribution if the other distribution can be obtained from the first by iteratively shifting probability mass from better to worse outcomes. The approach provides a way of making comparisons of multidimensional welfare that is robust to different weighting schemes. That is, it allows comparisons without making assump-

[^2]tions about utility functions and/or social welfare functions other than nondecreasingness. Thus, ordinal indicators can be used for population welfare comparisons (Arndt et al., 2012); i.e., it is only required that outcomes can be ranked from worse to better within each dimension. ${ }^{2}$

Previous applications of FOD for welfare comparisons have used only binary indicators (see, e.g., Arndt and Tarp, 2017, for a collection of studies applying FOD in developing countries using binary indicators). In this paper, we use multi-level indicators; i.e., within each indicator, more than two levels are allowed. In our comparisons of EU member states, we include three multi-level indicators of the welfare dimensions income, health, and education, which have four, five, and three ordered levels, respectively, yielding a total of sixty different outcomes. We make spatial analyses of countries relative to each other within a given year as well as temporal analyses of countries over time. We find between 40 and 45 multidimensional spatial dominances in each of the three years analyzed (2005, 2009, and 2013) out of 276 potential dominances each year. We find only a few temporal dominances.

In the multidimensional spatial analyses, the countries that are most often dominated are southern and eastern European countries (Hungary, Italy, Latvia, Lithuania, Poland, Portugal, Slovenia, and Spain), whereas the dominant countries are most often northern and western European countries (Austria, Germany, Ireland, the Netherlands, Sweden, and the United Kingdom). ${ }^{3}$ In the multidimensional temporal analyses, only a few countries have experienced a FOD improvement in welfare and no country has experienced a FOD deterioration in welfare over the period. When a multidimensional analysis that includes more than just income is conducted, the financial crisis thus did not lower broadly defined population welfare. Different patterns often appear from the separate one-dimensional analyses. These findings highlight the importance of a multidimensional view in welfare analyses.

[^3]The paper is structured as follows. Section 2 briefly reviews related empirical literature on multidimensional welfare in Europe. Section 3 describes the concept of FOD in a multidimensional welfare setting, followed by a description of a method for identifying dominances empirically and the Copeland (1951) method for providing a ranking based on pairwise dominance comparisons. Section 4 describes the EU-SILC data applied. The results are shown in Section 5 and discussed in Section 6. Section 7 concludes.

## 2 Related empirical literature

To the authors knowledge, there is no previous literature that applies multidimensional dominance concepts in a country comparison setting in Europe (with the exception of Hussain, 2016, discussed later in this section). The empirical literature most closely related to the present paper is the group of studies which has calculated summary indicators of welfare for European population groups.

The United Nations Development Program (UNDP) publishes the Human Development Index (HDI) for most of the world's sovereign countries and states as a measure of progress in a given country (UNDP, 2014). Their focus is on three dimensions of welfare, namely a long and healthy life, being knowledgeable, and having a decent standard of living. The indicators of each dimension are life expectancy at birth, mean years of schooling and expected years of schooling, and purchasing power parity (PPP) adjusted gross national income (GNI) per capita, respectively. Both the two intra-education indicators making up the education dimension and the three dimensions in the HDI are weighted equally by $\frac{1}{2}$ and $\frac{1}{3}$, respectively. ${ }^{4}$ All EU member states in the present study have "very high human development" in the years analyzed in the present study (UNDP, 2011, 2014). ${ }^{5}$ western and northern European member states are typically ranked higher (i.e., they have a higher HDI) than southern and eastern European member states.

[^4]The UNDP also publishes the Multidimensional Poverty Index (MPI, sometimes denoted $M_{0}$ ), but only for developing countries. Furthermore, the year of the surveys used to calculate the MPI in a given year differs significantly (e.g., UNDP, 2014, Table 6, pages 180-181). However, in a recent paper, Alkire and Apablaza (2016) (building on Alkire et al., 2014) explore multidimensional poverty in Europe using MPI on EU-SILC data in 2006, 2009, and 2012 following the Alkire-Foster methodology (Alkire and Foster, 2011a,b, Foster et al., 1984, and see also Alkire et al., 2015). The MPI applied in Alkire and Apablaza (2016) includes twelve binary indicators in six dimensions: income, employment, material deprivation, education, environment, and health. ${ }^{6}$ As in UNDP (2011, 2014), western and northern European member states are typically ranked higher (i.e., they have a lower MPI) than southern and eastern European member states.

Other studies of multidimensional welfare and poverty in a European country comparison context are available in the literature. They generally consider dimensions and/or indicators that are very different from the ones used in the present paper (see Section 4 for a description). For example, Bossert et al. (2013) use a weighting scheme and a deprivation approach on EU-SILC data of arrears, inability to keep the home adequately warm, lack of capacity to face unexpected required expenses, and inability to afford a meal with meat, chicken, or fish (or a vegetarian protein equivalent) every other day, a one-week annual holiday away from home, a car, a washing mashine, a color TV, and a telephone. Whelan et al. (2014) apply a deprivation approach and the Alkire-Foster methodology on basic deprivation, consumption deprivation, health, and neighbourhood environment using EU-SILC data. Hussain (2016) applies the HDI, MPI, FOD, and more on EU-SILC data of deprivations similar to those used in Bossert et al. (2013). This is the only other empirical application of FOD on European countries. As other previous applications of FOD for welfare comparisons, Hussain (2016) uses binary indicators. Permanyer and Hussain (2017) combine multiple scenario simulated data with observed

[^5]data from 48 Demographic and Health Surveys around the developing world to provide a methodological comparison of FOD with other multidimensional measures, including the MPI.

## 3 Methodology

Suppose that welfare is measured in $N$ dimensions and let $X \subseteq \mathbb{R}^{N}$ be a finite set of multidimensional outcomes. A distribution of welfare is described by a probability mass function $f$ over $X$ (i.e. $\sum f(x)=1$ and $f(x) \geq 0$ for all $x \in X$ ). We refer to $f$ as a distribution. A subset $Y \subseteq X$ is a lower comprehensive set (LCS) if $x \in Y, y \in X$, and $y \leq x$ implies $y \in Y$ (for an illustrative example, see Appendix 2). A distribution $f$ first order dominates distribution $g$ if
(i) $\sum_{x \in Y} g(x) \geq \sum_{x \in Y} f(x)$ for all $Y \subseteq X$.

It is well-known that condition (i) of multidimensional FOD is equivalent to the following two definitions: (ii) $g$ can be obtained from $f$ by a finite number of shifts of probability mass from one outcome to another that is worse, and (iii) social welfare is weakly higher for $f$ than for $g$ for any nondecreasing additively separable social welfare function; i.e., $\sum_{x \in X} f(x) w(x) \geq \sum_{x \in X} g(x) w(x)$ for any weakly increasing real function $w(\cdot) . .^{7,8}$

Note that FOD only requires ordinal data and that it is absent of assumptions about the strength of preferences for each dimension, the relative desirability of changes among

[^6]levels within or between dimensions, and the substitutability/complementarity among the dimensions (Arndt et al., 2012) as mentioned in Section 1. This makes FOD applicable to a wide range of indicators, whereas, e.g., dominance concepts following Atkinson and Bourguignon (1982) and Atkinson and Bourguignon (1987) are not suited for the analysis of ordinal data. ${ }^{9}$

Using definition (ii), Mosler and Scarsini (1991) and Dyckerhoff and Mosler (1997) show that identifying FOD corresponds to checking if a certain linear program has a feasible solution. The first empirical implementation of this approach was provided by Arndt et al. (2012) in a study of child poverty in Mozambique and Vietnam with multiple binary indicators (see also Arndt and Tarp, 2017). In this paper, we identify dominances using definition (i), which is an exact test of dominance. To the authors' knowledge, this approach has not previously been applied to identify multidimensional population welfare. ${ }^{10}$

When we test for FOD using definition (i), one challenge is that the number of LCSs increases drastically when more dimensions and/or levels are included in the analysis. ${ }^{11}$ One has to carefully consider the number of dimensions as well as the number of levels of each indicator, as there is a trade-off between adequate characterization of welfare and increasing computational complexity of checking dominances. We first identify all LCSs using an iterative algorithm available from the authors. After the identification of all LCSs, checking for FOD using definition (i) is straightforward. ${ }^{12}$

[^7]When comparing two populations it may be the case that none of them dominates the other. Thus, generally we are unable to obtain a complete ranking of all populations by way of FOD comparisons. However, the Copeland (1951) method can be used as a measure of the tendency to outperform other populations as an overall relative indicator of population well-being (Arndt et al., 2016, Siersbæk et al., 2017), which can be applied to the spatial analyses to obtain a ranking of the compared populations. The Copeland method involves counting, for each of the $n$ populations, how many of the $n-1$ other populations it dominates and subtracting the number of times it is dominated by the other populations. The corresponding Copeland score is in the interval $[-(n-1) ; n-1]$, which is normalized to $[-1 ; 1]$.

## 4 Data

The data applied are from the EU Statistics on Income and Living Conditions (EU-SILC) database. We focus on the years 2005, 2009, and 2013 (i.e., before, during, and after the financial and economic crisis). As of 2005, the EU-SILC data cover all of the 25 member states at the time. The EU member states Bulgaria, Croatia, and Romania are therefore not included due to entry into the EU in 2007, 2013, and 2007 respectively. Furthermore, Malta is omitted due to insufficient data. The sample sizes for the member states range from 5,429 to 47,311 respondents in a given year (a complete overview is shown in Table A3 in Appendix 3). All the data are collected based on the same (translated) questions in all EU member states using representative samples. We include three key dimensions of welfare: income, health, and education. Income is used in most measures of welfare. Health and education are prevalent in many measures of multidimensional welfare (e.g., World Bank, 1990, Alkire, 2002, UNDP, 2014, Alkire and Apablaza, 2016), and they have

[^8]Table 1 Description of welfare dimensions and indicators

| Dimension | Indicator | Level | Construction |
| :--- | :--- | :--- | :--- |
| Income | Equivalized | 1 | First quartile $^{b}$ |
|  | annual net $^{\text {income }^{a}}$ | 2 | Second quartile $^{b}$ |
|  |  | 3 | Third quartile $^{b}$ |
|  | 4 | Fourth quartile $^{b}$ |  |
| Health | Self-reported | 1 | Very bad |
|  | health | 2 | Bad |
|  |  | 3 | Fair |
|  |  | 4 | Good |
|  |  | 5 | Very good |
| Education | Highest ISCED | 1 | Pre-primary, primary, and lower secondary |
|  | level obtained | 2 | Upper secondary and post-secondary |
|  |  | 3 | First $^{c}$ and second ${ }^{d}$ stage tertiary |

Notes: a) Net income after transformation using equivalence scale weights. b) Quartiles are based on the EU distribution of PPP-adjusted real income in 2005. c) Not leading to an advanced research qualification. d) Leading to an advanced research qualification.
both been affected by the financial and economic crisis (e.g., Stuckler et al., 2009, Kentikelenis et al., 2011, OECD, 2013a). The inclusion of these three particular dimensions also enables us to make interesting comparisons between our findings and welfare indices such as the HDI (UNDP, 2014) and the MPI (Alkire and Apablaza, 2016). Both of these indices use a weighting scheme and include indicators of health, education, and some measure of standard of living, typically income. The dimensions and indicators used in the present paper are described below and briefly summarized in Table 1.

