

Random-Key Cuckoo Search for the Travelling Salesman Problem

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Abstract

Combinatorial optimization problems are typically NP-hard, and thus very challenging to solve. In this paper, we present the random key cuckoo search (RKCS) algorithm for solving the famous Travelling Salesman Problem (TSP). We used a simplified random-key encoding scheme to pass from a continuous space (real numbers) to a combinatorial space. We also consider the displacement of a solution in both spaces using Lévy flights. The performance of the proposed RKCS is tested against a set of benchmarks of symmetric TSP from the well-known TSPLIB library. The results of the tests show that RKCS is superior to some other metaheuristic algorithms.

1 Introduction

Many combinatorial optimization problems are NP-hard, and thus very challenging to solve. In fact, they cannot be solved efficiently by any known algorithm in a practically short time scale when the size of the problem is moderately large [9]. The main difficulty arises with the number of combinations which increases exponentially with the size of the problem. Searching for every possible combination is extremely computationally expansive and unrealistic. An example of these problems is the travelling salesman problem [10] in which a salesperson has to visit a list of cities exactly once, and returning to the departure city, with the aim of minimizing the total travelled distance or the overall cost of the trip.

Despite the challenges, TSP remains one of the most widely studied problems in combinatorial optimization. It is often used for testing optimization algorithms. Problems such as TSP do not have an efficient algorithm to solve them. It is practically very difficult to get a solution of optimal quality and in a reduced runtime simultaneously. This requires some heuristic algorithms that can find good (not necessarily optimal) solutions in a good runtime by trial and error. Approximate algorithms such as metaheuristics [2] are actually the best choice to solve many combinatorial optimization problems. They are characterized by their

simplicity and flexibility while demonstrating remarkable effectiveness. Metaheuristics are usually simple to implement; however, they are often capable to solve complex problems and can thus be adapted to solve problems with diverse objective function properties, either continuous, discrete or mixed, including many real-world optimization problems, from engineering to artificial intelligence [22].

In essence, metaheuristics are optimization algorithms that adopt some strategies to explore and exploit a given solution space with the aim to find the best solution. They balance their search concentration between some promising regions and the all space regions. They generally begin with a set of initial solutions, and then, examine step by step a sequence of solutions to reach (hopefully) the optimal solution of the problem.

Some issues may arise when solving a combinatorial optimization problem with a metaheuristic, and a key issue is how to define neighbourhood solutions for such problems. Several metaheuristics are designed in principle for continuous optimization problems. So, the question is how to treat combinatorial problems properly without losing the good performance of these metaheuristics. In this paper, we propose the random-key cuckoo search (RKCS) algorithm using the random-key encoding scheme to represent a position, found by the cuckoo search algorithm, in a combinatorial space.

TSP is solved with random keys by various metaheuristics [19, 4]. This work presents a novel approach using Cuckoo Search algorithm (CS)[25], based on random keys [1], with a simple local search procedure to solve TSP. CS is a nature-inspired metaheuristic algorithm which was developed by Yang and Deb in 2009 to solve continuous optimization problems. With this approach, we aim to formulate the transition between a continuous search space and a combinatorial search space without passing through traditional adaptation operators that may affect the performance of the algorithm, and to ensure a direct interpretation of various operators used by metaheuristics in continuous search space.

The rest of this paper is organized as follows: Sect. 2, first, introduces the standard cuckoo search. Section 3 introduces briefly the TSP. Section 4 presents the random-key encoding scheme, while Sect. 5 describes the discrete CS to solve symmetric TSP using Random key. Then, Sect. 6 presents results of numerical experiments on a set of benchmarks of symmetric TSP from the TSPLIB library [18]. Finally, Sect. 7 concludes with some discussions and future directions.

2 Cuckoo Search Algorithm

Some cuckoo species can have the so-called brood parasitism as an aggressive reproduction strategy. This most studied and discussed feature is that cuckoos lay eggs in a previously observed nest of another species to let host birds hatch and brood their young cuckoo chicks. From the evolutionary point of view, cuckoos aim to increase the probability of survival and reduce the probability of abandoning eggs by the host birds[16]. The behaviour of cuckoos is mimicked successfully in the cuckoo search algorithm, in combination with Lévy flights to effectively search better and optimal survival strategy. Lévy flights [3], named by the French mathematician Paul Lévy, represent a model of random walks characterized by their step lengths which obey a power-law distribution. Several scientific studies have shown that the search for preys by hunters follows typically the same characteristics of Lévy flights. This model is commonly presented by small random steps followed occasionally by large jumps [3, 20].

