PATHS OF HOMOMORPHISMS FROM STABLE KNESER GRAPHS

CARSTEN SCHULTZ

ABSTRACT. We denote by $SG_{n,k}$ the stable Kneser graph (Schrijver graph) of stable *n*-subsets of a set of cardinality 2n + k. For $k \equiv 3 \pmod{4}$ and $n \geq 2$ we show that there is a component of the χ -colouring graph of $SG_{n,k}$ which is invariant under the action of the automorphism group of $SG_{n,k}$. We derive that there is a graph G with $\chi(G) = \chi(SG_{n,k})$ such that the complex $\operatorname{Hom}(SG_{n,k}, G)$ is non-empty and connected. In particular, for $k \equiv 3 \pmod{4}$ and $n \geq 2$ the graph $SG_{n,k}$ is not a test graph.

1. INTRODUCTION

For graphs G and H, the complex Hom(G, H) is a cell complex whose vertices are the graph homomorphisms from G to H and whose topology captures global properties of the set of these homomorphisms. Research on these complexes in recent years has been driven by the concept of a test graph. In this work we present a result in this area which can be formulated naturally in the category of graphs without mentioning complexes.

Definition ([BK06]). A graph T is a *test graph* if for all graphs G and $r \ge 0$ such that the cell complex Hom(T, G) is (r - 1)-connected we have $\chi(G) \ge r + \chi(T)$.

A cell complex is said to be 0-connected if it is non-empty. If there is a graph homomorphism from T to G, then the chromatic number of G is at least as large as that of T. Therefore the condition for r = 0 is always satisfied.

A cell complex is said to be 1-connected if it is non-empty and path-connected. If $f, g: T \to G$ are graph homomorphism, then by definition there is an edge from f to g in Hom(T, G) if and only if f and g differ at exactly one vertex of T. This is all that is needed to understand the condition for r = 1, and since that is all that we will be interested in in this work, we omit the description of the higher dimensional cells of Hom(T, G).

The seminal result regarding test graphs is that K_2 is a test graph. This is a translation of a result by Lovász [Lov78] which predates the definiton of the complex Hom(T, G). While some effort has gone into proving that certain graphs are test graphs [BK06, BK07, Sch09, DS10, Sch10], it was not known from the beginning if possibly every graph is a test graph. An example of a graph which is not a test graph was given by Hoory and Linial.

Theorem ([HL05]). There is a graph T such that $\operatorname{Hom}(T, K_{\chi(T)})$ is connected.

The graph T in this example fails to be a test graph already for r = 1. Since $\operatorname{Hom}(T, K_{\chi(T)})$ is non-empty by definition, we would otherwise have $\chi(K_{\chi(T)}) \geq \chi(T) + 1$, which is absurd.

Date: 2nd June 2010.

CARSTEN SCHULTZ

So this graph T fails to be a test graph in the most fundamental way. Furthermore, the fact that $\operatorname{Hom}(T, K_{\chi(T)})$ is connected also implies that $\operatorname{Hom}(K_2, T)$ cannot be $(\chi(T) - 3)$ -connected, i.e. the chromatic number of T is not detected by the test graph K_2 . The proof of this uses some easy topology and functorial properties of Hom. One might therefore ask if there are graphs T whose chromatic numbers are detected by K_2 and which still fail to be test graphs for r = 1. We will find such examples among the stable Kneser graphs.

The stable Kneser graphs, first introduced by Schrijver, form a two parameter family of graphs $SG_{n,k}$, see Definition 4.1. We list some facts that are known of them.

- ▷ The chromatic number of $SG_{n,k}$ equals k+2 (Schrijver [Sch78], extending work of Lovasz [Lov78] and Bárány [Bár78]).
- ▷ Indeed, $\operatorname{Hom}(K_2, SG_{n,k})$ is homotopy equivalent to a k-sphere as shown by Björner and de Longueville [BDL03] (a simplified proof can be found in [Sch10]).
- ▷ The graph $SG_{n,k}$ is vertex critical, i.e. every induced subgraph on a proper subset of its set of vertices is (k + 1)-colourable [Sch78].
- ▷ The stable Kneser graph $SG_{1,k}$ is a complete graph and hence a test graph by work of Babson and Kozlov [BK06]. The stable Kneser graph $SG_{n,1}$ is a cycle on 2n+1 vertices and hence also a test graph by their work [BK07].
- ▷ For $n \ge 2$ and $k \ge 1$, Braun [Bra09] has shown the automorphism group of $SG_{n,k}$ to be the symmetry group of a (2n + k)-gon.

