# Discovering Compressing Serial Episodes from Event Sequences 

Ibrahim A ${ }^{1}$, Shivakumar Sastry ${ }^{2}$, P.S. Sastry ${ }^{1}$<br>${ }^{1}$ Department of Electrical Engineering, Indian Insitute of Science, Bangalore<br>${ }^{2}$ Department of Electrical and Computer Engineering Unversity of Akron


#### Abstract

Most pattern mining methods yield a large number of frequent patterns and isolating a small, relevant subset of patterns is a challenging problem of current interest. In this paper we address this problem in the context of discovering frequent episodes from symbolic time series data. Motivated by the Minimum Description Length principle, we formulate the problem of selecting relevant subset of patterns as one of searching for a subset of patterns that achieves best data compression. We present algorithms for discovering small sets of relevant non-redundant episodes that achieve good data compression. The algorithms employ a novel encoding scheme and use serial episodes with inter-event constraints as the patterns. We present extensive simulation studies with both synthetic and real data, comparing our method with the existing schemes such as GoKrimp and SQS. We also demonstrate the effectiveness of these algorithms on event sequences from a composable conveyor system; this system represents a new application area where use of frequent patterns for compressing the event sequence is likely to be important for decision-support and control.


Keywords: Frequent Episodes, Serial Episodes, Mining Event Sequences, Discovering Compressing Patterns, MDL, Inter-Event-Time constraints.

## 1. Introduction

Frequent pattern mining is an important problem in the area of data mining that has diverse applications in a variety of domains [11. Even though many algorithms have been proposed for frequent pattern mining, most of these methods produce a large number of frequent patterns. In addition, the patterns found are often redundant in the sense that many patterns are very similar. The redundancy and the large volume of the patterns discovered makes it difficult to use the mined patterns to gain useful insights into the data or to use them to extract rules which are effective for prediction, classification etc., in the application domain. Thus, finding a small set of non-redundant, relevant and informative pat-
terns that succinctly characterize the data, is an important problem of current interest.

There are many methods that are proposed for reducing the number of extracted frequent patterns. Many such methods concentrate on eliminating patterns that are deemed to be non-informative given the other frequent patterns. For example, in the context of transaction datasets, concepts such as closed [19, 26], non-derivable [5] and maximal 4, 15] itemsets were suggested to reduce the number of frequent itemsets extracted. Similarly, closed sequential patterns were proposed for sequence datasets $6,25,30$. Even though such methods result in some reduction in the number of patterns returned by the algorithm, the number of patterns still remains substantial. Also, the redundancy in the final set of patterns is, often, still large.

Recently, there have been other efforts for finding a small set of informative patterns that best describes the data. For example, (7] proposes a method for summarization of transaction datasets based on some ideas from information theory. They propose a method of selecting a subset of frequent itemsets to achieve a good lossy summarization of the database. Here each transaction is summarized by one itemset with as little loss of information as possible. In [27], which also proposes a lossy summarization, each transaction is covered, partially, by the largest frequent itemset. In contrast to these methods, 22,24 propose lossless summarization of transaction datasets using the Minimum Description Length (MDL) principle. A related approach called Tiling was used by 9, 29, again for a lossless summarization of the data.

In this paper, we address the problem of discovering a set of patterns that can achieve succinct lossless representation of temporal sequence data. We present algorithms that discover a small set of relevant patterns (which are special forms of serial episodes) which summarize the data well. We use the MDL principle 10 to define what we mean by summarizing the data well. The basic idea is that a set of patterns characterizes or summarizes the data sequence well, if the set of patterns can be used as a model to encode the data to achieve good compression.

As mentioned above, the MDL principle has been used earlier to obtain relevant and non-redundant subsets of frequent patterns. The idea was first explored by the Krimp algorithm $[24$ in the context of transaction data. This algorithm selects a subset of frequent itemsets which, when used for encoding the database, achieves good compression. Each selected itemset is assigned a code with shorter code lengths assigned to higher frequency itemsets. The algorithm tries to encode each transaction with the codes of itemsets which have minimal code lengths and which cover maximum number of items.

Similar strategies have been proposed for sequence data also [12, 13, 23]. For sequential data, unlike in the case of transaction data, the temporal ordering is important and this presents additional complications while encoding the data. For example, consider a single transaction, $t=A B C D$ from a transaction database and two itemsets $A C$ and $B D$. The codewords for $A C$ and $B D$ can encode the transaction $t$ (since the transaction is just a set of items). Now consider a sequence $s=A B C D$ and two serial episodes $A \rightarrow C$ and $B \rightarrow D$. Even though the occurrences of $A \rightarrow C$ and $B \rightarrow D$ would cover the sequence $s$, this information alone is insufficient for encoding the sequence. Since the order of events is important in sequential data, in order to get back the exact sequence, one needs to specify where exactly the occurrences of the episodes happen in the sequence. For example, we need to know that the $A$ and $C$ in the occurrence of $A \rightarrow C$ are not contiguous and that there is a $B$ in the gap between them. One
needs to have some way of taking care of such gaps while encoding the data with the occurrences of some frequent episodes. In general, the events in a sequence constituting an episode occurrence need not be contiguous, and different occurrences can have arbitrary temporal overlaps. An encoding scheme should be able to properly take care of this.

The previous approaches for using the MDL principle to summarize sequence data 121323 , explicitly record such gaps while encoding data, thus significantly increasing the encoding length. While the methods presented in 13,23 consider only sequential data without time stamps, the method in 12 does encode event sequences with time stamps also; but the encoding scheme needs to individually encode each event time stamp. In some cases, the resulting encoding may become even longer than the raw data $\sqrt{12}$. For the problem of identifying a relevant subset of frequent patterns, we are using the encoded length (of the data encoded with a subset of patterns), only as a figure of merit to compare different subsets. Hence, the fact of the encoding length becoming more than the raw data is, per se, not disallowed. However, the underlying philosophy of MDL principle suggests that one needs a good level of data compression to have confidence in a model. For example, even if the sequence data is iid noise and has no temporal structure, there would be some subset of patterns that would achieve lower encoded data length than other subsets. However, one expects that even the best such subset here would not achieve any appreciable level of data compression, thus suggesting that there are no significant temporal regularities in the data. On the other hand, for a sequence with significant temporal regularities, one expects good compression of the data sequence, if the method is able to discover the best temporal patterns and encode the sequence with them. In general, if we can discover some long episodes which occur many times, then their occurrences can encode many events in the data sequence thus giving rise to the possibility of data compression.

In this paper, we consider summarizing event sequences (having time stamps on events) using a pattern class consisting of serial episodes with fixed interevent times. We present algorithms for discovering a small subset of relevant frequent episodes that result in good compression of the data sequence. The novelty of our approach is that, in contrast to the existing schemes in 12, 13, 23, our method does not need to explicitly encode gaps in episode occurrences and the encoding scheme is such that we can retrieve the full data sequence with the time stamps on events, from the encoded sequence. The encoding of the data consists of only the start times of occurrences of various episodes; the gaps are determined from the fixed inter-event time constraints of the episodes. We show through simulations that our method results in better data compression. We also show, through empirical experiments, that the episodes that result in good data compression are also highly relevant for the dataset.

We also illustrate the benefits of our algorithm using an application, where it is important to both find relevant patterns and achieve good data compression. We consider streams of sensor-data from a composable conveyor system (CCS) [3, 21] that is useful for materials handling. In this system, several conveying units are dynamically composed to achieve the application objectives; consequently, utilizing the data streams to diagnose or reconfigure the system is important. The data consists of a sequence of predefined events such as, package entered a unit, package exited a unit, package arrived at an input port, etc. Such events occur at various units in the conveyor system during its routine operation. On this data stream, frequent serial episodes represent the routes (sequence of
units) over which packages were transported in the conveyor system. The interevent times corresponds to the various physical constraints such as time required for a package to move through a specific unit, the time required for two adjacent units to complete a handshake protocol to transfer packages between them etc. Thus a small set of relevant episodes can provide a good summary of the events in the conveyor system. We can use the discovered set of relevant episodes to achieve a lossless compression of the original temporal event sequence to support remote monitoring, diagnostics and visualization activities. We explain the system in more detail in Section 5.1.1. Using data obtained from a high-fidelity discrete event simulator of such conveyor systems, we demonstrate that our algorithms: (a) unearth a small set of relevant episodes that capture the essence of the transport through the system, and (b) our scheme achieves good data compression.

Even though our method is motivated by the above application, we show that our method is effective with other general sequential data as well. Apart from conveyor system data streams, we show the effectiveness of our methods with text data as well as on a few other real data sequences. These are the data sets that are used to illustrate the effectiveness of the algorithms presented in $12,13,23$. We compare the performance of our algorithm with these methods on these data sets as well as on the composable conveyor system data.

The rest of the paper is organized as follows. In Section 2 , we briefly review the formalism of episodes, introduce the new subclass of serial episodes and formally state the problem. Section 3 describes our encoding scheme for temporal data using our episodes. The various algorithms for mining and subset selection are explained in Section 4 and the experimental results are given in Section 5. We conclude the paper in Section 6 .

## 2. Problem Statement

### 2.1. Fixed Interval Serial Episodes

The data we consider is a sequence of $n$ events denoted as $\mathcal{D}=\left\langle\left(E_{1}, t_{1}\right),\left(E_{2}, t_{2}\right)\right.$, $\left.\ldots,\left(E_{n}, t_{n}\right)\right\rangle$, where $t_{i} \leq t_{i+1}$, and if $t_{i}=t_{i+1}$, then $E_{i} \neq E_{i+1}$, where $E_{i} \in \Sigma$, is the event-type, $\Sigma$ is the alphabet and $t_{i} \in \mathbb{Z}^{+}$is the time stamp of the $i^{t h}$ event. Note that we can have multiple events (of different types) all occurring at the same time instant. A $k$-node serial episode $\alpha$ is denoted as $e_{1} \rightarrow e_{2} \rightarrow \cdots \rightarrow e_{k}$ where $e_{i} \in \Sigma, \forall i$. An occurrence of $\alpha$ in $\mathcal{D}$ is a mapping $h:\{1 \ldots k\} \rightarrow\{1 \ldots n\}$, such that $e_{i}=E_{h(i)}, 1 \leq i \leq k$ and $t_{h(i)}<t_{h(j)}$, for $i<j$. An occurrence can be denoted by $\left(t_{h(1)}, \ldots, t_{h(k)}\right)$, the event times of the events constituting the occurrence. We call the interval $\left[t_{h(1)}, t_{h(k)}\right]$ as the occurrence window of this occurrence. (If $k=1$, then for the 1-node episode, the occurrence window is essentially a number which is the event time of that event). Consider an example event sequence

$$
\begin{array}{r}
\mathcal{D}_{1}=\langle(A, 1),(A, 2),(B, 3),(E, 4),(A, 5),(B, 6),(C, 6),(B, 7),(D, 8) \\
(C, 10),(E, 11)\rangle \tag{1}
\end{array}
$$

In the data sequence given in (1), a few occurrences of episode $A \rightarrow B \rightarrow C$ are $(1,3,6),(2,3,6),(5,6,10)$.