Inspired by these behaviours and strategies, the Cuckoo Search (CS) algorithm was

developed by Xin-She Yang and Suash Deb in 2009, which was initially designed for solving multimodal functions. It has been shown that CS can be very efficient in dealing with highly nonlinear optimization problems [24, 27, 26, 13, 12].

CS is summarized as the following ideal rules: (1) each cuckoo lays one egg at a time and selects a nest randomly; (2) the best nest with the highest quality of egg can pass onto the new generations; (3) the number of host nests is fixed, and the egg laid by a cuckoo can be discovered by the host bird with a probability $p_a \in [0, 1]$.

CS uses a balanced combination of a local random walk and the global explorative random walk, controlled by a switching parameter p_a . The local random walk can be written as

$$x_i^{t+1} = x_i^t + \alpha s \otimes H(p_a - \epsilon) \otimes (x_j^t - x_k^t), \quad (1)$$

where x_j^t and x_k^t are two different solutions selected randomly by random permutation, $H(u)$ is a Heaviside function, ϵ is a random number drawn from a uniform distribution, and s is the step size. On the other hand, the global random walk is carried out using Lévy flights

$$x_i^{t+1} = x_i^t + \alpha L(s, \lambda), \quad (2)$$

where

$$L(s, \lambda) = \frac{\lambda \Gamma(\lambda) \sin(\pi\lambda/2)}{\pi} \frac{1}{s^{1+\lambda}}, \quad (s \gg s_0 > 0). \quad (3)$$

Here $\alpha > 0$ is the step size scaling factor, which should be related to the scales of the problem of interest. Lévy flights have an infinite variance with an infinite mean [25].

In this approach we have taken as a basis an improved version of CS [14]. This improvement considers a new category of cuckoos that can engage a kind of surveillance on nests likely to be a host. These cuckoos use mechanisms before and after brooding such as the observation of the host nest to decide if the chosen nest is the best choice or not. So, from the current solution, this portion p_c of cuckoos searches in the same area a new better solution via Lévy flights.

The goal of the improvement is to strengthen intensive search around the current solutions, using the new fraction p_c . The process of this fraction can be introduced in the standard algorithm of CS as shown in Algorithm 1.

3 Travelling Salesman Problem

To simplify the statement of the travelling salesman problem, we can assume that we have a list of m cities that must be visited by a salesperson and returning to the departure city. To calculate the best tour in term of distance, some rules or assumptions can be used before starting the trip. Each city on the list must be visited exactly once, and for each pair of cities, given that the distance between any two cities is known. This problem is commonly called as the ‘‘The travelling salesman problem’’.

The TSP can be stated formally [7] as: Let $C = \{c_1, \dots, c_m\}$ be a set of m distinct cities, $E = (c_i, c_j) : i, j \in \{1, \dots, m\}$ be the edge set, and $d_{c_i c_j}$ be a cost measure associated with the edge $(c_i, c_j) \in E$. The objective of the TSP is to find the minimal length of a closed tour that visits each city once. Cities $c_i \in C$ are represented by their coordinates (c_{i_x}, c_{i_y}) and $d_{c_i c_j} = \sqrt{(c_{i_x} - c_{j_x})^2 + (c_{i_y} - c_{j_y})^2}$ is the Euclidean distance between c_i and c_j .