We use individual equivalized annual net income as an indicator in the income dimension and correct it using PPP to facilitate cross-country comparisons. ${ }^{13}$ Using EU quartiles in 2005 as thresholds, a four-level indicator is constructed. All incomes in 2009 and 2013 are deflated using the consumer price index (CPI). The indicator in the health dimension is self-reported health ranging from 1 (Very bad) to 5 (Very good); i.e., a fivelevel indicator. It includes different aspects of subjective health including physical, social,

[^9]and emotional function and biomedical signs and symptoms. In the education dimension, we use the highest ISCED level obtained in three levels from 1 to 3 , where 3 is best. ${ }^{14}$

The three indicators imply $4 \cdot 5 \cdot 3=60$ different outcomes. To identify dominances using definition (i), it is required to check 116,424 inequalities (Sampson and Whitaker, 1988). An illustration of the data setup that enables identification of FOD using the code available from the authors is shown in Table A4 in Appendix 4 using sample data for Germany in 2005 for all outcomes.

## 5 Results

Tables 2, 3, and 4 show the one-dimensional and multidimensional analyses in 2005, 2009, and 2013 respectively. " I " indicates that the row country dominates the column country in the income dimension. Similarly "H" indicates dominance in the health dimension and "E" indicates dominance in the education dimension. The absence of the indicator(s) implies that there is no dominance in the relevant dimension(s). A gray cell indicates multidimensional dominance (MD), which is tested using definition (i) in Section 3. Note that the column totals for I, H, E, and MD yield the total number of times the column country is dominated by another country in each dimension (I, H, and E) and in the multidimensional analysis, respectively. ${ }^{15}$ The corresponding row totals yield the total number of times a country is dominant in the three different dimensions and in the multidimensional analysis.

Table 5 shows the temporal FOD results in both the one-dimensional and the multidimensional analyses. For each row country, an "I", "H", and/or "E" in column 2 indicates

[^10]Table 2 Spatial first order dominances, 2005

|  | AT | BE | CY | CZ | DE | DK | EE | EL | ES | FI | FR | HU | IE | IT | LT | LU | LV | NL | PL | PT | SE | SI | SK | UK | $\mathrm{I}^{a}$ | $\mathrm{H}^{\text {b }}$ | $\mathrm{E}^{c}$ | $\mathrm{MD}^{\text {d }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AT | - | I | I | IH | I | I | IH | IE | IH | I | I | IHE | I | IHE | IH |  | IHE | I | IHE | IHE | I | IHE | IH |  | 21 | 11 | 7 | 6 |
| BE |  | - |  | IH |  |  | IH | IE | IHE | I | IE | IH | E | IE | IH | E | IH |  | IH | IHE | I | IH | IH |  | 15 | 10 | 7 | 2 |
| CY |  |  | - | IH |  | E | IH | IE | IHE | H | E | IH | E | IE | IH | E | IH |  | IH | IHE |  | IH | IH |  | 12 | 11 | 8 | 2 |
| CZ |  |  |  | - |  |  | IH |  |  |  |  | IH |  | E | IH |  | IH |  | IH | HE |  | HE | I |  | 6 | 7 | 3 | 0 |
| DE | E | IE | IE | IE | - | E | IHE | IE | IE | IE | IE | IHE | E | IHE | IHE | E | IHE | E | IE | IHE | IE | IHE | IE | E | 17 | 7 | 23 | 7 |
| DK |  | H |  | IH |  | - | IH | IE | IH | I | E | IH |  | IHE | IH | E | IH |  | IH | IHE | I | IH | IH |  | 14 | 12 | 5 | 1 |
| EE | E |  | E |  |  | E | - | E | E |  | E | IE | E | E | IE | E | IHE |  | IE | HE |  | E | I |  | 5 | 2 | 15 | 1 |
| EL |  |  |  | I |  |  | IH | - |  | H |  | IH |  | E | IH |  | IH |  | IH | IHE |  | H | IH |  | 8 | 9 | 2 | 1 |
| ES |  |  |  | IH |  |  | IH | I | - |  |  | IH |  |  | IH |  | IH |  | IH | IHE |  | H | I |  | 9 | 8 | 1 | 1 |
| FI |  |  | E | I |  | E | I | IE | IE | - | E | IHE | E | E | I | E | IH |  | IH | IHE |  | IH | IH |  | 11 | 6 | 10 | 2 |
| FR |  |  |  | IH |  |  | IH | IE | IH | I | - | IH |  | IE | IH |  | IH |  | IH | IHE |  | IH | IH |  | 13 | 10 | 3 | 1 |
| HU |  |  |  |  |  |  |  |  |  |  |  | - |  | E |  |  |  |  |  | E |  |  | I |  | 1 | 0 | 2 | 0 |
| IE | H | H |  | IH | H | H | IH | IE | IHE | H | H | IH | - | IHE | IH | H | IH | H | IH | IHE | H | IH | IH | H | 12 | 21 | 4 | 3 |
| IT |  |  |  | I |  |  | IH | I | I |  |  | IH |  | - | IH |  | IH |  | I | IH |  | IH | I |  | 11 | 6 | 0 | 0 |
| LT |  |  |  |  |  |  |  | E |  |  | E | E |  | E | - |  | HE |  |  | E |  | E | 1 |  | 1 | 1 | 7 | 0 |
| LU | I | IH | I | IH | IH | I | IH | IE | IH | I | IH | IH | I | IHE | IH | - | IH | I | IH | IHE | I | IH | IH | I | 23 | 14 | 3 | 2 |
| LV |  |  |  |  |  |  |  |  |  |  |  | E |  | E |  |  | - |  |  | E |  | E | I |  | 1 | 0 | 4 | 0 |
| NL |  | I | IE | IH | H | IE | IH | IE | IHE | I | IE | IH | E | IHE | IH | E | IH | - | IH | IHE | I | IH | I |  | 18 | 11 | 9 | 3 |
| PL |  |  |  |  |  |  |  |  |  |  |  | H |  | E | I |  | IH |  | - | HE |  | HE | I |  | 3 | 4 | 3 | 0 |
| PT |  |  |  | I |  |  | I |  |  |  |  | I |  |  | I |  | I |  | I | - |  |  | I |  | 7 | 0 | 0 | 0 |
| SE |  | H | E | IH |  | E | IH | IE | IHE |  | E | IHE | E | E | IHE | E | IHE |  | IHE | IHE | - | IHE | IH |  | 11 | 11 | 14 | 6 |
| SI |  |  |  | I |  |  | I |  |  |  |  | I |  |  | I |  | IH |  | I | H |  | - | I |  | 7 | 2 | 0 | 0 |
| SK |  |  |  |  |  |  |  |  |  |  |  | E |  | E |  |  | H |  | E | HE |  | E | - |  | 0 | 2 | 5 | 0 |
| UK |  | HE | IE | IH | H | E | IH | IE | IHE | IE | HE | IHE | IE | IHE | IHE | E | IHE | E | IH | IHE |  | IHE | IH | - | 15 | 14 | 16 | 7 |
| $\mathrm{I}^{a}$ | 1 | 4 | 5 | 17 | 2 | 3 | 18 | 14 | 13 | 8 | 5 | 19 | 3 | 10 | 20 | 0 | 20 | 2 | 19 | 15 | 6 | 13 | 23 | 1 | 241 | - | - | - |
| $\mathrm{H}^{\text {b }}$ | 1 | 5 | 0 | 11 | 4 | 1 | 15 | 0 | 10 | 3 | 3 | 17 | 0 | 7 | 15 | 1 | 21 | 1 | 14 | 20 | 1 | 17 | 11 | 1 | - | 179 | - | - |
| $\mathrm{E}^{c}$ | 2 | 2 | 6 | 1 | 0 | 7 | 1 | 14 | 9 | 2 | 10 | 9 | 8 | 20 | 4 | 9 | 6 | 2 | 5 | 21 | 1 | 10 | 1 | 1 | - | - | 151 | - |
| $\mathrm{MD}^{\text {d }}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 5 | 0 | 0 | 5 | 0 | 6 | 3 | 0 | 5 | 0 | 2 | 14 | 0 | 4 | 0 | 0 | - | - | - | 45 |

Notes: See Table A1 in Appendix 1 for an abbreviation list of EU member states' names. "I" indicates that the row country dominates the column country in the income dimension; i.e., when only income is considered. Similarly, "H" indicates dominance in the health dimension and "E" indicates dominance in the education dimension. The absence of the indicator(s) implies that there is no dominance in the relevant dimension(s). A gray cell indicates multidimensional dominance; i.e., when all three dimensions are considered simultaneously. For example, Austria dominates the Czech Republic in the income (I) and health (H) dimensions in 2005, but neither in the education (E) dimension nor multidimensionally (absence of gray cell). a) Income. b) Health. c) Education. d) Multidimensional.
Table 3 Spatial first order dominances, 2009

|  | AT | BE | CY | CZ | DE | DK | EE | EL | ES | FI | FR | HU | IE | IT | LT | LU | LV | NL | PL | PT | SE | SI | SK | UK | $\mathrm{I}^{a}$ | $\mathrm{H}^{\text {b }}$ | $\mathrm{E}^{c}$ | $\mathrm{MD}^{\text {d }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AT | - | I | I | IH | I | I | I | I | I | I |  | IH |  | IHE | IH |  | IH |  | IH | IHE | I | IH | IH | I | 19 | 9 | 2 | 2 |
| BE |  | - |  | IH |  | I | I | IE | IE | I |  | IH | E | IHE | IH | E | IH |  | IH | IHE |  | IH | IH |  | 14 | 9 | 6 | 2 |
| CY |  | 1 | - | IH | I | I | I | IE | IE | I | E | IH |  | IHE | IH | E | IH |  | IH | IHE | I | IH | IH | I | 18 | 9 | 6 | 1 |
| CZ |  |  |  | - |  |  | I |  |  |  |  | IH |  | E | IH |  | IH |  | H | HE |  | H | I |  | 5 | 6 | 2 | 0 |
| DE | E | E | E | IE | - | E | IHE | IE | IE | E | E | IE | E | IHE | IHE | E | IHE | E | IHE | IHE | E | IHE | IE | IE | 13 | 7 | 23 | 7 |
| DK |  |  |  | I |  | - | I | IE | I | I | E | IH |  | IE | IH | E | IH |  | IH | IHE |  | IH | IH |  | 13 | 7 | 5 | 1 |
| EE | E |  | E |  |  | E | - | E | E | E | E | IE | E | E | IE | E | IHE | E | E | E | E | E |  | E | 3 | 1 | 19 | 1 |
| EL |  |  |  | I |  |  | I | - |  |  |  | IH |  | E | IH |  | I |  | I | IHE |  | H | IH |  | 8 | 5 | 2 | 1 |
| ES |  |  |  | I |  |  | I | I | - |  |  | I |  | H | IH |  | IH |  | I | IHE |  | IH | I |  | 10 | 5 | 1 | 1 |
| FI |  |  | E | IH |  | E | IH | IE | IE | - | E | IH | E | IHE | IH | E | IH |  | IH | IHE |  | IH | I | E | 12 | 9 | 10 | 2 |
| FR |  | I |  | IH | I | I | IH | IE | I | I | - | IH |  | IHE | IH | E | IH |  | IH | IHE | I | IH | IH | I | 18 | 10 | 4 | 2 |
| HU |  |  |  |  |  |  |  |  |  |  |  | - |  | E |  |  |  |  |  | HE |  |  |  |  | 0 | 1 | 2 | 0 |
| IE | H | IH | IH | IH | IH | IH | IH | IE | IHE | IH | IH | IH | - | IHE | IH | HE | IH | H | IH | IHE | IH | IH | IH | IH | 20 | 22 | 5 | 3 |
| IT |  |  |  | I |  |  | I | I |  |  |  | 1 |  | - | IH |  | IH |  | I | IHE |  | I | I |  | 10 | 3 | 1 | 1 |
| LT |  |  |  |  |  |  |  | E |  |  | E | IE |  | E | - | E | IE |  |  | E |  |  |  |  | 2 | 0 | 7 | 0 |
| LU | IH | IH | I | IH | I | IH | IH | IE | IH | IH | I | IH | 1 | IHE | IH | - | IH | I | IH | IHE | I | IH | IH | I | 23 | 15 | 3 | 2 |
| LV |  |  |  |  |  |  |  | E |  |  |  | IE |  | E |  |  | - |  |  | E |  |  |  |  | 1 | 0 | 4 | 0 |
| NL |  | I | E | IH | IH | IE | IH | IE | IHE | IH | IE | IH | E | IHE | IH | E | IH |  | IH | IHE | I | IH | IH | IE | 19 | 13 | 10 | 3 |
| PL | E |  |  |  |  |  | I |  |  |  |  | I |  | E | IH |  | I |  | - | HE |  |  | I |  | 5 | 2 | 3 | 0 |
| PT |  |  |  |  |  |  | I |  |  |  |  | I |  |  | I |  | I |  | I | - |  |  | 1 |  | 6 | 0 | 0 | 0 |
| SE | HE | IH | E | IH | H | IHE | IH | IE | IHE | IH | E | IHE | E | IHE | IHE | HE | IHE |  | IHE | IHE | - | IHE | IH | IE | 16 | 17 | 16 | 8 |
| SI |  |  |  | 1 |  |  | I |  |  |  |  | IE |  | E | I |  | I |  | I | HE |  | - | 1 |  | 7 | 1 | 3 | 0 |
| SK | E |  |  | E |  |  |  |  |  |  |  | IH |  | E |  |  |  |  | E | HE |  |  | - |  | 1 | 2 | 5 | 0 |
| UK | H | H | E | IH | H | HE | IH | IE | IHE | H | HE | IH | E | IHE | IH | HE | IH |  | IH | IHE | H | IH | IH | - | 12 | 19 | 9 | 3 |
| $\mathrm{I}^{a}$ | 1 | 7 | 3 | 16 | 6 | 8 | 19 | 14 | 12 | 9 | 3 | 23 | 1 | 12 | 20 | 0 | 21 | 1 | 17 | 15 | 6 | 14 | 19 | 8 | 255 | - | - | - |
| $\mathrm{H}^{\text {b }}$ | 4 | 4 | 1 | 10 | 4 | 4 | 8 |  | 5 | 5 | 2 | 14 | 0 | 12 | 17 | 3 | 16 | 1 | 13 | 20 | 2 | 15 | 11 | 1 | - | 172 | - | - |
| $\mathrm{E}^{c}$ | 5 | 1 | 6 | 2 | 0 | 6 | 1 | 14 | 9 | 2 | 9 | 6 | 7 | 21 | 3 | 12 | 4 | 2 | 4 | 23 | 2 | 3 | 1 | 5 | - | - | 148 |  |
| $\mathrm{MD}^{\text {d }}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 4 | 0 | 0 | 1 | 0 | 10 | 2 | 0 | 3 | 0 | 2 | 15 | 0 | 2 | 0 | 0 | - | - | - | 40 |

