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Some connections between BCK-algebras and $n-$ary block codes

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Abstract. In the last time some papers were devoted to the study of the connections between binary block codes and BCK-algebras. In this paper, we try to generalize these results to n-ary block codes, providing an algorithm which allows us to construct a BCK-algebra from a given n-ary block code.

Keywords: BCK-algebras; n-ary block codes.

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0. Introduction

Y. Imai and K. Iseki introduced BCK-algebras in 1966, through the paper [Im, Is; 66], as a generalization of the concept of set-theoretic difference and propositional calculi. This class of BCK-algebras is a proper subclass of the class of BCI-algebras and has many applications to various domains of mathematics.

One of the recent applications of BCK-algebras was given in the Coding Theory. In the paper [Ju,So; 11], the authors constructed a finite binary block-codes associated to a finite BCK-algebra. In [Fl; 15], the author proved that, in some circumstances, the converse of the above statement is also true and in the paper [B,F; 15] the authors proved that binary block codes are an important tool in providing orders with which we can build algebras with some asked properties. For other details regarding BCK-algebras, the reader is referred to [Is, Ta; 78].

In general, the alphabet on which are defined block codes are not binary. It is used an alphabet with n elements, $n \geq 2$, identified usually with the set $A_n = \{0, 1, 2, ..., n-1\}$. These codes are called $n-$ary block codes. In the present paper, we will generalize this construction of binary block codes to $n-$ary block codes. For this purpose, we will prove that to each $n-$ary block code V we can associate a BCK-algebra X such that the $n-$ary block-code generated by X, V_X, contains the code V as a subset.

1. Preliminaries

Definition 1.1. An algebra $(X,*,\theta)$ of type $(2,0)$ is called a *BCI-algebra* if the following conditions are fulfilled:
1) $((x*y)*(x*z))*(z*y)=\theta$, for all $x,y,z\in X$;
2) $(x*(x*y))*y=\theta$, for all $x,y\in X$;
3) $x*x=\theta$, for all $x\in X$;
4) For all $x,y,z\in X$ such that $x*y=\theta, y*x=\theta$, it results $x=y$.

If a BCI-algebra X satisfies the following identity:
5) $\theta*x=\theta,$ for all $x\in X$, then X is called a *BCK-algebra*.

A BCK-algebra X is called *commutative* if $x*(x*y)=y*(y*x)$, for all $x,y\in X$ and *implicative* if $x*(y*x)=x$, for all $x,y\in X$. A BCK-algebra $(A,*,0)$ is called *positive implicative* if and only if

$$(x*y)*z=(x*z)*(y*z), \text{ for all } x,y,z\in A.$$

The partial order relation "$\leq$" on a BCK-algebra is defined such that $x\leq y$ if and only if $x*y=\theta$.

An equivalent definition of BCK-algebra was gave in the following proposition.

Proposition 1.2. ([Me, Ju; 94], Theorem 1.6) *An algebra $(X,*,\theta)$ of type $(2,0)$ is a BCK-algebra if and only if the following conditions are satisfied:*
1) $((x*y)*(x*z))*(z*y)=\theta$, *for all* $x,y,z\in X$;
2) $x*(0*y)=x$,*for all* $x,y\in X$;
3) *For all $x,y,z\in X$ such that $x*y=\theta, y*x=\theta$, it results $x=y$.*

Let $(X,*,\theta)$ be a finite BCK-algebra with n elements and A be a finite nonempty set. A map $f:A\rightarrow X$ is called *a BCK-function.* Let $A_n=\{0,1,2,...,n-1\}$. In the following, we will consider BCK algebra X and the set A under the form: $X=\{r_0,r_1,...,r_{n-1}\}, A=\{x_0,x_1,...,x_{m-1}\}, m\leq n$. A *cut function* of f is a map $f_{r_j}:A\rightarrow A_n$, $r_j\in X$, such that $f_{r_j}(x_i)=k$ if and only if $r_j*f(x_i)=r_k$, for all $r_j,r_k\in X, x_i\in A, i,j,k\in\{0,1,2,...,n-1\}$. For each BCK-function $f:A_n\rightarrow X$, we can define an $n-$ary block-code with codewords of length m. For this purpose, we consider to each element $r\in X$ the cut function $f_r:A\rightarrow A_n, r\in X$. To each such a function, will correspond the codeword w_r, with symbols from the set A_n. We have $w_r=w_0w_1...w_{n-1}$, with $w_i=j,\ j\in A_n,$ if and only if $f_r(x_i)=j$, that means $r*f(i)=r_j$. We denote this code with V_X. In this way, we can associate to each BCK-algebra an $n-$ary block code.