Since $\text{Hom}(K_2, SG_{n,k})$ is topologically as nice as one might hope, one might have thought that all stable Kneser graphs are test graphs. It turns out, however, that very few of them are.

Theorem ([Sch10, 10.5–10.10]). If $k \notin \{0, 1, 2, 4, 8\}$ then there is an N(k) such that for all $n \geq N(k)$ the graph $SG_{n,k}$ is not a test graph. For $k \equiv 3 \pmod{4}$ the graph $SG_{n,k}$ fails to be a test graph for r = 1.

The proof of this theorem studies the action of the automorphism group of $SG_{n,k}$ on the space $\operatorname{Hom}(K_2, SG_{n,k})$ using methods from algebraic topology. The current work gives an elementary proof for the case $k \equiv 3 \pmod{4}$, which yields the stronger result that in these cases we can actually set N(k) = 2. Its main result is thus the following.

Theorem. Let $k \equiv 3 \pmod{4}$ and $n \geq 2$. There is a graph G with $\chi(G) = \chi(SG_{n,k}) = k + 2$ and such that $\operatorname{Hom}(SG_{n,k}, G)$ is non-empty and connected.

In the proof, which is the combination of the following two theorems, the automorphism group of $SG_{n,k}$ again plays an important role. However, we only have to study its action on the set of path components of $\text{Hom}(SG_{n,k}, K_{k+2})$. While we know from the above discussion that the complex $\text{Hom}(SG_{n,k}, K_{k+2})$ cannot be connected, we will show in Section 4 the following weaker result.

Theorem (4.3). Let $k \equiv 3 \pmod{4}$ and $n \geq 2$. Then there is a component of $\operatorname{Hom}(SG_{n,k}, K_{k+2})$ which is invariant under the action of the automorphism group of $SG_{n,k}$.

This of course relies on Braun's result on the structure of $\operatorname{Aut}(SG_{n,k})$. In Section 3 we give a self-contained proof the following general criterion from [Sch10].

Theorem (3.1). Let T be a finite, vertex critical graph. If there is a component of $\operatorname{Hom}(T, K_{\chi(T)})$ which is $\operatorname{Aut}(T)$ -invariant, then there exists a graph G such that $\operatorname{Hom}(T, G)$ is non-empty and connected and $\chi(G) = \chi(T)$.

That every endomorphism of $SG_{n,k}$ is an automorphism follows immediately from vertex criticality. In this sense, our proof also relies on Schrijver's result that $SG_{n,k}$ is vertex critical.

2. Constructions in the category of graphs

We recall some definitions related to the category of graphs. Details can be found in [Doc09a].

A graph G consists of a vertex set V(G) and a symmetric binary relation $E(G) \subset V(G) \times V(G)$. The relation is called *adjacency*, adjacent vertices are also called *neighbours* and elements of E(G) edges. We also write $u \sim v$ for $(u, v) \in E(G)$. We point out that we allow loops, i.e. edges of the form (v, v). A graph with a loop at every vertex is called reflexive.

A graph homomorphism $f: G \to H$ is a function $f: V(G) \to V(H)$ between the vertex sets which preserves the adjacency relation, $(f(u), f(v)) \in E(H)$ for all $(u, v) \in E(G)$. When discussing the structure of the set of graph homomorphisms from G to H, it will be useful to not only consider the cell complex Hom(G, H), but also the closely related graph [G, H]. This graph, sometimes also written H^G , is defined by

$$\begin{split} V([G,H]) &= V(H)^{V(G)}, \\ E([G,H]) &= \{(f,g) \colon (f(u),g(v)) \in E(H) \text{ for all } (u,v) \in E(G)\}\,. \end{split}$$

In particular, we have $f \sim f$ if and only if f is a graph homomorphism. Furthermore, we have the following.

2.1. **Lemma.** Let $f, g: G \to H$ be graph homomorphisms and G loopless. Then $f \sim g$ in [G, H] if and only if each $h: V(G) \to V(H)$ with $h(u) \in \{f(u), g(u)\}$ for all $u \in V(G)$ is a graph homomorphism.