[^11]Table 4 Spatial first order dominances, 2013

|  | AT | BE | CY | CZ | DE | DK | EE | EL | ES | FI | FR | HU | IE | IT | LT | LU | LV | NL | PL | PT | SE | SI | SK | UK | $\mathrm{I}^{a}$ | $\mathrm{H}^{\text {b }}$ | $\mathrm{E}^{c}$ | $\mathrm{MD}^{\text {d }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AT | - | I | I | IH | I | I | IH | I | I | I |  | IH | I | IHE | IH |  | IH |  | IH | IHE | I | IH | IH | I | 20 | 10 | 2 | 2 |
| BE |  | - | I | IH |  |  | IH | IE | IE | I |  | IH |  | IHE | IH | E | IH |  | IH | IHE |  | IH | IH | I | 15 | 10 | 5 | 2 |
| CY | H | H | - | IH | H | H | IH | IE | IHE |  |  | IH |  | IHE | IH | HE | IH |  | IH | IHE |  | IH | IH | IH | 13 | 17 | 5 | 3 |
| CZ |  |  |  | - |  |  | IH | I |  |  |  | IH |  | E | IH |  | IH |  | I | HE |  |  | I |  | 7 | 5 | 2 | 0 |
| DE | E | E | IE | I | - | E | IHE | IE | IE | E | E | IHE |  | IE | IHE | E | IHE | E | IHE | IHE | E | IE | IE | IE | 14 | 6 | 21 | 6 |
| DK |  |  | IE | IH |  | - | IH | IE | IE | I | E | IH |  | IHE | IH | E | IH |  | IH | IHE |  | IH | I | I | 15 | 9 | 7 | 2 |
| EE | E |  | E |  |  | E | - | E | E | E | E | IE |  | E | - | E | IHE | E | E | HE |  | E |  | E | 3 | 3 | 17 | 1 |
| EL |  |  |  |  |  |  | I | - |  |  |  | IH |  | HE | IH |  | IH |  | I | HE |  |  | I |  | 6 | 5 | 2 | 0 |
| ES |  |  |  | IH |  |  | IH | I | - |  |  | IH |  | H | IH |  | IH |  | IH | IHE |  | I | I |  | 10 | 8 | 1 | 1 |
| FI |  |  | E | I |  | E | IH | IE | IE | - | E | IH |  | IE | IH | E | IH | E | I | IHE |  | I | I | IE | 13 | 5 | 10 | 1 |
| FR |  | I | I | IH |  | I | IH | IE | I | I | - | IH |  | IHE | IH | E | IH |  | IH | IHE | I | IH | IH | I | 18 | 10 | 4 | 2 |
| HU |  |  |  |  |  |  |  |  |  |  |  | - |  | E |  |  |  |  |  | HE |  |  |  |  | 0 | 1 | 2 | 0 |
| IE | H | IH | I | IH | H | H | IH | IE | IHE | I | H | IH | - | IHE | IH | HE | IH | H | IH | IHE |  | IH | IH | IH | 16 | 19 | 5 | 3 |
| IT |  |  |  | I |  |  | I | I |  |  |  | I |  | - | IH |  | IH |  | I | IH |  | I | I |  | 10 | 3 | 0 | 0 |
| LT | E |  | E |  |  |  |  | E | E |  | E | IE |  | E | - | E | IE |  |  | E |  |  |  |  | 2 | 0 | 10 | 0 |
| LU | I | I | I | IH | I | I | IH | I | IH | I | I | IH | I | IHE | IH | - | IH | I | IH | IHE | I | IH | IH | I | 23 | 11 | 2 | 2 |
| LV |  |  |  |  |  |  |  | E |  |  |  | E |  | E |  | E | - |  |  | E |  |  |  |  | 0 | 0 | 5 | 0 |
| NL |  |  | IE | IH | H | I | IH | IE | IHE | I | E | IH |  | IHE | IH | E | IH | - | IH | IHE |  | I | I | IE | 16 | 10 | 8 | 3 |
| PL | E |  |  |  |  |  | IH |  |  |  |  | IHE |  | E | IH |  | IH |  | - | HE |  |  |  |  | 4 | 5 | 4 | 1 |
| PT |  |  |  |  |  |  | I | I |  |  |  | I |  |  | I |  | I |  | I | - |  |  | I |  | 7 | 0 | 0 | 0 |
| SE | HE | IH | IE | IH | H | IHE | IHE | IE | IHE | IHE | HE | IHE |  | IHE | IHE | HE | IHE | HE | IHE | IHE | - | IHE | IH | IE | 17 | 19 | 18 | 9 |
| SI | E |  |  | I |  |  | IH | IE |  |  |  | IHE |  | E | IH | E | IHE |  | I | IHE |  | - | I |  | 9 | 5 | 7 | 3 |
| SK | E |  |  | H |  |  | H |  |  |  |  | IH |  | E | H |  | IH |  | H | HE |  | H | - |  | 2 | 8 | 3 | 0 |
| UK | H |  | E | IH |  | H | IH | IE | IHE |  | E | IH |  | IHE | IH | E | IH |  | IH | IHE |  | IH | IH |  | 12 | 13 | 7 | 2 |
| $\mathrm{I}^{a}$ | 1 | 5 | 9 | 15 | 2 | 5 | 19 | 17 | 12 | 8 | 1 | 22 | 2 | 12 | 20 | 0 | 22 | 1 | 18 | 15 | 3 | 14 | 18 | 11 | 252 | - | - | - |
| $\mathrm{H}^{\text {b }}$ | 4 |  | 0 | 12 | 4 | 4 | 17 | 0 | 6 | 1 | 2 | 18 | 0 | 12 | 20 | 3 | 20 | 2 | 13 | 21 | 0 | 10 | 8 | 2 | - | 182 | - | - |
| $\mathrm{E}^{c}$ | 7 | 1 | 8 | 0 | 0 | 4 | 2 | 14 | 11 | 3 | 8 | 7 | 0 | 21 | 3 | 14 | 5 | 4 | 3 | 22 | 1 | 3 | 1 | 5 | - | - | 147 | - |
| $\mathrm{MD}^{\text {d }}$ | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 4 | 0 | 0 | 4 | 0 | 10 | 2 | 0 | 4 | 0 | 2 | 14 | 0 | 1 | 0 | 0 | - | - | - | 43 |

[^12]Table 5 Temporal first order dominances

|  | Deteriorations over time |  |  | Improvements over time |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 05 dom 09 | 05 dom 13 | 09 dom 13 | 09 dom 05 | 13 dom 05 | 13 dom 09 |
| AT | IH | IH | I |  | E | E |
| BE |  |  |  | IE | IE | HE |
| CY |  |  | I | IE | E | HE |
| CZ |  |  |  | IHE | IE | IE |
| DE |  |  |  | HE | HE | E |
| DK | H | H |  | E | E | IE |
| EE |  |  | I | IE | IE | E |
| EL | H | IH | I | E | E | E |
| ES |  |  | I | IE | IE | HE |
| FI |  |  |  | IE | IE | E |
| FR |  |  |  | IE | IE | IE |
| HU |  |  | I | IHE | HE | E |
| IE |  | H | IH | IE | IE | E |
| IT |  | I | I | IE | E | E |
| LT |  |  |  | IE | IE | E |
| LU | I | IH | I | E | E | E |
| LV |  |  | I | IHE | IHE | E |
| NL |  |  | IH | IHE | E | E |
| PL |  |  | I | IHE | IHE | HE |
| PT |  |  | I | HE | HE | E |
| SE |  |  |  | IHE | IE | E |
| SI |  | I | I | HE | HE | HE |
| SK |  |  |  | I | IE | E |
| UK | I | I | IH | H |  |  |
| $\mathrm{I}^{a}$ | 3 | 6 | 15 | 16 | 12 | 3 |
| $\mathrm{H}^{\text {b }}$ | 3 | 5 | 3 | 10 | 6 | 5 |
| $\mathrm{E}^{c}$ | 0 | 0 | 0 | 21 | 23 | 23 |
| $\mathrm{MD}^{\text {d }}$ | 0 | 0 | 0 | 3 | 2 | 0 |

Notes: See Table A1 in Appendix 1 for an abbreviation list of EU member states' names. For each row country, an "I" in column 2 indicates that 2009 dominates 2005 in the income dimension; i.e., when only income is considered. An "I" in column 3 indicates that 2013 dominates 2005 and so forth. Similarly, "H" indicates dominance in the health dimension and "E" indicates dominance in the education dimension. The absence of the indicator(s) implies that there is no dominance in the relevant dimension(s). A gray cell indicates multidimensional dominance; i.e., when all three dimensions are considered simultaneously. For example, Belgium in 2013 dominates 2009 in the health (H) and education (E) dimensions, but not in the income
(I) dimension nor multidimensionally (absence of gray cell). a) Income. b) Health. c) Education.
d) Multidimensional.
that 2009 dominates 2005 in the relevant dimension(s). A gray cell indicates multidimensional dominance (MD) of 2009 over 2005. Similarly, the presence of one or more of these in column 3 indicates, for each row country, that 2013 dominates 2005, and so forth.