A fixed interval serial episode is a serial episode with fixed inter-event gaps.

A fixed interval serial episode is denoted as $\beta=e_{1} \xrightarrow{\Delta_{1}} e_{2} \xrightarrow{\Delta_{2}} \cdots \xrightarrow{\Delta_{k-1}} e_{k}$. We will be considering the class of fixed interval serial episodes, where $\Delta_{i} \leq T_{g}$, $\forall i$, with $T_{g}$ being a user specified upper bound on allowable gap. An occurrence of $\beta$ in $\mathcal{D}$ is a mapping $h:\{1 \ldots k\} \rightarrow\{1 \ldots n\}$, such that $e_{i}=E_{h(i)}, 1 \leq i \leq k$ and $t_{h(i+1)}-t_{h(i)}=\Delta_{i}>0$, for $1 \leq i<k$. For example, in sequence $\mathcal{D}_{1}$ in (1), there are two occurrences of episode $A \xrightarrow{2} B \xrightarrow{3} C$, namely $(1,3,6)$ and $(5,7,10)$. Note that the time of the first event of an occurrence completely specifies the entire occurrence. This property of the fixed interval serial episodes allows us to design a coding scheme that results in data compression. A $k$-node fixed interval serial episode $\alpha=e_{1} \xrightarrow{\Delta_{1}} e_{2} \xrightarrow{\Delta_{2}} \cdots \xrightarrow{\Delta_{k-1}} e_{k}$ is called injective if $e_{i} \neq e_{j}, \forall i, j, i \neq j$.

In the literature, different notions of frequency are defined for episodes depending on the type of occurrences we count. (For a discussion on various frequencies see $[2]$ ). An episode is said to be frequent if its frequency is above a given threshold. In this paper, we consider the number of distinct occurrences as the frequency. Two occurrences are distinct if none of the events of one occurrence is among events of the other. More formally, a set of occurrences, $\left\{h_{1}, h_{2}, \ldots, h_{m}\right\}$ of an episode $\alpha$ are distinct if for any $k \neq k^{\prime}, h_{k}(i) \neq h_{k^{\prime}}(j), \forall i, j$. This is a natural notion of frequency for an injective fixed interval serial episode because any pair of its occurrences with different start times will always be distinct.

In this paper, we consider injective fixed interval serial episodes and from now on we refer to injective fixed interval serial episodes simply as episodes whenever there is no scope for confusion.

### 2.2. Selecting a Subset of Episodes Using the MDL Principle

Pattern mining algorithms often output a large number of frequent episodes. Our goal is to isolate a small subset of them which are non-redundant and are relevant for the data. To formalize this goal, we use the MDL principle which views learning as data compression. The idea is that if we can discover all the relevant regularities in the data, then an encoding based on these would result in data compression 10. Thus, the goal is to find a model which allows us to encode the data in a compact fashion.

Given any model, $H$, let $L(H)$ denote the length for encoding the model $H$ and let $L(\mathcal{D} \mid H)$ be the length of the data when encoded using the model $H$. Given an encoding scheme, under the MDL principle our goal is to find a model $H$ that minimizes total encoded length, $L(H, \mathcal{D})=L(H)+L(\mathcal{D} \mid H)$.

For us, different models correspond to different subsets of the set of frequent fixed interval serial episodes. As mentioned earlier, an occurrence of such an episode is uniquely specified by its start time. Hence, by giving the code for the identity of the episodes and a list of start times, we can code all the events constituting the occurrences of this episode. (We explain our encoding scheme in the next section). Thus, large episodes with many occurrences would account for a large number of events in the data sequence thus decreasing $L(\mathcal{D} \mid H)$. Another advantage of our use of the MDL principle is that it inherently takes care of redundancy. Selecting episodes with minimal overlap among their occurrences would help reduce the final encoded length.

Under MDL, we are looking at lossless coding and hence the occurrences of the selected subset of episodes have to cover the entire dataset; i.e., every event in the data sequence should be part of an occurrence of (at least) one of

Table 1. A data sequence and its encoding

$$
\mathcal{D}_{2}=\langle(D, 1)(A, 2)(C, 3)(E, 3)(A, 4)(B, 4)(C, 5)(D, 5)
$$

$(B, 6)(C, 7)(E, 7)(C, 8)(C, 9)\rangle$

| Size of <br> Episode | Episode <br> Name | No. of <br> Occurrences | List of <br> Occurrences |
| :---: | :---: | :---: | :---: |
| 3 | $A \xrightarrow{2} B \xrightarrow{1} C$ | 2 | $\langle 2,4\rangle$ |
| 3 | $D \xrightarrow{2} E \xrightarrow{2} C$ | 2 | $\langle 1,5\rangle$ |
| 1 | $C$ | 2 | $\langle 3,8\rangle$ |

the selected set of episodes. We can always ensure this by adding a few 1-node episodes, as needed. We will give details of our encoding in the next section. Our main problem can now be stated as below

Problem 1. Given a data sequence $\mathcal{D}$ and a set of (frequent) fixed interval serial episodes, $\mathcal{C}=\left\{C_{1}, C_{2}, \ldots, C_{N}\right\}$, find the subset $H^{*} \subseteq \mathcal{C}$ such that

$$
H^{*}=\underset{H \subseteq \mathcal{C}}{\arg \min }\{L(H)+L(\mathcal{D} \mid H)\}
$$

## 3. The Encoding Scheme for Data

In this section, we explain our encoding scheme and derive the expression for encoded data length.

### 3.1. Encoding

Each model $H$ is a set of some fixed interval serial episodes whose occurrences cover the data. Given such an $H$, which forms the dictionary, the data is then encoded by specifying the start times of selected occurrences of the episodes.

We explain our encoding scheme through an example. Table 1 shows a data sequence $\mathcal{D}_{2}$ and its encoding using three arbitrarily selected episodes. Each row in the table describes one of the episodes used and the encoding for the part of the data, covered by the occurrences of that episode. There are four columns in the table. Column 1 gives the size of the episode in that row and the second column specifies the episode. The third column gives the number of occurrences of that episode (used for encoding) and the last column gives a list of start times. Hence the first row of Table 1 specifies: a 3-node episode, namely, $A \xrightarrow{2} B \xrightarrow{1} C$ and two of its occurrences starting at times 2 and 4 . Thus, the first row of Table 1 accounts for the events $(A, 2),(B, 4),(C, 5)$ and $(A, 4),(B, 6),(C, 7)$ in the data, which are the events constituting the two occurrences of the episode $A \xrightarrow{2} B \xrightarrow{1} C$ starting at time stamps 2 and 4 respectively. We can think of the first two columns of the table as our dictionary and the last two columns of the table as the encoding of the data. Note that the seventh event in $\mathcal{D}_{2},(C, 5)$, is part of the occurrence of $A \xrightarrow{2} B \xrightarrow{1} C$ starting at 2 and of $D \xrightarrow{2} E \xrightarrow{2} C$ starting at 1 . While this is allowed in our encoding, minimizing such overlaps would improve total encoded length. In fact, we use injective fixed interval serial episodes in order to avoid overlap of different occurrences of the same episode, since, as we mentioned earlier, occurrences of injective fixed interval serial episodes starting at different
time stamps are distinct. occurrences In Table 1, the first two episodes account for all but two events in the data and hence we added a 1-node episode (in row 3 of the table) to ensure that we cover the full data sequence. Our final encoded sequence would be a table like this. Each entry in the table is essentially a series of integers and our final encoded data would consists of a series of integers obtained by stringing together the rows in order.

### 3.2. Decoding

In this section we discuss how to decode the encoded data. The encoded data consists of rows of a table, with each row specifying an episode and its occurrences. In each row, we read the first value, which is the size of the pattern. If this value is $k$, the next $2 k-1$ integers correspond to the codes of the event types ( $k$ units) followed by the inter event gaps ( $k-1$ units). The next value in the encoded sequence corresponds to the number of occurrences of the episode. We then need to read that many values to obtain all the occurrence start times of that episode and complete reading the current row. Since we know when the row is complete, the next integer would be the first entry of the next row and we repeat the same process as above. From each start time of occurrence of an episode, we can roll out the corresponding events because we know the event types and the inter-event gaps. Once we are done with rolling out all the occurrences of all the episodes, we have to just sort the events based on the time stamps and delete duplicate occurrence ${ }^{11}$ to retrieve back the original data sequence.

### 3.3. Length of the Encoding

We have seen that once the dictionary is fixed, the data encoding is just a series of integers denoting the start times of occurrences of the patterns in the dictionary. Even though, we could use bit level integer encoding schemes like Elias codes 13, 28 and Universal codes 20, 23] for encoding integers (and have the size of encoding dependent on the value of the integer), we use the notion of fixed memory units instead. The reason being that, the MDL principle looks at utilizing the regularity in data to compress the data and hence the level of compression should not depend on the magnitude of data item. For example, the value of a time stamp, per se, does not have any regularity and the compression achieved by the encoding scheme, hence, should not be dependent on the values of time stamps. Therefore, for calculating the total encoded lengths we consider event types and times to be integers and assume that each such integer accounts for one unit in the encoded length. Since our aim is to compare different models, keeping all lengths in terms of one unit per integer is sufficient for us. (Here we are assuming that, in describing episodes in the dictionary, all event-types take the same amount of memory. We could, of course, use codes such as Hamming codes, to reduce expected length of representation of episodes. We do not consider such extra compression here). Later on, while comparing our method with the various

[^0]other methods, we use bit level encoding for calculating lengths of integers so that we can easily compare with the results of other algorithms.