Example 1.3. We consider the following BCK-algebra $(X, *, \theta)$, with the multiplication given in the following table (see [Ju,So; 11] , Example 4.2).

$*$	θ	a	b	c
θ	θ	θ	θ	θ
a	a	θ	θ	a
b	b	a	θ	b
c	c	c	c	θ

We have $X = \{\theta, a, b, c\}$ $A = A_4 = \{0, 1, 2, 3\}$.We consider $f : A \to X, f(0) = \theta, f(1) = a, f(2) = b, f(3) = c$ and $f_r : A_4 \to A_4, r \in X$, a cut function.

To $r = \theta$, corresponds the codeword $w_\theta = 0000$. For $r = a$, we obtain the codeword 1001. Indeed, $f_a(0) = 1$, since $a * f(0) = a * \theta = a = f(1)$; $f_a(a) = 0$ since $a * f(1) = a * a = \theta = f(0)$; $f_a(b) = 0$ and $a * f(2) = a * b = \theta = f(0)$; $f_a(c) = 1$, also $a * f(3) = a * c = a = f(1)$;

We wonder if and in what circumstances the converse is also true?

In the following, we will try to find answers at this question.

2. Main results

Let $A'_n = \{1, 2, ..., n-1\}$ be a finite set and $V = \{w_1, w_2, ..., w_m\}$ be $n-$ary codewords, ascending ordered after lexicographic order. We consider $w_i = w_{i1}w_{i2}...w_{iq}, w_{ij} \in A'_n$, $j \in \{1, 2, ..., q\}$, with w_{ij} descending ordered such that

$$w_{iw_{ik}} \leq k, \quad i \in \{1, 2, ..., m\}, \quad k \in \{1, 2, ..., \min\{n-1, q\}\}$$

and $w_{ij} = 1$ in the rest.

Definition 2.1. Let V be the $n-$ary codeword, defined above. To this code we associate a matrix $M = (\alpha_{st})_{s,t \in \{0,1,...,r-1\}}$, $M \in \mathcal{M}_r(A_n)$, where r is defined in the following.

Case 1. $q < n$. Let $r = n - 1 + m$. We define $\alpha_{ss} = 0$, $\alpha_{s0} = s, \alpha_{0s} = 0$, $s \in \{0, 1, 2, ..., r-1\}$. For $1 \leq s \leq n-1$, put $\alpha_{st} = 1$, if $t \leq s$, $\alpha_{st} = 0$, if $t \geq s$. For $s \geq n-1$, define $\alpha_{st} = w_{it}$, for $t \in \{1, 2, ..., q\}$ and $\alpha_{sq+j} = 1$, for $q + j < s$. We have $\alpha_{st} = 0$, for $t \geq s$.

Case 2. $q \geq n$. Let $r = m + q + 1$. We define $\alpha_{ss} = 0$, $\alpha_{s0} = s, \alpha_{0s} = 0, s \in \{0, 1, 2, ..., r-1\}$. For $1 \leq s \leq q$, define $\alpha_{st} = 1$, if $t \leq s$, $\alpha_{st} = 0$, if $t \geq s$. For $s > q$, put $\alpha_{st} = w_{it}$, for $t \in \{1, 2, ..., q\}$ and $\alpha_{sq+j} = 1$, for $q + j < s$. We have $\alpha_{st} = 0$, for $t \geq s$.

The matrix M is called *the matrix associated to the $n-$ary block code* $V = \{w_{1,}w_2, ..., w_m\}$ and is a lower triangular matrix. Example of such a matrix can be found in Section 3.