## Spatial FOD comparisons

As seen from Tables 2 through 4, several spatial multidimensional dominances are identified. In 2005, 45 dominances are found (Table 2), whereas 40 and 43 dominances are found in 2009 and 2013, respectively (Tables 3 and 4, respectively). ${ }^{16}$ The multidimensional dominances are largely driven by a few countries that either dominate several others or are dominated often. For example, in 2005 Germany dominates seven countries, Austria dominates six countries, and Portugal is dominated by 14 countries (Table 2). The dominated countries are most often southern and eastern European countries (Hungary, Italy, Latvia, Lithuania, Poland, Portugal, and Slovenia), whereas the dominant countries are most often northern and western European countries (Austria, Germany, Ireland, the Netherlands, Sweden, and the United Kingdom). As mentioned, both the HDI (UNDP, 2014) and the MPI (Alkire and Apablaza, 2016) yield rankings where northern and western European countries are ranked higher than southern and eastern European countries. This is generally consistent with our findings. ${ }^{17}$

The following dominances are persistent in all the spatial analyses (i.e., dominance in 2005, 2009, and 2013 is found): Austria, the United Kingdom, and Luxembourg persistently dominate Italy and Portugal; Germany dominates Estonia, Lithuania, Latvia, and Portugal; Estonia dominates Latvia; Ireland and the Netherlands both dominate Spain, Italy, and Portugal; and Sweden dominates Hungary, Latvia, Lithuania, Poland, Portugal, and Slovenia. However, several dominances do change with the year of spatial analysis. For example, Belgium dominates Italy in both 2009 and 2013 but not in 2005, as is also the case for Sweden dominating Spain, the United Kingdom dominating Latvia in 2005

[^13]but not in 2009 and 2013, and so on.
The importance of multidimensional analyses of population welfare is well illustrated by considering Table 2 and noting that, for example, Sweden dominates Spain in all three dimensions analyzed separately in 2005. However, in the multidimensional analysis, no dominance is found (as indicated by the absence of a gray cell). The same is the case in 2009 for Cyprus dominating Italy and Sweden dominating Denmark, and in 2013 for the United Kingdom dominating Spain in all three dimensions but not multidimensionally. This illustrates that dominance in all the included dimensions analyzed separately does not imply multidimensional dominance.

The Copeland scores (normalized to the interval $[-1 ; 1]$ ) associated with the spatial FOD analyses are shown in Table 6 with the countries being ranked accordingly. We observe that almost no northern or western European countries are in the bottom half of the ranking and that almost no southern and eastern European countries are in the top half of the ranking. In addition, the rankings seem largely consistent over time. For example, Germany and Sweden are consistently ranked first, second, or third, the Czech Republic and Slovakia have a Copeland score of zero in all three years, and Latvia, Italy, and Portugal are consistently ranked 21st, 23rd, and 24th, respectively.

Despite the importance of a multidimensional approach to welfare comparisons, some information can still be gained by the one-dimensional analyses, since one-dimensional FOD is a necessary (though insufficient) condition for multidimensional FOD. The onedimensional analyses can therefore give an indication about within which dimensions(s) a country is lagging behind. In the income dimension, the most dominant countries are clearly northern and western European countries; e.g., Luxembourg dominating all of the 23 other countries in all three years, Austria dominating between 19 and 21 countries in the three years, and so on. The southern and in particular eastern European countries are most often dominated in the income dimension; e.g., Hungary, Latvia, and Lithuania being dominated by 20 or more countries in 2013. When considering the health dimension, the pattern is mostly similar. For example, Latvia, Lithuania, and Portugal are all dominated by more than 20 countries in 2013. The countries most often dominated in the education

Table 6 Copeland score and corresponding ranking of EU member states

| 2005 |  |  |  |  | 2009 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Rank | Country | Score | Rank | Country | Score | Rank | Country | Score |
| 1 | DE | 0.304 | 1 | SE | 0.348 | 1 | SE | 0.391 |
| 1 | UK | 0.304 | 2 | DE | 0.304 | 2 | DE | 0.261 |
| 3 | AT | 0.261 | 3 | IE | 0.130 | 3 | CY | 0.130 |
| 3 | SE | 0.261 | 3 | NL | 0.130 | 3 | IE | 0.130 |
| 5 | IE | 0.130 | 3 | UK | 0.130 | 3 | NL | 0.130 |
| 5 | NL | 0.130 | 6 | AT | 0.087 | 6 | AT | 0.087 |
| 7 | BE | 0.087 | 6 | BE | 0.087 | 6 | BE | 0.087 |
| 7 | CY | 0.087 | 6 | FI | 0.087 | 6 | DK | 0.087 |
| 7 | FI | 0.087 | 6 | FR | 0.087 | 6 | FR | 0.087 |
| 7 | LU | 0.087 | 6 | LU | 0.087 | 6 | LU | 0.087 |
| 11 | DK | 0.043 | 11 | CY | 0.043 | 6 | SI | 0.087 |
| 11 | EL | 0.043 | 11 | DK | 0.043 | 6 | UK | 0.087 |
| 11 | FR | 0.043 | 11 | EL | 0.043 | 13 | FI | 0.043 |
| 14 | CZ | 0 | 14 | CZ | 0 | 14 | CZ | 0 |
| 14 | EE | 0 | 14 | EE | 0 | 14 | EL | 0 |
| 14 | SK | 0 | 14 | SK | 0 | 14 | SK | 0 |
| 17 | PL | -0.087 | 17 | HU | -0.043 | 17 | EE | -0.043 |
| 18 | LT | -0.130 | 18 | LT | -0.087 | 17 | PL | -0.043 |
| 19 | ES | -0.174 | 18 | PL | -0.087 | 19 | LT | -0.087 |
| 19 | SI | -0.174 | 18 | SI | -0.087 | 20 | ES | -0.130 |
| 21 | HU | -0.217 | 21 | ES | -0.130 | 21 | HU | -0.174 |
| 21 | LV | -0.217 | 21 | LV | -0.130 | 21 | LV | -0.174 |
| 23 | IT | -0.261 | 23 | IT | -0.391 | 23 | IT | -0.435 |
| 24 | PT | -0.609 | 24 | PT | -0.652 | 24 | PT | -0.609 |

Notes: The Copeland scores are normalized to the interval $[-1 ; 1]$. If two or more countries have the same Copeland score, they are ordered alphabetically.
dimension seem to be particularly southern European and only some eastern European countries, though Luxembourg is dominated 14 times in 2013. For example, Greece, Spain, Italy, and Portugal are dominated between 11 and 20 times in 2013 whereas Germany, Estonia, Finland, Lithuania, and Sweden all dominate ten or more countries in the same year.

## Temporal FOD comparisons

The temporal FOD analyses yield five multidimensional dominances (Table 5), namely that 2009 dominates 2005 for the Czech Republic, and both 2009 and 2013 dominate 2005 for Latvia and Poland. Latvia and Poland have thus experienced a dominance improve-
ment in multidimensional welfare in both 2009 and 2013 compared to 2005 , whereas the improvement in the Czech Republic from 2005 to 2009 is not persistent when comparing 2005 and 2013. Noticeably, no countries have experienced a multidimensional dominance deterioration in welfare over time (i.e., over the course of the financial and economic crisis). This is consistent with the HDI (UNDP, 2014) where no European country has experienced a lower HDI in 2013 compared to 2005.

The one-dimensional temporal results yield several dominances. In the income dimension, 2005 dominates 2009 and 2009 dominates 2013 (and, hence, 2005 dominates 2013 due to transitivity) for Austria, Luxembourg, and the United Kingdom. This implies that these countries have experienced an unambiguous dominance deterioration of the income distribution over the entire time period. On the contrary, the Czech Republic and France have both experienced an unambiguous dominance improvement in the income distribution over the entire time period, since 2013 dominates 2009 and 2009 dominates 2005 (again, this implies that 2013 dominates 2005 due to transitivity). Several changes in the income distribution between these two extremes occur - for example, Estonia experiencing an improvement in 2009 and 2013 compared to 2005 (2009 and 2013 dominate 2005) and a deterioration in 2013 compared to 2009 (2009 dominates 2013). As a last example, the Netherlands has experienced an improvement between 2005 and 2009 (2009 dominates 2005) but a deterioration between 2009 and 2013 (2009 dominates 2013). No clear pattern is present as to which parts of Europe have experienced a dominance improvement or deterioration of the income distribution over the time period considered. Importantly, but not surprisingly, 15 countries have experienced a deterioration in the income distribution from 2009 to 2013, whereas only the Czech Republic, Denmark, and France have experienced an improvement. This is in contrast to the two other comparisons (2005 with 2009 and 2013, respectively), where 12 to 16 countries have experienced improvements and three to six countries have experienced a deterioration.

In the health dimension, Poland and Slovenia have experienced an unambiguous improvement, since 2013 dominates 2009 and 2009 dominates 2005. No countries have experienced an unambiguous deterioration over the entire time period. There is no clear
geographical pattern with respect to improvements or deteriorations in the health distribution over the time period. The overall result for education is clearer: no country has experienced a deterioration in the education distribution, only improvements have occurred, with 21 out of the 24 countries experiencing unambiguous improvements over the entire time period. 2009 does not dominate 2005 for Austria and Slovakia; however, 2013 dominates both 2005 and 2009. The United Kingdom is the only exception since no improvements were found in the entire time period analyzed.

In general, there seems to have been a significant dominance deterioration in the income distribution in European countries between 2009 and 2013, which is not surprising considering the financial and economic crisis. The number of countries experiencing improvements in health is largely constant, yet with a small decline between 2009 and 2013, and the same (large) number of countries are consistently experiencing an improvement in the distribution of education.

## 6 Discussion

A country bias in self-reported health has been found in the literature. For example, Jürges (2007) finds that Denmark and Sweden tend to overrate their self-assessed health, whereas particularly France, Germany, Spain, and Italy tend to underrate it compared to a constructed index of the prevalence of chronic conditions and physical health measures. Focusing on dominances in the health dimension in Tables 2 through 4, we cannot rule out that, for example, Denmark dominating Spain in 2005 in the health dimension is due to Danes overrating their self-assessed health (and/or Spaniards underrating theirs). Whether or not this has an impact on the results is not evident. An underrating of health in, say, Italy may not necessarily mean that Italy is dominated by, say, Sweden in the health dimension. However, it is worth noting that Denmark and Sweden do not consistently dominate France, Germany, Spain, and Italy in the health dimension. These results do not yield clear evidence about whether or not the self-reported health measure is adequate in describing population health. It does, however, indicate that no clear trend is found across all countries.

Though the FOD approach is theoretically well founded, a few empirical limitations are worth noting. First, as discussed in Section 3, the FOD approach, and other robust methods that do not rely on a weighting scheme, may yield an indeterminate result where no dominance is found when comparing two countries. For example, 43 out of the potential 276 dominances are found in 2013 (Table 4). This provides limited information about the relative welfare of all the populations and makes us unable to obtain a complete ranking of all EU member states, unlike what can be found using the HDI and the MPI. As shown in Section 5, the Copeland (1951) method can be used as a measure of the tendency to outperform other countries as an overall relative indicator of population well-being (Arndt et al., 2016). However, this does not guarantee a complete ranking as in the present paper, where some countries have the same Copeland score. But the dominances we do observe are the only comparisons that provide unambiguous proof that the dominant country is "better off" than the dominated. A complete ranking obtained by alternative methods, although convenient, would be obtained due to the additional more restrictive assumptions underlying these methods and/or the assumptions about the dimensions.