It follows that $f \sim g$ if an edge joins the two graph homomorphism f and g in Hom(G, H). If on the other hand $f \sim g$, then we can get from f to g in Hom(G, H) by changing the values at the vertices of G in any order. Therefore, for questions of connectivity it does not matter whether we work in Hom(G, H) or in the induced subgraph of looped vertices of [G, H]. That two graph homomorphisms are in the same component can now be reformulated as follows.

2.2. **Definition.** For $n \ge 0$ we define a reflexive graph I_n by $V(I_n) = \{0, \ldots, n\}$, $i \sim j \iff |i - j| \le 1$. For graph homomorphisms $f, g: G \to H$ we write $f \simeq g$ if and only if there is an $n \ge 0$ and a graph homomorphism $p: I_n \to [G, H]$ with p(0) = f, p(n) = g.

The construction $[\bullet, \bullet]$ is an *inner hom*, intimately related to products. The product of two graphs in the category \mathcal{G} of graphs and graph homomorphisms is given by

$$V(G \times H) = V(G) \times V(H),$$

$$E(G \times H) = \{((u, u'), (v, v')) \colon (u, v) \in E(G), (u', v') \in E(H)\}.$$

CARSTEN SCHULTZ

For every graph G, the functor $[G, \bullet]$ is a right adjoint to $\bullet \times G$, i.e. there is a natural equivalence

(1)
$$\mathcal{G}(Z \times G, H) \cong \mathcal{G}(Z, [G, H]).$$

4

For example, that looped vertices of [G, H] correspond to graph homomorphisms can be derived as a formal consequence of this adjunction. If **1** denotes the terminal object of \mathcal{G} , we obtain

$$\mathcal{G}(G,H) \cong \mathcal{G}(\mathbf{1} \times G,H) \cong \mathcal{G}(\mathbf{1},[G,H]),$$

and since 1 is a graph consisting of one vertex and one loop, the graph homomorphisms from 1 to some other graph correspond to the looped vertices of that graph.

3. A CRITERION FOR NOT BEING A TEST GRAPH

If T is a vertex critical finite graph, then every graph homomorphism $T \to T$ will have to be surjective and hence bijective. In other words, every endomorphism of T is an automorphism. For graphs with this property we obtain the following criterion to decide whether they satisfy the test graph property for r = 1.

3.1. **Theorem.** Let T be a finite graph such that $\operatorname{End}(T) = \operatorname{Aut}(T)$. If there is a component of $\operatorname{Hom}(T, K_{\chi(T)})$ which is $\operatorname{Aut}(T)$ -invariant, then there exists a graph G with $\chi(G) = \chi(T)$ such that $\operatorname{Hom}(T, G)$ is non-empty and connected.

3.2. **Remark.** This is the case r = 1, $s = \chi(T) + 1$ of the implication (iv) \Rightarrow (ii) of [Sch10, Thm 10.1]. The proof there uses results from [Doc09b] and [DS10]. Here we are only interested in dimension 1, and for this easier case we can give a self-contained proof which follows the same lines.

3.3. **Remark.** The converse holds in general without conditions on T. If $f: T \to G$ and $c: G \to K_{\chi(T)}$, then $c \circ f$ is a vertex of $\operatorname{Hom}(T, K_{\chi(T)})$, and if $\operatorname{Hom}(T, G)$ is connected, then the component of that vertex will be invariant under $\operatorname{Aut}(T)$: If $\gamma \in \operatorname{Aut}(T)$ then $f \circ \gamma \simeq f$ implies $c \circ f \circ \gamma \simeq c \circ f$.

Proof of Theorem 3.1. Let $c: T \to K_{\chi(T)}$ be a colouring which lies in the invariant component. Then for every $\gamma \in \operatorname{Aut}(T)$ there is an $n \ge 0$ and a graph homomorphism $I_n \to [T, K_{\chi(T)}]$ with $0 \mapsto c$ and $n \mapsto c\gamma$. We can assemble these into a graph homomorphism $X \to [T, K_{\chi(T)}]$, where X is a reflexive graph consisting of a vertex u and for every $\gamma \in \operatorname{Aut}(T)$ a path from u to a vertex v_{γ} and the graph homomorphism such that $u \mapsto c$ and $v_{\gamma} \mapsto c\gamma$. We will later require that the paths have a certain minimal length, which we can arrange.

