

Modeling the capacitated *p*-cable trench problem with facility costs

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Abstract

In this work, two network design concepts which have been proven to be relevant with respect to telecommunication applications, are joined within a unified approach. First, the cable trench problem searches for cost-minimizing network structures that take into account two types of edge costs appearing in the installation of wire-based networks, namely trenching costs and cable costs. Second, the facility location problem considers the placement of shared telecommunication equipment, like switches or concentrators, together with an assignment of entities to demand nodes. Following practical needs, we join these concepts within the new capacitated cable trench problem with facility costs and service capacity in terms of number of customers that can be served by facilities. Within this setting, facility location decisions in wire-based networks can be taken under a more realistic cost scenario. A mixed-integer linear program and valid inequalities are proposed. Experiments indicate a positive impact of the valid inequalities on computational time and integrality gap.

Keywords Cable-trench problem \cdot Facility location \cdot Valid inequalities \cdot Mixed-integer linear program

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1 Introduction

Facility location models study the optimal placement of service entities and are of particular interest in telecommunication network design, e.g., in the placement of concentrators (Yaman 2005; Gourdin et al. 2002). In such models, expenses for connecting demand to source nodes are typically defined through a single cost type. However, this modeling approach relaxes an aspect which is relevant in the design of wire-based networks, namely the inclusion of two cost components that have to be treated separately. First, cable cost occur for purchasing and installation of cable and are typically dependent on the length of the installed cables. Second, trenching costs are generated for building trenches that contain the cables. This includes the expenses for opening the ground and establishing construction sites if trenches are prepared along roads or pavements in urban areas. Trenching costs, alike cable costs, depend on the total distance of realized trenches. However, as a trench can contain several cables, i.e., network edges contain at most one trench, but can contain more than one cable, both cost types have to be considered separately. When designing a cable-trench network, one (extreme) option is to only minimize the trenching effort by reducing the total distance covered by trenches. This consequently leads to a minimum spanning tree (MST) solution. In such a solution, however, the cables connecting demand nodes to the source(s) may need to take detours which in turn increases the cost for installing cables. On the other hand, with respect to pure cable costs, shortest connections between source(s) and demand nodes are preferred. This leads to shortest path tree (SPT) solutions. The cable trench problem (CTP) considers both cost types in a combined approach, i.e., the concepts of MST and SPT are merged.

The CTP in its basic version was introduced by Vasko et al. (2002). The authors proposed a mathematical model for the case of a single source node and a single cable type. Moreover, they provided a heuristic solution approach. The relevance for application in telecommunication network design was proved by Nielsen (2008) by evaluating the CTP based on a real-world dataset (approx. 440,000 households) from regions of Denmark. Jeng et al. (2007, 2006) applied the CTP to demonstrate the behavior of a DNA-based evolutionary algorithm on small-size instances of six nodes and eight edges. Moreover, several extensions of the CTP are available. Marianov et al. (2012) formulated the p-cable trench problem (p-CTP) that searches for CTPsolutions enabling exactly p source nodes. A mathematical model was proposed and a heuristic based on Lagrangean Relaxation was presented. A matheuristic approach for solving the *p*-CTP was introduced by Lalla-Ruiz et al. (2016). A further extension of the *p*-CTP is the *p*-cable trench problem with covering (*p*-CTPC), (Gutiérrez-Jarpa et al. 2015; Marianov et al. 2015). In this approach, demand nodes are connected through a hierarchical two-level system where only the primary and secondary servers are forming a wire-based network. The demand nodes, however, connect through a wireless system and need to be located in the covering area of at least one server node. The capacitated p-cable trench problem with covering (Cp-CTPC), addressed by Calik et al. (2016), incorporates customers' demands and capacity constraints to the primary and secondary servers. To enable a joint network design for different network operators, Schwarze (2015) extends the CTP to multiple cable types. Under the assumption of a single source node, the multi-commodity cable trench problem

Finally, the CTP and its variants are applied outside the telecommunication field. First, motivated by a question from medical image processing, Vasko et al. (2016) provides the generalized cable trench problem (GCTP) that assigns two length parameters to edges, a cable length and a trench length. Based on the GCTP, the generalized Steiner cable trench problem (GSCTP) is proposed by Landquist et al. (2018). In the GSCTP only a subset of nodes has to be supplied, however, the full set of nodes can be used for creating the solution network. Consequently, this setting enables nodes that are part of the solution tree although network traffic is only routed through those nodes, i.e., Steiner nodes.

Zyma et al. (2017) consider a problem from radio astronomy for which lowfrequency antennas have to be connected within a network and model it as a GCTP. In the context of wind farm construction, Wędzik et al. (2016) propose an extended setting where in addition to cable and trenching cost also the impact of lifecycle energy loss in the network infrastructure is included. Finally, with respect to the design of power transmission in a metro depot, Jamili and Ramezankhani (2015) apply a modified version of the CTP, introducing virtual nodes as new elements of a two-dimensional grid.

In this work, we generalize the problem approach of Marianov et al. (2012) and present the capacitated cable trench problem with facility costs (cCTP-FL), where opened facilities have a cost and limitation on the number of customers associated to them. To meet the requirements of basic facility location approaches, the *p*-CTP is extended regarding three major points.