Let model $H$ contain the episodes $\left\{F_{1}, F_{2}, \ldots, F_{K}\right\}$, with $\left|F_{i}\right|$ denoting the size of episode $F_{i}$. Each episode needs one integer to represent its size, $\left|F_{i}\right|$ integers for representing the event types and $\left(\left|F_{i}\right|-1\right)$ integers for inter-event gaps. Hence the first two columns of the table need $\sum_{i=1}^{K}\left(1+\left|F_{i}\right|+\left|F_{i}\right|-1\right)=$ $\sum_{i=1}^{K} 2\left|F_{i}\right|$ integers. This is $L(H)$.

Let $f_{i}$ be the number of occurrences of $F_{i}$ listed in the column 4 of our table. Then, columns 3 and 4 together need $\sum_{i=1}^{K}\left(f_{i}+1\right)$ integers. This is $L(\mathcal{D} \mid H)$. Thus for the model $H$, the total encoded length is

$$
\begin{equation*}
L(H, \mathcal{D})=L(H)+L(\mathcal{D} \mid H)=\sum_{i=1}^{K} 2\left|F_{i}\right|+\sum_{i=1}^{K}\left(f_{i}+1\right) \tag{2}
\end{equation*}
$$

For the encoding given by Table 1 , the length for the first row is $1+(3+2)+1+2=$ 9 and it is easy to see that the total encoded length is $9+9+5=23$.

The length of raw data can be taken to be $2|\mathcal{D}|$ where $|\mathcal{D}|$ is the number of events in the data. However, taking this as the length of uncompressed data may result in higher value for the compression achieved by an algorithm. This is because, even without finding any patterns, we can represent the raw data more compactly by simply using only 1 -node episodes in our encoding. If we have $M$ event types then we use $M$ 1-node episodes for encoding. The total encoded length, using Equation (2), and taking $K=M$ would be $2 M+|\mathcal{D}|+M$. (Note that $\sum_{i=i}^{M} f_{i}=|\mathcal{D}|$, because all occurrences of the $M$ 1-node episodes, together would exactly cover the data; also no event in the data would be part of occurrences of two different 1-node episodes). We call such an encoding trivial encoding. For the data sequence $\mathcal{D}_{2}$, the length for trivial encoding would be $5 \times 2+13+5=28$. Even though in this example the length of the trivial encoding is more than $2|\mathcal{D}|$, for real datasets we would have $|\mathcal{D}| \gg M$ and hence total encoded length of trivial encoding would be less than $2|\mathcal{D}|$. Hence in calculating data compression with our method, we compare the length of the trivial encoding with the length of the encoding using selected episodes.

## 4. Algorithms

In this section, we consider algorithms for discovering a subset of episodes which achieves good compression. Finding the optimal subset of episodes to minimize total encoded length is known to be NP-Hard 12,23 . Hence the methods we present here are approximation algorithms to Problem 1. We begin by presenting algorithm CSC-1 (CSC is for Constrained Serial episode Coding), which is a two phase method. This consists of discovering all frequent episodes through a depth-first search algorithm followed by a greedy method of selecting a subset based on maximum coverage and minimum overlap. We then present algorithm CSC-2, which directly mines for relevant fixed interval serial episodes from the data without first discovering all frequent episodes.

```
Algorithm 1 MineEpisodes \(\left(\mathcal{D}, T_{g}, f_{t h}\right)\)
Input: Sequence data \(\mathcal{D}\), the maximum inter-event gap \(T_{g}\) and frequency thresh-
old, \(f_{t h}\)
Output: The set of frequent episodes, \(\mathcal{C}\)
    \(\mathcal{A} \leftarrow\) Set of all frequent 1-node episodes in \(\mathcal{D}\), along with occurrence lists.
    for all \(A \in \mathcal{A}\) do
        \(\operatorname{ExploreDFS}\left(A, \mathcal{A}, T_{g}, f_{t h}\right) \quad \triangleright\) All the frequent episodes
                        \(\triangleright\) are added to the global list \(\mathcal{C}\)
    end for
```

```
Algorithm 2 ExploreDFS \(\left(\alpha, \mathcal{A}, T_{g}, f_{t h}\right)\)
Input: Episode \(\alpha\) with its occurrence list; \(\mathcal{A}\) : the set of frequent one node
episodes with its occurrence lists; \(T_{g}\) : Maximum inter-event gap; \(f_{t h}\) : frequency
threshold
Output: The set of frequent episodes \(\mathcal{C}\).
    for all \(A \in \mathcal{A} \backslash\{\) set of event-types in \(\alpha\}\) do
        occurrlist-for-delta \(\leftarrow\) find-lists \(\left(\alpha, A, T_{g}\right)\)
        for \(j=1 \rightarrow T_{g}\) do
            if \(\mid\) occurrlist-for-delta \((j)\left|\geq f_{t h} \times|\mathcal{D}|\right.\) then
                    \(\beta \leftarrow(\alpha \xrightarrow{j} A)\)
            \(\beta\).occurrencelist \(\leftarrow\) occurrlist-for-delta \((j)\)
            \(\mathcal{C} \leftarrow \mathcal{C} \cup \beta\)
            \(\operatorname{ExploreDFS}\left(\beta, \mathcal{A}, T_{g}\right)\)
            end if
        end for
    end for
```


### 4.1. First Algorithm: CSC-1

We first explain our depth-first mining algorithm and the basis for the greedy strategy for subset selection before describing the full CSC-1 algorithm (which is listed as Algorithm (4).

### 4.1.1. Mining

To obtain all frequent injective fixed interval serial episodes, we use a depth-first (also known as pattern-growth) strategy using occurrence windows. See 1, 17] for more details on depth-first strategies using occurrence windows. The basic idea is as follows. First we find all 1-node frequent episodes (which are eventtypes that occur often enough) and for each frequent 1-node episode, keep its occurrence list which is a list of event times where the 1-node episode occurs in the data. Let $\alpha$ be an episode and suppose we are given a list of all its occurrence windows (also called occurrence list). Recall that the occurrence window of an episode $\alpha$ is an interval $\left[t_{s}, t_{e}\right]$, where $t_{s}$ and $t_{e}$ are the times of the first and last events of $\alpha$ in this occurrence. If we know all occurrence windows of 1-node episode $A$, then we can easily check whether $\left[t_{s}, t_{e}+j\right]$ is an occurrence window of $\alpha \xrightarrow{j} A$. Thus we can easily calculate the occurrence windows of episodes such as $\alpha \xrightarrow{j} A$, for all allowed $(A, j)$, and hence find frequent episodes of next size.

```
Algorithm 3 find-lists \(\left(\alpha, A, T_{g}\right)\)
Input: Episode \(\alpha\) and one node episode \(A\) with their occurrence lists.
Output: The array, occurrlist-for-delta storing the occurrence lists with different
gaps.
    for all \(\left[t_{s}^{\alpha}, t_{e}^{\alpha}\right] \in \alpha\).occurrencelist do
            Let \(t^{A}\) be the first occurrence of \(A\) after \(t_{e}^{\alpha} \quad \triangleright\) NULL if no such
    occurrence
    while \(t^{A} \neq N U L L\) and \(t^{A}-t_{e}^{\alpha} \leq T_{g}\) do
            \(j \leftarrow t^{A}-t_{e}^{\alpha} ;\)
            Add \(\left[t_{s}^{\alpha}, t^{A}\right]\) to occurrlist-for-delta \((j) \quad\) Corresponding to \(\alpha \xrightarrow{j} A\)
            \(t^{A} \leftarrow\) next occurrence of \(A \quad \triangleright\) NULL if there is no next occurrence
            end while
    end for
    return occurrlist-for-delta
```

By doing this recursively, we find all frequent episodes. The implementation of this idea is described in Algorithms 1 to 3 .

The main function is MineEpisodes, listed as Algorithm 1. This is a wrapper function, which finds, for each event-type $A \in \mathcal{A}$, where $\mathcal{A}$ is the set of frequent 1-node episodes, all the frequent fixed interval serial episodes with $A$ as the prefix, using the ExploreDFS function (in line 3). The frequency threshold value, $f_{t h}$ is user specified, and is given as a fraction of the data length (see line 4).

The procedure ExploreDFS, listed as Algorithm 2, is a recursive function. Given an input episode $\alpha$, it finds all the frequent right extensions of $\alpha$, i.e., all the frequent episodes with $\alpha$ as prefix. For each $A$, the procedure initially finds the occurrence windows for the episodes $\alpha \xrightarrow{j} A, 1 \leq j \leq T_{g}$, where $T_{g}$ is the maximum allowed inter-event gap (line 2 , Algorithm 2 , by calling the procedure find-lists, which is explained below. The function then recursively goes deeper for each frequent episode, $\alpha \xrightarrow{j} A$ (line 8).

The find-lists procedure (listed as Algorithm 3), takes as input, episode $\alpha$ and event type $A$ (which is also a 1-node episode), and finds the occurrence windows for all the episodes $\alpha \xrightarrow{j} A, j \leq T_{g}$. For each occurrence window $\left[t_{s}^{\alpha}, t_{e}^{\alpha}\right]$ of $\alpha$, it looks for all the occurrences $t^{A}$ of $A$ such that the new occurrence window satisfies the maximum inter-event gap constraint $T_{g}$ (the condition for while loop in line 3). An occurrence window satisfying the constrain is then added to the occurrence list corresponding to the episode $\alpha \xrightarrow{j} A$, where $j=t^{A}-t_{e}^{\alpha}$ (line 5).

Using these algorithms, we get all injective frequent fixed interval serial episodes. We then go on to select the best representative subset.