Definition 2.2. With the above notations, let $M \in \mathcal{M}_r(A_n)$ be the matrix associated to the $n-$ary block code $V = \{w_1, w_2, ..., w_m\}$ defined on A'_n and $A_r = \{0, 1, ..., r-1\}$ be a nonempty set. We define on A_r the following multiplication

$$i * j = \alpha_{ij} = w_{ij} = k.$$

Theorem 2.3. *With the above notations, we have that* $(A_r, *, 0)$ *is a BCK-algebra.*

Proof. Since conditions 2), 3) from Proposition 1.2 are satisfied using Definition 2.1, we will only prove that $((i*j)*(i*k))*(k*j) = 0$, for all $i, j, k \in \{0, 1, ..., r-1\}$.

Case 1: $\mathbf{j = 0, k \neq 0}$. We will prove that $(i*(i*k))*k = 0$. For $i = 0$ it is clear.

For $k = 0$, we obtain $(i*(i*0))*0 = (i*i)*0 = 0$.

For $k \neq 0$, $i \geq r-m, k \in \{1, 2, ..., q\}$,we have $(i*(i*k)) = w_{iw_{ik}} \leq k$, therefore $(i*(i*k))*k = 0$.

For $k \neq 0$, $i \geq r-m, k \geq q+1, i \geq k$,we have $(i*(i*k))*k = 0$, since $i*k = 1, i*1 \leq n-1 < k$.

For $i < r-m$, $k \leq q+1$, we have $(i*(i*k))*k = 0$ since $i*k = 1, i*1 = 1$ and $1*k = 0$.

For $i < r-m$, $k > q+1$, we have $(i*(i*k))*k = 0$ since $i*k = 0$, we obtain $(i*0)*k = i*k = 0$.

Case 2: $\mathbf{k = 0, j \neq 0}$. We will prove that $(i*j)*i = 0$. We always have that $i*j \leq i$, therefore $(i*j)*i = 0$.

Case 3: $\mathbf{k \neq 0, j \neq 0}$. We will prove that $((i*j)*(i*k))*(k*j) = 0$. For $i = 0$, it is clear. We suppose that $i \neq 0$.

For $i \geq r-m$ and $j, k < r-m, j < k$. We have $n-1 \geq (i*j) \geq (i*k)$, therefore $((i*j)*(i*k)) = 1$. We also obtain $k*j = 1$, therefore $((i*j)*(i*k))*(k*j) = 1*1 = 0$.

For $i \geq r-m$ and $j, k < r-m, k < j$. We have $n-1 \geq (i*j) \leq (i*k)$, therefore $((i*j)*(i*k)) = 0$. It results that $((i*j)*(i*k))*(k*j) = 0$.

For $i \geq r-m$ and $j, k \geq r-m, j < k$. We can have $i*j = 1$ and $i*k = 1$,therefore $(i*j)*(i*k) = 0$. We can also have $i*j = 1, i*k = 0$ and $k*j = 1$, since $j < k$. It results that $((i*j)*(i*k))*(k*j) = (1*0)*1 = 1*1 = 0$. Or, we can have $i*j = 0, i*k = 0$, therefore the asked relation is zero.

For $i \geq r-m$ and $j, k \geq r-m, k < j$. We can have $i*j = 1$ and $i*k = 1$,therefore $(i*j)*(i*k) = 0$. Or, we can have $i*k = 1, i*j = 0$ and $k*j = 0$, therefore we obtain zero. We also can have $i*j = 0, i*k = 0$, therefore the asked relation is zero.

For $i \geq r-m$ and $k < r-m < j$. We can have $i*j = 0$, therefore the asked relation is zero. We can have $i*j = 1$. It results $((i*j)*(i*k))*(k*j) = (1*(i*k))*0 = 1*\beta = 0$, since $k < j$ and $\beta \geq 0$.

For $i \geq r-m$ and $j < r-m < k$. We have $i*j = 1$.If $i*k = 1$, we obtain zero. If $i*k = 0$, it results $((i*j)*(i*k))*(k*j) = (1*0)*(k*j) = 1*(k*j) = 0$, since $k*j \geq 1$.

For $i < r-m$ and $j,k < r-m, j < k$. We have $i*j = 1, i*k = 1$, therefore we obtain zero.

For $i < r-m$ and $j,k < r-m, k < j$. We can have $((i*j)*(i*k))*(k*j) = (1*1)*0 = 0$. Or, we can have $(i*j) = 0$, therefore we obtain zero.