Second, the FOD approach provides no information about whether a dominant distribution is marginally or substantially better than the dominated distribution. For example, our finding that the Netherlands dominates Italy in 2013 provides no information about whether the welfare distribution in the Netherlands is much better or only slightly better than the welfare distribution in Italy. One can use bootstrapping to obtain an empirical probability of observing dominances under re-sampling to mitigate this limitation (Arndt et al., 2012).

Third, some dimensions that have been shown to have an impact on individuals' wellbeing (and hence on population welfare) cannot be included in a FOD analysis. For example, Delhey (2004) shows that besides income, education, and health, dimensions such as partnership and employment status are significant in explaining life satisfaction for individuals in European countries after controlling for characteristics such as gross domestic product (GDP) per capita, political freedom, and more. However, partnership may not be suitable in a FOD analysis since the dimension is not ordinal in nature. One
cannot say that being single is worse (or better) than having a partner.
As with any measure of welfare using non-continuous indicators, FOD is sensitive to the threshold(s) between levels. ${ }^{18}$ However, the inclusion of multi-level indicators mitigates this sensitivity. ${ }^{19}$ As an example, consider Greece and Spain and the single indicator in the health dimension in Table 2. Greece does not dominate Spain, nor does Spain dominate Greece. However, consider aggregating the health dimension's five levels into a binary indicator. Suppose that we I) aggregate being in very bad, bad, and fair health into ill health, and being in good and very good health into decent health, or II) aggregate being in very bad, and bad health into ill health, and being in fair, good, and very good health into decent health. The only difference between I and II is thus that fair health is included in ill health in the former and in decent health in the latter. When we use the aggregation in I, the shares of the population in the two categories are 0.2318 in ill health and 0.7682 in decent health in Greece and 0.2564 in ill health and 0.7436 in decent health in Spain. Hence, Greece dominates Spain. On the contrary, using the aggregation in II, the share of the population in the two categories are 0.0877 in ill health and 0.9123 in decent health in Greece and 0.0680 in ill health and 0.9320 in decent health in Spain. Hence, Spain dominates Greece; i.e., the conclusion is reversed. Different threshold(s) between levels can thus alter conclusions about population welfare rankings. A finer subdivision of indicators because of the inclusion of multi-level indicators (as opposed to binary indicators) will thus lower the risk of threshold choices impacting results. Appendix 5 shows the multidimensional FOD analyses using binary indicators rather than the multi-level ones used in Section 5.

[^14]
## 7 Conclusion

We compare multidimensional welfare in EU member states before, during, and after the financial crisis both spatially and temporally using first order dominance (FOD) on multi-level indicators. Implicitly or explicitly, weighting schemes are used in most multidimensional analyses of welfare. Our approach enables us to make comparisons of multidimensional population welfare that are robust to different weighting schemes. We add to the existing literature by using multi-level indicators of dimensions thus avoiding simplified welfare comparisons relying on binary indicators. In addition, the use of multi-level indicators of dimensions mitigates one of the challenges common to all welfare methods using non-continuous indicators, namely that they are sensitive to the thresholds between levels. We add to the scarce literature applying FOD on developed countries, and we stress the importance of multidimensional welfare analyses since dominance in each single dimension is merely a necessary but insufficient condition for multidimensional dominance.

Several dominances between European member states are found in all the three years analyzed. These are largely driven by relatively few countries which either dominate or are dominated by quite a few other countries. In particular, the dominated countries are most often southern and eastern EU member states, whereas the dominant countries are most often northern and western European member states. This is consistent with the existing literature. The ranking of countries using the Copeland method does not vary much in the three years analyzed. northern and western European countries are consistently ranked higher than southern and eastern European countries. We find that only a few countries have experienced temporal multidimensional improvements in welfare, namely the Czech Republic, Latvia, and Poland, and that no countries have experienced a dominance deterioration from 2005 through 2013. Thus, while the financial and economic crisis has had major impacts on especially income both at the individual and country level, the EU member states' broadly defined multidimensional welfare has not unambigously deteriorated during this period.

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## Appendix 1 Abbreviation list

Table A1 Abbreviations for included EU member states and regional groupings

| Abbreviation $^{a}$ | Country | Region $^{d}$ |
| :--- | :--- | :--- |
| AT | Austria | Western Europe |
| BE | Belgium | Western Europe |
| CY | Cyprus | Southern Europe |
| CZ | Czech Republic | Eastern Europe |
| DE | Germany | Western Europe |
| DK | Denmark | Northern Europe |
| EE | Estonia | Eastern Europe |
| EL | Greece | Southern Europe |
| ES | Spain | Southern Europe |
| FI | Finland | Northern Europe |
| FR | France | Western Europe |
| HU | Hungary | Eastern Europe |
| IE | Ireland | Northern Europe |
| IT | Italy | Southern Europe |
| LT | Lithuania | Eastern Europe |
| LU | Luxembourg | Western Europe |
| LV | Latvia | Eastern Europe |
| NL | Netherlands | Western Europe |
| PL | Poland | Eastern Europe |
| PT | Portugal | Southern Europe |
| SE | Sweden | Northern Europe |
| SI | Slovenia | Eastern Europe |
| SK | Slovakia | Eastern Europe |
| UK ${ }^{c}$ | United Kingdom | Northern Europe |

Notes: a) In line with the EU abbreviations rules, we use the two letter ISO code (ISO 3166 alpha-2) as abbrevations except for b) EL instead of GR for Greece, and c) UK instead of GB for the United Kingdom (Great Britain and Northern Ireland) (European Union, 2011, section 7.1.1.). d) We use the United Nations' M49 standard (UNSD, 1999) for the grouping of countries to specific regions. However, for linguistic simplicity, we include the Baltic member states (Estonia, Latvia, and Lithuania) in the group of eastern European countries, i.e., separately from the other included northern member states (Denmark, Finland, Ireland, Sweden, and the United Kingdom).

## Appendix 2 Illustration of LCSs

Table A2 illustrates all LCSs in the bivariate case with binary indicators. Let dimension A be the row dimension and dimension $B$ be the column dimension. In each dimension an individual can either be in outcome 0 or 1 , where 1 is best. This yields four LCSs in total: $\operatorname{LCS}_{1}=\{(0,0)\}, \operatorname{LCS}_{2}=\{(0,0),(0,1)\}, \operatorname{LCS}_{3}=\{(0,0),(1,0)\}$, and $\mathrm{LCS}_{4}=\{(0,0),(0,1),(1,0)\} .{ }^{20}$ To check for FOD between two distributions $f$ and $g$ using definition (i), one simply has to check that the following four inequalities are satisfied:
$\left.i_{1}\right) g(0,0) \leq f(0,0)$
$\left.i_{2}\right) g(0,0)+g(0,1) \leq f(0,0)+f(0,1)$
iz) $g(0,0)+g(1,0) \leq f(0,0)+f(1,0)$
$\left.i_{4}\right) g(0,0)+g(0,1)+g(1,0) \leq f(0,0)+f(0,1)+f(1,0)$

Table A2 Illustrating all LCSs, bivariate and binary

| $\mathrm{LCS}_{1}$ | $\mathrm{LCS}_{2}$ | $\mathrm{LCS}_{3}$ | $\mathrm{LCS}_{4}$ |
| :---: | :---: | :---: | :---: |
| B | B | B | B |
| 01 | 01 | 01 | $0 \quad 1$ |
| $\begin{array}{ll} \mathrm{A} & 1 \\ 0 \end{array} .$ | AA <br> 1 <br> 0 | A1 $\cdot$ <br> 0 . | A1 $\cdot$  <br> 0 $\cdot$  <br> 0 $\cdot$  |

Note: A gray dotted cell indicates that the outcome is part of the relevant LCS.

[^15]
## Appendix 3 Sample sizes

Table A3 Samples sizes for EU-SILC data

| 2005 |  |  |  | 2009 |  | 2013 |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Country $^{a}$ | No. obs. | Percent | No. obs. | Percent | No. obs. | Percent |  |
| AT | 10,413 | 2.95 | 11,049 | 3.06 | 10,938 | 3.15 |  |
| BE | 9,966 | 2.82 | 11,652 | 3.23 | 11,592 | 3.34 |  |
| CY | 8,997 | 2.55 | 7,553 | 2.09 | 10,980 | 3.16 |  |
| CZ | 7,826 | 2.21 | 16,829 | 4.67 | 11,602 | 3.34 |  |
| DE | 24,976 | 7.08 | 23,824 | 6.61 | 22,540 | 6.49 |  |
| DK | 5,956 | 1.68 | 5,866 | 1.62 | 5,429 | 1.56 |  |
| EE | 9,643 | 2.73 | 8,724 | 2.42 | 10,106 | 2.91 |  |
| EL | 12,381 | 3.51 | 15,045 | 4.17 | 15,318 | 4.41 |  |
| ES | 30,276 | 8.58 | 30,418 | 8.44 | 26,429 | 7.61 |  |
| FI | 10,904 | 3.09 | 9,952 | 2.76 | 10,756 | 3.1 |  |
| FR | 18,749 | 5.31 | 20,102 | 5.58 | 20,563 | 5.92 |  |
| HU | 14,663 | 4.15 | 20,380 | 5.65 | 21,270 | 6.13 |  |
| IE | 12,030 | 3.41 | 9,898 | 2.74 | 9,442 | 2.72 |  |
| IT | 47,311 | 13.41 | 42,657 | 11.84 | 36,612 | 10.55 |  |
| LT | 9,919 | 2.81 | 9,518 | 2.64 | 8,195 | 2.36 |  |
| LU | 7,525 | 2.13 | 8,623 | 2.39 | 7,996 | 2.30 |  |
| LV | 7,913 | 2.24 | 12,066 | 3.35 | 12,112 | 3.49 |  |
| NL | 9,347 | 2.65 | 9,724 | 2.69 | 10,102 | 2.91 |  |
| PL | 37,671 | 10.68 | 29,229 | 8.11 | 27,804 | 8.01 |  |
| PT | 10,702 | 3.03 | 11,101 | 3.08 | 14,008 | 4.03 |  |
| SE | 6,035 | 1.71 | 7,538 | 2.09 | 6,084 | 1.75 |  |
| SI | 8,287 | 2.34 | 9,282 | 2.57 | 9,001 | 2.59 |  |
| SK | 12,877 | 3.65 | 13,773 | 3.82 | 13,220 | 3.81 |  |
| UK | 18,282 | 5.18 | 15,350 | 4.26 | 14,855 | 4.28 |  |
| Total ${ }^{b}$ | 352,649 | 100.00 | 360,153 | 100.00 | 346,954 | 100.00 |  |
|  |  |  |  |  |  |  |  |

Notes: For each row country, the numbers in columns 2, 4, and 6 indicate the sample sizes (number of observations) in the EU-SILC data in 2005, 2009, and 2013, respectively. Similarly, for each row country, columns 3,5 , and 7 indicate the percent of total observations in 2005 , 2009, and 2013, respectively. a) See Table A1 in Appendix 1 for an abbreviation list of EU member states. b) Difference in total of percentages due to rounding.