### 4.1.2. Selection Strategy

Given a data sequence $\mathcal{D}$, let $\alpha$ be an $N$-node fixed interval serial episode with frequency $f_{\mathcal{D}}^{\alpha}$. We define the score of $\alpha$ in $\mathcal{D}$ as

$$
\begin{equation*}
\operatorname{score}(\alpha, \mathcal{D})=f_{\mathcal{D}}^{\alpha} N-\left(2 N+f_{\mathcal{D}}^{\alpha}+1\right) \tag{3}
\end{equation*}
$$

Recall that $2 N+f_{\mathcal{D}}^{\alpha}+1$ is the total encoded length for encoding all the events that constitute the $f_{\mathcal{D}}^{\alpha}$ occurrences of $\alpha$. (Recall from Section 3.3 that $2 N$ is
the length for encoding $\alpha$ and $f_{\mathcal{D}}^{\alpha}+1$ is the total length for encoding all $f_{\mathcal{D}}^{\alpha}$ occurrences of $\alpha$.). It is easy to see that $f_{\mathcal{D}}^{\alpha} N$ is a lower bound on the encoded data length for trivially encoding all the events in the $f_{\mathcal{D}}^{\alpha}$ occurrences of (the $N$-node episode) $\alpha$ with 1 -node episodes ${ }^{2}$.

If $\operatorname{score}(\alpha, \mathcal{D})>0$, then $\alpha$ is called a useful candidate since selecting it can improve encoding length by at least the value of $\operatorname{score}(\alpha, \mathcal{D})$, in comparison to trivial encoding. From Equation (3), we can easily see that $\operatorname{score}(\alpha, \mathcal{D})>0$, if $f_{\mathcal{D}}^{\alpha}>\frac{2 N+1}{N-1}$. Thus the episode $\alpha$ will be a useful candidate if $f_{\mathcal{D}}^{\alpha}>5$ for $|\alpha|=2$ and $f_{\mathcal{D}}^{\alpha}>3$ for $|\alpha| \geq 3$. But selecting any useful candidates would not lead to efficient encoding. For any pair of selected episodes, we also want their occurrences to have least number of events in common. Our subset selection procedure for encoding the data is based on greedy selection of episodes whose occurrences cover large number of events in the data and have low level of overlap with the occurrences of other selected episodes.

Let $\mathcal{F}_{s}=\left\{\beta_{1}, \beta_{2}, \ldots, \beta_{S}\right\}$ be a set of episodes of size greater than one. Given any such $\mathcal{F}_{s}$, let Let $L\left(\mathcal{F}_{s}, \mathcal{D}\right)$ denote the total encoded length of $\mathcal{D}$, when we encode all the events which are part of the occurrences of episodes in $\mathcal{F}_{s}$, by using episodes in $\mathcal{F}_{s}$ as per our encoding scheme and encode the remaining events in data, if any, by episodes of size one.

Given any two episodes $\alpha, \beta$, let $O M(\alpha, \beta)$ denote the number of events in the data that are covered by occurrences of both $\alpha$ and $\beta$. We call OM the Overlap Matrix.

We define, for $\alpha \notin \mathcal{F}_{s}$

$$
\begin{equation*}
\operatorname{overlap-score}\left(\alpha, \mathcal{D}, \mathcal{F}_{s}\right)=f_{\mathcal{D}}^{\alpha} N-\sum_{\beta_{i} \in \mathcal{F}_{s}} O M\left(\alpha, \beta_{i}\right)-\left(2 N+f_{\mathcal{D}}^{\alpha}+1\right) \tag{4}
\end{equation*}
$$

Note that overlap-score $\left(\alpha, \mathcal{D}, \mathcal{F}_{s}\right)=\operatorname{score}(\alpha, \mathcal{D})-\sum_{\beta_{i} \in \mathcal{F}_{s}} O M\left(\alpha, \beta_{i}\right)$ and is another measure for the gain in encoding length, when we add $\alpha$ to $\mathcal{F}_{s}$. The measure has an interesting property as explained below.

Proposition 1. If overlap-score $\left(\alpha, \mathcal{D}, \mathcal{F}_{s}\right)>0$, then $L\left(\mathcal{F}_{s}, \mathcal{D}\right)>L\left(\mathcal{F}_{s} \cup\{\alpha\}, \mathcal{D}\right)$
Proof. First, note that the difference in encoding will only be in the section of the data, where the encodings using $\mathcal{F}_{s}$ and $\mathcal{F}_{s} \cup\{\alpha\}$ differ. As is easy to see, $\sum_{\beta_{i} \in \mathcal{F}_{s}} O M\left(\alpha, \beta_{i}\right)$ is an upper bound on the number of events of the occurrences of $\alpha$, that are shared with the occurrences of episodes in $\mathcal{F}_{s}$. Hence, $f_{\mathcal{D}}^{\alpha} N-$ $\sum_{\beta_{i} \in \mathcal{F}_{s}} O M\left(\alpha, \beta_{i}\right)$ is a lower bound on the number of events not covered by anyone in $\mathcal{F}_{s}$ and which are covered by the occurrences of $\alpha$. Hence if we use $\mathcal{F}_{s}$, it takes at least $f_{\mathcal{D}}^{\alpha} N-\sum_{\beta_{i} \in \mathcal{F}_{s}} O M\left(\alpha, \beta_{i}\right)$ units for encoding these events using size- 1 episodes. In contrast, by adding $\alpha$ to the $\mathcal{F}_{s}$, it takes $2 N+f_{\mathcal{D}}^{\alpha}+1$ units to encode those occurrences, independent of the number of events $\alpha$ shares with other episodes. Thus the reduction in encoding length between the two is at least $f_{\mathcal{D}}^{\alpha} N-\sum_{\beta_{i} \in \mathcal{F}_{s}} O M\left(\alpha, \beta_{i}\right)-\left(2 N+f_{\mathcal{D}}^{\alpha}+1\right)$, which is the overlap-score $\left(\alpha, \mathcal{D}, \mathcal{F}_{s}\right)$. Hence the result.

[^1]Proposition 1 says that, with respect to an already selected set of episodes $\mathcal{F}_{s}$, adding an episode $\alpha$ with overlap-score $\left(\alpha, \mathcal{D}, \mathcal{F}_{s}\right)>0$ to the set $\mathcal{F}_{s}$, would only reduce the total length of encoded data. Our greedy heuristic is to select the episode with the maximum overlap-score. (Note that, by definition, overlap-score $\left(\alpha, \mathcal{D}, \mathcal{F}_{s}\right)=\operatorname{score}(\alpha, \mathcal{D})$, if $\left.\mathcal{F}_{s}=\emptyset\right)$.

### 4.1.3. $C S C-1$

The CSC-1 algorithm selects a best subset of the frequent episodes based on minimizing encoding length. The algorithm takes as input, $K$, the maximum number of episodes (of size greater than 1) in the final selected subset. Thus, it can be used a method to select the 'best- $K$ ' episodes or to select the best subset to achieve maximum compression (by choosing a very large value of $K$ ).

The CSC-1 algorithm for selecting a good subset of (maximum $K$ ) episodes is listed as Algorithm 4. The algorithm runs in iterations of mining frequent fixed interval serial episodes from the data sequence, then selecting, one by one, a set of good encoding episodes from the mined set and finally deleting the occurrences of the selected episodes from the sequence. The process is repeated until we found $K$ good episodes or we cannot find any episode that can give any gain in encoding.

In each iteration of the while loop (lines 3 18), we first mine the set of frequent fixed interval serial episodes, $\mathcal{C}$, using the MineEpisodes procedure, explained in Section 4.1.1 (line 5). We next calculate the $O M$ matrix (line 6) and then calculate the overlap-score. Each iteration of the repeat loop (lines 7 7 15) looks for an episode in the current candidates set, $\mathcal{C}$, which has the highest positive overlap-score. If such an episode is found, it is added to the set $\mathcal{F}_{s}$. This greedy strategy is justified by Proposition 1. The set $\mathcal{F}_{s}$, thus contains all the episodes selected in the repeat loop. The repeat loop is broken, when no episode with positive overlap-score exists in the current candidate set or we have selected $K$ episodes(line 15). Then all the events in the occurrences of the episodes in $\mathcal{F}_{s}$ are deleted from the data (line 16). We then once again repeat the process of finding frequent episodes from this modified data and selecting a subset of episodes from this episode set.

The while loop runs as long as the selected set size is less than $K$ and it finds at least one episode that increases encoding efficiency. This condition is checked in lines 12,13 . When we cannot find any more episodes with positive overlap-score, we encode the remaining events in the data with 1-node episodes (lines 19.20 .

The only remaining part in the CSC-1 algorithm is the calculation of the matrix $O M$ (line 6), which we explain now. The procedure FindOverlapMatrix, listed as Algorithm 5, utilizes the occurrence lists for all the frequent episodes (obtained from Algorithms 2 and 3), to calculate $O M$ matrix by one more pass over data using the standard Finite State Automata(FSA) based method for tracking episode occurrences $[2,14,16]$. FSAs in $[2,14,16$ are used to track occurrences of episodes. Algorithm 5 uses FSAs for a different purpose since we already have the occurrences of the episodes. Here, the FSAs associated with episodes, and hence some times called episode automata, are used to find, for each event $\left(E_{i}, t_{i}\right)$ in the sequence, the set of episodes for which this event is part of one of their occurrences. For all such pairs of episodes in that set, the $O M$ matrix is incremented by 1 .

