Effectiveness Evaluation Method of Constellation Satellite Communication System with Acceptable Consistency and Consensus Under Probability Hesitant Intuitionistic Fuzzy Preference Relationship

Yanjun Wang (yjwang199378@hotmail.com)
Hefei University of Technology https://orcid.org/0000-0003-3702-5693
Xiaoxuan Hu
Hefei University of Technology
Lidong Wang
Dalian Maritime University

Research Article

Keywords: Constellation satellite communication system, Effectiveness evaluation, Probabilistic hesitant intuitionistic fuzzy preference relation, Approximate consistency, Group consensus

Posted Date: January 10th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1016226/v1

Noname manuscript No.
(will be inserted by the editor)

Effectiveness evaluation method of constellation satellite communication system with acceptable consistency and consensus under probability hesitant intuitionistic fuzzy preference relationship

Yanjun Wang[*,1,2] · **Xiaoxuan Hu**[1,2] · **Lidong Wang** [3]

Received: date / Accepted: date

Abstract System effectiveness evaluation is an important part of constellation satellite communication system research, with applications in project verification and optimization as well as tactical and technical measurement argumentation. This paper presents a systematic and comprehensive effectiveness evaluation method for a constellation satellite communication system under a probabilistic hesitant intuitionistic fuzzy preference relationship (PHIFPR), aiming to better address the fuzziness and uncertainty in effectiveness evaluation. First, a proposed definition of PHIFPR describes the hesitancy of evaluators, provides hesitancy distribution information, and depicts the worst negative information and risk preferences in effectiveness evaluation. Then, we deduce the approximate consistency index of PHIFPR and establish a mathematical programming model to increase individual consistency when the approximate consistency index does not reach a predetermined level. In the sequel, a proposed group consensus index uses the PHIFPR-based Hausdorff distance to measure the closeness between evaluators' judgements. Afterwards, a consistency and consensus improvement model is designed to retain the original opinions of evaluators to make the consistency and consensus of PHIFPRs acceptable. Moreover, a goal programming model is established to gain the reliable scheme priority weights by regarding the approximate consistency condition of a PHIFPR as a fuzzy constraint. Finally, an experimental example is offered to highlight the practicability and feasibility of the proposed method, and some comparative analyses with other methods offer insights into the designed method.

Keywords Constellation satellite communication system · Effectiveness evaluation · Probabilistic hesitant intuitionistic fuzzy preference relation · Approximate consistency · Group consensus

1 Introduction

Terrestrial cellular wireless networks have developed rapidly, making it easier for people to exchange information. However, due to geographical factors, about 98% of the earth's surface cannot be effectively covered. Therefore, no structure exists to transfer information within these regions. Fortunately, the constellation satellite communication system can effectively solve this problem, enabling seamless coverage

[*]Yanjun Wang
yjwang199378@hotmail.com
[1] School of Management, Hefei University of Technology, Hefei 230009, People's Republic of China
[2] Key Laboratory of Process Optimization and Intelligent Decision Making, Ministry of Education, Hefei 230009, People's Republic of China
[3] Department of Mathematics, Dalian Maritime University, Dalian 116026, People's Republic of China

Abbreviation	
GDM	group decision making
AHP	analytic hierarchy process
PHIFPR	probability hesitant intuitionistic fuzzy preference relationship
IFPR	intuitionistic fuzzy preference relationship
ILV	intuitionistic fuzzy value
HIFS	hesitant intuitionistic fuzzy set
HIFN	hesitant intuitionistic fuzzy number
PHFPR	probabilistic hesitant fuzzy preference relationship
PHFE	probabilistic hesitant fuzzy element
PHIFN	probabilistic hesitant intuitionistic fuzzy number
HFE	hesitant fuzzy element
FPR	fuzzy preference relationship
GA	genetic algorithm

of the earth's surface, anytime and anywhere, to complement the terrestrial wireless network. The constellation satellite communication system is characterized by complex technology, a long development period, and huge investment, involving a large number of indicator parameter configurations, the constellation design scheme strengths and weaknesses are directly related to mission application performance. If a designed scheme is wrongly chosen or cannot meet the application requirements, it will cause huge losses, and can even affect military information system construction. Effectiveness evaluation can fully demonstrate the extent to which constellation satellite designed schemes meet application demand, and determine the efficiency of schemes to provide an important basis for their selection. At the same time, the pros and cons of designed schemes can be found, and decision-making information can be obtained so as to improve them and formulate schemes.

In summary, a valid and viable evaluation is a necessary prerequisite to select a scheme and guide its design, and a scientific method is needed for obtaining an accurate evaluation. In the evaluation, multiple evaluators provide their judgements on a finite set of constellation satellite designed schemes and integrate their judgements into collective intelligence aiming to find a common solution, thus, it can be treated as a GDM, which is a powerful tool to address complex evaluation problems [7, 20]. In the past years, several scholars have conducted in-depth research on effectiveness evaluation methods based on the GDM theory to support the practical requirements of developing satellite communication systems. For example, Qin et al. [26] proposed to evaluate space information applications based on the AHP, which can provide a reference for researching effectiveness evaluation. Zhang et al. [50] aimed to evaluate military satellite communication systems using a dual hesitancy fuzzy value and index weights calculated by geometric means and least-squares. Liu et al. [19] combined the unilateral chain and fuzzy evaluation methods to improve the validity and accuracy of an evaluation and provide a reference for effectiveness evaluation of satellite communication systems. To evaluate communication interference in a geostationary satellite orbit system layout, Dong et al. [4] modeled interference estimation and analytical calculation under different scenarios and proposed an evaluation method based on the interference function extremum, which has certain reference value for communication interference evaluation and avoidance measure formulation. Based on the above literature review, most of the existing evaluation methods use a single precise value to model the uncertainty arising from human cognitive behavior. In the actual evaluation process, the satellite communication's related material and evaluators' knowledge are two main information sources for evaluation, the single precise value is often too restrictive and difficult for evaluators to precisely express their individual preferences due to the inherent ambiguity and the complexity of the evaluation environment. Therefore, it is necessary to develop a powerful and effective method aiming to provide a more logical, easier to express, and more informative way of expressing information.

The evaluation process of the constellation satellite communication system usually requires evaluators to compare and rank the schemes. Preference relation [1,6,23] is an effective tool to describe this evaluation problem, in which evaluator can express his or her individual preferences by comparing each pair of constellation satellite designed schemes. Up to now, much attention has been paid to the study of its theory and application [15,21,34]. Since traditional preference relations require evaluators to provide accurate values [30], however, the complexity of the environment and problems, as well as differences in thinking and knowledge, restrict the application of preference relations. To address this problem, scholars introduced Zadeh's [47] fuzzy sets to preference relation to constructing fuzzy preference relations and generalized it in the evaluation situation. Furthermore, some extended forms of fuzzy preference relations have been successively proposed to allow evaluators to comprehensively and intuitively describe their individual preferences information from multiple dimensions, including interval-valued fuzzy preference relationship [2], triangular fuzzy preference relationship [37], incomplete interval fuzzy preference relationship [38,41], hesitant fuzzy preference relationship [55], and incomplete hesitant fuzzy preference relationship [12].

In the above-mentioned fuzzy preference relationships, the basic element of fuzzy preference relationships only provides the membership degree, which represents that a designed scheme is preferred over another, but ignores the corresponding non-membership degree. In addition, the hesitant fuzzy preference relationship can describe the membership degree of an element by a finite set of all possible values, but it overlooks considering the occurrence possibilities of every uncertain element. To deal with these issues, the IFPR was introduced by Xu [39], which makes the results of the preference process exhibit the characteristic of affirmation, negation, and hesitation by simultaneously considering the membership degree and non-membership degrees. In the real evaluation, evaluators cannot give explicit preferences over the scheme due to the complexity of the evaluation problem, it is suitable to make evaluators express their evaluation information from three aspects: "preferred", "not preferred" and "indeterminate", aiming to improve the accuracy of expressing a preference for the scheme. At present, many scholars devoted IFPR to group evaluation [5,22,42,46]. And the PHFE was first proposed by Zhu [54], as an extension of the hesitant fuzzy element [33], which describes not only the hesitancy of evaluators when making the evaluation, but also the hesitancy distribution information. Because the PHFE is an important extension of the HFE, it is natural to develop the PHFPR and apply it to the evaluation process [14,17,18]. To more fully simulate human perception and cognition, this study synthesizes the advantages of the above two preference relationships by cultivating a more general fuzzy preference relationship, called a PHIFPR, which is more convenient and flexible than current fuzzy preference relationships and can more accurately reflect the evaluators' individual preferences by considering three aspects of the judgement and the occurrence probabilities of the uncertain elements.

In addition, the consistency index of preference relationship plays a crucial part in effectiveness evaluation, as it is concerned with the reasonableness of the result, which can be misleading if the result does not meet the consistency requirement. However, it is difficult to obtain a fully consistent preference relationship, in this circumstances, evaluators can strive for seeking an admissible consistent preference relationship to guarantee the rationality of evaluation result. With the growing complexity of the effectiveness evaluation problem, requiring an evaluator to consider all aspects of an evaluation is becoming impossible, therefore, there is always more than one evaluator making an evaluation. Hence, group consensus is another critical issue that cannot be ignored in the process of effectiveness evaluation, since group consensus is the inevitable course for multiple evaluators with different views necessary to reach a consistent decision. To solve the effectiveness evaluation problem of constellation satellite communication systems based on individual consistency and group consensus involves three processes [16,44,49]: (1) checking the individual consistency, i.e., when the consistency level has not yet been achieved, individual consistency must be improved; (2) judging the group consensus among evaluators, i.e., if it cannot satisfy the requirement, group consensus must be improved so as to select the scheme with the highest satisfaction as much as possible; and (3) determining the priority weight of scheme based on evaluation information that meets individual consistency, and the ranking order of schemes can be generated by the priority weight of the scheme. The improvements of individual consistency and group consensus are most important among the above three processes, which is a challenging task for many researchers. To date, a large number of scholars have conducted fruitful research on the topic of evaluation problems based on individual consistency and group consensus [10,11,13,31,32,35,36,48]. Generally, the individual consistency and group consensus improvement processes usually involve either mathematical programming modeling or iterative algorithms, and the former can save time and can be more practical in an emergency evaluation. From this point of view, we build a mathematical programming model to improve individual consistency and group consensus and shorten evaluation time in this paper.

The effectiveness evaluation of constellation satellite communication systems is influenced by many criteria, including coverage, quality of service, adaptability, interrupt performance, and protection performance. The description of some criteria may be uncertain and difficult to quantify, which directly intensifies the uncertainty and hesitation of the evaluation team in the evaluation process. Therefore, it is necessary to determine the optimal constellation satellite designed scheme by selecting an appropriate preference relationship description and establishing a suitable evaluation method. This paper focuses on an effectiveness evaluation method with acceptable consistency and consensus under PHIFPR, which can provide technical support for the development of the constellation satellite communication system. The main novelties and contributions of the proposed method are highlighted as follows.

1. We define PHIFPR, which expresses evaluators' views, includes the hesitancy of evaluators and hesitancy distribution information, and depicts the worst negative evaluation information and the evaluator's risk preference, which can be used to describe the evaluation information as completely as possible to avoid substantial information loss.
2. A Hausdorff distance formula, which does not require normalization, is designed to calculate the distance between two PHIFNs. The Hausdorff distance can effectively determine the distance between any two PHIFNs with different numbers of elements. It is not influenced by the subjective preferences of evaluators.
3. We define an approximate consistency matrix and appropriate consistency index to measure the evaluation information's rationality given by evaluators, and define a group consensus index to measure the closeness between evaluators' judgements.
4. Several programming models are formulated to revise PHIFPRs with unacceptable approximate consistency and group consensus, and to determine scheme priority weights according to the revised PHIFPRs. These models have a uniform mathematical structure to limit the amount of calculation.

The rest of this article is organized as follows. Section 2 reviews basic concepts of the IFPR, HIFS, and PHFPR. The PHIFPR and Hausdorff distance between any two probability hesitant intuitionistic fuzzy elements are defined in section 3. In section 4, the approximate consistency matrix, approximate consistency index, and group consensus index of PHIFPRs are defined, and some programming models are designed to improve the individual consistency and group consensus among evaluators. In section 5, we develop a modeling method to derive priority weights of schemes based on repaired PHIFPRs. Section 6 introduces a method to address the effectiveness evaluation problem of constellation satellite communication system with acceptable consistency and consensus under PHIFPRs. Section 7 describes an experiment to verify the practicality of the proposed method by ranking the constellation satellite designed schemes, and some comparisons are made to current evaluation methods. Some conclusions are presented in section 8.

2 Preliminaries

This section recalls several basic concepts, such as the IFPR, HIFS, and PHFPR.

Definition 1 [29] Let $Y = \{y_1, y_2, \ldots, y_n\}$ be a fixed nonempty universe set, the IFPR is denoted by a matrix $\tilde{A} = (\tilde{a_{ij}})_{n\times n}$, where $\tilde{a_{ij}} = \langle (y_i, y_j), \mu(y_i, y_j), \vartheta(y_i, y_j)\rangle$ $(i, j = 1, 2, \ldots, n)$. For convenience, let $\tilde{a_{ij}} = \langle \mu_{ij}, \vartheta_{ij}\rangle$, where μ_{ij} is the degree to which object y_i is preferred to object y_j, and ϑ_{ij} is the degree to which y_i is not preferred to y_j, with conditions

$$\mu_{ij},\ \vartheta_{ij} \in [0, 1],\ \mu_{ij} + \vartheta_{ij} \le 1,\ \mu_{ij} = \vartheta_{ji},\ \mu_{ii} = \vartheta_{ii} = 0.5,\ \forall i, j = 1, 2, \ldots, n.$$

The elements of IFPR are ILVs, which consider both the membership degree μ_{ij} and non-membership degree ϑ_{ij}. Evaluators are often too restrictive and provide precise (crisp) evaluation opinions with difficulty for some schemes due to incompleteness or lack of evaluation information. Zhou et al. [53] proposed the hesitant intuitionistic fuzzy set to overcome this drawback.