First, the *p*-CTP does not consider expenses for opening facilities. However, in particular in the telecommunications sector, equipment is expensive and corresponding costs should not be neglected. In this sense, the cost of opening a facility might depend on the chosen location. The preparation of a sufficient environment for a facility may include rental fees or investments in suitable infrastructure. Thus, those costs may depend on the chosen region and can differ, e.g., due to local conditions. Moreover, the inclusion of facility setup expenses is a common component in facility location problems; see, e.g., Klose and Drexl (2005) and Yaman (2005) for telecommunication applications in particular. To integrate this aspect within the cCTP-FL, node-dependent facility opening costs f_i are introduced for each node *i*. Thus, opening costs depend on the chosen location which enables to map, e.g., differences in rental fees. From a modeling perspective, it is recommended to avoid zero facility opening costs $f_i = 0$ as in this case, the objective function value of an optimal solution is decreasing with increasing p. In the extreme case of opening a facility at each node, i.e., if p = nholds, optimal solutions yield zero cost. Thus, it would not be possible to answer strategic questions regarding an optimal number of facilities p. Summarizing, we assume $f_i > 0$ for each node *i*.

Second, the *p*-CTP requires to open exactly *p* facilities. However, in the cCTP-FL, opening a facility causes positive expenses. Thus, in order to obtain practical relevant solutions, it will be useful to relax the fixed number of facilities. Consequently, in the cCTP-FL, we define p^{lb} and p^{ub} as lower and upper bounds on the number of opened facilities *p*.

	<i>p</i> -CTP	cCTP-FL
# of opened facilities	р	$[p^{lb}, p^{ub}]$
Facility <i>i</i> opening cost	0	f_i
# of customers served by <i>i</i>	Unbounded	$\leq m_i$
	# of opened facilities Facility <i>i</i> opening cost # of customers served by <i>i</i>	p-CTP# of opened facilitiespFacility i opening cost0# of customers served by iUnbounded

Third, a capacitated version approach is enabled by adding $m_i \ge 0$ as an upper bound on the number of customers that can be served by facility *i*. In telecommunication applications, capacities play a role as the number of demand nodes that can be served by a particular equipment can be limited due to technical reasons. Furthermore, by fixing $m_i = 0$ for particular nodes *i*, it is possible to define a subset of nodes that are forbidden to be facilities, e.g., if the area is not feasible for establishing server devices. Table 1 summarizes the differences between the models.

Given the relation between *p*-CTP and cCTP-FL as presented in Table 1, it is possible to convert any *p*-CTP instance to a cCTP-FL instance by setting $p^{lb} = p^{ub} = p$, $f_i = 0$ for all nodes *i*, and m_i sufficiently large for all *i*. Thus, the *p*-CTP provides particular cases of the more general cCTP-FL.

The remainder of this paper is organized as follows. A mixed-integer linear programming formulation is introduced in Sect. 2. Additional constraints devoted to improve the formulation are proposed in Sect. 3. Computational experiments that validate the positive impact of the valid inequalities are presented in Sect. 4. The paper closes with conclusions in Sect. 5.

2 Mathematical model

Subsequently, a mathematical model for the cCTP-FL, extending the formulation given by Vasko et al. (2002), is presented. Let $\overline{G} = (\overline{V}, \overline{E})$ be a connected, not necessarily complete graph with nodes $i \in \overline{V}$, directed edges $(i, j) \in \overline{E}$ and positive edge lengths $d_{ij} > 0$ for all $(i, j) \in \overline{E}$. Let $n = |\overline{V}|$ be the number of nodes. Self-loops are not allowed, i.e., for all $i \in \overline{V}$ it holds that $(i, i) \notin \overline{E}$. If a trench can be prepared on (i, j), then this trench can contain cables that are installed in both directions, from *i* to *j*, or, from *j* to *i*. That is, it is assumed that if $(i, j) \in \overline{E}$ holds for $i \neq j$, then $(j, i) \in \overline{E}$ is satisfied, too. In terms of distance, the direction of the cable in the trench does not matter, therefore it is assumed that $d_{ij} = d_{ji}$ holds for all $(i, j) \in \overline{E}$.

The cost for installing cable is proportional to the cable length and is denoted by $\gamma \ge 0$ per unit of length. Similarly, the trenching costs are proportional to the length of a trench and $\tau \ge 0$ denotes the per unit trenching costs. The cost for opening a facility on node *i* is given as $f_i \ge 0$.

Finally, there are three types of bounds. First, $p^{lb} \in \mathbb{Z}$ and $p^{ub} \in \mathbb{Z}$ are lower and upper bounds on the number of opened facilities. Second, for each node $i \in \overline{V}$, if *i* is opened as a facility, the number of served costumers is bounded by $m_i \in \mathbb{N}$.

Following the modeling approach of the *p*-CTP (Marianov et al. 2012), an artificial source node 0 is introduced within the cCTP-FL and added to the node set: $V = \overline{V} \cup \{0\}$.

For all nodes $i \in \overline{V}$, zero-cost artificial edges (0, i) are added such that one obtains $E = \overline{E} \cup \{(0, i)\}_{i \in \overline{V}}$ with $d_{0i} = 0$ for all $i \in \overline{V}$.