For $i < r-m$ and $j,k < r-m, j < n-1+\max\{q,m\}-m \leq k$. We have $i*j = 1, i*k = 0$ and $k*j = 1$. It results $((i*j)*(i*k))*(k*j) = (1*0)*1 = 1*1 = 0$.

For $i < r-m$ and $k < r-m, k < r-m \leq j$. We can have $((i*j)*(i*k))*(k*j) = (1*1)*0 = 0$. Or, we can have $(i*j) = 0$, therefore we obtain zero.

For $i < r-m$ and $j,k \geq r-m, j < k$. We have $(i*j) = 0$, therefore we obtain zero.

For $i < r-m$ and $j,k \geq r-m, j > k$. We have $(i*j) = 0$, therefore we obtain zero. $\square$

Remark 2.4.

1) BCK-algebra $(A_r, *, 0)$ obtained in Theorem 2.3 is unique up to an isomorphism.

2) From Theorem 2.3, let $(A_r, *, 0)$ be the obtained BCK-algebra, with $A_r = \{0,1,2,...r-1\}$. If $X = \{a_0 = \theta, a_1, a_2, ..., a_{r-1}\}$, with multiplication "$\circ$" given by the relation $a_i \circ a_j = a_k$ if and only if $i*j = k$, for $i, j.k \in \{0,1,2,...,r-1\}$, then $(X, \circ, \theta)$ is a BCK-algebra.

3) If we consider $A_q = \{0,1,2,...q-1\}$, the map $f : A_q \to X, f(i) = a_i$, gives us a code V_X, associated to the above BCK-algebra $(X, \circ, \theta)$, which contains the code V as a subset.

Definition 2.5. Let $(X, *, \theta)$ be a BCK-algebra, and $I \subseteq X$. We say that I is a *right-ideal* for the algebra X if $\theta \in I$ and $x \in I, y \in X$ imply $x * y \in I$. An ideal I of a BCK-algebra X is called *a closed ideal* if it is also *a subalgebra* of X (i.e. $\theta \in I$ and if $x, y \in I$ it results that $x*y \in I$).

Let V be an $n-$ary block code. From Theorem 2.3 and Remark 2.4, we can find a BCK-algebra X such that the obtained $n-$ary block-code V_X contains the $n-$ary block-code V as a subset.

Let V be a binary block code with m codewords of length q. With the above notations, let X be the associated BCK-algebra and $W = \{\theta, w_1, ..., w_r\}$ the associated $n-$ary block code which include the code V. We consider the codewords $\theta, w_1, w_2, ..., w_r$ lexicographically ordered, $\theta \geq_{lex} w_1 \geq_{lex} w_2 \geq_{lex} ... \geq_{lex} w_r$. Let $M \in \mathcal{M}_r(A_n)$ be the associated matrix with the rows $\theta, w_1, ..., w_r$, in this order. Let L_{w_i} and C_{w_j} be the lines and columns in the matrix M. We consider the sub-matrix M' of the matrix M with the rows $L_{w_1}, ..., L_{w_m}$ and the columns $C_{w_{m+1}}, ..., C_{w_{m+q}}$, which is the matrix associated to the code C.

Proposition 2.6. *With the above notations, we have that* $\{\theta, w_1, w_{r-m}, w_{r-m+1}, ..., w_r\}$ *determines a closed right ideal in the algebra* X.

Proof. Let $Y = \{\theta, w_1, w_{r-m}, w_{r-m+1}, ..., w_r\}$. We will prove that $y \in Y, x \in X$ imply $y * x \in Y$. From the definition of the multiplication in the algebra X, we have that $y * x \in \{\theta, w_1\}$. In the same time, if $x, y \in Y$, it results that $x * y \in Y$, since $y * x \in \{\theta, w_1\}$.