Definition 2 [53] Let $Y = \{y_1, y_2, \ldots, y_n\}$ be a fixed set, the HIFS h on Y is

$$h_i = \{y_i, \langle \mu_{h_i}(y_i), \vartheta_{h_i}(y_i)\rangle | y_i \in Y\},$$

where $\mu_{h_i}(y_i)$ and $\vartheta_{h_i}(y_i)$ are respectively the membership and non-membership degrees of the element y_i to h_i, $\mu_{h_i}(y_i)$ is a hesitant fuzzy number with $\mu_{h_i}(y_i) = \{r^1_{h_i}(y_i), r^2_{h_i}(y_i), \ldots, r^{L_{h_i}}_{h_i}(y_i)\}$, $r^l_{h_i}(y_i) \subseteq [0, 1]$ $(l = 1, 2, \ldots, L_{h_i})$, and $\vartheta_{h_i}(y_i) = 1 - \max\{\mu_{h_i}(y_i)\}$ represents the maximum risk and the worst negative information on the basis of the greatest membership degree.

Definition 3 [53] Let $h_1 = \langle \mu_1, 1-\max\{\mu_1\}\rangle$ with $\mu_1 = \{r^l_1 | l = 1, 2, \ldots, L_{h_1}\}$ and $h_2 = \langle \mu_2, 1-\max\{\mu_2\}\rangle$ with $\mu_2 = \{r^t_2 | t = 1, 2, \ldots, L_{h_2}\}$ be two HIFNs. Their basic operations are as follows:

1. $h_1 \oplus h_2 = \left\langle \cup_{r^l_1 \in \mu_1,\ r^t_2 \in \mu_2} \{r^l_1 + r^t_2 - r^l_1 r^t_2\}, ((1 - \max\{\mu_1\})(1 - \max\{\mu_2\}))\right\rangle.$
2. $h_1 \otimes h_2 = \left\langle \cup_{r^l_1 \in \mu_1,\ r^t_2 \in \mu_2} \{r^l_1 r^t_2\}, (1 - \max\{\mu_1\} \max\{\mu_2\})\right\rangle.$
3. $\lambda h_1 = \left\langle \cup_{r^l_1 \in \mu_1} \{1 - (1 - r^l_1)^\lambda\}, (1 - \max\{\mu_1\})^\lambda \right\rangle,\ (\lambda > 0).$
4. $h^\lambda_1 = \left\langle \cup_{r^l_1 \in \mu_1} \{(r^l_1)^\lambda\}, 1 - (\max\{\mu_1\})^\lambda \right\rangle,\ (\lambda > 0).$

To make a final choice based on the results of aggregation requires one to rank the HIFNs. Zhou et al. [53] introduced the score and accuracy functions to compare HIFNs.

Definition 4 [53] Let $h = \langle \mu, 1 - \max\{\mu\}\rangle$ be a HIFN with $\mu = \{r^l | l = 1, 2, \ldots, L_h\}$. Its score function is

$$SC(h) = \sum_{r^l \in \mu} \frac{r^l}{L_h} - 1 + max\{\mu\}.$$

Definition 5 [53] Let $h = (\mu, 1 - \max\{\mu\})$ be a HIFN with $\mu = \{r^l | l = 1, 2, \ldots, L_h\}$. Its accuracy function is

$$AC(h) = 1 + \sum_{r^l \in \mu} \frac{r^l}{L_h} - \max\{\mu\}.$$

Definition 6 [53] Let $h_1 = \langle \mu_1, 1-\max\{\mu_1\}\rangle$ and $h_2 = \langle \mu_2, 1-\max\{\mu_2\}\rangle$ be two HIFNs. $SC(h_i)$ $(i = 1,2)$ and $AC(h_i)$ $(i = 1,2)$ are score and accuracy functions, respectively, with the following characteristics:

$$\begin{aligned}
&(1)\ \text{If } SC(h_1) > SC(h_2),\ \text{then } h_1 > h_2,\\
&(2)\ \text{If } SC(h_1) < SC(h_2),\ \text{then } h_1 < h_2,\\
&(3)\ \text{When } SC(h_1) = SC(h_2),\\
&\quad (a)\ \text{if } AC(h_1) < AC(h_2),\ \text{then } h_1 < h_2,\\
&\quad (b)\ \text{if } AC(h_1) > AC(h_2),\ \text{then } h_1 > h_2,\\
&\quad (c)\ \text{if } AC(h_1) = AC(h_2),\ \text{then } h_1 = h_2.
\end{aligned}$$

μ_1 in $h_1 = \langle \mu_1, 1-\max\{\mu_1\}\rangle$ is a hesitant fuzzy number, as proposed by Torra [33], which can model the uncertainty by the hesitancy of evaluators to act on the evaluation. But the elements in the hesitant fuzzy number occur with equal probabilities, which is obviously problematic. To address this, Xu and Zhou [45] defined the PHFE, and then the PHFPR was developed by Li and Wang [13] to realize pairwise comparisons.

Definition 7 [13] Let $Y = \{y_1, y_2, \ldots, y_n\}$ be a fixed set. A PHFPR is denoted by a matrix $\tilde{H} = (\tilde{h_{ij}}(p_{ij}))_{n\times n}$, where $\tilde{h_{ij}}(p_{ij}) = \{r_{ij}^l(p_{ij}^l) | l = 1,2,\ldots,L_{\tilde{h_{ij}}}\}$ is a PHFE, indicating all possible degrees of y_i preferred to y_j, with conditions:

1. $r_{ij}^l + r_{ji}^{L_{\tilde{h_{ij}}}-l+1} = 1$, $p_{ij}^l = p_{ji}^{L_{\tilde{h_{ij}}}-l+1}$,
2. $r_{ii} = 0.5$, $p_{ii} = 1$,
3. $r_{ij}^l \le r_{ij}^{l+1}$, $(i,j = 1,2,\ldots,n)$,

where r_{ij}^l is the lth possible value in $\tilde{h_{ij}}$, and p_{ij}^l is the corresponding probability of r_{ij}^l.

3 Probabilistic hesitant intuitionistic fuzzy preference relationship

The IFPR can describe the negative information in effectiveness evaluation, and the PHFPR considers hesitancy distribution information given by evaluators. The PHIFPR synthesizes these two preference relationships to more precisely describe evaluators' preference information. In this paper, we introduce a simplified PHIFPR based on the definition of the HIFS [53] to avoid providing non-membership information in the complex evaluation process. In the simplified form, the non-membership degree is obtained from the maximum membership degree, which denotes that the non-membership degree will be an extreme value.

Definition 8 If $Y = \{y_1, y_2, \ldots, y_n\}$ is a fixed set, then the PHIFPR is defined by a matrix $\tilde{Q} = (\tilde{q_{ij}})_{n\times n}$, where $\tilde{q_{ij}} = \langle (y_i, y_j), r(y_i, y_j)p(y_i, y_j), 1-\max\{r(y_i, y_j)\}\rangle$ $(i,j = 1,2,\ldots,n)$. For convenience, let $\tilde{q_{ij}} = \langle r_{ij}(p_{ij}), \vartheta_{ij}\rangle$, where $r_{ij}(p_{ij}) = \{r_{ij}^l(p_{ij}^l) | l = 1,2,\ldots,L_{r_{ij}}\}$ denotes the possible priority degrees to which the object y_i is preferred to the object y_j, and $\vartheta_{ij} = 1-\max\{r_{ij}\}$ is the degree to which the object y_i is not preferred to the object y_j, with conditions:

1. $r_{ij}^l,\ \vartheta_{ij} \in [0,1],\ r_{ij}^l + \vartheta_{ij} \le 1,\ \forall l = 1,2,\ldots,L_{r_{ij}}$.
2. $r_{ij}^{\sigma(l)} + r_{ji}^{\sigma(L_{r_{ji}}-l+1)} = 1$, $p_{ij}^{\sigma(l)} = p_{ji}^{\sigma(L_{r_{ji}}-l+1)}$, where $r_{ij}^{\sigma(l)}$ is the $l-$largest element in r_{ij} and $p_{ij}^{\sigma(l)}$ is its probability of occurrence.
3. $r_{ii} = \vartheta_{ii} = 0.5, p_{ii} = 1, \text{and } \tilde{q}_{ii} = \langle 0.5(1), 0.5\rangle$.
4. $p_{ij}^l \in [0,1]$ is the probability of r_{ij}^l in r_{ij}, and $\sum_{l=1}^{L_{r_{ij}}} p_{ij}^l = 1$.

From Definition 8, we find that the PHIFPR, which considers the worst negative information and represents the risk preference by the non-membership degree, is composed of PHIFNs.

Definition 9 Let $q_1 = \langle r_1(p_1), 1-\max\{r_1\}\rangle$ with $r_1(p_1) = \{r_1^l(p_1^l)|l = 1, 2, \ldots, L_{r_1}\}$ and $q_2 = \langle r_2(p_2), 1-\max\{r_2\}\rangle$ with $r_2(p_2) = \{r_2^t(p_2^t)|t = 1, 2, \ldots, L_{r_2}\}$ be two PHIFNs, where $\sum_{l=1}^{L_{r_1}} p_1^l = 1$ and $\sum_{t=1}^{L_{r_2}} p_2^t = 1$. Then the operation laws of the PHIFNs are defined as follows,

1. $q_1 \oplus q_2 = \left\langle \cup_{r_1^l\in r_1,\ r_2^t\in r_2} \left\{r_1^l + r_2^t - r_1^l r_2^t\right\} \left(p_1^l p_2^t\right), (1-\max\{r_1\})(1-\max\{r_2\})\right\rangle$.
2. $q_1 \otimes q_2 = \left\langle \cup_{r_1^l\in r_1,\ r_2^t\in r_2} \left\{r_1^l r_2^t\right\} \left(p_1^l p_2^t\right), (1-\max\{r_1\}\max\{r_2\})\right\rangle$.
3. $\lambda q_1 = \left\langle \cup_{r_1^l\in r_1} \left\{1-(1-r_1^l)^\lambda\right\} \left(p_1^l\right), (1-\max\{r_1\})^\lambda\right\rangle,\ (\lambda > 0)$.
4. $q_1^\lambda = \left\langle \cup_{r_1^l\in r_1} \left\{\left(r_1^l\right)^\lambda\right\} \left(p_1^l\right), 1-(\max\{r_1\})^\lambda\right\rangle,\ (\lambda > 0)$.

Definition 10 Let $q_1 = \langle r_1(p_1),\ 1-\max\{r_1\}\rangle$, $r_1(p_1) = \{r_1^l(p_1^l)|l = 1, 2, \ldots, L_{r_1}\}$ be a PHIFN. Its score function can be represented as

$$S(q_1) = \sum_{l=1}^{L_{r_1}} r_1^l p_1^l - 1 + \max\{r_1\}.$$

Definition 11 Let $q_1 = \langle r_1(p_1),\ 1-\max\{r_1\}\rangle$, $r_1(p_1) = \{r_1^l(p_1^l)|l = 1, 2, \ldots, L_{r_1}\}$ be a PHIFN. Its accuracy function can be represented as

$$A(q_1) = \sum_{l=1}^{L_{r_1}} r_1^l p_1^l + 1 - \max\{r_1\}.$$

We define a new comparison method for PHIFNs based on Definitions 10 and 11.

Definition 12 Let $q_1 = \langle r_1(p_1),\ 1-\max\{r_1\}\rangle$ with $r_1(p_1) = \{r_1^l(p_1^l)|l = 1, 2, \ldots, L_{r_1}\}$ and $q_2 = \langle r_2(p_2), 1-\max\{r_2\}\rangle$ with $r_2(p_2) = \{r_2^t(p_2^t)|t = 1, 2, \ldots, L_{r_2}\}$ be two PHIFNs. Assume all probability hesitant membership degrees in PHIFNs are arranged in ascending order, and let $r_1^{\sigma(j)}p_1^{\sigma(j)}$ and $r_2^{\sigma(j)}p_2^{\sigma(j)}$ be the jth values in q_1 and q_2, respectively. We define the following comparison method.

1. $q_1 < q_2$ if $r_1^{\sigma(j)}p_1^{\sigma(j)} \le r_2^{\sigma(j)}p_2^{\sigma(j)}$, $r_1^{\sigma(L_{r_1})}p_1^{\sigma(L_{r_1})} \le r_2^{\sigma(L_{r_2})}p_2^{\sigma(L_{r_2})}$, and $r_1^{\sigma(L_{r_1})} \le r_2^{\sigma(L_{r_2})}$, where at least one of " $\le$ " should be " $<$," i.e., the equal signs of these inequalities cannot exist at the same time ($j = 1, 2, \ldots, l; l = min\{L_{r_1}, L_{r_2}\}$);
2. $q_1 \prec q_2$ if $q_1 < q_2$ is not satisfied and $S(q_1) < S(q_2)$, or if $q_1 < q_2$ is not satisfied, $S(q_1) = S(q_2)$, and $A(q_1) < A(q_2)$ (where " $\prec$ " means "inferior to");
3. $q_1 = q_2$ if $S(q_1) = S(q_2)$ and $A(q_1) = A(q_2)$.

The Hausdorff distance [8,9], which is the maximum distance from a set to the nearest point in another set, is a reliable way to calculate the distance between sets. We define the Hausdorff distance of PHIFNs as follows.

Definition 13 Let $q_1 = \langle r_1(p_1),\ 1-\max\{r_1\}\rangle$ with $r_1(p_1) = \{r_1^l(p_1^l)|l = 1, 2, \ldots, L_{r_1}\}$ and $q_2 = \langle r_2(p_2), 1-\max\{r_2\}\rangle$ with $r_2(p_2) = \{r_2^t(p_2^t)|t = 1, 2, \ldots, L_{r_2}\}$ be two PHIFNs, where $\sum_{l=1}^{L_{r_1}} p_1^l = 1$ and $\sum_{t=1}^{L_{r_2}} p_2^t = 1$. The Hausdorff distance between q_1 and q_2 is

$$\begin{aligned} &D(q_1, q_2) \\ &= \max\left\{\max_{r_1^l\in r_1}\min_{r_2^t\in r_2}\left|r_1^l p_1^l - r_2^t p_2^t\right|, \max_{r_2^t\in r_2}\min_{r_1^l\in r_1}\left|r_2^t p_2^t - r_1^l p_1^l\right|, |(1-\max\{r_1\})-(1-\max\{r_2\})|\right\}. \end{aligned} \quad (1)$$

Theorem 1 *Let Q be a set of PHIFNs. The Hausdorff distance between PHIFNs has the following properties:*

1. $0 \leq D(q_1, q_2) \leq 1, \forall q_1, q_2 \in Q$.
2. $D(q_1, q_1) = 0, \forall q_1 \in Q$.
3. $D(q_1, q_2) = D(q_2, q_1), \forall q_1, q_2 \in Q$.
4. $\forall q_1, q_2, q_3 \in Q$, *if* $\forall q_1 < q_2 < q_3$, *then* $D(q_1, q_3) \geq D(q_1, q_2)$ *and* $D(q_1, q_3) \geq D(q_2, q_3)$.