The following three classes of variables are included in the formulation. For each edge $(i, j) \in E$ one has $x_{ij} \in \mathbb{N}_0$ giving the number of cables that are installed on edge (i, j). Moreover, for each $(i, j) \in E$ such that i < j, if there is a cable installed either on edge (i, j) or on edge (j, i) one has $y_{ij} = 1$ and $y_{ij} = 0$ otherwise. As the direction of cable does not matter for a trench, y_{ij} is defined only for i < j. Finally, for each node $i \in \overline{V}$ variable z_i equals one if a facility is opened at node i. The mathematical formulation cCTP-FL is presented next.

$$\min\sum_{(i,j)\in E}\gamma d_{ij}x_{ij} + \sum_{(i,j)\in E:i$$

i

$$\sum_{i\in\bar{V}} x_{0j} = n \tag{2}$$

$$\sum_{\substack{\in V: (j,i)\in E}} x_{ji} - \sum_{\substack{j\in \bar{V}: (i,j)\in E}} x_{ij} = 1 \quad i\in \bar{V}$$
(3)

$$ny_{ij} \ge x_{ij} + x_{ji} \qquad i \in V, \, j > i : (i, j) \in E \tag{4}$$

$$z_i \ge y_{0i} \qquad i \in \bar{V} \tag{5}$$

$$\sum_{i\in\bar{V}} z_i \le p^{ub} \tag{6}$$

$$\sum_{i \in \bar{V}} z_i \ge p^{lb} \tag{7}$$

$$x_{0i} - 1 \le m_i \qquad i \in \bar{V} \tag{8}$$

$$\sum_{(j,i)\in E: \ j\neq 0,i} x_{ji} \le (n-p^{lb})(1-z_i) \quad \forall i \in \bar{V}$$

$$\tag{9}$$

$$x_{ii} \in \mathbb{N}_0 \quad (i, j) \in E \tag{10}$$

$$y_{ij} \in \{0, 1\}$$
 $(i, j) \in E : i < j$ (11)

$$z_i \in \{0, 1\} \quad i \in \bar{V} \tag{12}$$

The objective function (1) summarizes cable and trenching costs as well as the costs for opening facilities. Trenching costs are considered only for (i, j) with i < j, i.e., independent from the actual direction of the cable. This is feasible as one has by assumption $d_{ij} = d_{ji}$ for all $(i, j) \in E$. Constraints (2) ensure that *n* cables are leaving the artificial source node 0, i.e., each node $i \in \overline{V}$ can be served. By (3), a single cable is terminating at each node $i \in \overline{V}$. By (4), a trench has to be installed on edge (i, j)if cables are using edges (i, j) or (j, i). Note that constraints (2)–(4) are modified versions of constraints (5.2), (5.3), and (5.5) from Vasko et al. (2002). Constraints (5) ensure that whenever a trench is built from the artificial source 0 to a node *i*, then *i* is opened as facility, i.e., $z_i = 1$ has to hold. By (6) and (7), the number of facilities has to lie between p^{lb} and p^{ub} . By (8), the number of customers that are served by



Fig. 1 cCTP-FL problem instance with 11 nodes

a node *i* is bounded above by m_i . Note that x_{0i} gives the number of cables that enter node *i*. As a single cable is terminating in *i*, see (3), the remaining $x_{0i} - 1$ cables are used to serve customers in the subtree induced by node *i*. Finally, by (9) it is ensured that if a node $i \in \overline{V}$ is a facility, then ingoing cables are only allowed to come from the source node 0. This constraint forbids facility nodes to act as "transition points" for cables that come from different facilities and thus to maintain the upper bound m_i on the number of connected customers. In general, transition nodes (of non-facilities) may be part of an optimal solution. A more detailed discussion is provided together with an example next. For illustration of the cCTP-FL, a small example with n = 11nodes is used. Consider the instance given in Fig. 1. Each edge (i, j) is labeled with its distances d_{ij} . For each node i = 1, ..., 11 the cost for opening a facility is fixed to $f_i = 1000$ and the number of customers is bounded by $m_i = 10$, i.e., for a total of n = 11 nodes, $m_i = 10$ does not reduce the solution space.

Figures 2, 3 and 4 report the optimal solutions for different choices of γ and τ and for an unlimited number of facilities ($p^{lb} = 0$, $p^{ub} = 11$). Node zero, indicated by a dashed circle, is the artificial source node and edges (0, *i*), depicted in dashed lines, are the corresponding artificial edges. The flow on edge (0, *i*) represents the number of nodes that are served by facility *i*. Facility nodes are depicted by bold circles and for each edge (*i*, *j*) the label gives the number x_{ij} of installed cables. It can be observed that for increasing network building cost, i.e., for increasing γ and τ , the number of facilities increases, too. I.e., the cost for opening extra facilities pays off as cost for network rollout is saved.

Similar solutions can be enforced by adjusting the bounds on the number of opened facilities. For instance, if network installation cost is low ($\gamma = \tau = 30$) but bounds are fixed as $p^{lb} = 3$ and $p^{ub} = 11$, the solution given in Fig. 4 is obtained. On the other hand, for $\gamma = \tau = 60$ and $p^{lb} = 0$ and $p^{ub} = 2$, the solution with two facilities given in Fig. 3 is obtained.