Algorithm 1 Improved CS Algorithm

- 1: Objective function $f(x), x = (x_1, \dots, x_m)^T$
 - 2: Generate initial population of n host nests $x_i (i = 1, \dots, n)$
 - 3: **while** ($t < \text{MaxGen}$) or (stop criterion) **do**
 - 4: **Start searching with a fraction (p_c) of smart cuckoos**
 - 5: Get a cuckoo randomly by Lévy flights
 - 6: Evaluate its quality/fitness F_i
 - 7: Choose a nest among n (say, j) randomly
 - 8: **if** ($F_i > F_j$) **then**
 - 9: replace j by the new solution;
 - 10: **end if**
 - 11: A fraction (p_a) of worse nests are abandoned and new ones are built;
 - 12: Keep the best solutions (or nests with quality solutions);
 - 13: Rank the solutions and find the current best
 - 14: **end while**
 - 15: Postprocess results and visualization
-

A tour can be represented as a cyclic permutation [8] $\pi = (\pi(1), \pi(2), \dots, \pi(m))$ of cities from 1 to m if $\pi(i)$ is interpreted to be the city visited in step $i, i = 1, \dots, m$. The cost of a permutation (tour) is defined as:

$$f(\pi) = \sum_{i=1}^{m-1} d_{\pi(i)\pi(i+1)} + d_{\pi(m)\pi(1)} \quad (4)$$

If $d_{c_i c_j} \neq d_{c_j c_i}$ for at least one (c_i, c_j) , we say that it is an Asymmetric Euclidean TSP, while if $d_{c_i c_j} = d_{c_j c_i}$ then the TSP becomes a Symmetric Euclidean TSP, which is adopted in this paper. Given m as the number of cities to visit in the list, the total number of possible tours covering all cities can be seen as a set of feasible solutions of the TSP and is given as $m!$.

The challenges of solving a TSP have motivated many researchers to design various algorithms. An effective search should be able to detect the best solution for the majority of its instances in a reasonable runtime. The good thing about TSP is that the statement is simple and requires no mathematical background to understand, though it is difficult to produce a good solution. However, TSP is crucially important in both academia and applications. Lenstra et. al. and Reinelt [11, 17] gave some reviews of direct and indirect applications of TSP in several industrial and technological fields, such as drilling problem of printed circuit boards (PCBs), overhauling gas turbine engines, X-ray crystallography, computer wiring, order-picking problem in warehouses, vehicle routing, and mask plotting in PCB production.

4 Random-Key Encoding Scheme

Random-key encoding scheme [1] is a technique that can be used to transform a position in a continuous space and convert it into a combinatorial one. It uses a vector of real numbers by associating each number with a weight. These weights are used to generate one combination as a solution.

The random real numbers drawn uniformly from $[0, 1)$ compose a vector showed in Fig. 4. On the other hand, the combinatorial vector is composed of integers ordered according to the weights of real numbers in the first vector, illustrated as follows (Fig. 4):

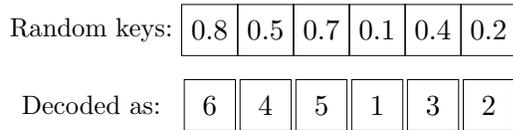


Figure 1: Random key encoding scheme.

5 Random Key CS for TSP

Random keys are an encoding scheme which was used early with genetic algorithms for sequencing and optimization problems by Bean [1]. It is based on random real numbers in a continuous space to encode solutions in a combinatorial space. These random numbers, presented as tags, are generated from $[0, 1)^m$ space, where m is the size of the TSP instance to be solved (or the dimension of this space). Our approach here extends this idea by performing Lévy flights distribution to generate the random numbers. This allows an improved way to balance the search for solutions in local areas as well as global areas. In this work, we will thus use both random walks and Lévy flights [23] whose step lengths are chosen from a probability distribution with a power-law tail.

5.1 Solution Representation

Figure 2 and Algorithm 2 present the steps of generating a TSP solution using random keys. First, agents are randomly positioned according to their real values in $[0, 1)$. Each agent has an integer index regardless of his ascending order among other agents in the linked list (see Fig. 2). All agents are ordered according to their weights (real numbers) and their indices form together an initial solution of the TSP instance. So, this essentially means that the integers/agent indices, here, correspond to the city index and the order of agents is the visiting order of the cities.