Proof: The proof of theorem 1 is proved in the Appendix.

4 Consistency and consensus-improving models for PHIFPR

As is well known, the consistency of preference relation reflects the strict logical relations among each evaluation entry. The idea of fuzzy set theory is that everything is elastic to some extent [51]. Thus, the logic of strict consistency is incompatible with the uncertainty of fuzzy sets, which means that the concept of consistency is not applicable for PHIFPR. In this section, the approximate consistency matrix and approximate consistency index are first defined, and the corresponding mathematical programming model is built to improve the consistency level when PHIFPRs are not acceptably approximate consistency. Then, the consensus index of evaluators is used to check whether the group consensus level is achieved. A mathematical programming model is built to repair PHIFPRs when they are at an unacceptable consistency or consensus level.

4.1 Consistency-improvement process of PHIFPRs

This subsection first defines the approximate consistency matrix, and then the concept of approximate consistency index is introduced to test the consistency level of a PHIFPR based on the approximate consistency matrix and the additive transitivity. In addition, we devise a programming model to repair PHIFPR that is not acceptably consistent.

Definition 14 Let $\tilde{Q} = (\tilde{q}_{ij})_{n\times n}$ be a PHIFPR, where $\tilde{q}_{ij} = \langle r_{ij}(p_{ij}),\ 1 - \max\{r_{ij}\}\rangle$ with $r_{ij}(p_{ij}) = \{r_{ij}^l(p_{ij}^l)|l = 1, 2, \ldots, L_{r_{ij}}\}$ and score function $S(\tilde{q}_{ij}) = \sum_{l=1}^{L_{r_{ij}}} r_{ij}^l p_{ij}^l - 1 + \max\{r_{ij}\}$. Then $E_{\tilde{Q}} = (eq_{ij})_{n\times n}$ represents the approximate consistency matrix of $\tilde{Q} = (\tilde{q}_{ij})_{n\times n}$, where

$$eq_{ij} = \begin{cases} r_{ij}, & if \quad L_{r_{ij}} = 1 \\ a_{ij}, & elseif \quad L_{r_{ij}} \neq 1 \ and \ a_{ij} \in [0, 1] \\ \lceil a_{ij} \rceil - a_{ij}, & else \end{cases},$$

$a_{ij} = S(\tilde{q}_{ij}) + \frac{1 - max\{r_{ij}\} + min\{r_{ij}\}}{2}$, and $\lceil a_{ij} \rceil$ denotes the smallest integer greater than a_{ij}.

Based on Definition 14, we have

$$E_{\tilde{Q}} = eq_{ij} = \begin{bmatrix} 0.5 & eq_{12} & \cdots & eq_{1n} \\ eq_{21} & 0.5 & \cdots & eq_{2n} \\ \cdots & \cdots & \vdots & \cdots \\ eq_{n1} & eq_{n2} & \cdots & 0.5 \end{bmatrix}.$$

For $\forall i, j$, we have $eq_{ij} + eq_{ji} = 1$, and

(1) if $\tilde{q}_{ij} \succ \tilde{q}_{ji}$, which represents that scheme i is superior to scheme j, then $eq_{ij} > 0.5$ and $eq_{ji} < 0.5$.

(2) if $\tilde{q}_{ij} \prec \tilde{q}_{ji}$, which represents that scheme j is superior to scheme i, then $eq_{ij} < 0.5$ and $eq_{ji} > 0.5$.

(3) if $\tilde{q}_{ij} = \tilde{q}_{ji}$, which indicates no difference between schemes i and j, then $eq_{ji} = eq_{ij} = 0.5$.

From the analysis above, we can find that the approximate consistency measure matrix $E_{\tilde{Q}} = (eq_{ij})_{n\times n}$ is a FPR. Consistency analysis in preference relation is the key to avoid evaluators not providing their opinions randomly and obtaining reliable results. Tanino [30] defined the additive transitive property of a FPR as follows,

$$(eq_{ij} - 0.5) + (eq_{j\kappa} - 0.5) = (eq_{i\kappa} - 0.5),\ \forall\ i, j, \kappa = 1, 2, \ldots, n. \tag{2}$$

If the above additive transitivity is satisfied, we can infer that $E_{\tilde{Q}}$ is a consistency FPR.

Definition 15 Suppose $\tilde{Q} = (\tilde{q}_{ij})_{n\times n}$ is a PHIFPR whose approximate consistency measure matrix is $E_{\tilde{Q}} = (eq_{ij})_{n\times n}$. $E_{\tilde{Q}}$ is an additively consistent FPR if and only if:

$$eq_{ij} = eq_{i\kappa} + eq_{\kappa j} - 0.5,\ \forall\ i, j, \kappa = 1, 2, \ldots, n. \tag{3}$$

Given that the approximate consistency measure matrix $E_{\tilde{Q}} = (eq_{ij})_{n\times n}$ is a FPR, we have $eq_{ij} = 1 - eq_{ji}$, i.e., the lower triangle elements of $E_{\tilde{Q}}$ can be deduced from the upper triangle elements of $E_{\tilde{Q}}$, and we can write eq. (3) as

$$eq_{ij} = eq_{i\kappa} + eq_{\kappa j} - 0.5,\ \forall\ i < j < \kappa. \tag{4}$$

Due to the limited rational behavior of evaluators together with the complexity of the evaluation scenarios, the inconsistency in the evaluation information given by evaluators is usually existing. In the following, the approximate consistency index for PHIFPR is defined to measure the consistency level of the PHIFPR.

Definition 16 Let $\tilde{Q} = (\tilde{q}_{ij})_{n\times n}$ be a PHIFPR with approximate consistency measure matrix $E_{\tilde{Q}} = (eq_{ij})_{n\times n}$. The approximate consistency index of $\tilde{Q}$ is

$$CI(\tilde{Q}) = 1 - \frac{4}{n(n-1)(n-2)} \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{\kappa=j+1}^{n} \left| eq_{ij} + eq_{j\kappa} - eq_{i\kappa} - 0.5 \right|. \tag{5}$$

Note that $0 \leq |eq_{ij} + eq_{j\kappa} - eq_{i\kappa} - 0.5| \leq \frac{3}{2}$ $(i, j, \kappa = 1, 2, \ldots, n)$. To standardize the summation $\sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{\kappa=j+1}^{n} |eq_{ij} + eq_{j\kappa} - eq_{i\kappa} - 0.5|$ into a unit closed interval $[0, 1]$, we divide by $\frac{6}{\frac{3}{2} * n(n-1)(n-2)}$. If $\tilde{Q}$ is approximately perfectly consistent, then its consistency index $CI(\tilde{Q}) = 1$. Conversely, if the consistency index $CI(\tilde{Q}) = 1$, then $\tilde{Q}$ is approximately perfectly consistent. In addition, the greater the value of $CI(\tilde{Q})$, the higher the consistency level of PHIFPR.

The consistency of a preference relation is concerned with rationality, and an inconsistent evaluation often leads to misleading solutions. Therefore, a method to improve the consistency is necessary if the PHIFPRs furnished by evaluators show unacceptable consistency. We will construct a mathematical programming model to improve individual consistency in effectiveness evaluation to revise evaluators' preferences.

Let $\tilde{Q} = (\tilde{q_{ij}})_{n\times n}$ $(i, j = 1, 2, \ldots, n)$ be a PHIFPR, where $\tilde{q}_{ij} = \langle r_{ij}(p_{ij}),\ 1 - \max\{r_{ij}\}\rangle$, $r_{ij}(p_{ij}) = \{r_{ij}^{l}(p_{ij}^{l}) | l = 1, 2, \ldots, L_{r_{ij}}\}$, and its approximate consistency measure matrix is $E_{\tilde{Q}} = (eq_{ij})_{n\times n}$. If $\tilde{Q} = (\tilde{q_{ij}})_{n\times n}$ is unacceptable, then a mathematical programming model is constructed to derive $\hat{Q}^{*} = (\hat{q}_{ij}^{*})_{n\times n}$, where $\hat{q}_{ij}^{*} = \langle r_{ij}^{*}(p_{ij}^{*}),\ 1 - \max\{r_{ij}^{*}\}\rangle$, $r_{ij}^{*}(p_{ij}^{*}) = \{r_{ij}^{*l}(p_{ij}^{*l}) | l = 1, 2, \ldots, L_{r_{ij}^{*}}\}$, and its approximate consistency measure matrix $E_{\hat{Q}}^{*} = (eq_{ij}^{*})_{n\times n}$ can be calculated according to Definition 14. The programming model is constructed to minimize the necessary adjustment, as follows:

$$\text{(Model 1)} \quad \min \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left| eq_{ij} - eq_{ij}^{*} \right|$$

$$s.t. \begin{cases} 1 - \frac{4}{n(n-1)(n-2)} \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{\kappa=j+1}^{n} \left| eq_{ij}^{*} + eq_{j\kappa}^{*} - eq_{i\kappa}^{*} - 0.5 \right| \geq CI_0, \\ r_{ij}^{*l} \in [0,1], \ (i = 1,2,\ldots,n-1; \ i < j; \ l = 1,2,\ldots,L_{r_{ij}^{*}}), \\ p_{ij}^{*1} + p_{ij}^{*2} + \ldots + p_{ij}^{*L_{r_{ij}^{*}}} = 1, \ (i = 1,2,\ldots,n-1; \ i < j). \end{cases}$$

Let CI_0 be a preset threshold, which can be determined according to the situation and evaluators' preferences. How to determine an appropriate threshold requires further research. Saaty [27] and Cutello et al. [3] conducted in-depth research on this topic.

In Model 1, the first constraint is the approximate consistency index from Eq. (5), which ensures that $\hat{Q}^{*}$ satisfies the acceptable approximate consistency condition. The remaining constraints ensure that $\hat{Q}^{*}$ is a PHIFPR. An acceptably consistent PHIFPR $\hat{Q}^{*}$ can be obtained from Model 1 and applied to improve the consistency level of a PHIFPR.

We simplify Model 1 by removing the absolute value in the objective function and first constraint. We adopt the following notations. Let $\zeta_{ij} = eq_{ij} - eq_{ij}^{*}$, $\zeta_{ij}^{+} = \frac{(|\zeta_{ij}| + \zeta_{ij})}{2}$ and $\zeta_{ij}^{-} = \frac{(|\zeta_{ij}| - \zeta_{ij})}{2}$. Then we use $|\zeta_{ij}| = \zeta_{ij}^{+} + \zeta_{ij}^{-}$ to remove the absolute value condition from the objective function. Letting $\delta_{ij\kappa} = eq_{ij}^{*} + eq_{j\kappa}^{*} - eq_{i\kappa}^{*} - 0.5$, $\delta_{ij\kappa}^{+} = \frac{(|\delta_{ij\kappa}| + \delta_{ij\kappa})}{2}$ and $\delta_{ij\kappa}^{-} = \frac{(|\delta_{ij\kappa}| - \delta_{ij\kappa})}{2}$, we can use $|\delta_{ij\kappa}| = \delta_{ij\kappa}^{+} + \delta_{ij\kappa}^{-}$ to remove the absolute value from the first constraint to obtain Model 2,

$$\text{(Model 2)} \quad \min \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (\zeta_{ij}^{+} + \zeta_{ij}^{-})$$

$$s.t. \begin{cases} 1 - \frac{4}{n(n-1)(n-2)} \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{\kappa=j+1}^{n} (\delta_{ij\kappa}^{+} + \delta_{ij\kappa}^{-}) \geq CI_0, \\ eq_{ij}^{*} + eq_{j\kappa}^{*} - eq_{i\kappa}^{*} - 0.5 + \delta_{ij\kappa}^{-} - \delta_{ij\kappa}^{+} = 0, \ (i = 1,2,\ldots,n-2; i < j < \kappa), \\ eq_{ij} - eq_{ij}^{*} + \zeta_{ij}^{-} - \zeta_{ij}^{+} = 0, \ (i = 1,2,\ldots,n-1; i < j), \\ r_{ij}^{*l} \in [0,1], \ (i = 1,2,\ldots,n-1; \ i < j; \ l = 1,2,\ldots,L_{r_{ij}^{*}}), \\ p_{ij}^{*1} + p_{ij}^{*2} + \ldots + p_{ij}^{*L_{r_{ij}^{*}}} = 1, \ (i = 1,2,\ldots,n-1; \ i < j) \\ \zeta_{ij}^{-}, \zeta_{ij}^{+} > 0, \ (i = 1,2,\ldots,n-1; \ i < j), \\ \delta_{ij\kappa}^{+}, \delta_{ij\kappa}^{-} > 0, \ (i = 1,2,\ldots,n-2; i < j < \kappa). \end{cases}$$

4.2 Consistency and consensus-improving process of PHIFPRs

Evaluators can have diverse experience and knowledge that causes their evaluation information to differ. The purpose of effectiveness evaluation is to balance their decision preferences and find a viable and acceptable scheme for the whole group. We introduce a group consensus index that can reflect the closeness degree among evaluations, using the Hausdorff distance to check the group consensus level. We also design a mathematical programming model to increase group consensus and individual approximate consistency when these are unacceptable.