All optimal solutions presented in Figs. 2, 3 and 4 are trees rooted at the artificial source node 0 with subtrees induced by facility nodes. However, it is important to



Fig. 2 $\tau = \gamma = 30$; cost 3970



Fig. 3 $\tau = \gamma = 50$; cost 5400

note that in general, cyclic structures are possible, i.e., optimal solutions of the cCTP-FL are not necessarily trees. For instance, consider the problem depicted in Fig. 1 and keep $\tau = \gamma = 60$. Fix the upper bounds on the number of served nodes to $m_8 = 1, m_{11} = 8$ and $m_i = 0$ for all $i \notin \{8, 11\}$. Moreover, set the facility opening cost as $f_8 = f_{11} = 1000$ and $f_i = 100,000$ for all remaining $i \notin \{8, 11\}$. As a result, nodes 8 and 11 are chosen as facilities, see the optimal solution depicted in Fig. 5. Non-facility node 5 receives incoming cables from two nodes, thus, this solution is not a tree. This non-tree solution is optimal as nodes 8 and 11 are the best candidates for facilities. However, to reach node 2 from facility 11, the best option is to choose a path over node 5, although node 5 is already served by facility 8. Thus, in the subtree induced by facility 11, node 5 is acting as a "transit node". Note that constraints (9) prevent facility nodes to become transit nodes.



Fig. 5 Non-tree opt. sol.

For positive trenching costs of $\tau > 0$, the existence of non-tree optimal solutions is an exclusive property of the cCTP-FL, as the CTP and the *p*-CTP produce trees as optimal solutions in this case. Apparently, in the latter models, cyclic structures do not pay off as there are no capacity restrictions nor facility opening costs.

3 Valid inequalities

The performance of the mathematical formulation provided in Sect. 2 can be enhanced by adding valid inequalities. After providing the formal representation of five new valid inequalities, examples are given that illustrate that the valid inequalities indeed cut off non-integral solutions.

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The first class of valid inequalities, see also Schwarze (2015), is based on the fact that each node $i \in \overline{V}$ has to be connected to the trench network. That is, at least one trench is adjacent to each *i*. Note that it is not feasible to consider only edges ingoing to node *i* as trench variables y_{ij} are declared only for i < j but can be used by cables in bidirectional fashion. We obtain the following constraint

$$\sum_{(j,i)\in E: j(13)$$

The second valid inequality exploits the fact that at least n trenches are required to build a network with n + 1 nodes.

$$\sum_{(i,j)\in E:i< j} y_{ij} \ge n \tag{14}$$

For the cCTP-FL, it would be too restrictive to assume equality in (14) as nontree solutions can be optimal, see Fig. 5. This is in contrast to the CTP, for which $\sum_{(i,j)\in E:i < j} y_{ij} = n$ is valid, see (5.4) in Vasko et al. (2002).

A third class of valid inequalities prevents the inclusion of dummy trenches that contain no cable, i.e., positive values of y_{ij} on edges (i, j) that have $x_{ij} = x_{ji} = 0$.

$$x_{ij} + x_{ji} \ge y_{ij} \qquad \forall i \in V, \, j > i : (i, j) \in E \tag{15}$$

Inequality (15) is particularly relevant for the LP-relaxation of cCTP-FL (and thus, for the performance of branch-and-cut methods) because the integrality of variables z_i is dropped there. To save facility opening costs, low values for z_i can be realized by choosing low values for y_{0i} . This leads to opened facilities *i* with low trenching effort on edges (0, *i*). On the other hand, to satisfy (13), a minimum of *n* trenches is required, which then in turn results in dummy trenches. An illustration is provided in the example below.

Finally, inequalities (16) and (17) reduce the facility allocation to the combination of selected nodes that can provide a cable service to the complete network.

$$\sum_{i\in\bar{V}}m_i z_i \ge n - \sum_{i\in\bar{V}}z_i \tag{16}$$

$$(m_i + 1)z_i \ge x_{0i} \quad \forall i \in \bar{V}$$

$$\tag{17}$$

The impact of the valid inequalities is illustrated by studying the LP-relaxation of cCTP-FL in the subsequent example. Consider the cCTP-FL instance given in Fig. 1 and assume $f_i = 1000$ for all $i \in V$, $m_1 = m_2 = m_3 = 5$ and $m_i = 4$ for i = 4, ..., 10. Moreover, we fix $\gamma = \tau = 60$ and $p^{lb} = 0$, $p^{ub} = 11$. Recall that the objective function value of the integer problem is $g^* = 6060$; see Fig. 4. We consider the LP-relaxation of this problem, i.e., choose $x_{ij}, y_{ij}, z_i \in \mathbb{R}^+_0$ to illustrate the impact of the valid inequalities (13)–(15).

The LP-relaxation of (FL-CTP) as described in (1)–(12) delivers an optimal objective function value of $g^{LP} = 1000$. That is, the observed integrality gap is given as



Fig. 6 LP-relaxation, trench network, incl. (13), IntGap = 3.14



Fig. 7 LP-relaxation, trench network, incl. (13), (14), IntGap = 2.12

 $g^*/g^{LP} = 6060/1000 = 6.06$. In this solution, each node is opened as facility with $z_i = 0.091$, and $y_{01} = 0.091$ holds for all $i \in \overline{V}$; no other trenches are installed. Clearly, this solution violates (13) where it is required that the trenches adjacent to a node sum up to at least a value of one. By adding (13) to cCTP-FL, the objective function value of the LP-relaxation increases to $g^{LP(13)} = 1927.27$ which leads to an improved integrality gap of $g^*/g^{LP(13)} = 6060/1927.27 = 3.14$. In this solution, see Fig. 6, still each node acts as facility with $z_i = y_{01} = 0.091$. However, additional trenches are added in order to satisfy (13). On the other hand, the values of the *y*-variables are too low and sum up only to 6.45 < n = 11. That is, valid inequality (14) is violated in the solution of Fig. 6.