Algorithm 2 Initial Solution Algorithm

- 1: Set of m agents a_i ($i = 1, \dots, m$);
 - 2: **for** $i = 1$ **to** m **do**
 - 3: Assign a random real number in $[0, 1)$ for agent a_i ;
 - 4: **end for**
 - 5: **for** $i = 1$ **to** m **do**
 - 6: Get the order of agent a_i according to his weight;
 - 7: **end for**
 - 8: **for** $i = 1$ **to** m **do**
 - 9: The agent index is the city index;
 - 10: The agent order is the city visiting order;
 - 11: **end for**
 - 12: Return a solution of cities' visiting order and weights;
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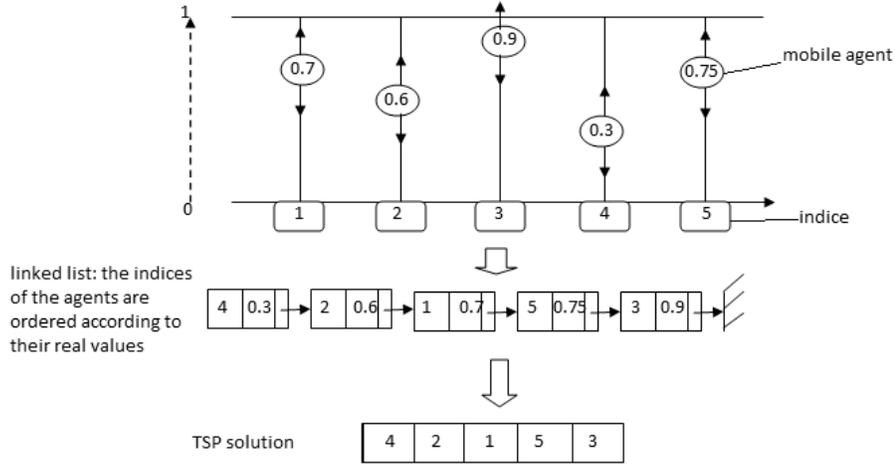


Figure 2: Procedure of random-keys to generate a TSP solution.

5.2 Displacement

The procedure of generating new solutions by a perturbation in the real space can lead to some issues when these agents start to move in $[0, 1)$. Such moves can affect the order of agents in the linked list and therefore a new TSP solution can be generated as shown in Fig. 3. The displacement of each agent is guided directly by Lévy flights. The order of each city is changed by small perturbations or big jumps according to the values generated in their weights via Lévy flights.

Our approach here is mainly based on two types of search moves: 1) global search carried out on solution areas guided by the movements (following Lévy flights) of agents; 2) local search which detects the best solutions in the areas found by the agents. The combination of both local and global search moves can improve the performance. Briefly, RKCS begins with a search for new promising areas. It combines intensification and diversification via small steps and large jumps to distant areas. When pointing on an area found by Lévy flights, the best solution in this area is detected and another search is triggered to generate a new one via Lévy flights. So, we can summarize these steps by the following Algorithm 3:

Algorithm 3 Generating New Solution

- 1: Solution S of m cities/agents a_i ($i = 1, \dots, m$);
 - 2: Select randomly l agents ($1 \leq l \leq m$).
 - 3: **for** $i = 1$ **to** l **do**
 - 4: Assign new position via Lévy flights (Equation 2) for agent a_i ;
 - 5: Reposition a_i in the linked list (see Figure 3);
 - 6: Update S ;
 - 7: **end for**
 - 8: Return the new generated solution S ;
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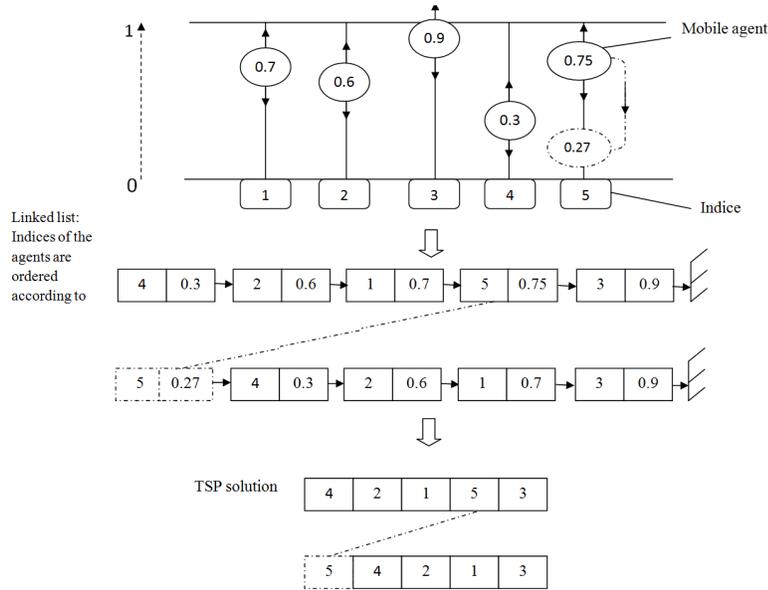