The graph homomorphism which we have just constructed gives rise via (1) to a graph homomorphism $f: X \times T \to K_{\chi(T)}$ with $f(v_{\gamma}, t) = c(\gamma(t)) = f(u, \gamma(t))$ for all $\gamma \in \operatorname{Aut}(T)$ and $t \in V(T)$. We define an equivalence relation on $V(X \times T)$ such that $(v_{\gamma}, t) \sim (u, \gamma(t))$ and obtain a commutative diagram



with q the quotient map. Now let $j: T \to G$ be the inclusion at u, more formally $j = q \circ (\text{const}_u, \text{id}_T)$. Now we already know that $\text{Hom}(T, G) \neq \emptyset$ and $\chi(G) = \chi(T)$. We will proceed to show that Hom(T, G) is connected.

We first note that $\operatorname{const}_u \simeq \operatorname{const}_{v_\gamma} \colon T \to X$ for all $\gamma \in \operatorname{Aut}(T)$, and hence

$$j \simeq q \circ (\operatorname{const}_{v_{\gamma}}, \operatorname{id}_T) = q \circ (\operatorname{const}_u, \gamma) = j\gamma.$$

Now let $g: T \to G$ be an arbitrary graph homomorphism. Let \sim' be the equivalence relation on V(X) given by $v_{\gamma} \sim' u$. The reflexive graph X/\sim' is a bouquet of circles, and there is a natural surjection $G \to X/\sim'$. Since we may assume to haven chosen the graph X large enough (this depending on T, not on g), the image of the composition $T \xrightarrow{g} G \to X/\sim'$ will miss at least one vertex of each of the circles. Let X' be the graph obtained from X/\sim' by removing one these vertices from each circle. We denote the vertex corresponding to u by u'. The preimage of X' in G is isomorphic to $X' \times T$. Let $h: X' \times T \to G$ be the corresponding embedding such that $j = h \circ (\operatorname{const}_{u'}, \operatorname{id}_T)$. This defines a commutative diagram



Now $\tilde{g} = (\tilde{g}_1, \tilde{g}_2)$. Since X' is a reflexive tree with loops, we have $\tilde{g}_1 \simeq \text{const}_{u'}$. Since End(T) = Aut(T), there is a $\gamma \in \text{Aut}(T)$ such that $\tilde{g}_2 = \gamma$. Therefore

$$q \simeq h \circ (\text{const}_{u'}, \gamma) = h \circ (\text{const}_{u'}, \text{id}_T) \circ \gamma = j\gamma \simeq j.$$

Since g was arbitrary, this shows that Hom(T, G) is connected.

4. Paths of colourings of stable Kneser graphs

4.1. **Definition.** Let $n \ge 1$, $k \ge 0$ and m = 2n + k. The Kneser graph $KG_{n,k}$ is the graph whose vertices are the *n*-element subsets of $\{0, \ldots, m-1\}$ and in which two of them are adjacent if and only if they are disjoint. We call a subset S of $\{0, \ldots, m-1\}$ semi-stable, if $\{i, i+1\} \not\subset S$ for all $0 \le i \le m-2$, and stable, if additionally $\{0, m-1\} \not\subset S$. The stable Kneser graph $SG_{n,k}$ is the induced subgraph of $KG_{n,k}$ on the set of stable sets. The semi-stable Kneser graph $\overline{SG}_{n,k}$ is the induced subgraph of $KG_{n,k}$ on the set of semi-stable sets.

4.2. **Definition.** We call the graph homomorphism

$$c_{n,k} \colon SG_{n,k} \to K_{k+2},$$
$$S \mapsto \min S$$

the canonical colouring of $\overline{SG}_{n,k}$ and its restriction the canonical colouring of $SG_{n,k}$.

We will prove the following.

4.3. **Theorem.** Let $k \equiv 3 \pmod{4}$ and $n \geq 2$. Then for any $\gamma \in \operatorname{Aut}(SG_{n,k})$ and $c \colon SG_{n,k} \to K_{k+2}$ the canonical colouring there is a path in $\operatorname{Hom}(SG_{n,k})$ from c to $c\gamma$.

We consider automorphisms of K_{k+2} before turning to automorphisms of $SG_{n,k}$.

CARSTEN SCHULTZ

4.4. **Proposition.** Let $n \ge 2$, $k \ge 1$. Let $\pi \in A_{k+2}$ be an even permutation of the vertices of K_{k+2} . Then $c_{n,k} \simeq \pi \circ c_{n,k} : \overline{SG}_{n,k} \to K_{k+2}$.