Adding valid inequalities (13) and (14) to cCTP-FL increases the optimal value of the LP-relaxation to $g^{LP(13,14)} = 2860$, leading to a new intregrality gap of $g^*/g^{LP(13,14)} = 6060/2860 = 2.12$. Figure 7 illustrates the trench network of this



Fig. 8 LP-relaxation, trench network, incl. (13)–(15), IntGap = 1.28



Fig. 9 LP-relaxation, cable network, incl. (13)–(15), IntGap = 1.28

solution. Still each node is chosen as facility with $z_i = y_{01} = 0.091$. That is, for each $i \in \overline{V}$, one cable is installed from the source to *i* and we have $x_{0i} = 1$. Furthermore, no other cables are installed. As a consequence, all trenches originating from nodes $i \neq 0$ are "dummy trenches" that are installed only to satisfy (14) but that do not contain cable.

To prevent the installation of dummy trenches, (15) is added, yielding an optimal LP-relaxation value of $g^{LP(13,14,15)} = 4720$ and an integrality gap of $g^*/g^{LP(13,14,15)} = 6060/4720 = 1.28$. In this solution, see Fig. 8, only six facility nodes (1, 2, 3, 5, 8, 10) are established. Figure 9 gives the cable network of this solution. Cable loops are part of the solution, e.g., there is 0.5 cable on edges (5, 7) and (7, 5), each. In order to allow small z_i -values, small y_{0i} -values are chosen. In turn, extra trenches are required to satisfy (14) which can, due to (15), only be realized if they contain cable, which leads to the observed cable loops.



Fig. 10 LP-relaxation, trench network, incl. (13)–(17), IntGap = 1.16



Fig. 11 LP-relaxation, cable network, incl. (13)–(17), IntGap = 1.16

Figure 8 illustrates that the values of y_{0i} are small, which in turn allows low values of z_i , see (5) and thus low facility opening costs in the objective function. To increase those values, valid inequalities (16) and (17) provide lower bounds on variables z_i . The optimal objective function value of the LP-relaxation, see Figs. 10 and 11 is $g^{LP(13-17)} = 5240$, leading to an improved integrality gap of $g^*/g^{LP(13-17)} = 1.16$. Moreover, the cable network in this case is integral and thus provides a feasible (although not optimal) solution for the considered instance.

4 Numerical experiments

This section is dedicated to test the performance of the optimization model introduced in Sect. 2 as well as the contribution of the valid inequalities proposed in Sect. 3.

All the computational experiments have been conducted over new problem instances generated in this work.¹ They are based on the problem instances collected by Beasley (2015) for the *p*-median problem. The sizes of the instances corresponds to n = 10, 20, 50, 100, and 200 nodes and five instances per group are generated and tested. The remaining parameters p^{lb} and p^{ub} are selected as a portion of the total number of nodes $p^{lb} = r^{lb}n$, $p^{ub} = r^{ub}n$ and are evaluated for the following three combinations $(r^{lb}, r^{ub}) \in \{(0, 0.25), (0, 0.5), (0.25, 0.75)\}$. Notice that since p^{lb} and p^{ub} are integers, the result of such multiplication is set to integer. Moreover, m_i is randomly generated using a discrete uniform distribution U(0, n).

The mathematical model has been executed with CPLEX 12.6, set to all-default and restricted to one thread. The executions have been done in a computer equipped with an AMD Opteron Processor 6172 (32 CPUs) with 2.1 GHz and 64 GB of RAM.

To evaluate the impact of the new valid inequalities, optimal solutions are computed twice for each instance. Table 2 reports averages grouped by instance size whereas the detailed results for each single instance are given in Tables 3, 4 and 5. The reported values are divided into the following columns: the upper bound (UB), lower bound (LB), relative tolerance on the gap (Gap) calculated as (UB-LB)/UB), the node count (Nodes), and time measured in seconds (Time). First, results are provided for the standard cCTP-FL formulation, see column 'Standard cCTP-FL (1)-(12)'. Second, results are given for cCTP-FL when all valid inequalities are added to the standard formulation, see column 'Standard + valid inequalities (13)-(17)'. For each model setting, upper and lower bounds as well as the corresponding gaps are reported. These values are provided by CPLEX after expiration of the maximal running time of 3600 s. For $n \leq 100$, optimal solutions are detected within the allotted time such that gaps are zero for these instance classes, see, Table 2. For n = 200, an average gap of 0.11 results from the standard cCTP-FL model and is reduced to 0.07. More detailed, in those cases where a positive gap is reported for the standard cCTP-FL, i.e., for instances n = 200, this gap is in no case increased, but is reduced for twelve of 15 cases by including the valid inequalities, when adding the valid inequalities; see Tables 3, 4 and 5. Also, Table 2 reports the number of nodes evaluated in CPLEX during the branch and bound and the required time in seconds. As can be seen, for each instance size, the average computational time is reduced by introducing the valid inequalities. Considering the instances in detail, see Tables 3, 4 and 5, for n = 200, none of the 15 tested instances could be solved to optimality using the standard cCTP-FL. However, by including the valid inequalities, for three instances under the parameter setting $p^{lb} = 0.25n$; $p^{ub} = 0.75n$, see Table 5, optimal solutions have been found. Moreover, for eight of the remaining twelve instances, the gap is reduced and the upper bound is decreased, i.e., improved integral solutions are detected.