Figure 3: Procedure of random keys move to generate new TSP solution

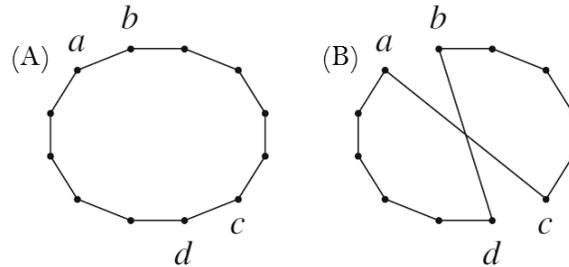


Figure 4: 2-opt move, (A) Initial tour. (B) The tour created by 2-opt move [the edges (a, b) and (c, d) are removed, while the edges (a, c) and (b, d) are added.

5.3 Local Search

After finding a new area by Lévy flights, a local search is performed to detect the best solution in this area. For this local search, 2-opt move [6] is used where it removes two edges in the TSP solution and reconnects the new two created paths, in a different possible way as showed in Fig. 4. In the minimization case, it is done only if the new tour is shorter than the current one. Obviously, this process is repeated until no further improvement is possible or when a given number of steps is reached.

Steepest Descent (Algorithm 4) is a simple local search method that can be easily trapped in a local minimum, and generally, it can not find a good quality solutions. We choose this simplified local search “Steepest Descent” method to show the performance of CS combined with RK. It allowed us to generate solutions of good quality, without introducing an advanced local search method.

Algorithm 4 Steepest Descent Algorithm

```
1: Objective function  $f(x), x = (x_1, \dots, x_m)^T$ ;  
2: Initial TSP solution  $S_0$  of  $m$  ordered cities;  
3: Current solution  $S, S \leftarrow S_0$ ;  
4: Choose stop boolean value,  $stop \leftarrow FALSE$ ;  
5: while  $stop = FALSE$  do  
6:   Choose the best neighbour  $S_v$  of  $S$  via 2-Opt moves;  
7:   if  $f(S_v) < f(S)$  then  
8:     Replace  $S$  by  $S_v$ ;  
9:   else  
10:     $stop \leftarrow TRUE$   
11:   end if  
12: end while  
13: Return  $S$ ;
```

5.4 RKCS Algorithm

Using the same steps of Improved CS [14] and as summarized in Algorithm 1, before starting the search process, RKCS generates the random initial solution or population as explained in Fig. 2 and Algorithm 2. Generating a random initial population is to show how RKCS can find good solutions in the search space without using an enhancement pre-processes.

The second phase is triggering the p_c portion of smart cuckoos. These cuckoos begin by exploring new areas from the current solutions. As shown in Fig. 3, they use Lévy flights to move in the real space and interpreting this move to have a new TSP solution in the new area. The second step is to find a good solution in this area following Algorithm 4.

After p_c portion phase, RKCS employs one cuckoo to search for a new good solution, starting from the best solution of the population. It proceeds, like the second phase (p_c portion phase), in two steps. Firstly, it locates a new area, from the best solution, via Lévy flights and then finds a good solution in this area. The found solution is compared with a random selected solution in the population. The best one of the both solutions earns its place in the population.

The last phase is for the worst and abandoned solutions that will be replaced by new ones. They start searching, for a new good solution, far from the best solution in the population by a big jump. In this case a big jump is perturbing more agents via Lévy flights. All these phases are illustrated in the flowchart of RKCS (Fig. 5).

6 Experimental results

We have implemented the proposed random key cuckoo search and tested it using the well-known TSPLIB library [18]. For each instance, 30 independent runs have been carried out. The properly selected parameter values used for the experiments of RKCS algorithm are shown in Table 1.