4.2.1 Consensus index of PHIFPRs

Suppose the consensus index among k evaluators is calculated according to Eq. (6). The similarity degree $SIM(e_{m1}, e_{n1})$ between any two evaluators e_{m1} and e_{n1} is obtained by the Hausdorff distance in Definition 13. The proximity degree $PRO(e_{m1})$ between evaluator e_{m1} and other evaluators can be obtained based on the similarity degree $SIM(e_{m1}, e_{n1})$, and the group consensus index $CON(e)$ among $\{e_1, e_2, \ldots, e_k\}$ based on the proximity degree can be obtained.

Suppose $e_{m1} = (\tilde{q}_{ij}^{m1})_{n\times n}$, where $\tilde{q}_{ij}^{m1} = \langle r_{ij}^{m1}(p_{ij}^{m1}), 1-\max\{r_{ij}^{m1}\}\rangle$ with $r_{ij}^{m1}(p_{ij}^{m1}) = \{r_{ij}^{m1,l}(p_{ij}^{m1,l})|l = 1, 2, \ldots, L_{r_{ij}^{m1}}\}$, and $e_{n1} = (\tilde{q}_{ij}^{n1})_{n\times n}$, where $\tilde{q}_{ij}^{n1} = \langle r_{ij}^{n1}(p_{ij}^{n1}), 1-\max\{r_{ij}^{n1}\}\rangle$ with $r_{ij}^{n1}(p_{ij}^{n1}) = \{r_{ij}^{n1,t}(p_{ij}^{n1,t})|t = 1, 2, \ldots, L_{r_{ij}^{n1}}\}$ are the evaluation opinions of two evaluators. To capture the agreement degree between each pair of evaluators, we first define the similarity degree between them. The smaller the distance between e_{m1} and e_{n1}, the more similar they are. From this, we can define the similarity degree as,

$$
\begin{aligned}
&SIM(e_{m1}, e_{n1})\\
&= 1 - \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} D(\tilde{q}_{ij}^{m1}, \tilde{q}_{ij}^{n1})\\
&= 1 - \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \max\Bigg\{ \max_{r_{ij}^{m1,l}\in r_{ij}^{m1}} \min_{r_{ij}^{n1,t}\in r_{ij}^{n1}} \left| r_{ij}^{m1,l} p_{ij}^{m1,l} - r_{ij}^{n1,t} p_{ij}^{n1,t} \right|, \max_{r_{ij}^{n1,t}\in r_{ij}^{n1}} \min_{r_{ij}^{m1,l}\in r_{ij}^{m1}} \Big| r_{ij}^{n1,t} p_{ij}^{n1,t} -\\
&\quad r_{ij}^{m1,l} p_{ij}^{m1,l} \Big|, \left|(1-\max\{r_{ij}^{m1}\}) - (1-\max\{r_{ij}^{n1}\})\right| \Bigg\}.
\end{aligned}
$$

Let $\sum_{n1=1,n1\neq m1}^{k} SIM(e_{m1}, e_{n1})$ be the overall similarity degree associated with e_{m1} and other evaluators. Then $PRO(e_{m1})$ $(m1 = 1, 2, \ldots, k)$ can be calculated between e_{m1} and other evaluators as,

$$
\begin{aligned}
&PRO(e_{m1})\\
&= \frac{1}{k-1} \sum_{n1=1,n1\neq m1}^{k} SIM(e_{m1}, e_{n1})\\
&= \frac{1}{k-1} \sum_{n1=1,n1\neq m1}^{k} \left(1 - \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} D(\tilde{q}_{ij}^{m1}, \tilde{q}_{ij}^{n1})\right)\\
&= \frac{1}{k-1} \sum_{n1=1,n1\neq m1}^{k} \Bigg(1 - \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \max\Bigg\{ \max_{r_{ij}^{m1,l}\in r_{ij}^{m1}} \min_{r_{ij}^{n1,t}\in r_{ij}^{n1}} \left| r_{ij}^{m1,l} p_{ij}^{m1,l} - r_{ij}^{n1,t} p_{ij}^{n1,t} \right|,\\
&\quad \max_{r_{ij}^{n1,t}\in r_{ij}^{n1}} \min_{r_{ij}^{m1,l}\in r_{ij}^{m1}} \left| r_{ij}^{n1,t} p_{ij}^{n1,t} - r_{ij}^{m1,l} p_{ij}^{m1,l} \right|, \left|(1-\max\{r_{ij}^{m1}\}) - (1-\max\{r_{ij}^{n1}\})\right| \Bigg\} \Bigg).
\end{aligned}
$$

The sum of proximity degrees of all evaluators to all other evaluators is $\sum_{m1=1}^{k} PRO(e^{m1})$. The consensus index $CON(e)$ among all evaluators $\{e_1, e_2, \ldots, e_k\}$ can be defined as,

$$
\begin{aligned}
&CON(e)\\
&= \frac{1}{k(k-1)} \sum_{m1=1}^{k} \sum_{n1=1,n1\neq m1}^{k} SIM(e_{m1}, e_{n1})\\
&= \frac{1}{k(k-1)} \sum_{m1=1}^{k} \sum_{n1=1,n1\neq m1}^{k} \left(1 - \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} D(\tilde{q}_{ij}^{m1}, \tilde{q}_{ij}^{n1})\right)\\
&= \frac{1}{k(k-1)} \sum_{m1=1}^{k} \sum_{n1=1,n1\neq m1}^{k} \Bigg(1 - \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \max\Bigg\{ \max_{r_{ij}^{m1,l}\in r_{ij}^{m1}} \min_{r_{ij}^{n1,t}\in r_{ij}^{n1}} \left| r_{ij}^{m1,l} p_{ij}^{m1,l} - r_{ij}^{n1,t} p_{ij}^{n1,t} \right|,\\
&\quad \max_{r_{ij}^{n1,t}\in r_{ij}^{n1}} \min_{r_{ij}^{m1,l}\in r_{ij}^{m1}} \left| r_{ij}^{n1,t} p_{ij}^{n1,t} - r_{ij}^{m1,l} p_{ij}^{m1,l} \right|, \left|(1-\max\{r_{ij}^{m1}\}) - (1-\max\{r_{ij}^{n1}\})\right| \Bigg\} \Bigg).
\end{aligned}
$$

Since $SIM(e_{m1}, e_{n1}) = SIM(e_{n1}, e_{m1})$, we can simplify the above formula to

$$
\begin{aligned}
&CON(e)\\
&=\frac{2}{k(k-1)}\sum_{m1=1}^{k-1}\sum_{n1=m1+1}^{k}\left(1-\frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\max\left\{\max_{r_{ij}^{m1,l}\in r_{ij}^{m1}}\min_{r_{ij}^{n1,t}\in r_{ij}^{n1}}\left|r_{ij}^{m1,l}p_{ij}^{m1,l}-r_{ij}^{n1,t}p_{ij}^{n1,t}\right|,\right.\right.\\
&\left.\left.\max_{r_{ij}^{n1,t}\in r_{ij}^{n1}}\min_{r_{ij}^{m1,l}\in r_{ij}^{m1}}\left|r_{ij}^{n1,t}p_{ij}^{n1,t}-r_{ij}^{m1,l}p_{ij}^{m1,l}\right|,\left|(1-\max\{r_{ij}^{m1}\})-(1-\max\{r_{ij}^{n1}\})\right|\right\}\right). \qquad (6)
\end{aligned}
$$

It is obvious that $0 \leq CON(e) \leq 1$, and the larger the consensus index $CON(e)$, the greater the consensus among PHIFPRs. Moreover, $CON(e) = 1$ if and only if $e_1 = e_2 = \ldots = e_k$, i.e., when the PHIFPRs given by evaluators are all the same, which indicates full consensus. If CON_0 is a predefined consensus threshold, then the evaluation information of all evaluators has an acceptable consensus level if $CON(e) > CON_0$. Otherwise, it must be improved.

Next, we next build a mathematical programming model to increase group consensus and individual approximate consistency when these are unacceptable.

4.2.2 New goal program to improve approximate consistency and group consensus of PHIFPRs

We design a mathematical programming model to improve individual approximate consistency and group consensus in effectiveness evaluation.

Suppose $\tilde{Q}^{m1} = (\tilde{q}_{ij}^{m1})_{n\times n}$ $(m1 = 1,2,\ldots,k)$ represent some PHIFPRs obtained from k evaluators, where $\tilde{q}_{ij}^{m1} = \langle r_{ij}^{m1}(p_{ij}^{m1}),\ 1-\max\{r_{ij}^{m1}\}\rangle$ with $r_{ij}^{m1}(p_{ij}^{m1}) = \{r_{ij}^{m1,l}(p_{ij}^{m1,l})|l = 1,2,\ldots,L_{r_{ij}^{m1}}\}$, and their approximate consistency measure matrices are $E_{\tilde{Q}}^{m1} = (eq_{ij}^{m1})_{n\times n}$. If a PHIFPR is not with acceptable approximate consistency or all evaluators cannot reach consensus, then the PHIFPRs must be repaired and a group of PHIFPRs $\hat{Q}^{m1} = (\hat{q}_{ij}^{m1})_{n\times n}$, where $\hat{q}_{ij}^{m1} = \langle \hat{r}_{ij}^{m1}(\hat{p}_{ij}^{m1}),\ 1-\max\{\hat{r}_{ij}^{m1}\}\rangle$ with $\hat{r}_{ij}^{m1}(\hat{p}_{ij}^{m1}) = \{\hat{r}_{ij}^{m1,l}(\hat{p}_{ij}^{m1,l})|l = 1,2,\ldots,L_{\hat{r}_{ij}^{m1}}\}$, must be obtained by the mathematical programming model. The approximate consistency measure matrices of $\hat{Q}^{m1} = (\hat{q}^{m1})_{n\times n}$ are $E_{\hat{Q}}^{m1} = (\hat{eq}_{ij}^{m1})_{n\times n}$. The model is

$$
\text{(Model 3)}\quad \min\ \frac{2}{kn(n-1)}\sum_{m1=1}^{k}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left|eq_{ij}^{m1}-\hat{eq}_{ij}^{m1}\right|
$$

$$
s.t.\begin{cases}
1-\frac{4}{n(n-1)(n-2)}\sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1}\sum_{\kappa=j+1}^{n}\left|\hat{eq}_{ij}^{m1}+\hat{eq}_{j\kappa}^{m1}-\hat{eq}_{i\kappa}^{m1}-0.5\right|\geq CI_0,\ (m1=1,2,\ldots,k),\\
\frac{2}{k(k-1)}\sum_{m1=1}^{k-1}\sum_{n1=m1+1}^{k}\left(1-\frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}D(\hat{q}_{ij}^{m1},\hat{q}_{ij}^{n1})\right)\geq CON_0,\\
\hat{r}_{ij}^{m1,l}\in[0,1],\ (i=1,2,\ldots,n-1;\ i<j;\ m1=1,2,\ldots,k;\ l=1,2,\ldots,L_{\hat{r}_{ij}^{m1}}),\\
\hat{p}_{ij}^{m1,1}+\hat{p}_{ij}^{m1,2}+\ldots+\hat{p}_{ij}^{m1,L_{\hat{r}_{ij}^{m1}}}=1,\ (i=1,2,\ldots,n-1;\ i<j;\ m1=1,2,\ldots,k).
\end{cases}
$$

The first constraint guarantees that the obtained preference relationship is of acceptable approximate consistency, the second constraint guarantees that the obtained preference relationship can reach the group consensus level, and the remaining constraints ensure that $\hat{Q}^{m1}$ $(m1 = 1,2,\ldots,k)$ are PHIFPRs.

We remove the absolute value condition from Model 3 to simplify the presentation. Let $\eta_{ij}^{m1} = eq_{ij}^{m1} - \hat{eq}_{ij}^{m1}$, $\eta_{ij}^{+,m1} = \frac{(|\eta_{ij}^{m1}|+\eta_{ij}^{m1})}{2}$, and $\eta_{ij}^{-,m1} = \frac{(|\eta_{ij}^{m1}|-\eta_{ij}^{m1})}{2}$. Then we use $|\eta_{ij}^{m1}| = \eta_{ij}^{+,m1} + \eta_{ij}^{-,m1}$ to remove the absolute value from the objective function. Let $\xi_{ij\kappa}^{m1} = \hat{eq}_{ij}^{m1} + \hat{eq}_{j\kappa}^{m1} - \hat{eq}_{i\kappa}^{m1} - 0.5$, $\xi_{ij\kappa}^{+,m1} = \frac{(|\xi_{ij\kappa}^{m1}|+\xi_{ij\kappa}^{m1})}{2}$, and $\xi_{ij\kappa}^{-,m1} = \frac{(|\xi_{ij\kappa}^{m1}|-\xi_{ij\kappa}^{m1})}{2}$. Then we use $|\xi_{ij\kappa}^{m1}| = \xi_{ij\kappa}^{+,m1} + \xi_{ij\kappa}^{-,m1}$ to remove the absolute value from the first constraint. Let $o_{ij}^{m1,n1} = \hat{r}_{ij}^{n1,t}\hat{p}_{ij}^{n1,t} - \hat{r}_{ij}^{m1,l}\hat{p}_{ij}^{m1,l}$, $o_{ij}^{+,m1,n1} = \frac{(|o_{ij}^{m1,n1}|+o_{ij}^{m1,n1})}{2}$, and $o_{ij}^{-,m1,n1} = \frac{(|o_{ij}^{m1,n1}|-o_{ij}^{m1,n1})}{2}$, then $|o_{ij}^{m1,n1}| = o_{ij}^{+,m1,n1} + o_{ij}^{-,m1,n1}$. Let $\gamma_{ij}^{m1,n1} = max\{\hat{r}_{ij}^{n1}\} - max\{\hat{r}_{ij}^{m1}\}$, $\gamma_{ij}^{+,m1,n1} = \frac{(|\gamma_{ij}^{m1,n1}|+\gamma_{ij}^{m1,n1})}{2}$, and $\gamma_{ij}^{-,m1,n1} = \frac{(|\gamma_{ij}^{m1,n1}|-\gamma_{ij}^{m1,n1})}{2}$, then $|\gamma_{ij}^{m1,n1}| = \gamma_{ij}^{+,m1,n1} + \gamma_{ij}^{-,m1,n1}$. In this way, we can get Model 4,