To give a more concise illustration of the time saving after inclusion of the valid inequalities as well as the impact on the gap, boxplots are provided in Figs. 12 and 13, respectively. Here, the length of the whiskers is bounded by 1.5 times the Interquartile Range (IQR). Outliers are indicated by circles. Note that the infeasible instances 10n - i04 and 10n - i05 ($p^{lb} = 0n$; $p^{ub} = 0.25n$), see Table 3, are neglected for both illustrations. Moreover, to have a fair comparison of computational times, instances

¹ https://github.com/elalla/pCableTrench.

u	Standard cCT	P-FL (1)–(12)				Standard + vi	alid inequalities (13)-(17)		
	UB	LB	Gap	Nodes	Time (s)	UB	LB	Gap	Nodes	Time (s)
10	233.06	233.06	0.00	5.62	0.04	233.06	233.06	0.00	8.15	0.03
20	538.29	538.29	0.00	244.80	0.56	538.29	538.29	0.00	125.80	0.30
50	1307.59	1307.59	0.00	698.13	9.47	1307.59	1307.59	0.00	931.80	9.19
100	2740.43	2731.94	0.00	8992.60	666.94	2740.43	2740.43	0.00	4709.53	209.49
200	4392.61	3911.64	0.11	3901.40	3600.36	4372.73	4058.80	0.07	13,786.80	3368.61
Average	1842.40	1744.51	0.02	2768.51	855.47	1838.42	1775.63	0.01	3912.42	717.52

 Table 2
 Average numerical results for instances proposed in this work

Table 3 Nui	merical results for	the instance set of	$\operatorname{fr}(p^{lb} = 0n)$	$p^{ub} = 0.25n$						
Instance	Standard cC	TP-FL (1)-(12)				Standard + v	alid inequalities ((13)-(17)		
	UB	LB	Gap	Nodes	Time (s)	UB	LB	Gap	Nodes	Time (s)
10n-i01	406.80	406.80	0.00	0	0.03	406.80	406.80	0.00	38	0.09
10n-i02	592.40	592.40	0.00	51	0.09	592.40	592.40	0.00	45	0.11
10n-i03	420.90	420.90	0.00	0	0.03	420.90	420.90	0.00	23	0.03
10n-i04 ^a	I	I	ļ	I	I	I	I	I	I	I
10n-i05 ^a	I	I	I	I	I	I	I	I	I	I
20n-i01	750.30	750.30	0.00	139	0.75	750.30	750.30	0.00	82	0.61
20n-i02	702.40	702.40	0.00	56	0.68	702.40	702.40	0.00	44	0.48
20n-i03	678.40	678.40	0.00	364	1.01	678.40	678.40	0.00	754	0.77
20n-i04	684.80	684.80	0.00	948	2.86	684.80	684.80	0.00	281	0.71
20n-i05	1111.30	1111.30	0.00	2112	1.52	1111.30	1111.30	0.00	609	1.12
50n-i01	1621.00	1621.00	0.00	506	12.08	1621.00	1621.00	0.00	1225	21.38
50n-i02	1506.10	1506.10	0.00	1110	9.51	1506.10	1506.10	0.00	1184	12.74
50n-i03	1483.80	1483.80	0.00	1176	22.52	1483.80	1483.80	0.00	2010	27.75
50n-i04	1590.40	1590.40	0.00	554	34.22	1590.40	1590.40	0.00	803	23.11
50n-i05	1482.20	1482.20	0.00	1366	36.67	1482.20	1482.20	0.00	3349	39.11
100n-i01	3216.10	3088.86	0.04	48,549	3600.15	3216.10	3216.10	0.00	19,321	1237.44
100n-i02	3101.80	3101.80	0.00	27,931	840.69	3101.80	3101.80	0.00	17,050	548.88
100n-i03	2329.70	2329.70	0.00	1833	186.95	2329.70	2329.70	0.00	7978	264.32
100n-i04	2405.90	2405.90	0.00	33,358	1856.59	2405.90	2405.90	0.00	10,856	401.27
100n-i05	2900.50	2900.50	0.00	1088	62.32	2900.50	2900.50	0.00	3933	83.81

Table 3 contin	ned									
Instance	Standard cCT	P-FL (1)-(12)				Standard + v	alid inequalities (13)-(17)		
	UB	LB	Gap	Nodes	Time (s)	UB	LB	Gap	Nodes	Time (s)
200n-i01	4063.80	3523.76	0.13	2942	3600.35	4042.00	3620.56	0.10	5667	3600.45
200n-i02	4595.40	4037.20	0.12	2583	3600.36	4564.50	4130.44	0.10	9612	3600.45
200n-i03	4072.70	3609.40	0.11	6933	3600.39	4072.70	3876.77	0.05	13,858	3600.45
200n-i04	4317.20	3922.26	0.09	1576	3600.35	4304.30	3929.57	0.09	9692	3600.42
200n-i05	4645.80	3926.81	0.15	930	3600.35	4590.90	3919.47	0.15	6323	3600.44
Average	2116.51	1994.65	0.03	5917.61	1072.63	2111.27	2020.07	0.02	4988.57	898.52
^a For $p^{lb} = 0n$;	$p^{ub} = 0.25n, t$	hese instances dc	o not contain	any feasible solu	ution. Once p^{ub} i:	s increased (e.g.,	Tables 4, 5) a fea	isible solutior	ו can be found	