Table 2 shows the test results of running RKCS algorithm to solve some benchmark instances of the symmetric TSP from the TSPLIB library [18]. The first column corresponds to the name of instances with their optimum in parentheses. The column ‘Best’ shows the length of the best solution found by RKCS, the column ‘Average’ gives the average solution found by RKCS, the column ‘Worst’ shows the length of the worst solution length among

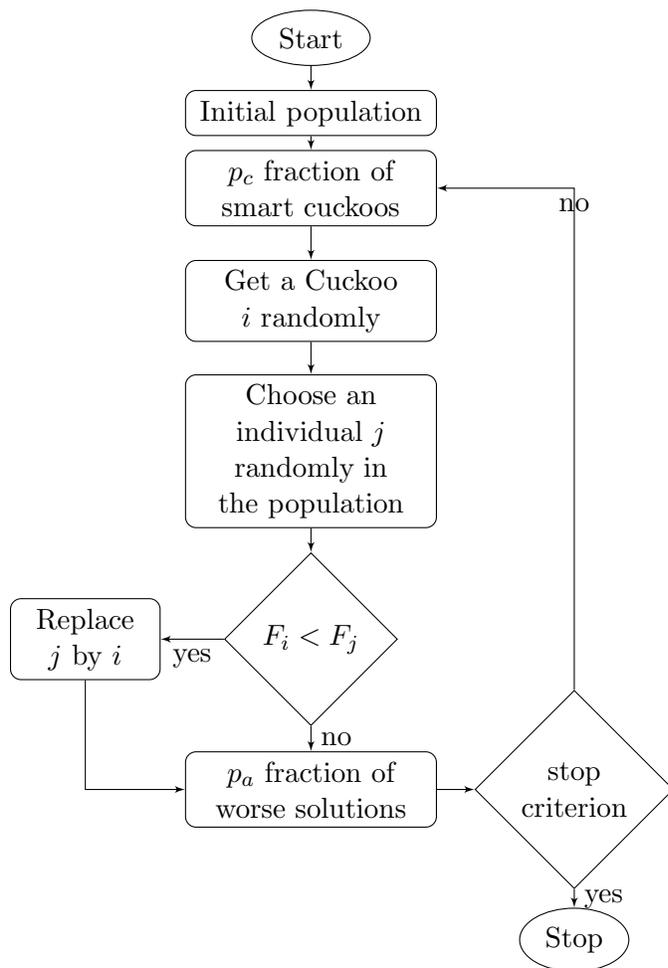


Figure 5: The flowchart of the RKCS algorithm.

Table 1: Parameter settings for RKCS algorithm.

Parameter	Value	Meaning
n	30	Population size
p_c	0.6	Portion of smart cuckoos
p_a	0.2	Portion of bad solutions
$MaxGen$	500	Maximum number of iterations
α	0.01	Step size
λ	1	Index

the 30 independent runs of the RKCS algorithm.

These results confirm that the proposed approach is able to find good or the optimum solution for the tested instances (‘bold’ in the Table 2 shows that RKCS reaches the optimal solution of the tested instance). Therefor, we can say that the random-key encoding scheme can be a very useful tool for switching from continuous to combinatorial spaces. It allows operators of the continuous space to behave freely, then projecting the changes made by these operators in the combinatorial space. It also facilitates a better control in balancing intensification and diversification through Lévy flights, which make intensified small steps in a limited region followed by a big explorative jump to a distant region. Using the real numbers, Lévy flights can easily act with the notion of distance and can define clearly small or big steps. Then RK projects these changes in the space of TSP solutions.

Table 2: Results of random-key cuckoo search for the travelling salesman problem

Instance(opt)	Best	Average	Worst
eil51(426)	426	426.9	430
berlin52(7542)	7542	7542	7542
st70(675)	675	677.3	684
pr76(108159)	108159	108202	109085
eil76(538)	538	539.1	541
kroA100(21282)	21282	21289.65	21343
eil101(629)	629	631.1	636
bier127(118282)	118282	118798.1	120773
pr136(96772)	97046	97708.9	98936
pr144(58537)	58537	58554.45	58607
ch130(6110)	6126	6163.3	6210

The RKCS experimental results are then compared with all other algorithms tested in ‘Hybrid Gravitational Search Algorithm with Random-key Encoding Scheme Combined with Simulated Annealing’ [4]. Table 3 and Fig. 6 show that RKCS outperforms all these algorithms in the tested instances. A good balance between exploration and exploitation of space areas and a simple local search technique are significant components to reach good results. This confirms that the proposed random-key encoding scheme can provide a good performance by combining global and local search strength in one entity within RKCS via Lévy flights.