Proof. Let m = 2n + k. We will assume that π is a cycle of the form $(i \ i+1 \ i+2)$ with $0 \le i < k$. This is possible, since these permutations generate A_{k+2} . The proof will be by induction on k. We distinguish three cases.

 $\underline{i} > 0$. We note that $c(S) \ge i$ if and only if $S \subset \{i, \ldots, m-1\}$. The induced subgraph on the set of these vertices is isomorphic to $\overline{SG}_{n,k-i}$ via $S \mapsto S - i$. Therefore a path from $c_{n,k-i}$ to $(0\ 1\ 2) \circ c_{m,k-i}$, which exists by induction, can be extended to a path from $c_{n,k}$ to $\pi \circ c_{n,k}$ by fixing all colours less than i.

i = 0, k > 1. We define

$$c' \colon \overline{SG}_{n,k} \to K_{k+2},$$
$$S \mapsto \begin{cases} k+1, & m-1 \in S, \\ \min S, & \text{otherwise.} \end{cases}$$

Obviously c' is a graph homomorphism. If $S \sim S'$ then $m-1 \notin \emptyset = S \cap S'$ and hence $c_{n,k}(S) = c'(S)$ or $c_{n,k}(S') = c'(S')$. This shows $c' \sim c_{n,k}$. The induced subgraph on those S for which c'(S) < k + 1 equals $\overline{SG}_{n,k-1}$. The restriction of c'to that subgraph equals $c_{n,k-1}$. Therefore a path from $c_{n,k-1}$ to $\pi \circ c_{n,k-1}$ extends to one from c' to $\pi \circ c'$. Hence $c_{n,k} \sim c' \simeq \pi \circ c' \sim \pi \circ c_{n,k}$.

i = 0, k = 1. There is a unique function $h: V(\overline{SG}_{n,1}) \to V(SG_{2,1})$ with $h(S) \cap \{0, \overline{1}, 2, 3\} = S \cap \{0, 1, 2, 3\}$ for all S. It satisfies $h(S) \subset S$ and is therefore a graph homomorphism. Also, $c_{n,1} = c_{2,1} \circ h$. It therefore suffices to show that $c_{2,1} \simeq (0 \ 1 \ 2) \circ c_{2,1}$, where $c_{2,1}$ is defined on $SG_{2,1}$. We note that $SG_{2,1}$ is a cycle of length 5. Homomorphisms between cycles have been studied in more detail by Čukić and Koylov [ČK06]. For our needs the following explicit construction suffices.

$\{0,3\}$	$\{1, 4\}$	$\{0, 2\}$	$\{1, 3\}$	$\{2, 4\}$
0	1	0	1	2
0	2	0	1	2
1	2	0	1	2
1	2	0	1	0
1	2	0	2	0
1	2	1	2	0

Each row of the table defines a graph homomorphism $SG_{2,1} \to K_3$. Adjacent rows define homomorphisms which are adjacent in $[SG_{2,1}, K_3]$.

4.5. Lemma. Let $n \ge 2$, $k \ge 1$, m = 2n + k. Let $\tau, \rho \in \operatorname{Aut}(SG_{n,k})$ be defined by $\tau S = S + 1$, $\rho S = k - S$,

with arithmetic modulo m, and $\bar{\tau}, \bar{\rho} \in \operatorname{Aut}(K_{k+2})$ by

$$\bar{\tau}x = x + 1, \qquad \qquad \bar{\rho}x = k - x,$$

with arithmetic modulo k + 2. Then $c\tau \sim \overline{\tau}c$ and $c\rho \sim \overline{\rho}c$, where c is the canonical colouring.

Proof. Let $S, S' \in V(SG_{n,k})$.

If $S \sim S'$, then S and S' cannot both contain m-1. Assume $m-1 \notin S$. Then $c\tau(S') \sim c\tau(S) = \overline{\tau}c(S)$ and $c\tau(S) = \overline{\tau}c(S) \sim \overline{\tau}c(S')$. This shows $c\tau \sim \overline{\tau}c$.

Assume $c\rho(S) \not\sim \bar{\rho}c(S')$, i.e. $c\rho(S) = i = \bar{\rho}c(S')$ for some $0 \le i < k+2$. If i = k+1 then $S = S' = \{k+1, k+3, \dots, k+2n-1\}$. If $0 \le i < k$, then $k - i \in S \cap S'$. In both cases $S \not\sim S'$. This shows $c\rho \sim \bar{\rho}c$.