## Appendix 4 Data structure

Table A4 Example of data structure using sample data for Germany in 2005.

| Education $=1$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Income | Health |  |  |  |  |  |  |
|  |  | 1 | 2 | 3 | 4 | 5 | Total |
|  | 1 | 0.06 | 0.28 | 0.73 | 0.63 | 0.28 | 1.98 |
|  | 2 | 0.13 | 0.61 | 1.78 | 1.95 | 0.79 | 5.27 |
|  | 3 | 0.13 | 0.52 | 1.79 | 2.55 | 1.10 | 6.09 |
|  | 4 | 0.14 | 0.30 | 1.09 | 1.95 | 1.17 | 4.65 |
|  | Total | 0.46 | 1.71 | 5.39 | 7.09 | 3.34 | 17.99 |
| Education $=2$ |  |  |  |  |  |  |  |
| Income | Health |  |  |  |  |  |  |
|  |  | 1 | 2 | 3 | 4 | 5 | Total |
|  | 1 | 0.11 | 0.49 | 1.15 | 1.33 | 0.28 | 3.36 |
|  | 2 | 0.18 | 1.14 | 3.82 | 4.70 | 1.18 | 11.01 |
|  | 3 | 0.21 | 1.20 | 5.68 | 8.47 | 2.14 | 17.70 |
|  | 4 | 0.14 | 0.87 | 4.84 | 8.71 | 2.96 | 17.53 |
|  | Total | 0.65 | 3.70 | 15.49 | 23.20 | 6.55 | 49.60 |
| Education $=3$ |  |  |  |  |  |  |  |
| Income | Health |  |  |  |  |  |  |
|  |  | 1 | 2 | 3 | 4 | 5 | Total |
|  | 1 | 0.05 | 0.20 | 0.43 | 0.65 | 0.14 | 1.47 |
|  | 2 | 0.13 | 0.41 | 1.44 | 1.83 | 0.45 | 4.26 |
|  | 3 | 0.10 | 0.62 | 2.93 | 4.54 | 1.02 | 9.22 |
|  | 4 | 0.11 | 0.79 | 4.36 | 9.40 | 2.80 | 17.46 |
|  | Total | 0.38 | 2.03 | 9.17 | 16.42 | 4.41 | 32.42 |
| Sum ${ }^{\text {a }}$ |  |  |  |  |  |  | 100.00 |

[^16]
## Appendix 5 Binary FOD analyses

As mentioned, applications of FOD in a welfare context have until now used binary indicators. To show the consequences of using multi-level indicators instead of binary ones, analyses similar to those in Section 5 but using binary indicators have been conducted. Instead of applying the multi-level indicators outlined in Table 1, we combine the levels into binary outcomes as shown in Table A5, where the second column from the right shows the multi-level indicators applied in the previously reported results, and the rightmost column shows how the multi-level indicators are aggregated into binary indicators. ${ }^{21}$

The results are shown in Tables A6 through A9. These are the analogous of Tables 2 through 5 , the only difference being that the results are obtained using the binary indicators described Table A5. Naturally, the multidimensional binary analyses yield several more dominances than the corresponding multidimensional multi-level analyses. ${ }^{22}$ In particular, we never obtain an indeterminate result in each dimension analyzed separately

Table A5 Description of welfare dimensions and binary indicators

| Dimension | Indicator | Multi-level | Binary |
| :---: | :---: | :---: | :---: |
| Income | Equivalized annual net income | 1 | $\left\{\begin{array}{l}1 \\ 2\end{array}\right.$ |
|  |  | 2 |  |
|  |  | 3 |  |
|  |  | 4 |  |
| Health | Self-reported health | 1 | $\left\{\begin{array}{l} 1 \\ 2 \end{array}\right.$ |
|  |  | 2 |  |
|  |  | 3 |  |
|  |  | 4 |  |
|  |  | 5 |  |
| Education | Highest ISCED level obtained | 1 | $\left\{\begin{array}{l}1 \\ 2\end{array}\right.$ |
|  |  | 2 |  |
|  |  | 3 |  |

Note: See Table 1 for further information.

[^17]since the distribution in a given dimension is fully described by a single number, e.g., those who are worse off. In the binary analyses, we obtain between 138 and 149 multidimensional dominances, i.e., around three and a half times more than we do in the multi-level analyses. The corresponding Copeland scores are shown in Table A10.

The overall results are similar to those obtained in the multilevel analyses, i.e., the countries most often dominated are southern and eastern European member countries, and the dominant countries are mostly northern and western European member states. The Copeland scores using binary indicators are also similar to the ones using multilevel indicators where almost no northern or western European countries are in the bottom half of the ranking and almost no southern and eastern European countries are in the top half of the ranking. Though the use of binary indicators enables us to more easily compare European countries and to obtain a more complete ranking, the trade-off is whether the binary indicators adequately describe the distribution of the population in the dimensions considered, or if we are willing to obtain (weakly) fewer dominances by more precisely dividing the population into multidimensional and multi-level outcomes.
Table A6 Spatial first order dominances using binary indicators, 2005

|  | AT | BE | CY | CZ | DE | DK | EE | EL | ES | FI | FR | HU | IE | IT | LT | LU | LV | NL | PL | PT | SE | SI | SK | UK | $\mathrm{I}^{a}$ | $\mathrm{H}^{\text {b }}$ | $\mathrm{E}^{c}$ | $\mathrm{MD}^{\text {d }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AT | - | I | I | IHE | IH | I | IH | IE | IH | IH | IH | IHE | I | IHE | IH |  | IHE | I | IHE | IHE | I | IHE | IHE | I | 22 | 14 | 9 | 8 |
| BE | HE | - | E | IHE | H | E | IHE | IE | IHE | IHE | IHE | IHE | E | IHE | IHE | E | IHE | E | IHE | IHE | IE | IHE | IHE |  | 15 | 15 | 21 | 13 |
| CY | HE | IH | - | IHE | H | IHE | IH | IE | IHE | IH | IHE | IHE | IE | IHE | IHE | HE | IHE | H | IHE | IHE | IH | IHE | IHE | H | 18 | 21 | 16 | 11 |
| CZ |  |  |  | - |  |  | IH |  |  |  |  | IH |  | HE | IH |  | IH |  | IH | HE |  | HE | IH |  | 6 | 9 | 3 | 0 |
| DE | E | IE | IE | IHE | - | IE | IHE | IE | IE | IE | IE | IHE | IE | IHE | IHE | E | IHE | IE | IHE | IHE | IE | IHE | IHE | IE | 21 | 10 | 23 | 10 |
| DK | HE | IH |  | IHE | H | - | IH | IE | IH | IH | IHE | IHE |  | IHE | IHE | HE | IHE | H | IHE | IHE | IH | IHE | IHE | H | 16 | 20 | 13 | 10 |
| EE | E |  | E | E |  | E | - | E | E |  | E | IHE | E | E | IHE | E | IHE |  | IHE | HE |  | HE | IHE |  | 5 | 7 | 17 | 4 |
| EL | H | H | H | IHE | H | H | IH | - | H | H | H | IHE |  | HE | IH | H | IHE | H | IHE | IHE | H | HE | IHE | H | 8 | 22 | 8 | 6 |
| ES | E |  |  | IHE | H | E | IH | IE | - | H | E | IHE |  | HE | IHE | E | IHE |  | IHE | IHE |  | IHE | IHE |  | 10 | 12 | 14 | 8 |
| FI | E |  | E | IHE | H | E | IHE | IE | IE | - | E | IHE | E | IHE | IHE | E | IHE |  | IHE | IHE | IE | IHE | IHE |  | 13 | 11 | 19 | 10 |
| FR | E |  |  | IHE | H |  | IH | IE | IH | IH | - | IHE |  | IHE | IH |  | IHE |  | IHE | IHE | I | IHE | IHE |  | 14 | 13 | 10 | 8 |
| HU |  |  |  | E |  |  |  |  |  |  |  | - |  | E | H |  | H |  | E | E |  | E | I |  | 1 | 2 | 5 | 0 |
| IE | HE | IH | H | IHE | H | IHE | IH | IHE | IHE | IH | IHE | IHE | - | IHE | IHE | HE | IHE | H | IHE | IHE | IH | IHE | IHE | H | 17 | 23 | 15 | 12 |
| IT |  |  |  | I |  |  | IH | I | I |  |  | IH |  | - | IH |  | IH |  | IH | IH |  | IHE | IH |  | 11 | 8 | 1 | 0 |
| LT | E |  |  | E |  |  |  | E |  |  | E | IE |  | E | - |  | HE |  | E | E |  | E | IE |  | 2 | 1 | 11 | 0 |
| LU | IHE | IH | I | IHE | IH | I | IH | IE | IH | IH | IHE | IHE | 1 | IHE | IHE | - | IHE | IH | IHE | IHE | I | IHE | IHE | I | 23 | 17 | 12 | 11 |
| LV |  |  |  | E |  |  |  |  |  |  |  | IE |  | E | I |  | - |  | E | E |  | E | IE |  | 3 |  | 7 | 0 |
| NL | HE | IH | IE | IHE | H | IE | IHE | IE | HE | IHE | IHE | IHE | IE | IHE | IHE | E | IHE |  | IHE | IHE | IE | IHE | IHE |  | 19 | 16 | 20 | 13 |
| PL |  |  |  | E |  |  |  |  |  |  |  | IH |  | E | IH |  | IH |  | - | HE |  | HE | IH |  | 4 | 6 |  | 0 |
| PT |  |  |  | I |  |  | I |  |  |  |  | IH |  | E | IH |  | IH |  | I |  |  | E | I |  | 7 | 3 | 2 | 0 |
| SE | HE | H | E | IHE | H | E | IHE | IE | IHE | H | HE | IHE | E | IHE | IHE | HE | IHE | H | IHE | IHE | - | IHE | IHE |  | 12 | 18 | 18 | 11 |
| SI |  |  |  | I |  |  | I | I |  |  |  | IH |  |  | IH |  | IH |  | , | IH |  | - | I |  | 9 | 4 | 0 | 0 |
| SK |  |  |  | E |  |  |  |  |  |  |  | HE |  | E | H |  | H |  | E | HE |  | HE | - |  | 0 | 5 | 6 | 0 |
| UK | HE | IHE | IE | IHE | H | IE | IHE | IE | IHE | IHE | IHE | IHE | IE | IHE | IHE | HE | IHE | IHE | IHE | IHE | IHE | IHE | IHE | - | 20 | 19 | 22 | 14 |
| $\mathrm{I}^{a}$ | 1 | 8 | 5 | 17 | 2 | 7 | 18 | 15 | 13 | 10 | 9 | 22 | 6 | 12 | 21 | 0 | 20 | 4 | 19 | 16 | 11 | 14 | 23 | 3 | 276 | - | - | - |
| $\mathrm{H}^{\text {b }}$ | 9 | 8 | 2 | 14 | 13 | 3 | 16 | 1 | 11 | 12 | 10 | 21 | 0 | 15 | 22 | 6 | 23 | 7 | 17 | 20 | 5 | 19 | 18 | 4 | - | 276 | - | - |
| $\mathrm{E}^{c}$ | 14 | 2 | 7 | 20 | 0 | 10 | 6 | 15 | 9 | 4 | 13 | 18 | 8 | 22 | 12 | 11 | 16 | 3 | 19 | 21 | 5 | 23 | 17 | 1 | - | - | 276 | - |
| $\mathrm{MD}^{\text {d }}$ | 1 | 0 | 0 | 14 | 0 | 0 | 6 |  | 6 | 3 | 7 | 15 | 0 | 12 | 12 | 0 | 15 | 0 | 14 | 14 | 1 | 13 | 15 | 0 | - | - | - | 149 |