In Algorithm 5, each state of an episode automaton specifies the state of a

```
Algorithm 4 CSC-1 ( \(\left.\mathcal{D}, T_{g}, f_{t h}, K\right)\)
\(\overline{\text { Input: Data sequence } \mathcal{D} \text {; maximum inter-event gap } T_{g} \text {; threshold } f_{t h} \text {; maximum }}\)
number of selected episodes \(K\).
Output: The set of selected frequent episodes \(\mathcal{F}\)
    \(\mathcal{F} \leftarrow \emptyset\)
    coveringexists \(\leftarrow\) true
    while coveringexists and \(|\mathcal{F}|<K\) do
        \(\mathcal{F}_{s} \leftarrow \emptyset\)
        \(\mathcal{C} \leftarrow \operatorname{MineEpisodes}\left(\mathcal{D}, T_{g}, f_{t h}\right)\)
        \(O M \leftarrow\) FindOverlapMatrix \((\mathcal{D}, \mathcal{C})\)
        repeat
            \(\alpha \leftarrow \arg \max _{\gamma \in \mathcal{C}}\) overlap-score \(\left(\gamma, \mathcal{D}, \mathcal{F}_{s}\right)\)
                if overlap-score \(\left(\alpha, \mathcal{D}, \mathcal{F}_{s}\right)>0\) then
                    \(\mathcal{F}_{s} \leftarrow \mathcal{F}_{s} \cup\{\alpha\}\)
                    \(\mathcal{C} \leftarrow \mathcal{C} \backslash \alpha\)
                else if overlap-score \(\left(\alpha, \mathcal{D}, \mathcal{F}_{s}\right) \leq 0\) and \(\mathcal{F}_{s}=\emptyset\) then
                    coveringexists \(\leftarrow\) false
                end if
        until overlap-score \(\left(\alpha, \mathcal{D}, \mathcal{F}_{s}\right) \leq 0\) or \(\left|\mathcal{F} \cup \mathcal{F}_{s}\right|=K\)
        \(\mathcal{D} \leftarrow \mathcal{D} \backslash\) (occurrences of \(\mathcal{F}_{s}\) )
        \(\mathcal{F} \leftarrow \mathcal{F} \cup \mathcal{F}_{s}\)
    end while
    \(\mathcal{A} \leftarrow\) Size-1 episodes in remaining \(\mathcal{D}\)
    \(\mathcal{F} \leftarrow \mathcal{F} \cup \mathcal{A}\)
    return \(\mathcal{F}\)
```

current occurrence and is denoted by $\left(\alpha, j, t_{s}\right)$, where $\alpha$ is the episode, $j$ is the state of the automaton to which it is expecting to transit (this means that for the current occurrence of the episode, it has seen events for $\alpha[1]$ to $\alpha[j-1]$ satisfying the inter-event constraints and is waiting for the event to occur with event-type $\left.\alpha[j]^{3}\right]$, and $t_{s}$ denotes the start time of the current occurrence. An episode automaton corresponding to a $N$ sized episode has $(N+1)$ states. An automaton for episode $\alpha$ is in state $j=1$ while it is waiting for the event with event-type $\alpha[1]$ and is in state $j=|\alpha|+1$ at the end of its occurrence.

Each event-type, $E$, is associated with a data structure called waits. waits $(E)$ contains the list of automaton waiting for the event with event-type $E$ to occur. Whenever an automaton corresponding to an episode $\alpha$ transits to a state $j$, in wait for an event-type $E$, that corresponding automaton state ( $\alpha, j, t_{s}$ ) is added to waits (E) (lines 10 and 16, Algorithm 5).

For each episode, $\alpha$, as we go along the sequence, the start of one of its occurrences (which we know apriori) initiates an automaton for the episode $\alpha$. As we parse through the events in $\mathcal{D}$ (for loop in line 4 ), each automaton corresponding to occurrences of the episodes will be waiting for a specific event to come up. On seeing the event $\left(E_{i}, t_{i}\right)$, those episodes (automaton) that were waiting for the event type, $E_{i}$, (which is obtained from waits $\left(E_{i}\right)$ (the inner for loop)) compares the start times with $t_{i}$ to see whether the event is part of the current occurrence (line 8 and line 13, Algorithm 5). If the constraints are satisfied, then the episode

[^2]```
Algorithm 5 FindOverlapMatrix \((\mathcal{D}, \mathcal{C})\)
Input: \(\mathcal{D}\), the sequence dataset; \(\mathcal{C}\), the set of frequent multiple node episodes in
\(\mathcal{D}\).
Output: OverlapMatrix, the \(|\mathcal{C}| \times|\mathcal{C}|\) matrix denoting the number of shared
events.
    for all \(\alpha \in \mathcal{C}\) do
    Add automate state \((\alpha, 1, \emptyset)\) to waits \((\alpha[1])\)
    end for
    for all event \(\left(E_{i}, t_{i}\right) \in \mathcal{D}\) do
        overlaplist \(\leftarrow \emptyset\)
        for all automata \(\left(\alpha, j, t_{s}\right) \in\) waits \(\left(E_{i}\right)\) do
            if \(j=1\) then
                    if \(\exists\) occurrence of \(\alpha\) starting at \(t_{i}\) then
                    Add \(\alpha\) to overlaplist
                    Add automata state \(\left(\alpha, j+1, t_{i}\right)\) to waits \((\alpha[2])\)
                    end if
                else
                    if \(\left(t_{i}-t_{s}\right)=\alpha . \Delta[j-1]\) then
                    Add \(\alpha\) to overlaplist
                    if \(j \neq|\alpha|\) then \(\quad \triangleright\) If \(j=|\alpha|\), we just retire the automaton
                            \(\operatorname{Add}\left(\alpha, j+1, t_{s}\right)\) to waits \((\alpha[j+1])\)
                    end if
                    Remove \(\left(\alpha, j, t_{s}\right)\) from waits \(\left(E_{i}\right)\)
                    end if
                end if
        end for
        Increment OverlapMatrix \([\alpha][\beta]\) by \(1, \forall \alpha, \beta \in\) overlaplist, \(\alpha \neq \beta\)
    end for
```

is noted as having accepted that event (in line 9 and line 14 Algorithm 5) and moves to the next state. Finally for those episodes that have accepted the current event, the corresponding $O M$ counts are incremented (in line 222).

### 4.1.4. Complexity of the Algorithm

There are three main steps in each iteration of Algorithm 4 the mining step (call to MineEpisodes in line 5), the calculation of the $O M$ matrix (line 6) and the selection step (the repeat loop). The mining step involves the generation of occurrence start times of the selected episodes. For each episode $\alpha$, this is done by a single pass over the occurrences of its prefix subepisode of size $(|\alpha|-1)$ and 1 -node suffix subepisode. The number of occurrences is of the order of $|\mathcal{D}|$ and hence for $\mathcal{C}$ selected episodes, the mining step takes $O(|\mathcal{C}||\mathcal{D}|)$.

The $O M$ matrix calculation depends on the number of episodes accepting each event in $\mathcal{D}$. Let $K_{i}$ denote the number of episodes in $\mathcal{C}$ containing eventtype $A_{i}$. Let $P_{i}$ denote the fraction of the events of type $A_{i}$ in $\mathcal{D}$. Then each event-type $A_{i}$ occurs $P_{i}|\mathcal{D}|$ times and each of its occurrence is associated with $O\left(K_{i}^{2}\right)$ updates of the $O M$ matrix. Hence, the runtime for the calculation of the $O M$ matrix is $O\left(\left(\sum_{i=1}^{M} P_{i} K_{i}^{2}\right)|\mathcal{D}|\right)=O\left(|\mathcal{C}|^{2}|\mathcal{D}|\right)$, since $K_{i}<|\mathcal{C}|$.

The repeat loop involves calculating the $\operatorname{overlap-score}\left(\gamma, \mathcal{D}, \mathcal{F}_{s}\right), \forall \gamma \in \mathcal{C}$, which takes $O\left(|\mathcal{C}|\left|\mathcal{F}_{s}\right|\right)$ and hence for $\left|\mathcal{F}_{s}\right|$ iterations takes $O\left(|\mathcal{C}|\left|\mathcal{F}_{s}\right|^{2}\right)$. Since

```
Algorithm 6 BestExtensions \(\left(\mathcal{D}, T_{g}\right)\)
Input: Data sequence \(\mathcal{D}\); maximum inter-event gap \(T_{g}\).
Output: Selected set of relevant candidate episodes \(\mathcal{C}\).
    \(\mathcal{A} \leftarrow\) Set of all 1-node episodes in \(\mathcal{D}\)
    for all \(A \in \mathcal{A}\) do
        patt \(\leftarrow A\)
        newpatt \(\leftarrow\) patt
        while \(\left(\right.\) patt \(\leftarrow \operatorname{Extensions}\left(\right.\) patt \(\left.\left.\left., \mathcal{A}, T_{g}\right)\right)\right) \neq N U L L\) do
            newpatt \(\leftarrow\) patt
        end while
        if score \((\) newpatt, \(\mathcal{D})>0\) then
            \(\mathcal{C} \leftarrow \mathcal{C} \cup\{\) newpatt \(\}\)
        end if
    end for
    return \(\mathcal{C}\)
```

$\left|\mathcal{F}_{s}\right|<|\mathcal{C}|$ the total computation for each iteration of our method is $O\left(|\mathcal{C}|^{2}|\mathcal{D}|+\right.$ $\left.|\mathcal{C}|^{3}\right)$. Hence the total execution is critically dependent on $|\mathcal{C}|$, the number of frequent episodes mined from the data.

### 4.2. An improved algorithm: CSC-2

The runtime of the CSC-1 method as shown in the previous section is $O\left(|\mathcal{C}|^{2}|\mathcal{D}|+\right.$ $|\mathcal{C}|^{3}$ ) and hence depends directly on the size of the mined candidate set of episodes, $|\mathcal{C}|$. Thus an increase in the size of $\mathcal{C}$ would increase the runtime by a cubic order. CSC-2 addresses exactly this problem. CSC-2 proceeds exactly as CSC-1 except that the set $\mathcal{C}$ (line 5, Algorithm (4) that it considers at each iteration is a small set rather than the full set of frequent fixed interval serial episodes. Earlier, in the MineEpisodes function, we have mined for all frequent episodes in a depth-first manner and gave that as the set $\mathcal{C}$. Now, instead of that, we obtain a set of episodes, each one of which is a best possible episode in one of the paths in the depth-first search tree. Here, the best episode is decided in a greedy fashion based on its contribution to coding efficiency. Note that the CSC-2 algorithm does not need any frequency threshold to be specified by the user because we do not mine for 'frequent' episodes.

The algorithm CSC-2 is same as Algorithm 4 except that the call to MineEpisodes (line 5. Algorithm (4) is replaced by a call to a new procedure BestExtensions. The rest of Algorithm 4 remains the same for both CSC-1 and CSC-2. Hence, we do not provide separate pseudo code for CSC-2. The procedure BestExtensions, which selects a set of candidate episodes, $\mathcal{C}$, is listed as Algorithm 6 .

For each event-type $A$, Algorithm 6 extends the 1-node episode $A$ with an event-type $B$ and gap $i$ to form the episode $A \xrightarrow{i} B$ such that

$$
<B, i>=\underset{\substack{<C, j \geq \\ C \neq A, j \leq T_{g}}}{\arg \max } \operatorname{score}(A \xrightarrow{j} C, \mathcal{D})
$$

The episodes are grown by right extension until none of the immediate extensions

