



## Correction to: Extended Formulations for Independence Polytopes of Regular Matroids

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### Correction to: Graphs and Combinatorics (2016) 32:1931–1944 <https://doi.org/10.1007/s00373-016-1709-8>

We report a logical error in our article that turns out to be fatal for the main result. The error lies in Lemma 3 for the case of a 3-sum, that is,  $k = 3$ . In fact, the claimed characterization of independent sets of the matroid  $\mathcal{M}$  in terms of those of its minors  $\mathcal{M}_1$  and  $\mathcal{M}_2$  is wrong in this case. The consequences for our main results are drastic in the sense that we currently see no way to prove Theorem 2 using the techniques from our paper. However, the corresponding polynomiality result was recently established by Manuel Aprile and Samuel Fiorini ([arXiv:1909.08539](https://arxiv.org/abs/1909.08539)).

The main statement in the original article, Theorem 2, claims that independence polytopes of regular matroids admit polynomial-size extended formulations. The proof of Theorem 2 relies on Lemma 3, which contains a wrong characterization of independent sets of a matroid  $\mathcal{M}$  that arises as a 3-sum. We elaborate on details below. We would like to thank Manuel Aprile for pointing out this error.

Unfortunately, the consequences for our main results are drastic in the sense the we currently see no way to prove Theorem 2 using the techniques from our paper. However, the polynomiality result of Theorem 2 was recently established by Manuel Aprile and Samuel Fiorini ([arXiv:1909.08539](https://arxiv.org/abs/1909.08539)).

**Error in Lemma 3.** The statement is concerned with the independent sets of a binary matroid  $\mathcal{M} = (E, \mathcal{I})$  that arises as the 3-sum of two binary matroids  $\mathcal{M}_1 = (E_1, \mathcal{I}_1)$  and  $\mathcal{M}_2 = (E_2, \mathcal{I}_2)$ , in which elements  $r_1$ ,  $p_1$  and  $q_1$  of  $\mathcal{M}_1$  (forming a circuit in

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$\mathcal{M}_1$ ) are identified with elements  $r_2$ ,  $p_2$  and  $q_2$  of  $\mathcal{M}_2$  (forming a circuit in  $\mathcal{M}_2$ ), respectively. The statement claims that the independent sets of  $\mathcal{M}$  are described by

$$\begin{aligned}\mathcal{I} = \{ & (I_1 \setminus \{r_1, p_1, q_1\}) \uplus (I_2 \setminus \{r_2, p_2, q_2\}) : I_1 \in \mathcal{I}_1, I_2 \in \mathcal{I}_2, \\ & |I_1 \cap \{r_1\}| + |I_2 \cap \{r_2\}| = 1, \\ & |I_1 \cap \{p_1\}| + |I_2 \cap \{p_2\}| = 1, \\ & |I_1 \cap \{q_1\}| + |I_2 \cap \{q_2\}| = 1\}.\end{aligned}$$

In the proof of the lemma we made the following mistake. We rewrote

$$\begin{aligned}\mathcal{I} = \{ & (J_1 \uplus J_2) : J_1 \subseteq E_1, J_2 \subseteq E_2, \\ & [J_1 \cup \{r_1\} \in \mathcal{I}_1 \text{ or } J_2 \cup \{r_2\} \in \mathcal{I}_2], \\ & [J_1 \cup \{p_1\} \in \mathcal{I}_1 \text{ or } J_2 \cup \{p_2\} \in \mathcal{I}_2], \\ & [J_1 \cup \{q_1\} \in \mathcal{I}_1 \text{ or } J_2 \cup \{q_2\} \in \mathcal{I}_2]\end{aligned}$$

as

$$\begin{aligned}\mathcal{I} = \{ & (I_1 \setminus \{r_1, p_1, q_1\}) \uplus (I_2 \setminus \{r_2, p_2, q_2\}) : I_1 \in \mathcal{I}_1, I_2 \in \mathcal{I}_2, \\ & |I_1 \cap \{r_1\}| + |I_2 \cap \{r_2\}| \geq 1, \\ & |I_1 \cap \{p_1\}| + |I_2 \cap \{p_2\}| \geq 1, \\ & |I_1 \cap \{q_1\}| + |I_2 \cap \{q_2\}| \geq 1\},\end{aligned}$$

which is clearly not true since the required supersets of  $J_1$  and  $J_2$  need not agree for the three cases. We now provide a counterexample to the case  $k = 3$  of Lemma 3.

**Counterexample.** We consider, for  $i = 1, 2$ , the binary matroids  $\mathcal{M}_i = (E_i, \mathcal{I}_i)$  with ground sets  $E_i := \{p_i, x_i, y_i, r_i, q_i, z_i\}$ , where the elements are associated to columns in the two totally unimodular matrices

$$\begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \\ x_1 \ y_1 \ r_1 \ p_1 \ q_1 \ z_1 \qquad p_2 \ x_2 \ y_2 \ r_2 \ q_2 \ z_2 \end{array}$$

and where a subset of  $E_i$  is independent in  $\mathcal{M}_i$  iff the associated columns are linearly independent (over  $\mathbb{R}$ ). Given the above matrices, the paper defines a 3-sum of  $\mathcal{M}_1$  and  $\mathcal{M}_2$  as the matroid  $\mathcal{M} = (E, \mathcal{I})$  with ground set  $E = \{x_1, y_1, x_2, y_2, z_1, z_2\}$  and where a subset of  $E$  is independent iff the associated columns in the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$x_1 \ y_1 \ x_2 \ y_2 \ z_1 \ z_2$$

are linearly independent.

Consider the set  $I := \{x_1, y_1, z_1\}$ , which is independent in  $\mathcal{M}_1$  and in  $\mathcal{M}$ . Suppose that the lemma was correct. This would imply the existence of  $I_1 \in \mathcal{I}_1$  and  $I_2 \in \mathcal{I}_2$  such that

$$I = (I_1 \setminus \{r_1, p_1, q_1\}) \uplus (I_2 \setminus \{r_2, p_2, q_2\})$$

and

$$\begin{aligned} |I_1 \cap \{r_1\}| + |I_2 \cap \{r_2\}| &\geq 1, \\ |I_1 \cap \{p_1\}| + |I_2 \cap \{p_2\}| &\geq 1, \\ |I_1 \cap \{q_1\}| + |I_2 \cap \{q_2\}| &\geq 1. \end{aligned}$$

Since  $I \cap E_2 = \emptyset$  we must have  $I \subseteq I_1$ , and since  $I$  is a basis in  $\mathcal{M}_1$  we even obtain  $I_1 = I$ . Since  $I_1 \cap \{r_1, p_1, q_1\} = \emptyset$ , the above inequalities imply that  $\{r_2, p_2, q_2\} \subseteq I_2$ . However, observe that the set  $\{r_2, p_2, q_2\}$  is dependent in  $\mathcal{M}_2$ , implying that  $I_2$  is also dependent, a contradiction.

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