	101 CITICATI 101	עווא ווופומוועע פעו ע		$h = \frac{1}{2}$						
Instance	Standard cC ⁷	TP-FL (1)-(12)				Standard + v	alid inequalities ((13)-(17)		
	UB	LB	Gap	Nodes	Time (s)	UB	LB	Gap	Nodes	Time (s)
10n-i01	146.00	146.00	0.00	0	0.04	146.00	146.00	0.00	0	0.02
10n-i02	197.00	197.00	0.00	0	0.02	197.00	197.00	0.00	0	0.02
10n-i03	89.80	89.80	0.00	0	0.01	89.80	89.80	0.00	0	0.02
10n-i04	188.10	188.10	0.00	0	0.04	188.10	188.10	0.00	0	0.02
10n-i05	315.00	315.00	0.00	22	0.04	315.00	315.00	0.00	0	0.02
20n-i01	419.60	419.60	0.00	15	0.40	419.60	419.60	0.00	61	0.23
20n-i02	440.50	440.50	0.00	0	0.14	440.50	440.50	0.00	0	0.06
20n-i03	366.00	366.00	0.00	14	0.19	366.00	366.00	0.00	0	0.06
20n-i04	406.40	406.40	0.00	8	0.20	406.40	406.40	0.00	56	0.11
20n-i05	578.20	578.20	0.00	7	0.12	578.20	578.20	0.00	0	0.13
50n-i01	1301.30	1301.30	0.00	06	1.10	1301.30	1301.30	0.00	1136	2.80
50n-i02	1282.90	1282.90	0.00	596	2.64	1282.90	1282.90	0.00	50	1.00
50n-i03	1072.40	1072.40	0.00	0	0.33	1072.40	1072.40	0.00	0	0.52
50n-i04	1192.30	1192.30	0.00	501	5.27	1192.30	1192.30	0.00	2351	2.84
50n-i05	1116.30	1116.30	0.00	2681	3.09	1116.30	1116.30	0.00	660	1.30
100n-i01	3078.80	3078.80	0.00	591	163.26	3078.80	3078.80	0.00	2397	126.43
100n-i02	3047.00	3047.00	0.00	710	139.85	3047.00	3047.00	0.00	834	69.94
100n-i03	2251.00	2251.00	0.00	501	39.01	2251.00	2251.00	0.00	615	29.93
100n-i04	2320.80	2320.80	0.00	496	161.17	2320.80	2320.80	0.00	1550	53.94

Table 4 Numerical results for the instance set of $(p^{lb} = 0n; p^{ub} = 0.50n)$

Table 4 continu	led									
Instance	Standard cCT	TP-FL (1)-(12)				Standard + v	alid inequalities (13)-(17)		
	UB	LB	Gap	Nodes	Time (s)	UB	LB	Gap	Nodes	Time (s)
100n-i05	2878.60	2878.60	0.00	496	34.08	2878.60	2878.60	0.00	1070	33.05
200n-i01	4054.00	3674.42	0.09	1052	3600.36	4033.20	3731.82	0.07	10,355	3600.41
200n-i02	4598.10	4070.93	0.11	1901	3600.34	4572.00	4046.61	0.11	9412	3600.47
200n-i03	4076.40	3796.70	0.07	2344	3600.35	4075.10	3825.25	0.06	13,799	3600.43
200n-i04	4308.90	3925.16	0.09	1274	3600.34	4311.50	3906.47	0.09	10,080	3600.42
200n-i05	4658.10	3926.72	0.16	1015	3600.35	4587.80	3925.17	0.14	5390	3600.47
Average	1775.34	1683.28	0.02	572.56	742.11	1770.70	1684.93	0.02	2392.64	732.98

	TICAL LESUIDS TOL		-10^{-1}	u, p = 0.00	(1)					
Instance	Standard cC1	TP-FL (1)-(12)				Standard + v	alid inequalities (13)-(17)		
	UB	LB	Gap	Nodes	Time (s)	UB	LB	Gap	Nodes	Time (s)
10n-i01	121.00	121.00	0.00	0	0.03	121.00	121.00	0.00	0	0.02
10n-i02	159.00	159.00	0.00	0	0.03	159.00	159.00	0.00	0	0.02
10n-i03	73.80	73.80	0.00	0	0.02	73.80	73.80	0.00	0	0.02
10n-i04	128.00	128.00	0.00	0	0.02	128.00	128.00	0.00	0	0.02
10n-i05	192.00	192.00	0.00	0	0.08	192.00	192.00	0.00	0	0.02
20n-i01	322.00	322.00	0.00	0	0.19	322.00	322.00	0.00	0	0.06
20n-i02	417.20	417.20	0.00	0	0.09	417.20	417.20	0.00	0	0.06
20n-i03	314.00	314.00	0.00	0	0.05	314.00	314.00	0.00	0	0.03
20n-i04	385.00	385.00	0.00	0	0.11	385.00	385.00	0.00	0	0.05
20n-i05	498.20	498.20	0.00	6	0.17	498.20	498.20	0.00	0	0.05
50n-i01	1301.30	1301.30	0.00	81	1.95	1301.30	1301.30	0.00	142	0.67
50n-i02	1282.90	1282.90	0.00	8	1.70	1282.90	1282.90	0.00	17	0.95
50n-i03	1072.40	1072.40	0.00	0	0.86	1072.40	1072.40	0.00	0	0.39
50n-i04	1192.30	1192.30	0.00	1110	6.05	1192.30	1192.30	0.00	1018	2.74
50n-i05	1116.30	1116.30	0.00	693	4.03	1116.30	1116.30	0.00	32	0.58
100n-i01	3078.80	3078.80	0.00	16,845	2086.27	3078.80	3078.80	0.00	1494	73.44
100n-i02	3047.00	3047.00	0.00	502	306.58	3047.00	3047.00	0.00	1169	86.18
100n-i03	2251.00	2251.00	0.00	497	185.48	2251.00	2251.00	0.00	534	27.89
100n-i04	2320.80	2320.80	0.00	498	247.39	2320.80	2320.80	0.00	1181	51.81
100n-i05	2878.60	2878.60	0.00	994	94.38	2878.60	2878.60	0.00	661	54.04