3. Examples

Example 3.1. Consider $A_7 = \{0,1,2,3,4,5,6\}$, $n = 7$, $q = 4$, $m = 3$, $r = 9$, $V = \{w_1, w_2, w_3\}$, with $w_1 = 3211, w_2 = 4221, w_3 = 4321$. The matrix M associated to the $n-$ary code V, is

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 6 & \mathbf{3} & \mathbf{2} & \mathbf{1} & \mathbf{1} & 1 & 0 & 0 & 0 \\ 7 & \mathbf{4} & \mathbf{2} & \mathbf{2} & \mathbf{1} & 1 & 1 & 0 & 0 \\ 8 & \mathbf{4} & \mathbf{3} & \mathbf{2} & \mathbf{1} & 1 & 1 & 1 & 0 \end{pmatrix}$$

and the corresponded BCK-algebra, $(X, *, \theta)$, where
$X = \{a_0 = \theta, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$,
with the following multiplication table

$*$	θ	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
θ	θ	θ	θ	θ	θ	θ	θ	θ	θ
a_1	a_1	θ	θ	θ	θ	θ	θ	θ	θ
a_2	a_2	a_1	θ	θ	θ	θ	θ	θ	θ
a_3	a_3	a_1	a_1	θ	θ	θ	θ	θ	θ
a_4	a_4	a_1	a_1	a_1	θ	θ	θ	θ	θ
a_5	a_5	a_1	a_1	a_1	a_1	θ	θ	θ	θ
a_6	a_6	$\mathbf{a}_3$	$\mathbf{a}_2$	$\mathbf{a}_1$	$\mathbf{a}_1$	a_1	θ	θ	θ
a_7	a_7	$\mathbf{a}_4$	$\mathbf{a}_2$	$\mathbf{a}_2$	$\mathbf{a}_1$	a_1	a_1	θ	θ
a_8	a_8	$\mathbf{a}_4$	$\mathbf{a}_3$	$\mathbf{a}_2$	$\mathbf{a}_1$	a_1	a_1	a_1	θ

If we consider $A = \{1,2,3,4\}$. The map $f : A \to X$, $f(1) = a_1, f(2) = a_2, f(3) = a_3, f(4) = a_4$ gives us the following block code
$V' = \{0000, 1000, 1100, 1110, 1111, \mathbf{3211}, \mathbf{4221}, \mathbf{4321}\}$, which contains V as a subset.

We remark that this algebra is not commutative since $a_7 * (a_7 * a_6) = a_7 * a_1 = a_4$ and $a_6 * (a_6 * a_7) = a_6 * \theta = a_6$. This algebra is not implicative since $a_6 * (a_7 * a_6) = a_6 * a_1 = a_3 \neq a_6$. This algebra is not positive implicative since $(x * y) * z \neq (x * z) * (y * z)$. Indeed, $(a_7 * a_6) * a_3 = a_1 * a_3 = \theta \neq (a_7 * a_3) * (a_6 * a_3) = a_2 * a_1 = a_1$.

Example 3.2. Let $A_4 = \{0,1,2,3\}$, $n = 4$, $q = 5$, $m = 3$, $r = 9$, $V = \{w_1, w_2, w_3\}$, with $w_1 = 21111, w_2 = 32111, w_3 = 33111$. We obtain the matrix M associated to the $n-$ary code V,

$$M=\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0\\
5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0\\
6 & \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 & 0 & 0\\
7 & \mathbf{3} & \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 1 & 0 & 0\\
8 & \mathbf{3} & \mathbf{3} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 1 & 1 & 0
\end{pmatrix}$$

and the corresponded BCK-algebra, $(X,*,\theta)$, where
$X=\{a_0=\theta,a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8\}$,
with the following multiplication table

$*$	θ	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
θ	θ	θ	θ	θ	θ	θ	θ	θ	θ
a_1	a_1	θ	θ	θ	θ	θ	θ	θ	θ
a_2	a_2	a_1	θ	θ	θ	θ	θ	θ	θ
a_3	a_3	a_1	a_1	θ	θ	θ	θ	θ	θ
a_4	a_4	a_1	a_1	a_1	θ	θ	θ	θ	θ
a_5	a_5	a_1	a_1	a_1	a_1	θ	θ	θ	θ
a_6	a_6	$\mathbf{a}_2$	$\mathbf{a}_1$	$\mathbf{a}_1$	$\mathbf{a}_1$	$\mathbf{a}_1$	θ	θ	θ
a_7	a_7	$\mathbf{a}_3$	$\mathbf{a}_2$	$\mathbf{a}_2$	$\mathbf{a}_1$	$\mathbf{a}_1$	a_1	θ	θ
a_8	a_8	$\mathbf{a}_3$	$\mathbf{a}_3$	$\mathbf{a}_1$	$\mathbf{a}_1$	$\mathbf{a}_1$	a_1	a_1	θ