Proof of Theorem 4.3. By a theorem of Braun [Bra09], every automorphism of $SG_{n,k}$ is induced by a permutation of the base set which preserves its cyclic adjacency relation. Therefore the elements τ and ρ of Lemma 4.5 generate Aut $(SG_{n,k})$, and it suffices to show that $c \simeq c\tau$ and $c \simeq c\rho$. By Lemma 4.5 this reduces to $c \simeq \bar{\tau}c$ and $c \simeq \bar{\rho}c$. By Proposition 4.4 these will be true if $\bar{\tau}$ and $\bar{\rho}$ are even permutations. Now sign $\bar{\tau} = (-1)^{k+1} = 1$ and sign $\bar{\rho} = (-1)^{\frac{k(k+1)}{2}} = 1$.

References

- [Bár78] BÁRÁNY, I. A short proof of Kneser's conjecture. J. Combin. Theory Ser. A, 25(3):325– 326, 1978.
- [BDL03] BJÖRNER, A. and DE LONGUEVILLE, M. Neighborhood complexes of stable Kneser graphs. Combinatorica, 23(1):23–34, 2003. Paul Erdős and his mathematics (Budapest, 1999).
- [BK06] BABSON, E. and KOZLOV, D. N. Complexes of graph homomorphisms. Israel J. Math., 152:285-312, 2006. http://dx.doi.org/10.1007/BF02771988.
- [BK07] BABSON, E. and KOZLOV, D. N. Proof of the Lovász conjecture. Ann. of Math. (2), 165(3):965-1007, 2007. http://dx.doi.org/10.4007/annals.2007.165.965.
- [Bra09] BRAUN. В. Symmetries of $_{\mathrm{the}}$ stableKneser graphs. AdvancesApplied Mathematics, Press, Corrected $\mathbf{Proof:}$ -, inIn 2009.
 - http://www.sciencedirect.com/science/article/B6W9D-4XVB6PF-3/2/08c3d7f1ce0a541c8ca13d25b37eb7db. (КОб] ČUКІĆ, S. L. and KOZLOV, D. N. Homotopy Type of Complexes of Graph
- [ČK06] ČUKIĆ, S. L. and KOZLOV, D. N. Homotopy Type of Complexes of Graph Homomorphisms between Cycles. Discrete Comput. Geom., 36(2):313–329, 2006. math.CO/0408015.
- [Doc09a] DOCHTERMANN, Α. Hom complexes and homotopy theory the in graphs. category of EuropeanJ. Combin., 30(2):490-509,2009. http://dx.doi.org/10.1016/j.ejc.2008.04.009
- [Doc09b] DOCHTERMANN, A. The universality of *Hom* complexes. *Combinatorica*, **29**(4):433–448, 2009.
- [DS10] DOCHTERMANN, A. and SCHULTZ, C. Topology of Hom complexes and test graphs for bounding chromatic number, 2010. Preprint, 42 pp., arXiv:0907.5079v2 [math.CO].
- [HL05] HOORY, S. and LINIAL, N. A counterexample to a conjecture of Björner and Lovász on the χ -coloring complex. J. Combin. Theory Ser. B, **95**(2):346–349, 2005. http://dx.doi.org/10.1016/j.jctb.2005.06.008.
- [Lov78] LovÁsz, L. Kneser's conjecture, chromatic number, and homotopy. J. Combin. Theory Ser. A, 25(3):319–324, 1978. http://dx.doi.org/10.1016/0097-3165(78)90022-5.
- [Sch78] SCHRIJVER, A. Vertex-critical subgraphs of Kneser graphs. Nieuw Arch. Wisk. (3), 26(3):454-461, 1978.
- [Sch09] SCHULTZ, C. Graph colorings, spaces of edges and spaces of circuits. Adv. Math., 221(6):1733-1756, 2009. http://dx.doi.org/10.1016/j.aim.2009.03.006.
- [Sch10] SCHULTZ, C. The equivariant topology of stable Kneser graphs, 2010. 34 pp., arXiv:1003.5688v1.

Institut für Mathematik, MA 6-2, Technische Universität Berlin, D-10623 Berlin, Germany

E-mail address: carsten@codimi.de