Table 3: Comparison of experimental results of RKCS with all algorithms cited in [4]

Instances	eil51	st70	rd100	pr124	rat195	Average
Best	426	675	7910	59030	2323	
GA	2.58%	2.35%	5.27%	2.74%	6.68%	3.92%
ACO	1.08%	1.98%	3.14%	1.23%	2.59%	2.00%
PSO	1.12%	2.32%	2.65%	1.98%	3.45%	2.30%
AIS	1.22%	1.79%	2.03%	1.45%	2.77%	1.85%
IWDA	4.08%	5.20%	4.97%	6.12%	5.34%	5.14%
BCO	2.19%	3.01%	2.44%	2.78%	3.43%	2.77%
EM	2.67%	3.05%	2.78%	3.45%	5.45%	3.48%
Hr-GSA	0.54%	0.34%	1.12%	1.05%	2.56%	1.12%
RKCS	0.0%	0.0%	0.0%	0.0%	0.38	0.07

In Table 3, the ‘Best’ denotes the best known-so-far optimal solution quality, and the other results recorded were the ratio of the solutions found by each algorithm to the optimal solution over 30 independent runs.

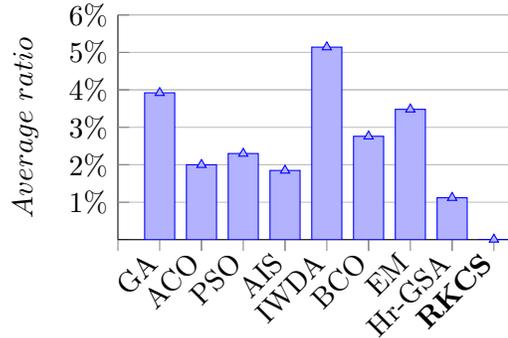


Figure 6: Average ratio of the solutions found to the optimal solution, for instances eil51, st70, rd100, pr124 and rat195

7 Conclusion

In this proposed approach, we used the random-key encoding scheme combined with the cuckoo search to develop the random-key cuckoo search (RKCS) algorithm for solving TSP. By testing the standard TSP instances’ library, we can confirm that the proposed approach is very efficient and can obtain good results.

Random keys have been used to switch between the continuous and the combinatorial search space, which enable cuckoo search to provide a good search mechanism with a fine balance between intensification and diversification.

However, RKCS can be altered or improved by many ways. For example, it can be useful to adapt the agent moves with some existing or new move operators in the combinatorial space. In addition, the tuning of algorithm-dependent parameters can be also fruitful [28], which may provide further research topics to see if the proposed approach can be further

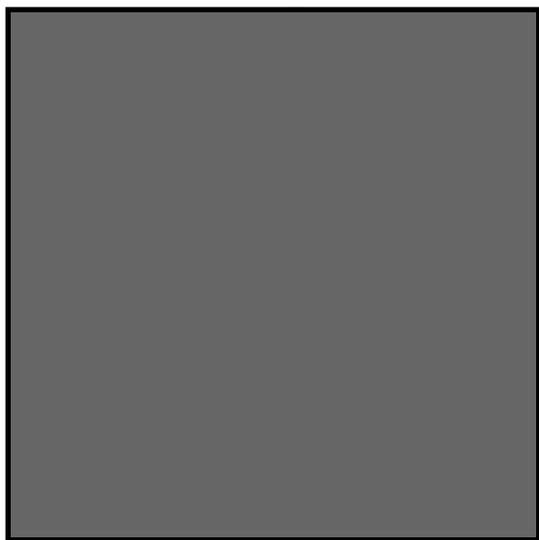
improved.

It can be expected that our approach can be used to solve other combinatorial optimization problems such as routing, scheduling and even mixed-integer programming problems. We can also generalize this work to solve some kinds of TSP problems such as Asymmetric [5], Spherical [15] and generalized [21] TSP.

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key:

0.8	0.5	0.2
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6	4	5
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