[^18]Table A7 Spatial first order dominances using binary indicators, 2009

|  | AT | BE | CY | CZ | DE | DK | EE | EL | ES | FI | FR | HU | IE | IT | LT | LU | LV | NL | PL | PT | SE | SI | SK | UK | $\mathrm{I}^{a}$ | $\mathrm{H}^{\text {b }}$ | $\mathrm{E}^{c}$ | $\mathrm{MD}^{\text {d }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AT | - | I | I | IHE | IH | IH | IH | I | I | IH | IH | IH | I | IHE | IH |  | IH | I | IH | IHE | I | IH | IH | I | 22 | 14 | 3 | 3 |
| BE | HE | - | E | IHE | H | IHE | IHE | IE | IHE | IHE | HE | IHE | E | IHE | IHE | E | IHE | E | IHE | IHE | E | IHE | IHE | IE | 15 | 16 | 22 | 12 |
| CY | HE | IH | - | IHE | IH | IHE | IH | IHE | IHE | IH | IHE | IHE |  | IHE | IHE | HE | IHE | H | IHE | IHE | I | IHE | IHE | I | 19 | 20 | 15 | 12 |
| CZ |  |  |  |  |  |  | IH |  |  |  |  | IH |  | E | IH |  | IH |  | IH | HE |  | H | I |  | 6 | 7 | 2 | 0 |
| DE | E | IE | E | IHE | - | IE | IHE | IE | IE | IE | E | IHE | E | IHE | IHE | E | IHE | E | IHE | IHE | IE | IHE | IHE | IE | 17 | 10 | 23 | 10 |
| DK | E |  |  | IHE | H | - | IH | IE | I | IH | HE | IHE |  | IHE | IHE | E | IHE |  | IHE | IHE |  | IHE | IHE | I | 14 | 13 | 13 | 9 |
| EE | E |  | E | E |  | E | - | E | E | E | E | IE | E | E | IHE | E | IHE | E | E | HE | E | E | IE | E | 4 | 3 | 21 | 2 |
| EL | HE | H |  | IHE | H | H | IH | - | H | H | H | IHE |  | HE | IH | H | IH | H | IHE | IHE |  | HE | IHE |  | 8 | 19 | 8 | 5 |
| ES | HE |  |  | IHE | H | HE | IH | IE | - | H | HE | IHE |  | IHE | IHE | E | IHE |  | IHE | IHE |  | IHE | IHE |  | 11 | 15 | 14 | 9 |
| FI | E |  | E | IHE | H | E | IH | IE | IE | - | E | IHE | E | IHE | IHE | E | IHE |  | IHE | IHE | E | IHE | IHE | E | 12 | 11 | 19 | 9 |
| FR | E | I |  | IHE | IH | I | IH | IE | I | IH | - | IHE |  | IHE | IH | E | IHE |  | IHE | IHE | I | IHE | IHE | I | 18 | 12 | 11 | 8 |
| HU | E |  |  | E |  |  | H |  |  |  |  | - |  | E | H |  | H |  | E | HE |  |  | E |  | 0 | 4 | 6 | 0 |
| IE | HE | IH | IHE | IHE | IH | IHE | IH | IHE | IHE | IH | IHE | IHE | - | IHE | IHE | HE | IHE | H | IHE | IHE | IH | IHE | IHE | IH | 20 | 23 | 16 | 14 |
| IT |  |  |  | IH |  |  | IH | I |  |  |  | IH |  | - | IH |  | IH |  | IH | IHE |  | IH | IH |  | 10 | 9 | 1 | 1 |
| LT | E |  |  | E |  |  |  | E |  |  | E | IE |  | E | - | E | IHE |  | E | HE |  | E | E |  | 2 | 2 | 12 | 1 |
| LU | IHE | IH | I | IHE | IH | IH | IH | IE | IH | IH | IH | IHE | I | IHE | IH | - | IHE | I | IHE | IHE | I | IHE | IHE | I | 23 | 17 | 10 | 9 |
| LV | E |  |  | E |  |  |  | E |  |  |  | IE |  | E |  |  | - |  | E | E |  | E | E |  | 1 | 0 | 9 | 0 |
| NL | HE | IH | IE | IHE | IH | IHE | IH | IE | IHE | IHE | IHE | IHE | IE | IHE | IHE | HE | IHE | - | IHE | IHE | IE | IHE | IHE | IE | 21 | 18 | 20 | 12 |
| PL | E |  |  | E |  |  | IH |  |  |  |  | IH |  | E | IH |  | IH |  | - | HE |  | H | I |  | 5 | 6 | 4 | 0 |
| PT |  |  |  | I |  |  | I |  |  |  |  | I |  |  | 1 |  | IH |  | I | - |  |  | I |  | 7 | 1 | 0 | 0 |
| SE | HE | IH | HE | IHE | H | IHE | IH | IHE | IHE | IH | HE | IHE | E | IHE | IHE | HE | IHE | H | IHE | IHE | - | IHE | IHE | IE | 16 | 21 | 18 | 12 |
| SI | E |  |  | IE |  |  | IH | I |  |  |  | IHE |  | E | IH |  | IH |  | IE | IHE |  | - | IE |  | 9 | 5 | 7 | 1 |
| SK | E |  |  | HE |  |  | H |  |  |  |  | IH |  | E | IH |  | IH |  | HE | HE |  | H | - |  | 3 | 8 | 5 | 0 |
| UK | HE | H | HE | IHE | H | HE | IH | IHE | IHE | IH | HE | IHE | E | IHE | IHE | HE | IHE | H | IHE | IHE | H | IHE | IHE | - | 13 | 22 | 17 | 11 |
| $\mathrm{I}^{a}$ | 1 | 8 | 4 | 17 | 6 | 9 | 19 | 15 | 12 | 11 | 5 | 23 | 3 | 13 | 21 | 0 | 22 | 2 | 18 | 16 | 7 | 14 | 20 | 10 | 276 |  |  |  |
| $\mathrm{H}^{\text {b }}$ | 9 | 7 | 3 | 16 | 13 | 10 | 20 | 4 | 8 | 12 | 11 | 19 | 0 | 14 | 21 | 6 | 23 | 5 | 17 | 22 | 2 | 18 | 15 | 1 | - | 276 | - | - |
| $\mathrm{E}^{c}$ | 20 | 1 | 8 | 21 | 0 | 10 | 2 | 15 | 9 | 4 | 12 | 17 | 7 | 22 | 11 | 13 | 14 | 3 | 19 | 23 | 5 | 16 | 18 | 6 | - | - | 276 | - |
| $\mathrm{MD}^{\text {d }}$ | 1 | 0 | 1 | 14 | 0 | 4 | 2 | 4 | 6 | 1 | 2 | 13 | 0 | 13 | 11 | 0 | 14 | 0 | 13 | 16 | 0 | 12 | 13 | 0 | - | - | - | 140 |

[^19]Table A8 Spatial first order dominances using binary indicators, 2013

|  | AT | BE | CY | CZ | DE | DK | EE | EL | ES | FI | FR | HU | IE | IT | LT | LU | LV | NL | PL | PT | SE | SI | SK | UK | $\mathrm{I}^{a}$ | $\mathrm{H}^{\text {b }}$ | $\mathrm{E}^{c}$ | $\mathrm{MD}^{\text {d }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AT | - | I | I | IHE | IH | I | IH | I | I | IH | IH | IH | I | IHE | IH |  | IH | I | IH | IHE | I | IH | IH | I | 22 | 13 | 3 | 3 |
| BE | HE | - | IE | IHE | H | HE | IHE | IHE | IHE | IHE | HE | IHE |  | IHE | IHE | HE | IHE | HE | IHE | IHE | E | IHE | IHE | IHE | 15 | 20 | 21 | 14 |
| CY | HE | H | - | IHE | H | H | IH | IHE | IHE | H | HE | IHE |  | IHE | IH | HE | IHE | H | IHE | IHE |  | IHE | IHE | IH | 13 | 21 | 13 | 10 |
| CZ |  |  |  | - |  |  | IH | I |  |  |  | IH |  | E | IH |  | IH |  | IH | HE |  |  | I |  | 7 | 6 | 2 | 0 |
| DE | E | IE | IE | IHE | - | IE | IHE | IE | IE | IHE | E | IHE | I | IE | IHE | E | IHE | E | IHE | IHE | IE | IHE | IE | IE | 19 | 9 | 22 | 9 |
| DK | HE | I | IE | IHE | H | - | IH | IE | IE | IH | HE | IHE | I | IHE | IHE | E | IHE |  | IHE | IHE |  | IHE | IHE | I | 17 | 14 | 15 | 9 |
| EE | E |  | E | E |  | E | - | E | E | E | E | IE |  | E | IHE | E | IHE | E | E | HE |  | E | E | E | 3 | 3 | 19 | 2 |
| EL | HE |  |  | HE | H | H | IH | - | H | H | H | IHE |  | HE | IH | H | IH | H | IHE | HE |  | H | IHE | H | 6 | 19 | 7 | 3 |
| ES | HE |  |  | IHE | H | H | IH | IE | - | H | HE | IHE |  | HE | IH | E | IHE |  | IHE | IHE |  | IHE | IHE |  | 10 | 15 | 12 | 7 |
| FI | E |  | IE | IHE |  | E | IH | IE | IE | - | E | IHE |  | IE | IHE | E | IHE | E | IHE | IHE |  | IHE | IE | IE | 14 | 8 | 18 | 7 |
| FR | E | I | I | IHE | IH | I | IH | IE | I | IH | - | IHE | I | IHE | IH | E | IHE | I | IHE | IHE | I | IHE | IHE | I | 21 | 12 | 11 | 8 |
| HU | E |  |  | E |  |  | H |  |  |  |  | - |  | E | H |  | H |  |  | HE |  |  | E |  | 0 | 4 | 5 | 0 |
| IE | HE | IHE | IHE | IHE | HE | HE | IHE | IHE | IHE | IHE | HE | IHE | - | IHE | IHE | HE | IHE | HE | IHE | IHE | HE | IHE | IHE | IHE | 16 | 23 | 23 | 16 |
| IT |  |  |  | IH | H |  | IH | I | I | H |  | IH |  | - | IH |  | IH |  | IH | IH |  | IH | IH |  | 11 | 11 | 0 | 0 |
| LT | E |  | E | E |  |  |  | E | E |  | E | IE |  | E | - | E | IHE |  | E | HE |  | E | E |  | 2 | 2 | 14 | 1 |
| LU | IHE | I | I | IHE | IH | IH | IH | IE | IH | IH | IH | IHE | I | IHE | IH | - | IH | I | IHE | IHE | I | IH | IHE | I | 23 | 16 | 8 | 7 |
| LV | E |  |  | E |  |  |  | E |  |  |  | IE |  | E |  | E | - |  | E | E |  |  | E |  | 1 | 0 | 9 | 0 |
| NL | HE | I | IE | IHE | IH | IHE | IH | IE | IHE | IH | HE | IHE | I | IHE | IHE | HE | IHE | - | IHE | IHE | I | IHE | IHE | IE | 20 | 17 | 17 | 11 |
| PL | E |  |  | E |  |  | IH |  |  |  |  | IHE |  | E | IH |  | IH |  | - | HE |  |  | IE |  | 5 | 5 | 6 | 1 |
| PT |  |  |  | I |  |  | I | I |  |  |  | I |  | E | 1 |  | IH |  | I | - |  |  | I |  | 8 | 1 | 1 | 0 |
| SE | HE | IH | IHE | IHE | H | IHE | IHE | IHE | IHE | IHE | HE | IHE | I | IHE | IHE | HE | IHE | HE | IHE | IHE | - | IHE | IHE | IHE | 18 | 22 | 20 | 15 |
| SI | E |  |  | IHE |  |  | IH | IE |  |  |  | IHE |  | E | IH | E | IHE |  | IHE | IHE |  | - | IE |  | 9 | 7 | 10 | 5 |
| SK | E |  |  | HE | H |  | IH |  |  | H |  | IH |  | E | IH |  | IH |  | H | HE |  | H | - |  | 4 | 10 | 4 | 0 |
| UK | HE |  | E | IHE | H | HE | IH | IE | IHE | H | HE | IHE |  | IHE | IHE | HE | IHE | H | IHE | IHE |  | IHE | IHE | - | 12 | 18 | 16 | 10 |
| $\mathrm{I}^{a}$ | 1 | 8 | 10 | 16 | 4 | 6 | 20 | 17 | 13 | 9 | 2 | 23 | 7 | 12 | 21 | 0 | 22 | 3 | 18 | 15 | 5 | 14 | 19 | 11 | 276 | - | - | - |
| $\mathrm{H}^{\text {b }}$ | 10 | 3 | 2 | 17 | 14 | 9 | 20 | 4 | 8 | 15 | 11 | 19 | 0 | 12 | 21 | 7 | 23 | 6 | 18 | 22 | 1 | 16 | 13 | 5 | - | 276 | - | - |
| $\mathrm{E}^{c}$ | 20 | 2 | 10 | 21 |  | 8 |  | 16 | 11 | 5 | 12 | 18 | 0 | 23 | 9 | 15 | 14 | 6 | 17 | 22 | 3 | 13 | 19 | 7 | - | - | 276 |  |
| $\mathrm{MD}^{\text {d }}$ | 1 | 1 | 2 | 14 | 0 | 2 | 4 | 4 | 6 | 3 | 0 | 15 | 0 | 10 | 9 | 0 | 14 | 0 | 14 | 14 | 0 | 11 | 11 | 3 | - | - | - | 138 |