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Instance	Standard cCT.	P-FL (1)–(12)				Standard + va	lid inequalities (13)-(17)		
	UB	LB	Gap	Nodes	Time (s)	UB	LB	Gap	Nodes	Time (s)
200n-i01	4260.60	3855.64	0.10	8695	3600.36	4260.60	4260.60	0.00	33,733	2838.25
200n-i02	4915.30	4584.20	0.07	6663	3600.35	4886.40	4886.40	0.00	34,954	2661.14
200n-i03	4204.00	3840.38	0.09	10,191	3600.42	4203.30	4203.30	0.00	22,561	1824.55
200n-i04	4327.80	3788.30	0.12	4992	3600.36	4320.70	4126.86	0.04	12,192	3600.40
200n-i05	4791.10	4192.78	0.12	5430	3600.37	4766.00	4492.66	0.06	9174	3600.43
Average	1786.02	1696.52	0.02	2288.32	837.49	1783.54	1764.86	0.00	4754.48	592.95





Fig. 13 Reduction of gap (p.p.)

are excluded where the computation has been stopped due to time expiration in both cases (with and without valid inequalities). In those cases, the reported time savings give no relevant information on the performance of the valid inequalities.

The time saving after inclusion of the valid inequalities is illustrated separately for the different instance sizes, see Fig. 12. For the chosen instances, it can be observed that the positive effect on the computational time becomes more stable with increasing number of nodes n. Although on average, the computational time decreases for each instance size, see Table 2, single cases are detected for $n \in \{10, 20, 50, 100\}$ where the computational time is even increasing when the valid inequalities are added. Moreover, positive as well as negative outliers are detected for those instance sets. However, for n = 200, an improvement of the computational time can be observed for each instance. In addition, no outliers appear in this case.

Regarding the reduction of the gap (in percentage points), Fig. 13 gives results only for n = 200 as for smaller instances, only zero gaps have been reported. It can be observed that there is no negative impact on the gap results from the inclusion of the valid inequalities. The lower limit of the IQR is 1 percentage point, the upper limit is 7 percentage points. At maximum 10 percentage points improvement of the gap is obtained. To verify the improvements obtained by the valid inequalities, two paired t-tests have been carried out to investigate whether the differences in computational time and whether the differences in the gap are significant. The tests are carried out over the full set of instances. However, along the same line as for the preparation of the boxplots, infeasible instances are neglected. Along the same line, to evaluate the effect of time savings, instances that experienced an excess of the time limit for both cases (with and without valid inequalities) are excluded. However, for the evaluation of the significance of the reduction of the gap, these instances stay included. As a result, the observed decrease of computational time obtained by adding the valid inequalities is statistically significant at a significance level of $\alpha = 0.05$ (t(60) = 2.6642, p = 0.009897). Similarly, the decreased gap provides statistical significance ($\alpha = 0.05$, t(72) = 3.164, p = 0.00228).

5 Conclusion

In order to enable a more realistic description of wire-based network design for multiple facilities, the novel cCTP-FL supports a flexible number of facilities as well as node-dependent facility opening costs and restrictions on the number of assigned demand nodes. Thus, the cCTP-FL extends the known models of CTP and p-CTP. A mathematical model is provided and improved by a set of new valid inequalities. The positive impact of these inequalities is validated within a numerical study. In particular, a significant reduction of computational time and integrality gap is observed.

Future steps and recommendations for further research on this problem are listed below:

- The analysis of the models' performance for larger instances is an open issue. This also points to the development of (meta-)heuristic approaches for the cCTP-FL as the gap in the large scenarios studied in this work is still relevant. Such development would be advisable for supporting cases and applications where the locations of facilities change over time in a short fashion, requiring thus to solve this problem frequently.
- Moreover, in order to make the approach more generic, we will study an extended version that allows multiple openings of facilities by generalizing $z_i \in \mathbb{N}$, e.g., if capacity at a certain node needs to be extended. Within this context, for real-world applications, a nonlinear cost representation will be needed as cost might increase in a nonlinear fashion, e.g., when new hardware is added to an existing facility.
- Considering the valid cuts proposed in this work, a more tailored branch-and-cut would also be a relevant research direction for this problem.
- Finally, a bi-objective study with regards to the contribution of the two main components in the objective function could be an interesting further step in this problem area.

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