$$(\text{Model 4})\quad \min\ \frac{2}{kn(n-1)}\sum_{m1=1}^{k}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left(\eta_{ij}^{+,m1}+\eta_{ij}^{-,m1}\right)$$

$$s.t.\begin{cases}
1-\frac{4}{n(n-1)(n-2)}\sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1}\sum_{\kappa=j+1}^{n}\left(\xi_{ij\kappa}^{+,m1}+\xi_{ij\kappa}^{-,m1}\right)\geq CI_0,\\
\hat{eq}_{ij}^{m1}+\hat{eq}_{j\kappa}^{m1}-\hat{eq}_{i\kappa}^{m1}-0.5+\xi_{ij\kappa}^{-,m1}-\xi_{ij\kappa}^{+,m1}=0,\ (i=1,2,\ldots,n-2;i<j<\kappa;\ m1=1,2,\ldots,k),\\
eq_{ij}^{m1}-\hat{eq}_{ij}^{m1}+\eta_{ij}^{-,m1}-\eta_{ij}^{+,m1}=0,\ (i=1,2,\ldots,n-1;i<j;\ m1=1,2,\ldots,k),\\
\frac{2}{k(k-1)}\sum_{m1=1}^{k-1}\sum_{n1=m1+1}^{k}\left(1-\frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\max\left\{\max_{\hat{r}_{ij}^{m1,l}\in\hat{r}_{ij}^{m1}}\min_{\hat{r}_{ij}^{n1,t}\in\hat{r}_{ij}^{n1}}(o_{ij}^{+,m1,n1}+o_{ij}^{-,m1,n1}),\right.\right.\\
\left.\left.\max_{\hat{r}_{ij}^{n1,t}\in\hat{r}_{ij}^{n1}}\min_{\hat{r}_{ij}^{m1,l}\in\hat{r}_{ij}^{m1}}(o_{ij}^{+,m1,n1}+o_{ij}^{-,m1,n1}),(\gamma_{ij}^{+,m1,n1}+\gamma_{ij}^{-,m1,n1})\right\}\right)\geq CON_0,\\
\hat{r}_{ij}^{n1,t}\hat{p}_{ij}^{n1,t}-\hat{r}_{ij}^{m1,l}\hat{p}_{ij}^{m1,l}+o_{ij}^{-,m1,n1}-o_{ij}^{+,m1,n1}=0,\ (i=1,2,\ldots,n-1;i<j;\ m1,n1=1,2,\ldots,k),\\
max\{\hat{r}_{ij}^{n1}\}-max\{\hat{r}_{ij}^{m1}\}+\gamma_{ij}^{-,m1,n1}-\gamma_{ij}^{+,m1,n1}=0,\ (i=1,2,\ldots,n-1;i<j;\ m1,n1=1,2,\ldots,k),\\
\eta_{ij}^{+,m1},\eta_{ij}^{-,m1},o_{ij}^{+,m1,n1},o_{ij}^{-,m1,n1},\gamma_{ij}^{+,m1,n1},\gamma_{ij}^{-,m1,n1}>0,(i=1,2,\ldots,n-1;\ i<j;\ m1,n1=1,2,\ldots\\
,k),\\
\xi_{ij\kappa}^{+,m1},\xi_{ij\kappa}^{-,m1}>0,\ (i=1,2,\ldots,n-2;i<j<\kappa;\ m1=1,2,\ldots,k),\\
\hat{r}_{ij}^{m1,l}\in[0,1],\ (i=1,2,\ldots,n-1;\ i<j;\ m1=1,2,\ldots,k;\ l=1,2,\ldots,L_{\hat{r}_{ij}^{m1}}),\\
\hat{p}_{ij}^{m1,1}+\hat{p}_{ij}^{m1,2}+\ldots+\hat{p}_{ij}^{m1,L_{\hat{r}_{ij}^{m1}}}=1,\ (i=1,2,\ldots,n-1;\ i<j;\ m1=1,2,\ldots,k).
\end{cases}$$

Due to the complexity of Model 4, directly using MATLAB can reduce the accuracy of results. The GA is a class of intelligent optimization algorithms proposed by Holland in 1962 [24], which is a stochastic search and optimization technique guided by the principles of evolution and natural genetics with substantial implicit parallelism. Drawing on the theory of biological evolution, the GA models the problem to be solved as a biological evolutionary process, generating the next generation of solutions through selection, crossover, mutation, etc., and gradually eliminating the solutions with low fitness function values and adding the solutions with high fitness function values. In this way, it is very possible to evolve individuals with high fitness function values after multiple generations. Thus, this paper adopts the GA to solve Model 4 to obtain the modified PHIFPRs with acceptable approximate consistency and consensus.

5 Deriving approximate priority weights from PHIFPRs

If the approximate consistency of PHIFPRs given by evaluators is unacceptable, then the rationality of priority weights derived by them cannot be guaranteed [32]. Evaluators' preferences are revised primarily for better consistency to obtain valid priority weights [28]. We develop a modeling method to derive the approximate priority weights of designed schemes from PHIFPRs based on the approximate consistency measure matrix in effectiveness evaluation.

Definition 17 [40] Let $Y=\{y_1,y_2,\ldots,y_n\}$ be a set of schemes, and $P=(p_{ij})_{n\times n}$ $(i,j=1,2,\ldots,n;p_{ij}\in[0,1])$ is a fuzzy preference matrix, where p_{ij} is the preference degree or intensity of schemes y_i and y_j, $p_{ii}=0.5$, and $p_{ij}+p_{ji}=1$. The fuzzy preference matrix P is said to be consistent, or additively consistent, if $p_{ij}=p_{i\kappa}-p_{j\kappa}+0.5$ $(i,j,\kappa=1,2,\ldots,n)$, i.e.,

$$p_{ij}=0.5+\frac{n-1}{2}(w_i-w_j),\forall\ i,j=1,2,\ldots,n,$$

where $w=\{w_1,w_2,\ldots,w_n\}$ are the priority weights of schemes, satisfying $w_i\in[0,1]$ and $\sum_{i=1}^{n}w_i=1$.

Similar to the above definition of consistency, the following definition of the approximate consistency measure matrix $E_{\tilde{Q}}$ for the PHIFPR $\tilde{Q}$ is developed.

Definition 18 Let $\tilde{Q} = (\tilde{q}_{ij})_{n\times n}$ be a PHIFPR, where $\tilde{q}_{ij} = \langle r_{ij}(p_{ij}),\ 1 - \max\{r_{ij}\}\rangle$ with $r_{ij}(p_{ij}) = \{r^l_{ij}(p^l_{ij})|l = 1, 2, \dots, L_{r_{ij}}\}$, and its approximate consistency measure matrix $E_{\tilde{Q}} = (eq_{ij})_{n\times n}$ is defined by Definition 14. Then $\tilde{Q} = (\tilde{q}_{ij})_{n\times n}$ is approximately consistent if $eq_{ij} = eq_{i\kappa} - eq_{j\kappa} + 0.5$ $(i, j, \kappa = 1, 2, \dots, n)$, i.e.,

$$eq_{ij} = 0.5 + \frac{n-1}{2}(w_i - w_j), \forall\ i, j = 1, 2, \dots, n, \tag{7}$$

where $w = \{w_1, w_2, \dots, w_n\}$ are the approximate priority weights of $\tilde{Q}$, satisfying $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$.

Remark 1 If $eq_{ij} = 0.5 + \frac{n-1}{2}(w_i - w_j)$, then $eq_{ji} = 0.5 + \frac{n-1}{2}(w_j - w_i)$.

Proof: The proof of Remark 1 is proved in the Appendix.

Remark 1 shows that if the preference relationship in the upper triangular matrix is approximately consistent, then the preference relationship in the lower triangular matrix is also approximately consistent. Therefore, it is only necessary to test the approximate consistency in the upper triangular matrix of PHIFPR.

In addition, Eq. (7) is true only for an approximately perfectly consistent PHIFPR, which is unrealistic for various reasons. In this case, Eq. (7) does not hold, i.e., there are some deviations between the two sides of Eq. (7). Generally, the higher the approximate consistency index of the PHIFPR, the more rational its preference information. Consequently, the approximate priority weights can be obtained by minimizing the absolute deviations between its two sides based on the following programming model:

$$\begin{aligned} &(\text{Model 5})\ \min\ \ \varsigma_{ij} = \left| eq_{ij} - 0.5 - \frac{n-1}{2}(w_i - w_j)\right| \\ &s.t. \begin{cases} \sum_{i=1}^n w_i = 1, \\ w_i \geq 0,\ \ (i = 1, 2, \dots, n). \end{cases} \end{aligned} \tag{8}$$

We discover that Eq. (8) is a multi-objective programming model, from which we derive the following remarks.

Remark 2 For Eq. (8), we have

$$\left| eq_{ij} - 0.5 - \frac{n-1}{2}(w_i - w_j)\right| = \left| eq_{ji} - 0.5 - \frac{n-1}{2}(w_j - w_i)\right|.$$

Remark 3 For Eq. (8), if $i = j$, then we have

$$\left| eq_{ij} - 0.5 - \frac{n-1}{2}(w_i - w_j)\right| = \left| eq_{ji} - 0.5 - \frac{n-1}{2}(w_j - w_i)\right| = 0.$$

Based on Remarks 2 and 3, Model 5 can be transformed to the following model, from which the approximate priority weights in a PHIFPR can be obtained:

$$\text{(Model 6)}\ \min\ \ P=\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left(b_{ij}d_{ij}^{+}+c_{ij}d_{ij}^{-}\right)$$

$$s.t.\begin{cases} eq_{ij}-0.5-\frac{n-1}{2}(w_i-w_j)-b_{ij}d_{ij}^{+}+c_{ij}d_{ij}^{-}=0,\ (i,j=1,2,\ldots,n;\ j>i),\\ d_{ij}^{+},d_{ij}^{-}\geq 0,\ (i,j=1,2,\ldots,n;\ j>i),\\ \sum_{i=1}^{n}w_i=1,\\ w_i\geq 0,\ (i=1,2,\ldots,n), \end{cases} \tag{9}$$

where d_{ij}^{+} and d_{ij}^{-} are the positive and negative deviations relative to the target of the goal ς_{ij}, respectively. b_{ij} and c_{ij} are the weights corresponding to d_{ij}^{+} and d_{ij}^{-}, respectively. Without loss of generality, we suppose that all goals ς_{ij} $(i,j=1,2,\ldots,n;\ j>i)$ are fair, i.e., $b_{ij}=c_{ij}=1$. Consequently, Eq. (9) can be written as

$$\text{(Model 7)}\ \min\ \ P=\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left(d_{ij}^{+}+d_{ij}^{-}\right)$$

$$s.t.\begin{cases} eq_{ij}-0.5-\frac{n-1}{2}(w_i-w_j)-d_{ij}^{+}+d_{ij}^{-}=0,\ (i,j=1,2,\ldots,n;\ j>i),\\ d_{ij}^{+},d_{ij}^{-}\geq 0,\ (i,j=1,2,\ldots,n;\ j>i),\\ \sum_{i=1}^{n}w_i=1,\\ w_i\geq 0,\ (i=1,2,\ldots,n). \end{cases}$$

The above model can be deemed as the selection process on the basis of the approximate consistency measure matrix, i.e., to choose the optimal constellation satellite designed scheme through the approximate priority weight of every designed scheme.

6 An effectiveness evaluation method of a constellation satellite communication system with PHIFPRs

For an effectiveness evaluation problem of a constellation satellite communication system, suppose there is a set of constellation satellite designed schemes $D=\{D_1,D_2,\ldots,D_n\}$, and $e=\{e_1,e_2,\ldots,e_k\}$ is a set of evaluators. Suppose $\omega=\{\omega_1,\omega_2,\ldots,\omega_k\}$ is the weight vector of evaluators, such that $0\leq\omega_{m1}\leq 1,\sum_{m1=1}^{k}\omega_{m1}=1$. Each evaluator e_{m1} provides a PHIFPR, $\tilde{Q}^{m1}=(\tilde{q}_{ij}^{m1})_{n\times n}$, where $\tilde{q}_{ij}^{m1}=\langle r_{ij}^{m1}(p_{ij}^{m1}),1-\max\{r_{ij}^{m1}\}\rangle$ with $r_{ij}^{m1}(p_{ij}^{m1})=\{r_{ij}^{m1,l}(p_{ij}^{m1,l})|l=1,2,\ldots,L_{r_{ij}^{m1}}\}$ $(m1=1,2,\ldots,k)$. The corresponding approximate consistency measure matrices $E_{\tilde{Q}}^{m1}=(eq_{ij}^{m1})_{n\times n}$ can be obtained according to Definition 14. We introduce a method of effectiveness evaluation using PHIFPRs, which follows and is described in Fig. 1.

Step 1: Identify PHIFPRs and obtain consistency threshold CI_0 and consensus threshold CON_0, based on the subjective preferences of evaluators and the evaluation environment. Evaluators e_{m1} $(m1=1,2,\ldots,k)$ provide their PHIFPRs, $\tilde{Q}^{m1}=(\tilde{q}_{ij}^{m1})_{n\times n}$, $(m1=1,2,\ldots,k)$.

Step 2: From Definition 14, calculate the approximate consistency measure matrices, $E_{\tilde{Q}}^{m1}$ $(m1=1,2,\ldots,k)$.

Step 3: Check approximate consistency and group consensus. The former can be tested through Eq. (5), and the group consensus level of $\{e_1,e_2,\ldots,e_k\}$ through Eq. (6). If the approximate consistency level of all individual PHIFPRs is higher than the threshold CI_0, and the group consensus of $\{e_1,e_2,\ldots,e_k\}$ also meets the threshold CON_0, then go to step 5. Otherwise, go to step 4.