[^20]Table A9 Temporal first order dominances using binary indicators

|  | Deteriorations over time |  |  | Improvements over time |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 05 dom 09 | 05 dom 13 | 09 dom 13 | 09 dom 05 | 13 dom 05 | 13 dom 09 |
| AT | IHE | IH | IH |  | E | E |
| BE |  |  |  | IHE | IHE | IHE |
| CY |  | I | I | IHE | HE | HE |
| CZ |  |  | H | IHE | IHE | IE |
| DE | I | I | IH | HE | HE | E |
| DK | H | H | H | IE | IE | IE |
| EE | H | H | I | IE | IE | HE |
| EL | H | IH | IH | IE | E | E |
| ES |  |  | I | IHE | IHE | HE |
| FI | H | H | IH | IE | IE | E |
| FR | H | H | H | IE | IE | IE |
| HU |  | I | I | IHE | HE | HE |
| IE |  | H | IH | IHE | IE | E |
| IT |  | 1 | I | IHE | HE | HE |
| LT |  |  | IH | IHE | IHE | E |
| LU | I | IH | IH | HE | E | E |
| LV |  |  | IH | IHE | IHE | E |
| NL |  | H | IH | IHE | IE | E |
| PL |  |  | I | IHE | IHE | HE |
| PT |  |  | IH | IHE | IHE | E |
| SE |  |  | I | IHE | IHE | HE |
| SI |  | I | I | IHE | HE | HE |
| SK |  |  |  | IHE | IHE | IHE |
| UK | IE | IH | IH | H | E | E |
| $\mathrm{I}^{a}$ | 4 | 9 | 19 | 20 | 15 | 5 |
| $\mathrm{H}^{\text {b }}$ | 6 | 10 | 14 | 18 | 14 | 10 |
| $\mathrm{E}^{c}$ | 2 | 0 | 0 | 22 | 24 | 24 |
| $\mathrm{MD}^{\text {d }}$ | 1 | 0 | 0 | 15 | 9 | 2 |

[^21]Table A10 Copeland scores and corresponding ranking of EU member states using binary indicators

| 2005 |  |  |  |  | 2009 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Rank | Country | Score | Rank | Country | Score | Rank | Country | Score |
| 1 | UK | 0.609 | 1 | IE | 0.609 | 1 | IE | 0.696 |
| 2 | BE | 0.565 | 2 | BE | 0.522 | 2 | SE | 0.652 |
| 2 | NL | 0.565 | 2 | NL | 0.522 | 3 | BE | 0.565 |
| 4 | IE | 0.522 | 2 | SE | 0.522 | 4 | NL | 0.478 |
| 5 | CY | 0.478 | 5 | CY | 0.478 | 5 | DE | 0.391 |
| 5 | LU | 0.478 | 5 | UK | 0.478 | 6 | CY | 0.348 |
| 7 | DE | 0.435 | 7 | DE | 0.435 | 6 | FR | 0.348 |
| 7 | DK | 0.435 | 8 | LU | 0.391 | 8 | DK | 0.304 |
| 7 | SE | 0.435 | 9 | FI | 0.348 | 8 | LU | 0.304 |
| 10 | AT | 0.304 | 10 | FR | 0.261 | 8 | UK | 0.304 |
| 10 | FI | 0.304 | 11 | DK | 0.217 | 11 | FI | 0.174 |
| 12 | EL | 0.217 | 12 | ES | 0.130 | 12 | AT | 0.087 |
| 13 | ES | 0.087 | 13 | AT | 0.087 | 13 | ES | 0.043 |
| 14 | FR | 0.043 | 14 | EL | 0.043 | 14 | EL | -0.043 |
| 15 | EE | -0.087 | 15 | EE | 0 | 15 | EE | -0.087 |
| 16 | IT | -0.522 | 16 | LT | -0.435 | 16 | SI | -0.261 |
| 16 | LT | -0.522 | 17 | SI | -0.478 | 17 | LT | -0.348 |
| 18 | SI | -0.565 | 18 | IT | -0.522 | 18 | IT | -0.435 |
| 19 | CZ | -0.609 | 19 | HU | -0.565 | 19 | SK | -0.478 |
| 19 | PL | -0.609 | 19 | PL | -0.565 | 20 | PL | -0.565 |
| 19 | PT | -0.609 | 19 | SK | -0.565 | 21 | CZ | -0.609 |
| 22 | HU | -0.652 | 22 | CZ | -0.609 | 21 | LV | -0.609 |
| 22 | LV | -0.652 | 22 | LV | -0.609 | 21 | PT | -0.609 |
| 22 | SK | -0.652 | 24 | PT | -0.696 | 24 | HU | -0.652 |

Notes: As in Table 6.


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[^2]:    ${ }^{1}$ Ravallion (2012) refers to this as "mashup indices".

[^3]:    ${ }^{2}$ The present paper focuses on comparisons of welfare in population distributions with ordinal multidimensional data. For comparisons of inequality across populations with (partially) ordinal multidimensional data, we refer to Gravel and Moyes (2012) and Sonne-Schmidt et al. (2016).
    ${ }^{3}$ See Tabel A1 in Appendix 1 for a grouping of countries to specific regions.

[^4]:    ${ }^{4}$ Note that there can be multiple indicators for the same welfare dimension as exemplified here, where both mean years of schooling and expected years of schooling are used as indicators in the education dimension. In this paper, we use one indicator for each welfare dimension included.
    ${ }^{5}$ In, e.g., 2013, the cut-off point for being in the "very high human development" category was 0.8.

[^5]:    ${ }^{6}$ For example, an indicator in the health dimension is that the respondent considers her own health as fair or above, and the indicator in the education dimension is whether or not the respondent has completed primary education.

[^6]:    ${ }^{7}$ The first proof of the equivalence between (i) and (iii) is usually attributed to Lehmann (1955) (however, see also Levhari et al., 1975). The first formulation and proof of the equivalence between (i) and (ii) is not easy to trace back to its roots, but Kamae et al. (1977) observed that the equivalence between (i) and (ii) is a corollary of Strassen's Theorem (Strassen, 1965). Østerdal (2010) provides a constructive direct proof of this for the finite case.
    ${ }^{8}$ In the one-dimensional case, $f$ first order dominates $g$ if and only if $F(x) \leq G(x)$ for all $x \in X$, where $F(\cdot)$ and $G(\cdot)$ are the cumulative distribution functions corresponding to $f$ and $g$, respectively. For a review of FOD in both a one-dimensional and multidimensional welfare setting using binary indicators, we refer to Siersbæk et al. (2017).

[^7]:    ${ }^{9}$ The less restrictive of these are instances of orthant stochastic orderings (see Dyckerhoff and Mosler, 1997) although the name "first order dominance" has sometimes been used synonymously for these.
    ${ }^{10}$ While linear programming is computationally faster, the approach may be challenged by numerical instability (see, e.g., Higham, 2002, for a general treatment). This may lead to the conclusion that a dominance exists when in fact there is no dominance (but "close").
    ${ }^{11}$ The number of LCSs is quantified by Sampson and Whitaker (1988) using the number of levels in each indicator. Strictly speaking, Sampson and Whitaker (1988) provide the number of upper comprehensive sets, which is, however, equal to the number of LCSs. For three dimensions with binary indicators, the total number of LCSs is 20. If the number of levels of each indicator is three, the total number of LCSs is 980 , and if four levels of each indicator are used, the total number of LCSs increases to 232,848 .
    ${ }^{12}$ The Matlab code for identifying all LCS and checking FOD is available on the following web-

[^8]:    page: https://sites.google.com/site/nikolajsiersbaek/code. The empty LCS and the full set of all outcomes are omitted in the code since the corresponding sums using definition (i) are 0 and 1 , respectively. Computationally efficient algorithms capable of handling several indicators and levels is grounds for further research. For the bivariate case, efficient algorithms are provided in Range and Østerdal (2017).

[^9]:    ${ }^{13}$ Equivalized total net income uses the OECD-modified scale (first proposed by Hagenaars et al., 1996). This assigns a weight of 1 to the first adult in the household, a weight of 0.5 to each additional member of the household aged 14 and over, and a weight of 0.3 to household members aged less than 14. The household's total net income is divided by this equivalized number of persons to get equivalized total net income (per person in the household). See OECD (2013b) for more information.

[^10]:    ${ }^{14}$ The ISCED (International Standard Classification of Education) is developed by UNESCO (United Nations Educational, Scientific and Cultural Organization) to facilitate cross-country comparisons of education systems since these vary in terms of structure. We use the ISCED 1997, which ranges from 0 (pre-primary education) to 6 (second stage of tertiary education); see UNESCO (2006).
    ${ }^{15}$ Since definition (i) in Section 3 uses weak inequalities rather than strict ones, a country will always dominate itself. For simplicity, these "self-dominances" are not included in Tables 2 through 4, nor in the remainder of the paper.

[^11]:    Notes: As in Table 2.

[^12]:    Notes: As in Table 2

[^13]:    ${ }^{16}$ The maximum number of potential dominances for $n$ countries is $\left(n^{2}-n\right) / 2 . n$ is raised to the second power to obtain all country combinations. The subtraction of $n$ in the nominator is to exclude self-dominance, whereas the 2 in the denominator is due to the fact that if country A dominates country B, B cannot dominate A. Since $n=24$ in this paper, the maximum number of potential dominances is $\left(24^{2}-24\right) / 2=276$.
    ${ }^{17}$ Note that the years of analysis in Alkire and Apablaza (2016) are 2006, 2009, and 2012, where only 2009 is somewhat directly comparable.

[^14]:    ${ }^{18}$ For example, measures such as the headcount ratio (see e.g., Sen, 1976 or Foster et al., 1984). Note that this is the case both if the indicator is ordinal in nature (as self reported health) or constructed from a cardinal variable (such as income).
    ${ }^{19}$ Though in a slightly different set-up, see also Hussain et al. (2016) for an example of refining dimensions to analyze the "depth" of FOD.

[^15]:    ${ }^{20}$ The fifth LCS that includes all outcomes is redundant since the probability mass functions both sum to 1 .

[^16]:    Notes: Percentage of the German population in the 60 different outcomes in 2005 as estimated using the sample from EU-SILC. a) Sum of the three sum of totals ( $17.99,49.60$, and 32.42 ). Differences in totals and sum are due to rounding.

[^17]:    ${ }^{21}$ Other aggregation thresholds have been used as well. While the results naturally are sensitive to the threshold choice, the ones shown in Table A5 have been chosen to exemplify the difference between multi-level and binary indicators.
    ${ }^{22}$ In general we will observe weakly more dominances as the number of outcome combinations decrease.

[^18]:    Notes: As in Table 2.

[^19]:    Notes: As in Table 2

[^20]:    Notes: As in Table 2.

[^21]:    Notes: As in Table 5.