```
Algorithm 7 Extensions \(\left(\alpha, \mathcal{A}, T_{g}\right)\)
Input: Episode \(\alpha\) with its occurrence list; \(\mathcal{A}\) : the set of frequent one node
episodes with its occurrence lists; \(T_{g}\) : Maximum inter-event gap.
Output: Episode extension with the highest frequency and minimum inter event
gap.
    \(\max\) freq \(\leftarrow \frac{2 *(|\alpha|+1)+1}{|\alpha|} \triangleright\) Min frequency for which coding efficiency is
                                    \(\triangleright\) achieved
    mingap \(\leftarrow T_{g}\)
    maxpatt \(\leftarrow N U L L\)
    for all \(A \in \mathcal{A} \backslash\{\) set of event-types in \(\alpha\}\) do
        occurrlist-for-delta \(\leftarrow\) find-lists \(\left(\alpha, A, T_{g}\right) \quad \triangleright\) find-lists in Algorithm 3
        bestgap \(\leftarrow \arg \max _{j \leq T_{g}} \mid\) occurrlist-for-delta \((j) \mid\)
        if \(\mid\) occurrlist-for-delta(bestgap) \(\mid \geq \max f r e q\) then
            if \(\mid\) occurrlist-for-delta(bestgap) \(\mid>\operatorname{maxfreq}\) OR bestgap \(<\) mingap
    then
            \(\beta \leftarrow(\alpha \xrightarrow{\text { bestgap }} A)\)
            及.occurrlist \(\leftarrow\) occurrlist-for-delta(bestgap)
            \(\beta\).freq \(\leftarrow \mid\) occurrlist-for-delta(bestgap) \(\mid\)
                    maxpatt \(\leftarrow \beta\)
                    maxfreq \(\leftarrow \beta\).freq
                    mingap \(\leftarrow\) bestgap
            end if
        end if
    end for
    return maxpatt
```

has a better score than the current episode. At this point we add the episode to the list of candidate episodes $\mathcal{C}$.

The extension algorithm is given in Algorithm 7. Among the immediate extensions, Algorithm 7 selects the one with the maximum frequency. If multiple extensions have the same frequency, then the one with the minimum gap is selected (lines 7 and 8 , Algorithm 7 ).

The number of candidates generated using this procedure is at most $M$, the size of the alphabet. In general, this is much smaller than the size of $\mathcal{C}$ in CSC-1. However, for large alphabet (i.e., large number of event types), even this could be costly. Then we can modify CSC-2 to keep only the 'best few' episodes in $\mathcal{C}$.

## 5. Experiments

In this section, we present the experimental results for our method and compare its performance with that of SQS $[23$ and GoKrimp $\sqrt{13}$. Since we observed that algorithm CSC-2 gives similar patterns as CSC-1, but is much more efficient time-wise, we present simulation results with CSC-2 only.

We consider three different types of data. The first type of data is the conveyor system data, since this application was the motivation for us to come up with the new subclass of fixed interval serial episodes and our novel encoding scheme. The set of data sequences that we consider are generated by a detailed simulator of composable conveyor systems. As we mentioned earlier, this is an application

Table 2. Summary of Datasets.

| Datasets | Events | Sequences | Classes | Alphabet Size, $M$ |
| :---: | :---: | :---: | :---: | :---: |
| jmlr | 75646 | 787 | NA | 3846 |
| aslbu | 36500 | 441 | 7 | 190 |
| aslgt | 178494 | 3493 | 40 | 47 |
| auslan2 | 1800 | 200 | 10 | 16 |
| context | 25832 | 240 | 5 | 56 |
| pioneer | 9766 | 160 | 3 | 92 |

area where compression achieved may be useful on its own. We explain more about the problem and the data in Section 5.1.1.

The second type of data that we consider is text data. We consider JMLR data set which contains 787 abstracts from the Journal of Machine Learning Research. This is a sequential symbolic data, but the time stamps, so to say, are just the serial number of the word in the sequence. This dataset is used to see how the various algorithms perform in unearthing relevant phrases (patterns) related to machine learning research. We show the top 20 patterns found by different methods on this dataset.

The final collection of data that we consider consists of five real-world data sets introduced in 18]. Each of these is a database of symbolic interval sequences with class labels; i.e., events in these databases are denoted by a symbol and an interval of its occurrence. As in 13], we consider the start and end of the interval to be two different events with different event-types. For example, the event interval ( $e, t 1, t 2$ ), where $e$ is the symbol and $[t 1, t 2]$ is the interval of its occurrence, would be considered as two events $(e-, t 1)$ and $(e+, t 2)$. Table 2 gives some relevant details regarding these data sets. The reason for using these datasets is that these are the only real world data sets on which results of performance by SQS and GoKrimp are reported. On all these data sets we compare the data compression and the classification accuracy of our method with that achieved by SQS and GoKrimp.

Our algorithms are implemented in $\mathrm{C}++$ and the experiments were executed single threaded on an Intel i7 4-core processor with 16 GB of memory running over a linux OS. The source code for our algorithms will be made available on request. The implementations of SQS and GoKrimp were obtained from the respective authors. For the GoKrimp algorithm, there is a significance level parameter (for a test called sign test), which is set to the default value of 0.01 and the minimum number of pairs needed to perform a sign test is set to the their default value of 25 . For our algorithm, the maximum inter-event gap was set to 5 for all the datasets. The $K$ value in the CSC- 2 algorithm, denoting the maximum number of patterns to be selected is set to infinity (which means maximum possible selection of episodes) unless otherwise stated.

### 5.1. Results on Composable Conveyor System data sequences

Before giving the results, we first give a brief explanation about composable conveyor systems 3,21.


Fig. 1. A two-input, two-output conveyor system. Packages enter via one of the inputs $I 1$ or $I 2$ and leave the system via one of the outputs as illustrated. The system is composed using instances of Segments and Turns 3. 31.

### 5.1.1. Conveyor System

Material handling conveyor systems move packages or material units from one or more inputs to specified outputs along predetermined paths. Such systems are used extensively in manufacturing, packaging, packet sorting etc. A typical conveyor system with two inputs and two outputs is shown in Fig. 1. This system is composed using instances of Segment and Turn units that each operate autonomously. A Segment moves a package over a predetermined length over its belt. A Turn is a unit that can serve as a merger or splitter for package flow. Each segment and turn unit has a predetermined maximum speed of operation and each unit has local sensors and actuators that are used to autonomously control its local behavior. The simulator we used produces a detailed event-trace of every change of state (event) in the sensors and actuators of each unit as packages move through the system.

The movement of the packages in the conveyor system is transformed to a temporal sequence as follows. We label the exit points of each input, output, turn and segment. We consider the transfer of a package from one conveyor unit to another as an event. The unit represents the type of the event and the time at which each event occurs is recorded as the event time. Hence, when a package moves along a path, an ordered sequence of events is recorded. All the events corresponding to one package may not be contiguous because during the same time other packages may be moving along other paths.

We consider three conveyor system datasets coming from three different topologies. Each topology can be identified by the various paths from the inputs to the outputs and the package input rates. We consider the package input arrival rates to be Poisson. The details of the three datasets is given in Table 3 and the structure of the topologies are given in Figures 1,2 and 3 .

For the conveyor system data, there is only one data sequence corresponding to each topology. The GoKrimp algorithm works using a statistical dependency test called SignTest, which does not work on single sequence datasets. The implementation of GoKrimp asks such sequences to be broken into a number smaller sequences. Hence, the datasets were broken into sequences of size 25 , 50 and 100 respectively for the $2 \mathrm{I}-2 \mathrm{O}, 3 \mathrm{I}-3 \mathrm{O}$ and the Package Sorter Topologies (creating longer sizes for sequences with longer and higher number of paths). Also, the conveyor system datasets are time stamped datasets. The SQS and


Fig. 2. Three-input, three-output conveyor system.


Fig. 3. Package Sorter topology.
Table 3. Various Conveyor system topologies, input rates, alphabet size $M$, number of events (in the final data set with the topology) and the paths in each topology.

| copology. |  |  |  |  |
| :---: | :---: | :---: | :--- | :--- |
| Topology | InputRate <br> (Poisson) | $M$ | Events | Paths |
| $2 \mathrm{I}-2 \mathrm{O}$ | 0.6 | 16 | 7659 | P1. I1 S1 T1 S4 T3 S7 T4 S8 O2 <br>  |
|  |  |  |  | $\mathbf{P 2 .}$ I2 S6 T3 S7 T4 S5 T2 S3 O1 |

Table 4. Patterns discovered by various algorithm in the Conveyor System data sets. The patterns for the CSC-2 is shown without the inter event gap. PS stands for Package Sorter.

| Method | Topology | Top Discovered Patterns |
| :---: | :---: | :---: |
| GOKRIMP | $2 \mathrm{I}-2 \mathrm{O}$ | [ S8 T4 S4 S2 T1 ], [ T3 I2 S3 S6 ], [ I1 O2 S8 ], [ S6 S4 S7 S5 ], [ S2 T1 S1 ], [ S8 T4 S4 ] |
|  | 3I-3O | [ T5 T9 ], [ S10 I1 S14 ], [ S2 T1 ], [ T3 T6 S18 ], [ S16 S12 ], [ S8 S6 ] |
|  | PS | [ T2 S4 ], [ S27 O2 ], [ S10 S12 ], [ S17 S18 ], [ S19 T10 ], [ S34 S35 ] |
| SQS | $2 \mathrm{I}-2 \mathrm{O}$ | [S3 S8 T2 S7 T4 T3 S6 I2 O1 O2 T4], [O1 S8 T4], [O1 S8 T2 T4], [S3 S8 T2 T4 T3 S6 I2 O1 O2], [T4 T3 S6][S5 S7 T4 T3] |
|  | 3I-3O | [T6 T9], [O2 S18], [O1 S11], [O3 S9], [T8 T7 S7], [T7 T3] |
|  | PS | [T16 S24 ], [S15 S36 ], [T11 S36 ], [T5 S24 ], [T10 T15 ], [S21 S36 ], [T12 S38 ] |
| CSC-2 | $2 \mathrm{I}-2 \mathrm{O}$ | [S6 T3 S7 T4 S5 T2 S3 O1], [S1 T1 S4 T3 S7 T4 S8 O2], [S4 T3 S7 T4 S8 O2] [I1 I2], [T3 S7 T4], [T1 I2] |