If we consider $A=\{1,2,3,4,5\}$. The map $f:A\to X$, $f(1)=a_1, f(2)=a_2, f(3)=a_3, f(a_4)=4, f(a_5)=5$, gives us the following block code $V_X=\{00000, 10000, 11000, 11100, 11110, \mathbf{21111}, \mathbf{32211}, \mathbf{33111}\}$, which contains V as a subset.

Example 3.3. We consider $A_4=\{0,1,2,3\}$, $n=4$, $q=5$, $m=5$, $r=11$, $V=\{w_1,w_2,w_3,w_4,w_5\}$, with $w_1=11111, w_2=21111, w_3=31111, w_4=32111, w_5=33111$. We obtain the matrix M associated to the $n-$ary code V,

$$M=\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 & 0 & 0 & 0 & 0\\
7 & \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 1 & 0 & 0 & 0 & 0\\
8 & \mathbf{3} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 1 & 1 & 0 & 0 & 0\\
9 & \mathbf{3} & \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 1 & 1 & 1 & 0 & 0\\
10 & \mathbf{3} & \mathbf{3} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 1 & 1 & 1 & 1 & 0
\end{pmatrix}$$

and the corresponded BCK-algebra, $(X,*,\theta)$, where
$X=\{a_0=\theta,a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10}\}$,
with the following multiplication table

$*$	θ	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
θ	θ	θ	θ	θ	θ	θ	θ	θ	θ	θ	θ
a_1	a_1	θ	θ	θ	θ	θ	θ	θ	θ	θ	θ
a_2	a_2	a_1	θ	θ	θ	θ	θ	θ	θ	θ	θ
a_3	a_3	a_1	a_1	θ	θ	θ	θ	θ	θ	θ	θ
a_4	a_4	a_1	a_1	a_1	θ	θ	θ	θ	θ	θ	θ
a_5	a_5	a_1	a_1	a_1	a_1	θ	θ	θ	θ	θ	θ
a_6	a_6	$\mathbf{a}_1$	$\mathbf{a}_1$	$\mathbf{a}_1$	$\mathbf{a}_1$	$\mathbf{a}_1$	θ	θ	θ	θ	θ
a_7	a_7	$\mathbf{a}_2$	$\mathbf{a}_1$	$\mathbf{a}_1$	$\mathbf{a}_1$	$\mathbf{a}_1$	a_1	θ	θ	θ	θ
a_8	a_8	$\mathbf{a}_3$	$\mathbf{a}_1$	$\mathbf{a}_1$	$\mathbf{a}_1$	$\mathbf{a}_1$	a_1	a_1	θ	θ	θ
a_9	a_9	$\mathbf{a}_3$	$\mathbf{a}_2$	$\mathbf{a}_1$	$\mathbf{a}_1$	$\mathbf{a}_1$	a_1	a_1	a_1	θ	θ
a_{10}	a_{10}	$\mathbf{a}_3$	$\mathbf{a}_3$	$\mathbf{a}_1$	$\mathbf{a}_1$	$\mathbf{a}_1$	a_1	a_1	a_1	a_1	θ

If we consider $A = \{1, 2, 3, 4, 5\}$. The map $f : A \to X$, $f(1) = a_1, f(2) = a_2, f(3) = a_3, f(a_4) = 4, f(a_5) = 5$, gives us the following block code $V' = \{00000, 10000, 11000, 11100, 11110, \mathbf{11111}, \mathbf{21111}, \mathbf{31111}, \mathbf{32111}, \mathbf{33111}\}$, which contains V as a subset.

Conclusions. In this paper, we proved that to each $n-$ary block code V we can associate a BCK-algebra X such that the $n-$ary block-code generated by X, V_X, contains the code V as a subset. This algebra is unique up to an isomorphism and X is not commutative, not implicative and not positive implicative BCK-algebra.

As a further research will be very interesting to study properties of the above constructed codes and how these codes in connections with their associated BCK-algebras.

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