Step 4: Deduce a set of modified PHIFPRs. The preference relationship should be modified by Model 4 if any PHIFPR fails to reach the acceptable consistency level or the group consensus does not meet the

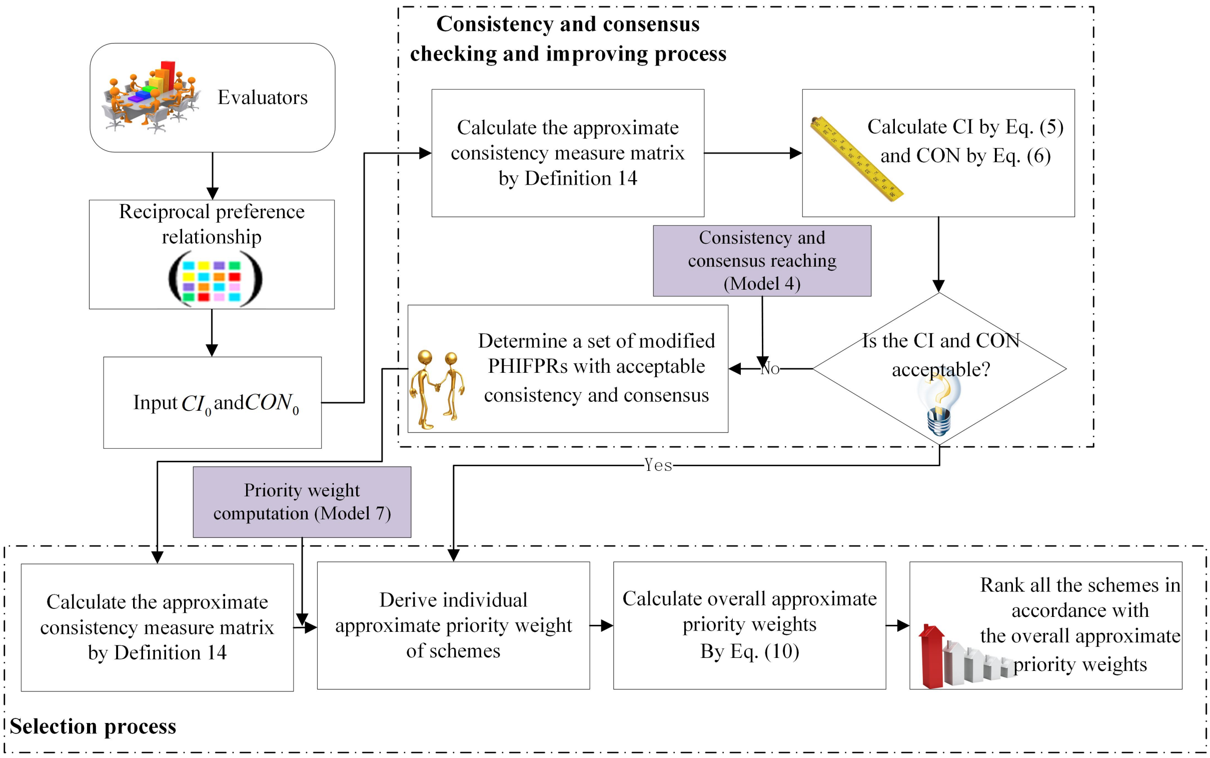

Fig. 1 Framework of effectiveness evaluation method with PHIFPR

threshold value. Modified PHIFPRs can then be obtained with acceptable approximate consistency and group consensus.

Step 5: Calculate approximate priority weights of schemes based on the evaluation opinions of every evaluator, calculated by Model 7.

Step 6: Calculate overall approximate priority weights of schemes as

$$w_i = (\omega_1, \omega_2, \ldots, \omega_k) \times \begin{pmatrix} w_1^1 & w_2^1 & \cdots & w_n^1 \\ w_1^2 & w_2^2 & \cdots & w_n^2 \\ \cdots & \cdots & \cdots & \cdots \\ w_1^k & w_2^k & \cdots & w_n^k \end{pmatrix}, \tag{10}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_k)$ is the weight vector of evaluators, and w_i^{m1} $(i = 1, 2, \ldots, n; m1 = 1, 2, \ldots, k)$ is the approximate priority weights of schemes for every evaluator.

Step 7: Rank schemes according to values calculated by Eq. (10). The scheme with the largest value is best.

7 A numerical example

Constellation scheme design is the primary concern of constellation satellite communication system construction, which largely determines a system's complexity, cost, and effectiveness. It has been much studied, and a variety of schemes have been proposed. Therefore, it is pivotal to design an evaluation method to select the optimal scheme from a finite set of designed schemes according to multiple influential criteria, with the aim of maximizing global real-time data communication and maximizing integrated information services for users. In this section, we provide an application example to demonstrate the feasibility and effectiveness of the designed method providing methodological support to constellation satellite communication system effectiveness evaluation.

Table 1 Evaluation information obtained from expert e_1

	D_1	D_2	D_3	D_4
D_1	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.4(0.3427),0.6(0.6573)\},0.4\rangle$	$\langle\{0.5(0.2838),0.6(0.2669),0.7(0.4493)\},0.3\rangle$	$\langle\{0.3(0.1276),0.5(0.8724)\},0.5\rangle$
D_2	$\langle\{0.6(0.3427),0.4(0.6573)\},0.4\rangle$	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.4(0.3788),0.7(0.6212)\},0.3\rangle$	$\langle\{0.3(0.2840),0.5(0.7160)\},0.5\rangle$
D_3	$\langle\{0.5(0.2838),0.4(0.2669),0.3(0.4493)\},0.5\rangle$	$\langle\{0.6(0.3788),0.3(0.6212)\},0.4\rangle$	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.15(0.4011),0.5(0.5989)\},0.5\rangle$
D_4	$\langle\{0.7(0.1276),0.5(0.8724)\},0.3\rangle$	$\langle\{0.7(0.2840),0.5(0.7160)\},0.3\rangle$	$\langle\{0.85(0.4011),0.5(0.5989)\},0.15\rangle$	$\langle\{0.5(1)\},0.5\rangle$

Table 2 Evaluation information obtained from expert e_2

	D_1	D_2	D_3	D_4
D_1	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.5(0.2320),0.65(0.7680)\},0.35\rangle$	$\langle\{0.45(1)\},0.55\rangle$	$\langle\{0.6(0.9846),0.8(0.0154)\},0.2\rangle$
D_2	$\langle\{0.5(0.2320),0.35(0.7680)\},0.5\rangle$	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.15(0.0574),0.35(0.9426)\},0.65\rangle$	$\langle\{0.4(0.5635),0.6(0.4365)\},0.4\rangle$
D_3	$\langle\{0.55(1)\},0.45\rangle$	$\langle\{0.85(0.0574),0.65(0.9426)\},0.15\rangle$	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.65(1)\},0.35\rangle$
D_4	$\langle\{0.4(0.9846),0.2(0.0154)\},0.6\rangle$	$\langle\{0.6(0.5636),0.4(0.4365)\},0.4\rangle$	$\langle\{0.35(1)\},0.65\rangle$	$\langle\{0.5(1)\},0.5\rangle$

There are three types of communication satellite coverage areas: hotspot, basic, and global, each with different coverage requirements. For example, a hotspot service area requires uninterrupted coverage. The constellation can have certain coverage interruptions, usually less than a minute, for a basic service area. A global service area requires the constellation to cover the whole world. According to coverage requirements, four constellation satellite designed schemes D_i $(i = 1, 2, 3, 4)$ are selected as alternatives for effectiveness evaluation in terms of coverage, quality of service, adaptability, interrupt performance, and protection performance. In the effectiveness evaluation, the relevant departments conducted a preliminary analysis of the basic problems of constellation satellite communication system construction and promptly invited four evaluators of experienced chief e_1, geological expert e_2, telecommunications expert e_3 and user of constellation satellite communication system e_4 to analyze and select the best constellation satellite designed scheme. First, we produce a corresponding questionnaire based on the fundamental data of the constellation satellite designed schemes and five criteria about coverage, quality of service, adaptability, interrupt performance, and protection performance. When filling in the questionnaire, four evaluators e_i $(i = 1, 2, 3, 4)$ are suggested using PHIFPR to express their individual preferences by pairwise comparisons of four schemes. Then we can obtain four PHIFPRs $\tilde{Q}^1, \tilde{Q}^2, \tilde{Q}^3, \tilde{Q}^4$, which are composed of PHIFNs. The weights of the evaluators are $\omega = (0.2, 0.3, 0.3, 0.2)$. The procedures for obtaining the optimal designed scheme by using the proposed method are described as follows,

Step 1: Identify PHIFPRs, the individual threshold $CI_0 = 0.9$, and the consensus threshold $CON_0 = 0.7$.

Four evaluators present their PHIFPRs $\tilde{Q}^1, \tilde{Q}^2, \tilde{Q}^3$, and $\tilde{Q}^4$, as shown in Tables 1, 2, 3, and 4, respectively.

Step 2: Calculate approximate consistency measure matrices.

Table 3 Evaluation information obtained from expert e_3

	D_1	D_2	D_3	D_4
D_1	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.8(1)\},0.2\rangle$	$\langle\{0.75(1)\},0.25\rangle$	$\langle\{0.45(1)\},0.55\rangle$
D_2	$\langle\{0.2(1)\},0.8\rangle$	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.4(0.5437),0.7(0.4563)\},0.3\rangle$	$\langle\{0.0751(0.5264),0.2751(0.4736)\},0.7249\rangle$
D_3	$\langle\{0.25(1)\},0.75\rangle$	$\langle\{0.6(0.5437),0.3(0.4563)\},0.4\rangle$	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.15(1)\},0.85\rangle$
D_4	$\langle\{0.55(1)\},0.45\rangle$	$\langle\{0.9249(0.5264),0.7249(0.4736)\},0.0751\rangle$	$\langle\{0.85(1)\},0.15\rangle$	$\langle\{0.5(1)\},0.5\rangle$

Table 4 Evaluation information obtained from expert e_4

	D_1	D_2	D_3	D_4
D_1	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.55(0.3547),0.7(0.6453)\},0.3\rangle$	$\langle\{0.6(1)\},0.4\rangle$	$\langle\{0.5(0.4631),0.7(0.3095),0.9(0.2274)\},0.1\rangle$
D_2	$\langle\{0.45(0.3547),0.3(0.6453)\},0.55\rangle$	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.25(0.5994),0.75(0.4006)\},0.25\rangle$	$\langle\{0.35(0.3733),0.6(0.6267)\},0.4\rangle$
D_3	$\langle\{0.4(1)\},0.6\rangle$	$\langle\{0.75(0.5994),0.25(0.4006)\},0.25\rangle$	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.3(0.0732),0.4(0.1087),0.6(0.8181)\},0.4\rangle$
D_4	$\langle\{0.5(0.4631),0.3(0.3095),0.1(0.2274)\},0.5\rangle$	$\langle\{0.65(0.3733),0.4(0.6267)\},0.35\rangle$	$\langle\{0.7(0.0732),0.6(0.1087),0.4(0.8181)\},0.3\rangle$	$\langle\{0.5(1)\},0.5\rangle$

Table 5 Modified evaluation information obtained from expert e_1

	D_1	D_2	D_3	D_4
D_1	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.4(0.3427),0.6(0.6573)\},0.4\rangle$	$\langle\{0.5(0.2838),0.6(0.2669),0.7(0.4493)\},0.3\rangle$	$\langle\{0.3(0.1276),0.5(0.8724)\},0.5\rangle$
D_2	$\langle\{0.6(0.3427),0.4(0.6573)\},0.4\rangle$	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.4(0.3788),0.7(0.6212)\},0.3\rangle$	$\langle\{0.3(0.2840),0.5(0.7160)\},0.5\rangle$
D_3	$\langle\{0.5(0.2838),0.4(0.2669),0.3(0.4493)\},0.5\rangle$	$\langle\{0.6(0.3788),0.3(0.6212)\},0.4\rangle$	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.15(0.4011),0.5(0.5989)\},0.5\rangle$
D_4	$\langle\{0.7(0.1276),0.5(0.8724)\},0.3\rangle$	$\langle\{0.7(0.2840),0.5(0.7160)\},0.3\rangle$	$\langle\{0.85(0.4011),0.5(0.5989)\},0.15\rangle$	$\langle\{0.5(1)\},0.5\rangle$

Table 6 Modified evaluation information obtained from expert e_2

	D_1	D_2	D_3	D_4
D_1	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.8987(0.4381),0.3292(0.5619)\},0.1013\rangle$	$\langle\{0.4513(1)\},0.5487\rangle$	$\langle\{0.8062(0.1476),0.5772(0.8524)\},0.1938\rangle$
D_2	$\langle\{0.1013(0.4381),0.6708(0.5619)\},0.3292\rangle$	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.4484(0.4050),0.2128(0.5950)\},0.5516\rangle$	$\langle\{0.2802(0.1548),0.5952(0.8452)\},0.4048\rangle$
D_3	$\langle\{0.5487(1)\},0.4513\rangle$	$\langle\{0.5516(0.4050),0.7872(0.5950)\},0.2128\rangle$	$\langle\{0.5(1)\},0.5\rangle$	$\langle\{0.6823(1)\},0.3177\rangle$
D_4	$\langle\{0.1938(0.1476),0.4228(0.8524)\},0.5772\rangle$	$\langle\{0.7198(0.1548),0.4048(0.8452)\},0.2802\rangle$	$\langle\{0.3177(1)\},0.6823\rangle$	$\langle\{0.5(1)\},0.5\rangle$

Table 7 Modified evaluation information obtained from expert e_3

	D_1	D_2	D_3	D_4
D_1	$\langle\{0.5(1)\}, 0.5\rangle$	$\langle\{0.8(1)\}, 0.2\rangle$	$\langle\{0.75(1)\}, 0.25\rangle$	$\langle\{0.45(1)\}, 0.55\rangle$
D_2	$\langle\{0.2(1)\}, 0.8\rangle$	$\langle\{0.5(1)\}, 0.5\rangle$	$\langle\{0.4(0.5437), 0.7 (0.4563)\}, 0.3\rangle$	$\langle\{0.1362(0.2337), 0.4394 (0.7663)\}, 0.5606\rangle$
D_3	$\langle\{0.25(1)\}, 0.75\rangle$	$\langle\{0.6(0.5437), 0.3 (0.4563)\}, 0.4\rangle$	$\langle\{0.5(1)\}, 0.5\rangle$	$\langle\{0.15(1)\}, 0.85\rangle$
D_4	$\langle\{0.55(1)\}, 0.45\rangle$	$\langle\{0.8638(0.2337), 0.5606 (0.7663)\}, 0.1362\rangle$	$\langle\{0.85(1)\}, 0.15\rangle$	$\langle\{0.5(1)\}, 0.5\rangle$