|  | 3I-3O | [S15 T7 S16 T8 S17 T9 S14 T6 S11 O2], [S6 T3 S7 T4 S5 T2 S3 T5 S9 O1] [S12 T7 S16 T8 S17 T9 S18 O3], [I1 S1 T1 T3 T7 S16 T8 S17 T9], [T7 S16 T8 S17 T9] |
|  | PS | $\begin{aligned} & \text { [T3 S5 S6 S7 S8 S9 T4 S10 T5], [S12 T6 S17 T7 T8 S19 T9 S23 S26 T15 S36 S37 S38 T16 S40 O3] } \\ & \text { [S39 T13 S32 S33 S34 T14 S28 S25 T8 S19 T9 S20 T10 T11 S24 T12 S27 O2] } \\ & \text { [T15 S36 S37 S38 T16 S40 O3], [T13 S32 S33 S34 T14] } \\ & {[\text { [S16 T6 S17 T7 T3 S5 S6 S7 S8 S9 T4 S10 T5], [S1 T1 S3 T2 T6 S17 T7] }} \end{aligned}$ |

GoKrimp algorithms do not take time stamped data. Hence experiments for SQS and GoKrimp on these datasets is carried out after removing the time stamps, while retaining the temporal ordering of the symbols.

### 5.1.2. Interpretability of Selected Patterns on conveyor system dataset

Since the Conveyor system dataset consists of movement of packages through specific paths, we expect the good patterns to be sub paths of these paths. The results on the three Conveyor system data sets is given in Table 4 In all the three data sequences, our algorithm retrieves episodes corresponding to the sub paths of package flow (see Table 3 for the actual paths). These patterns actually correspond to sub paths where the packages move along the segments and turns at a fixed speed, without any congestion. The patterns returned by SQS and GoKrimp did not have any particular significance with respect to the topologies. Even though SQS gave some long patterns, those did not give any information regarding the underlying system of transportation of packages.

### 5.1.3. Effectiveness and Efficiency of the Algorithms

In this section, we compare the effectiveness and efficiency of the methods in terms of compression achieved, the number of patterns returned and the run time on the conveyor system data set. Since the other methods, GoKrimp and SQS, employ a bit level coding scheme, for comparison purpose, we also calculate the exact bits needed for our encoding scheme, even though it doesn't affect the encoding efficiency as discussed earlier. The encoding scheme remains exactly the same except for the specification of the alphabet size $M$ in the beginning of the encoding. Since we only deal with positive integers, we assume that all the positive integers are encoded using the Elias codes [13. For a positive integer, $n$, the Elias Code Length for encoding $n$ is $2\left\lfloor\log _{2}(n)\right\rfloor+1$. Each event-type is

Table 5. Compression achieved by various algorithms on conveyor system data sequences.

| Datasets | SQS | GoKrimp | CSC-2 | CSC-2 <br> (Bitwise) |
| :---: | :---: | :---: | :---: | :---: |
| 2I-2O | 1.42 | 1.36 | $\mathbf{4 . 5 6}$ | $\mathbf{4 . 7 9}$ |
| 3I-3O | 1.13 | 1.11 | $\mathbf{4 . 6 3}$ | $\mathbf{4 . 8 9}$ |
| PackageSorter | 1.04 | 1.015 | $\mathbf{3 . 3 4}$ | $\mathbf{3 . 6}$ |

Table 6. Number of Patterns returned by various algorithms on conveyor system data sequences.

| Datasets | SQS | GoKrimp | CSC-2 |
| :---: | :---: | :---: | :---: |
| 2I-2O | 65 | 20 | 13 |
| 3I-3O | 114 | 22 | 27 |
| PackageSorter | 140 | 9 | 77 |

encoded using a fixed code size of $\lfloor\log M\rfloor+1$ bits. It is easy to see that this bit level encoding scheme can also be easily decoded as discussed earlier.

For each method, the compression achieved is calculated as the ratio of size of encoded data using singleton patterns to the size of encoded data using the selected patterns returned by the methods. Table 5 shows the compression ratios achieved by the three algorithms on the conveyor system data sets. Here, the column CSC-2 lists compression ratio in terms of memory units, while the last column CSC-2(Bitwise) gives the bit level compression ratio for CSC-2. As can be seen, our compression ratio is better by a factor of 3 . As mentioned earlier, remote visualization and monitoring of such composable conveyor system is a potential application area of temporal data mining where the compression achieved is also important. Our method achieves better compression and also returns a very relevant set of episodes.

Table 6 shows the number of patterns returned by the three methods and Table 7 shows run times of the three methods. Ideally for datasets with some inherent regularity in the sequence, a few significant patterns should explain the dataset well. In such cases, a good method should return a few relevant patterns. For the conveyor system datasets, where there is inherent regularity in the form of packet flows, CSC-2 returns a few highly relevant patterns, which happen to be sub paths of packet flows. GoKrimp also returns a small number of patterns, though as seen in Table 4, they are not relevant episodes. The PackageSorter topology is a complex topology with lots of intersecting paths. Hence CSC-2 captures more number of patterns here than from other topologies. As shown in Table 4, the patterns corresponded to sub paths relevant to the topology. But for the same topology, GoKrimp gives the minimum number of patterns. However none of these patterns are relevant in the sense that they capture good regularities in the underlying system. Hence GoKrimp algorithm fails to even see any inherent regularity in the datasets (which is also seen by the low compression achieved). SQS returns the highest number of patterns in all cases, which are all but irrelevant to these topologies.

Table 7. Run times in seconds on conveyor system data sequences.

| Datasets | SQS | GoKrimp | CSC-2 |
| :---: | :---: | :---: | :---: |
| 2I-2O | 6 | 1 | 1 |
| 3I-3O | 14 | 1 | 1 |
| PackageSorter | 20 | 1 | 2 |

Table 8. Patterns discovered by various algorithm in the JMLR data.

| Method | Top Discovered Patterns |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| GOKRIMP | support vector machin real world machin learn data set bayesian network | state art <br> high dimension <br> reproduc hilbert space <br> larg scale <br> independ compon analysi | neural network experiment result sampl size supervis learn support vector | well known special case solv problem signific improv object function |
| SQS | support vector machin machin learn <br> state art <br> data set bayesian network | larg scale nearest neighbor decis tree neural network cross valid | featur select graphic model real world high dimension mutual inform | sampl size learn algorithm princip compon analysi logist regress model select |
| CSC-2 | support vector machin <br> data set <br> learn algorithm <br> machin learn <br> bayesian network | reproduc kernel hilbert space <br> real world <br> model select <br> featur select <br> high dimension | classif problem optim problem loss function state art larg scale | experiment result gener error supervis learn graphic model propos method |

### 5.2. Results on other data sets

We now discuss the experimental results on the datasets given in Table 2, We first show the interpretability of the patterns output by different methods on the JMLR data. In section 5.2.2, we compare the methods for efficiency in terms of compression achieved, runtime and the number of patterns. In section 5.2.3, the effectiveness of the selected patterns from different methods is analyzed using the patterns as features for classification.

### 5.2.1. Interpretability of Selected Patterns

Of the six real world data sets, only the outputs from the JMLR data could be analyzed for interpretability. For the JMLR data set, we expect key phrases relevant to Machine Learning Research to pop up. We ran the CSC-2 algorithm with $K=20$ to find the top 20 selected patterns. For the other two algorithms we just selected the top 20 of the outputted patterns. Table 8 gives the patterns obtained on JMLR text data. As can be seen, the patterns returned by all three of the methods are relevant and almost identical. Also we couldn't see any redundancy in any of the pattern sets.

### 5.2.2. Efficiency of the algorithms

In this section, we analyze the efficiency of the methods in terms of compression achieved, the number of patterns returned and the run time on the data sets listed in Table 2. Table 9 presents the comparison of compression ratio for different methods. Again, the column named CSC-2 lists compression ratio in terms

Table 9. Compression achieved by various algorithms.

| Datasets | SQS | GoKrimp | CSC-2 | CSC-2 <br> (Bitwise) |
| :---: | :---: | :---: | :---: | :---: |
| jmlr | 1.039 | 1.008 | $\mathbf{1 . 0 7}$ | $\mathbf{1 . 1 1 7}$ |
| aslbu | 1.155 | 1.123 | $\mathbf{1 . 1 7}$ | $\mathbf{1 . 2 4}$ |
| aslgt | 1.308 | 1.156 | $\mathbf{1 . 9 8}$ | $\mathbf{1 . 9 9}$ |
| auslan2 | 1.571 | 1.428 | $\mathbf{1 . 8 8}$ | $\mathbf{1 . 9 6}$ |
| context | $\mathbf{2 . 7}$ | 1.7 | 1.95 | 1.98 |
| pioneer | 1.3 | 1.17 | $\mathbf{1 . 5 8}$ | $\mathbf{1 . 7 4}$ |

Table 10. Number of Patterns returned by various algorithms.

| Datasets | SQS | GoKrimp | CSC-2 |
| :---: | :---: | :---: | :---: |
| jmlr | 580 | 20 | 765 |
| aslbu | 195 | 67 | 453 |
| aslgt | 1095 | 68 | 105 |
| auslan2 | 13 | 4 | 7 |
| context | 138 | 33 | 39 |
| pioneer | 143 | 49 | 134 |

of memory units and the last column, CSC-2(Bitwise), gives the bit level compression ratio for CSC-2. As is seen in Table 9, CSC-2 offers better compression on almost all the datasets.

Table 10 shows the number of patterns returned by the three methods on real data. For the JMLR data, GoKrimp extracted only 20 patterns, which as we showed in Table 8, are all relevant. For SQS and CSC-2, the number of selected patterns were quite large. The initial phrases were all relevant to machine learning, as seen in Table 8, Later on, for the CSC-2 algorithm, most of the patterns returned were either relevant or slightly relevant machine learning, optimization and mathematical phrases with a bit of redundancy between the phrases. Never the less, we could make some sense out of most of the returned phrases. For the rest of the data sets, the size of the selected set of CSC-2 is in between that of the GoKrimp and SQS methods. We would like to point that the size of the selected set as such, without considering the relevance of the selected patterns does not have any particular significance. In Section 5.2.3, we discuss this again in relation to classification accuracy.

Table 11 compares the run times. Except for the JMLR text data, our method took less than 2 s for all the other data sets. For the JMLR data, the number of event types $M$ is high (see Table 2) and hence the candidate set $\mathcal{C}$ for CSC-2 is high, which results in more time for calculating the $O M$ matrix. On the data set aslgt, both SQS and GoKrimp take a lot of time because the number of events in the sequence is very large. However this does not affect our method much.

### 5.2.3. Classification Results

Here we show the classification results on the five labeled datasets in Table 2 (The JMLR data has no class labels). For each method, we select class specific patterns using the respective methods and merge them along with the set of all singletons (which are 1-node episodes) to form features/attributes for classification. Thus, for classification, each sequence is represented by a feature vector consisting of the number of occurrences of each of the (selected) patterns together with the counts

Table 11. Run times in seconds.

| Datasets | SQS | GoKrimp | CSC-2 |
| :---: | :---: | :---: | :---: |
| jmlr | 489 | $\mathbf{2}$ | 182 |
| aslbu | 196 | 28 | $\mathbf{5}$ |
| aslgt | $>10 \mathrm{Hrs}$ | 1440 | $\mathbf{4}$ |
| auslan2 | 1 | 1 | 1 |
| context | 70 | 36 | $\mathbf{1}$ |
| pioneer | 10 | 6 | $\mathbf{1}$ |

Table 12. Percentage Classification accuracy achieved by various algorithms on 20 runs.

| Datasets | SQS |  | GoKrimp |  | CSC-2 |  | CSC-2* |  | Singletons |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Accuracy | Accuracy |  | Accuracy |  | Accuracy |  | Accuracy |  |  |
|  | Mean | $\sigma$ | Mean | $\sigma$ | Mean | $\sigma$ | Mean | $\sigma$ | Mean | $\sigma$ |
| aslbu | 70.08 | 0.68 | 68.57 | 0.83 | 70.02 | 0.94 | $\mathbf{7 1 . 1 5}$ | 0.75 | 69.7 | 1.1 |
| aslgt | $\mathbf{8 6 . 1 2}$ | 0.18 | 85.55 | 0.23 | 85.9 | 0.15 | $\mathbf{8 6 . 1}$ | 0.19 | 82.17 | 0.19 |
| auslan2 | $\mathbf{3 5}$ | 1.23 | 32.7 | 1.6 | $\mathbf{3 5 . 1 8}$ | 1.2 | 34.05 | 1.13 | 32.58 | 1.7 |
| context | $\mathbf{9 4 . 0 2}$ | 0.94 | 93.83 | 0.61 | 93.7 | 0.89 | $\mathbf{9 4}$ | 0.65 | $\mathbf{9 4 . 0 2}$ | 0.58 |
| pioneer | 100 | 0 | 100 | 0 | 100 | 0 | 100 | 0 | 100 | 0 |

of the singletons in the sequence. We also show the results, when only singletons are used as features. We call this method as Singletons in the tables. In [13], GoKrimp was compared with closed mining episode method, BIDE [25], and one of their two step (mining and then subset selection) methods, SEQKrimp 12,13 . They used different classifiers and found that the linear SVM classifier was giving the best results for almost all datasets. And for all the datasets, they showed that the top patterns returned by the SeqKrimp and GoKrimp were giving better classification accuracy than the top patterns returned by the BIDE algorithm. For our study, we thus use the linear SVM as the classifier using the libsvm package 8]. Since SeqKrimp and GoKrimp were giving similar results, we only use GoKrimp for this comparison study as SeqKrimp is computationally very intensive.

The fixed interval serial episode patterns returned by CSC-2 are used as features in two different ways. In the first method, features are the counts of the selected fixed interval serial episodes along with the counts of the singletons. In the second approach, we drop the fixed inter-event constraints from the episodes and treat them as normal serial episodes (and care is taken to avoid multiple representations of the same serial episode). The counts of these serial episodes along with the singleton counts would be the feature representation of the sequences. The first method is called as CSC-2 and the second method is called $C S C$-2* in the results table.

For each dataset, the results were obtained by averaging 20 repetitions of 10 -fold cross-validation. The results are shown in Table 12 The table gives the mean and standard deviation $(\sigma)$ of the classification accuracy. We see that, SQS marginally outperforms the other methods for the aslgt and context datasets and the two CSC-2 methods have higher accuracies for the aslbu and auslan2 data sets, respectively. In all the datasets, however, the accuracies are similar for SQS and CSC-2. In comparison, the accuracies of the GoKrimp method are slightly lower than both the SQS and CSC-2 methods, except for the pioneer data, where none of the methods seem to misclassify. Table 13 shows the number of patterns

Table 13. The average number of features per data set for each method. The number (except for the Singletons) is the number of non-singleton extracted patterns which represent the feature set along with the singletons.

| Datasets | SQS | GoKrimp | CSC-2 | Singletons |
| :---: | :---: | :---: | :---: | :---: |
| aslbu | 70 | 17 | 23 | 263 |
| aslgt | 1154 | 418 | 923 | 94 |
| auslan2 | 15 | 0 | 6 | 23 |
| context | 150 | 64 | 42 | 107 |
| pioneer | 132 | 24 | 121 | 184 |

selected by each method as features and we see that the number of features selected by SQS is far higher than other methods. And we noticed that, even though the performance of SQS was only slightly better than CSC-2, the run time for the experiments was at least five times longer than that of CSC-2. On the other hand, GoKrimp selects comparatively the lowest number of patterns (except for the context data set) and also has the lowest accuracy among the three methods. For the auslan2 dataset, GoKrimp couldn't extract any pattern and hence the classification accuracy is similar to Singletons (the slight difference is due to difference in the Cross Validation splits for different runs). It is also interesting to see that the Singletons method, which consists of only the 1-node counts are always close to the best results. But nevertheless, the table shows that the selected patterns from different methods have contributed to the increase in accuracy.

### 5.3. Discussion

The results presented here show that CSC-2 is a good method for finding a subset of patterns that achieve good compression. It is also seen that these patterns that achieve good compression are also highly relevant for the problem. For the conveyor system datasets, our method was shown to perform extremely well in pulling out patterns representing the stable flow of items and achieving great compression. Both the aspects are of great importance for remote monitoring of such systems as discussed earlier. The other methods have failed in doing so. On these datasets, the other algorithms, namely SQS and GoKrimp failed to find patterns that capture the package flow and they also could not achieve much data compression.

We have also tested out method on some real world data sets. The CSC-2 algorithm discovered subsets of episodes that result in better data compression compared to the other methods and our algorithm also seems faster than other algorithms for most of the datasets. The subset of patterns identified by CSC-2 is also seen to be very effective in classification scenarios.

Recall that the patterns used by CSC-2 are fixed interval serial episodes. Such episodes, as we have seen, suited the conveyor system data sets, where sequential occurrences of events follow such a fixed gap mechanism. For the other sequential datasets used for classification, such constraints may not be really relevant. But even on these data sets, CSC-2 identifies a subset of patterns that result in both data compression as well as better performance in classification. This shows that our pattern structure is not particularly restrictive and it is useful on a variety of data sets.

## 6. Conclusion

Frequent episodes discovered from sequential data are supposed to give us good insights into the characteristics of the data source. However, in practice, most mining algorithms output a large number of highly redundant episodes. Isolating a small subset of episodes that succinctly characterize the data is a challenging problem. In this paper, we presented an MDL based approach for this problem. Using the interesting class of fixed interval serial episodes and a novel data encoding scheme, we presented a method to discover a subset of highly relevant episodes. In contrast to methods in [12, 13, 23, our method achieves good data compression, while being able to work with event sequences with time stamps.

We compared our method with SQS and GoKrimp on text data and also on a number of real world data sets which were used earlier in temporal data mining. On all these data sets, our method is good in comparison to others, both in terms of compression and run time. For the classification scenario, our method was only slightly less effective than SQS but better than GoKrimp. But we achieved it with far fewer patterns and very low run times.

In this paper, we also briefly discussed a novel application area for sequential pattern mining. This is the composable conveyor system. We presented empirical comparison of our method with that of others on three data sets from this problem domain to demonstrate both the effectiveness and efficiency of our method.

In this paper, we have not attempted any statistical analysis of our method so that we can relate the data compression to some measure of statistical significance of the pattern subset isolated by our method. This is an interesting and challenging direction to extend the work presented here. We would be exploring this in our future work.

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[^0]:    1 Note that while events of different types can occur with the same time stamp, events with same event-type cannot co-occur at the same time instant; hence we can easily spot duplicates while decoding.

[^1]:    ${ }^{2}$ We note that this is a lower bound because $\alpha$ is an injective episode. When $\alpha$ is an injective episode, no two occurrences of $\alpha$ can share an event and hence $f_{\mathcal{D}}^{\alpha}$ occurrences would contain $f_{\mathcal{D}}^{\alpha} N$ events in the data sequence. If the episodes were non-injective, then there is a possibility of events being shared by different occurrences of the same episode and hence $f_{\mathcal{D}}^{\alpha} N$ would not be the lower bound.

[^2]:    ${ }^{3} \alpha[i]$ denotes the $i$ th episode event of $\alpha$.