Table 8 Modified evaluation information obtained from expert e_4

	D_1	D_2	D_3	D_4
D_1	$\langle\{0.5(1)\}, 0.5\rangle$	$\langle\{0.55(0.3547), 0.7 (0.6453)\}, 0.3\rangle$	$\langle\{0.6(1)\}, 0.4\rangle$	$\langle\{0.5(0.4631), 0.7 (0.3095), 0.9(0.2274)\}, 0.1\rangle$
D_2	$\langle\{0.45(0.3547), 0.3 (0.6453)\}, 0.55\rangle$	$\langle\{0.5(1)\}, 0.5\rangle$	$\langle\{0.25(0.5994), 0.75 (0.4006)\}, 0.25\rangle$	$\langle\{0.35(0.3733), 0.6 (0.6267)\}, 0.4\rangle$
D_3	$\langle\{0.4(1)\}, 0.6\rangle$	$\langle\{0.75(0.5994), 0.25 (0.4006)\}, 0.25\rangle$	$\langle\{0.5(1)\}, 0.5\rangle$	$\langle\{0.3(0.0732), 0.4 (0.1087), 0.6(0.8181)\}, 0.4\rangle$
D_4	$\langle\{0.5(0.4631), 0.3 (0.3095), 0.1(0.2274)\}, 0.5\rangle$	$\langle\{0.65(0.3733), 0.4 (0.6267)\}, 0.35\rangle$	$\langle\{0.7(0.0732), 0.6 (0.1087), 0.4(0.8181)\}, 0.3\rangle$	$\langle\{0.5(1)\}, 0.5\rangle$

$$E_{\tilde{Q}}^1 = \begin{pmatrix} 0.5000 & 0.5315 & 0.7165 & 0.3745 \\ 0.4685 & 0.5000 & 0.6364 & 0.3432 \\ 0.2835 & 0.3636 & 0.5000 & 0.1846 \\ 0.6255 & 0.6568 & 0.8154 & 0.5000 \end{pmatrix}, \quad E_{\tilde{Q}}^2 = \begin{pmatrix} 0.5000 & 0.6902 & 0.4500 & 0.8031 \\ 0.3098 & 0.5000 & 0.0885 & 0.4873 \\ 0.5500 & 0.9115 & 0.5000 & 0.6500 \\ 0.1969 & 0.5127 & 0.3500 & 0.5000 \end{pmatrix},$$

$$E_{\tilde{Q}}^3 = \begin{pmatrix} 0.5000 & 0.8000 & 0.7500 & 0.4500 \\ 0.2000 & 0.5000 & 0.5869 & 0.1551 \\ 0.2500 & 0.4131 & 0.5000 & 0.1500 \\ 0.5500 & 0.8449 & 0.8500 & 0.5000 \end{pmatrix}, \quad E_{\tilde{Q}}^4 = \begin{pmatrix} 0.5000 & 0.7718 & 0.6000 & 0.8529 \\ 0.2282 & 0.5000 & 0.4503 & 0.4817 \\ 0.4000 & 0.5497 & 0.5000 & 0.5063 \\ 0.1471 & 0.5183 & 0.4937 & 0.5000 \end{pmatrix}.$$

Step 3: Check approximate individual consistency and group consensus.

From Eq. (5), the approximate consistency indices of evaluators are determined as $CI(\tilde{Q}^1) = 0.9837$, $CI(\tilde{Q}^2) = 0.8752$, $CI(\tilde{Q}^3) = 0.9544$, and $CI(\tilde{Q}^4) = 0.9178$. From Eq. (6), we obtain the consensus index among evaluators, $CON(e) = 0.6888$. By comparing CI_0 and CON_0, we can find that $\tilde{Q}^2$ is not with acceptable approximate consistency, $\tilde{Q}^1$, $\tilde{Q}^3$ and $\tilde{Q}^4$ are acceptably, and $\tilde{Q} = \{\tilde{Q}^1, \tilde{Q}^2, \tilde{Q}^3, \tilde{Q}^4\}$ is the unacceptable group consensus.

Step 4: Deduce a set of modified PHIFPRs.

We use the GA to obtain modified preference relationships $\hat{Q}^{m1}$ $(m1 = 1, 2, 3, 4)$, so that the modified PHIFPRs retain as much original information as possible while meeting the constraints. Fig. 2 shows the fitness function values based on 500 iterations in GA, from which we see that the fitness function is slowly minimized, with optimum value 0.0038. The modified preference relationships $\hat{Q}^{m1}$ $(m1 = 1, 2, 3, 4)$ can be obtained, as shown in Tables 5, 6, 7, and 8, respectively, and the approximate consistency measure matrices $E_{\hat{Q}}^{m1}$ $(m1 = 1, 2, 3, 4)$ are as follows:

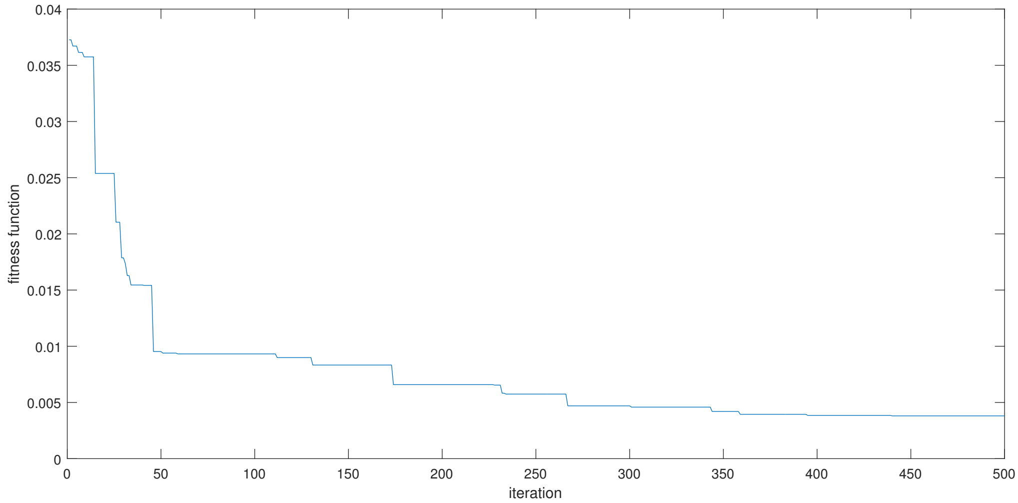

Fig. 2 Fitness function in successive GA iterations

$$E^1_{\hat{Q}} = \begin{pmatrix} 0.5000 & 0.5315 & 0.7165 & 0.3745 \\ 0.4685 & 0.5000 & 0.6364 & 0.3432 \\ 0.2835 & 0.3636 & 0.5000 & 0.1846 \\ 0.6255 & 0.6568 & 0.8154 & 0.5000 \end{pmatrix}, \quad E^2_{\hat{Q}} = \begin{pmatrix} 0.5000 & 0.6926 & 0.4513 & 0.8028 \\ 0.3074 & 0.5000 & 0.1388 & 0.4841 \\ 0.5487 & 0.8612 & 0.5000 & 0.6823 \\ 0.1972 & 0.5159 & 0.3177 & 0.5000 \end{pmatrix},$$

$$E^3_{\hat{Q}} = \begin{pmatrix} 0.5000 & 0.8000 & 0.7500 & 0.4500 \\ 0.2000 & 0.5000 & 0.5869 & 0.1563 \\ 0.2500 & 0.4131 & 0.5000 & 0.1500 \\ 0.5500 & 0.8437 & 0.8500 & 0.5000 \end{pmatrix}, \quad E^4_{\hat{Q}} = \begin{pmatrix} 0.5000 & 0.7718 & 0.6000 & 0.8529 \\ 0.2282 & 0.5000 & 0.4503 & 0.4817 \\ 0.4000 & 0.5497 & 0.5000 & 0.5063 \\ 0.1471 & 0.5183 & 0.4937 & 0.5000 \end{pmatrix}.$$

The approximate consistency index based on Eq. (5) can be obtained as follows: $CI(\hat{Q}^1) = 0.9837$, $CI(\hat{Q}^2) = 0.9037$, $CI(\hat{Q}^3) = 0.9544$, and $CI(\hat{Q}^4) = 0.9178$. The group consensus index among four evaluators is $CON(e) = 0.7055$. According to the threshold values $CI_0 = 0.9$ and $CON_0 = 0.7$, we can find that the modified preference relationships are of acceptable approximate consistency, and $\hat{Q} = \{\hat{Q}^1, \hat{Q}^2, \hat{Q}^3, \hat{Q}^4\}$ is the acceptable group consensus.

Step 5: Calculate approximate priority weights of schemes for every evaluator. From Model 7, we can obtain the approximate priority weights of the constellation satellite designed schemes:

$$\begin{aligned} w^1_1 &= 0.2661\ w^1_2 = 0.2450\ w^1_3 = 0.1393\ w^1_4 = 0.3496, \\ w^2_1 &= 0.3035\ w^2_2 = 0.1750\ w^2_3 = 0.3359\ w^2_4 = 0.1856, \\ w^3_1 &= 0.3406\ w^3_2 = 0.1448\ w^3_3 = 0.1406\ w^3_4 = 0.3740, \\ w^4_1 &= 0.3745\ w^4_2 = 0.1934\ w^4_3 = 0.2265\ w^4_4 = 0.2056. \end{aligned}$$

Step 6: Calculate overall approximate priority weights of designed schemes. From Eq. (10), we obtain:

$$w_1 = 0.3214,\ w_2 = 0.1836,\ w_3 = 0.2161,\ w_4 = 0.2789.$$

Step 7: Rank schemes. Given $w_1 > w_4 > w_3 > w_2$, i.e., $D_1 > D_4 > D_3 > D_2$, the optimal designed scheme is D_1. In the evaluation, we consider the two critical problems of group consensus and individual consistency, in which group consensus aims to help evaluators achieve agreement regarding the solution to a common evaluation problem, and individual consistency checking

Table 9 Comparison of evaluation methods

Evaluation method	Checking consistency	Improving consistency	Checking consensus	Improving consensus	Calculating the priority weight from consistent preference relationships
Our method	Yes	Yes	Yes	Yes	Yes
Evaluation method in [52]	Yes	Yes	No	No	Yes
Evaluation method in [53]	Yes	Yes	No	No	No

aims to ensure the rationality of evaluation information to avoid misleading priority weights of constellation satellite design schemes. In addition, to ensure the credibility of the obtained evaluation results, the fitting error of the approximate consistency measure matrix $E_{\hat{Q}}^{m1}$ $(m1 = 1, 2, 3, 4)$ is calculated by solving model 7, we can get $P^1 = 0.0490$, $P^2 = 0.2892$, $P^3 = 0.1372$, $P^4 = 0.2468$, where $P^{m1} = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left|\hat{eq}_{ij} - 0.5 - 1.5 * (w_i^{m1} - w_j^{m1})\right|$ $(m1 = 1, 2, 3, 4)$. As we all know, the higher the consistency of the PHIFPR, the more rational the preference information in that PHIFPR is, and the more reliable the priority weight vector of that PHIFPR is. Since the value of P^{m1} is relatively low in the case of retaining as much original information as possible, the ranking results obtained in this paper are of some reference value, which can provide the right direction for the selection of designed schemes.

To validate the effectiveness of the designed method, we compare it to other methods considering the same example. The method of Zhou and Xu [52] first uses the expected consistency to improve the individual consistency level by iterative optimization based on the PHFPR during the evaluation. Then they employ general operational laws to aggregate the evaluation information of every evaluator and sort the schemes. By their method, we can obtain a final ranking of $D_2 > D_4 > D_1 > D_3$, which is quite different from that obtained by our method. The reasons lead to this difference may contribute to two points: (1) the proposed method takes into account the max non-priority intensity, which represents the worst negative information and the maximum risk preference of evaluators during the effectiveness evaluation, which is more suitable and reasonable to describe psychological states of evaluators than the PHFPR in [52], (2) the group consensus is not considered in [52], which will result in lower satisfaction with the obtained result.

To further confirm the importance of the worst negative information and the risk preference of evaluators in the evaluation process, we employ the method of Zhou and Xu [53] based on the hesitant-intuitionistic fuzzy information with the same example. They propose an approximate consistency test to judge consistency level. Compared to the method of [52], they consider the worst negative information and the risk preference in the evaluation. Applying this methodology to address our example, we have $D_1 > D_4 > D_2 > D_3$, which implies that the best designed scheme proposed by the method in [53] is the same as ours, but the ranking orders of constellation satellite designed schemes are different. The reasons lead to this difference may contribute to two points: (1) the method in [53] also ignores group consensus problem, resulting in evaluators not achieving agreement regarding the solution to a common evaluation problem, (2) the method in [53] ignore the occurrence probability of each evaluation value, making it impossible for evaluators to accurately express their individual opinions.

Table 9 provides a comparative analysis of three evaluation methods. From Table 9, we can find that literature [52] and [53] both ignore the issue of group consensus in the evaluation process, the evaluation process involves many evaluators with different interest preferences, different educational backgrounds, different social status, and different understandings of the evaluation problem, there generally exists non-consensus among preference relations, thus the consensus reaching process is a necessary and important stage for the effectiveness evaluation. In addition, priority weight can be used to generate the ranking order of constellation satellite designed schemes, but the individual preferences provided by evaluators are usually inconsistent, and the priority weights obtained from the inconsistency matrix cannot guarantee the reliability of the evaluation results. Therefore, it is very important to determine the priority weights from the preference relations in the evaluation process, but the literature [53] failed to consider this problem. Given the above analysis, the method proposed in this paper can be more practical and reasonable compared to the existing methods.

In conclusion, our method can minimize the controversy caused by the optimal result and can more accurately describe the evaluation information of each evaluator than other methods. The main innovations of the proposed method are included as follows.

- We define a new preference relationship, PHIFPR, which can describe the hesitance of evaluators and the hesitance distribution information in the process of evaluation, consider the worst negative information, and relate the risk preference in the form of a non-membership degree. Therefore, PHIFPR can more appropriately describe evaluators' individual views, thereby avoiding the loss of information caused by inaccurate description of evaluation information.
- The approximate consistency and group consensus indices of PHIFPR are defined to check whether the evaluation matrices of evaluators are acceptable. These indices depend on the PHIFPR itself, and there is no need to construct a completely consistent PHIFPR.
- A goal programming model is established to derive modified PHIFPRs based on minimal information adjustment, by which the obtained judgment matrices have a higher individual consistency and group consensus. Furthermore, the model retains the original evaluation information as much as possible.
- The approximate priority weights of schemes can be objectively determined using the approximate consistency measure matrix of PHIFPRs. Generally, the higher the approximate consistency of a PHIFPR, the more rational its preference information and the more reliable its approximate priority weights.

Nevertheless, the proposed method has limitations. It is difficult to determine the threshold values of CI_0 and CON_0. To date, there has been no unified approach to set CI_0 and CON_0. The determination of the consistency or consensus threshold values depends on the the subjective preferences of evaluators and the actual evaluation environment. When the evaluation is urgent, smaller threshold values could be considered suitable; otherwise, higher threshold values should be chosen to determine a solution more suitable to the situation.

8 Conclusions

Individual consistency and group consensus are significant research topics in effectiveness evaluation, and are of increasing concern. This paper aims to introduce an effectiveness evaluation method for a constellation satellite communication system by examining and improving the approximate consistency and group consensus with PHIFPRs. The approximate consistency index for the PHIFPR is first deduced based on the proposed PHIFPR and designed approximate consistency measure matrix, and a programming model is established to improve the consistency of PHIFPRs when necessary. A group consensus index among evaluators is presented, and a programming model is introduced to simultaneously achieve the desired approximate consistency and group consensus. What's more, we propose a programming model based on the approximate consistency measure matrix to calculate the approximate priority weights of each constellation satellite designed scheme, from which a suitable optimal designed scheme can be obtained. Finally, we present some comparative analyses to illustrate the merits of the developed method through a case study. It is shown that our effectiveness evaluation method is more flexible and practical than the existing evaluation methods, and effective at evaluating and ranking constellation satellite designed schemes under an uncertain context.

Given the aforementioned conclusions, it is found that the whole evaluation process based on PHIFPR is given. As researchers point out, it is difficult to ask evaluators to give numerical judgments due to the complexity of the evaluation problem. Given that, this paper introduces a more general form of expressing preference called PHIFPR, where evaluators draw on PHIFPR to describe their individual preferences endowing evaluators with more rights to denote their qualitative judgments. In addition, to ensure the credibility of the obtained evaluation results, an approximate consistency checking method suitable for PHIFPR is designed, which can be used to determine the reliable ranking orders of constellation satellite designed schemes. The proposed method provides a basis for the comparative selection of schemes and provides technical support for the development of satellite communication technology.

The proposed effectiveness evaluation method also has some drawbacks that suggest directions for future research.

– Evaluators' weights are given in advance in the proposed method, which is excessively dependent on evaluators' knowledge, and may be dangerous and open to manipulation. Therefore, objective weighting methods should be developed to dynamically determine the weights of evaluators, depending on the environment and accumulated knowledge [25,43].
– We only considered comprehensive evaluation information based on our proposed evaluation criteria. To consider evaluation information under different criteria can provide more accurate and effective results. We wish to carry out an effectiveness evaluation of constellation satellite designed schemes under different criteria, and at the same time to analyze the interrelationships among multiple evaluation criteria.
– We consider the consensus problem of small group evaluation in this paper, but in practical evaluation, large-scale group evaluation (LSGE) is more worthy of attention as it can bring together more expert knowledge to help obtain better evaluation results. Therefore, in future work, we will carry out the evaluation of constellation satellite communication system based on LSGE.

In conclusion, the research of the PHIFPR and corresponding effectiveness evaluation method are still at an early stage, and a great deal of work remains to be done in the future in the constellation satellite communication system evaluation area.

Acknowledgements

The authors are grateful to the reviewers for their valuable and constructive comments to improve this paper. This work is supported by the Natural Science Foundation of China (Grant Nos. 71671059 and 71521001).

Compliance with ethical standards

Conflicts of interest The authors declare that they have no conflicts of interest.
Ethical approval This article does not contain any studies with human or animals performed by any of the authors.

Appendix

1. Proof of Theorem 1.

(1) As per Definition 8, it is known that $0 \le r_1^l p_1^l \le 1$, $0 \le r_2^t p_2^t \le 1$ $(l = 1, 2, \ldots, L_{r_1}, t = 1, 2, \ldots, L_{r_2})$; then, we can obtain $0 \le |r_1^l p_1^l| \le 1$, $0 \le |r_2^t p_2^t| \le 1$, and $0 \le |(1 - \max\{r_1\}) - (1 - \max\{r_2\})| \le 1$, and thus obtain $0 \le D(q_1, q_2) \le 1, \forall q_1, q_2 \in Q$.

(2)

$$\begin{aligned}
&D(q_1,q_1)\\
&=\max\left\{\max_{r_1^l\in r_1}\min_{r_1^z\in r_1}\left|r_1^lp_1^l-r_1^zp_1^z\right|,\max_{r_1^z\in r_1}\min_{r_1^l\in r_1}\left|r_1^zp_1^z-r_1^lp_1^l\right|,0\right\}\\
&=0.
\end{aligned}$$

(3)

$$\begin{aligned}
&D(q_1,q_2)\\
&=\max\left\{\max_{r_1^l\in r_1}\min_{r_2^t\in r_2}\left|r_1^lp_1^l-r_2^tp_2^t\right|,\max_{r_2^t\in r_2}\min_{r_1^l\in r_1}\left|r_2^tp_2^t-r_1^lp_1^l\right|,\left|(1-\max\{r_1\})-(1-\max\{r_2\})\right|\right\}\\
&=\max\left\{\max_{r_2^t\in r_2}\min_{r_1^l\in r_1}\left|r_2^tp_2^t-r_1^lp_1^l\right|,\max_{r_1^l\in r_1}\min_{r_2^t\in r_2}\left|r_1^lp_1^l-r_2^tp_2^t\right|,\left|(1-\max\{r_2\})-(1-\max\{r_1\})\right|\right\}\\
&=D(q_2,q_1).
\end{aligned}$$

(4) For any three PHIFNs, if $q_1<q_2<q_3$, then $0\le r_1^{\sigma(j)}p_1^{\sigma(j)}\le r_2^{\sigma(j)}p_2^{\sigma(j)}\le r_3^{\sigma(j)}p_3^{\sigma(j)}\le 1$, $r_1^{\sigma(L_{r_1})}p_1^{\sigma(L_{r_1})}\le r_2^{\sigma(L_{r_2})}p_2^{\sigma(L_{r_2})}\le r_3^{\sigma(L_{r_3})}p_3^{\sigma(L_{r_3})}$ and $0\le r_1^{\sigma(L_{r_1})}\le r_2^{\sigma(L_{r_2})}\le r_3^{\sigma(L_{r_3})}\le 1$ can be obtained by Definition 12. Therefore, it can be seen that

$$\begin{aligned}
&r_3^{\sigma(j)}p_3^{\sigma(j)}-r_1^{\sigma(j)}p_1^{\sigma(j)}\ge r_2^{\sigma(j)}p_2^{\sigma(j)}-r_1^{\sigma(j)}p_1^{\sigma(j)}\\
&\Rightarrow\left|r_3^{\sigma(j)}p_3^{\sigma(j)}-r_1^{\sigma(j)}p_1^{\sigma(j)}\right|\ge\left|r_2^{\sigma(j)}p_2^{\sigma(j)}-r_1^{\sigma(j)}p_1^{\sigma(j)}\right|\\
&\Rightarrow\min_{r_1^z\in r_1}\left\{\left|r_3^{\sigma(j)}p_3^{\sigma(j)}-r_1^zp_1^z\right|\right\}\ge\min_{r_1^z\in r_1}\left\{\left|r_2^{\sigma(j)}p_2^{\sigma(j)}-r_1^zp_1^z\right|\right\}\\
&\Rightarrow\max_{r_3^l\in r_3}\min_{r_1^z\in r_1}\left\{\left|r_3^lp_3^l-r_1^zp_1^z\right|\right\}\ge\max_{r_2^t\in r_2}\min_{r_1^z\in r_1}\left\{\left|r_2^tp_2^t-r_1^zp_1^z\right|\right\}.
\end{aligned}$$

Similarly, we can obtain $\max_{r_1^z\in r_1}\min_{r_3^l\in r_3}\left\{\left|r_1^zp_1^z-r_3^lp_3^l\right|\right\}\ge\max_{r_1^z\in r_1}\min_{r_2^t\in r_2}\{|r_1^zp_1^z-r_2^tp_2^t|\}$ and $|(1-\max\{r_3\})-(1-\max\{r_1\})|\ge|(1-\max\{r_2\})-(1-\max\{r_1\})|$, where $l=1,2,\ldots,L_{r_3}$, $t=1,2,\ldots,L_{r_2}$, $z=1,2,\ldots,L_{r_1}$. According to Definition 13, $D(q_1,q_3)\ge D(q_1,q_2)$ is obtained. Next, in a manner similar to certify $D(q_1,q_3)\ge D(q_2,q_3)$.

This completes the proof of Theorem 1.

2. Proof of Remark 1.

Let $\tilde{Q}=(\tilde{q}_{ij})_{n\times n}$ be a PHIFPR, where $\tilde{q}_{ij}=\langle r_{ij}(p_{ij}),\nu_{ij}\rangle$ with $r_{ij}(p_{ij})=\{r_{ij}^l(p_{ij}^l)|l=1,2,\ldots,L_{r_{ij}}\}$, and $\nu_{ij}=1-\max\{r_{ij}\}$. According to Definitions 8 and 14, when $L_{r_{ij}}$=1, we can easily get $eq_{ji}=0.5+\frac{n-1}{2}(w_j-w_i)$, so we will not elaborate on it here. When $L_{r_{ij}}>1$,

1) if $0<S(\tilde{q}_{ji})+\frac{1-\max\{r_{ji}\}+\min\{r_{ji}\}}{2}<1$, then

$$\begin{aligned}
eq_{ji}&=\sum_{l=1}^{L_{r_{ji}}}r_{ji}^lp_{ji}^l-1+\max\{r_{ji}\}+\frac{1-\max\{r_{ji}\}+\min\{r_{ji}\}}{2}\\
&=\sum_{l=1}^{L_{r_{ij}}}(1-r_{ij}^l)p_{ij}^l-\min\{r_{ij}\}+\max\{r_{ij}\}-\max\{r_{ij}\}-1+1+\frac{1-\max\{r_{ji}\}+\min\{r_{ji}\}}{2}\\
&=1+1-\sum_{l=1}^{L_{r_{ij}}}r_{ij}^lp_{ij}^l-\max\{r_{ij}\}+\frac{1-\max\{r_{ji}\}+2\cdot\max\{r_{ij}\}+\min\{r_{ji}\}-2\cdot\min\{r_{ij}\}-2}{2}\\
&=1-eq_{ij}\\
&=0.5+\frac{n-1}{2}(w_j-w_i).
\end{aligned}$$

2) if $S(r_{ji}) + \frac{1-\max\{r_{ji}\}+\min\{r_{ji}\}}{2} > 1$ or $S(r_{ji}) + \frac{1-\max\{r_{ji}\}+\min\{r_{ji}\}}{2} < 0$, then,

$$
\begin{aligned}
eq_{ji} = & \left\lceil \sum_{l=1}^{L_{r_{ji}}} r_{ji}^l p_{ji}^l - 1 + \max\{r_{ji}\} + \frac{1-\max\{r_{ji}\}+\min\{r_{ji}\}}{2} \right\rceil \\
& - \left(\sum_{l=1}^{L_{r_{ji}}} r_{ji}^l p_{ji}^l - 1 + \max\{r_{ji}\} + \frac{1-\max\{r_{ji}\}+\min\{r_{ji}\}}{2} \right) \\
= & \left\lceil \sum_{l=1}^{L_{r_{ij}}} (1-r_{ij}^l) p_{ij}^l - \min\{r_{ij}\} + \max\{r_{ij}\} - \max\{r_{ij}\} - 1 + 1 + \frac{1-\max\{r_{ji}\}+\min\{r_{ji}\}}{2} \right\rceil \\
& - \left(\sum_{l=1}^{L_{r_{ij}}} (1-r_{ij}^l) p_{ij}^l - \min\{r_{ij}\} + \max\{r_{ij}\} - \max\{r_{ij}\} - 1 + 1 + \frac{1-\max\{r_{ji}\}+\min\{r_{ji}\}}{2} \right) \\
= & \left\lceil 1 - \left(\sum_{l=1}^{L_{r_{ij}}} r_{ij}^l p_{ij}^l - 1 + \max\{r_{ij}\} + \frac{1-\max\{r_{ji}\}+\min\{r_{ji}\}}{2} \right) \right\rceil \\
& - \left(1 - \left(\sum_{l=1}^{L_{r_{ij}}} r_{ij}^l p_{ij}^l - 1 + \max\{r_{ij}\} + \frac{1-\max\{r_{ji}\}+\min\{r_{ji}\}}{2} \right) \right) \\
= & \lceil (1-a_{ij}) \rceil - (1-a_{ij}) + \lceil a_{ij} \rceil - \lceil a_{ij} \rceil \\
= & (\lceil (1-a_{ij}) \rceil - (1 - \lceil a_{ij} \rceil)) - (\lceil a_{ij} \rceil - a_{ij}) \\
= & 1 - (\lceil a_{ij} \rceil - a_{ij}) \\
= & 1 - eq_{ij} \\
= & 0.5 + \frac{n-1}{2}(w_j - w_i),
\end{aligned}
$$

where $a_{ij} = S(\tilde{q}_{ij}) + \frac{1-max\{r_{ij}\}+min\{r_{ij}\}}{2}$, and $\lceil a_{ij} \rceil$ denotes the smallest integer greater than a_{ij}.

The proof of Remark 1 is completed.

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