A Node Formulation for Multistage Stochastic Programs with Endogenous Uncertainty

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Abstract

This paper introduces a node formulation for multistage stochastic programs with endogenous (i.e., decision-dependent) uncertainty. Problems with such structure arise when the choices of the decision maker determine a change in the likelihood of future random events. The node formulation avoids an explicit statement of non-anticipativity constraints, and as such keeps the dimension of the model sizeable. An exact solution algorithm for a special case is introduced and tested on a case study. Results show that the algorithm outperforms a commercial solver as the size of the instances increases.

1 Introduction

Multistage stochastic programs offer a viable framework for modeling and solving problems involving a sequence of decisions interspersed with partial resolutions of some stochastic process. At each decision stage the decision maker knows the content of the uncertainty resolved until that stage and a probabilistic characterization of the remaining stochastic process. In the classical settings, it is assumed that decisions do not modify the stochastic process in any way (see e.g., Kall and Wallace, 1994; Birge and Louveaux, 1997). In other words, the uncertainty is entirely exogenous. While this description fits a large number of decision problems, several other can be found where decisions have an influence on the remainder stochastic process, e.g., by changing the likelihood of future realizations. This category of problem is referred to as *multistage stochastic programs with endogenous uncertainty* (MSPEU).

Following (Goel and Grossmann, 2006), there exist at least two ways in which decisions can influence the underlying stochastic process. The first possibility is that decisions alter the probability distribution of the stochastic process, thus changing the likelihood of the possible events. The second possibility is that decisions determine the time when the uncertainty is (partially) resolved. This article is concerned with multistage stochastic programs affected by the first type of endogenous uncertainty.

The research dealing with decisions influencing probability distributions is rather sparse. Jonsbråten et al. (1998) consider a case where decisions influence both the probability measure and the timing of the observation, i.e., at which stage certain random variables will be observed. The framework includes both two-stage and multistage problems, though the decisions influencing the uncertainty must be made at the first-stage only. Ahmed (2000) illustrates examples of problems with endogenous uncertainty, such as facility location, network design and server selection. The author presents an exact solution method for the resulting one-stage integer problems. Viswanath et al. (2004) consider the problem of investing in strengthening actions for the links of a network subject to disruptive events. The problem is modeled as a two-stage stochastic program where first-stage investment decisions influence the likelihood of disruptive events happening at upgraded links. The same problem is studied also by da Costa Flach (2010), Peeta et al. (2010) and Laumanns et al. (2014). Held and Woodruff (2005) consider the problem of interdicting a stochastic network, that is a network whose structure is unknown to the interdictor. In this problem, the probabilities of different future network configurations depend on previous interdiction actions. Tong et al. (2012) present an oil refinery planning problem considering that the uncertainty in product yield is

influenced by operation mode changeovers. Hellemo (2016) and Hellemo et al. (2018) discuss several ways of incorporating the influence of decision variables on the underlying probability distributions in two-stage stochastic programs. Particularly, the authors formulate two-stage models where prior probabilities are distorted through an affine transformation, or combined using a convex combination of several probability distributions. Furthermore, the authors present models which incorporate the parameters of the probability distribution as first-stage decision variables. Finally, Escudero et al. (2018) study the problem of mitigating the effects of natural disasters through preventive actions. The problem is formulated as a three-stage stochastic bilinear integer program with both exogenous and endogenous uncertainty. Particularly, decisions can influence both the probabilities and the intensity of future uncertain events.

When the second type of uncertainty is considered, the decision making process is such that the uncertainty is not resolved automatically at each decision stage as in classical stochastic programs with only exogenous uncertainty. Rather, the decisions made implicitly determine the time when the uncertainty is resolved. A typical example, provided by Goel and Grossmann (2004) and based on the petrochemical industry is as follows. A decision maker has to decide which gas reservoirs to explore, and when, by installing exploration facilities. The size and quality of the reservoirs is uncertain and can be known only when facilities have been installed. Thus, the time the uncertainty is resolved depends on installation decisions. The literature dealing with this type of stochastic programs includes Colvin and Maravelias (2008), Tarhan and Grossmann (2008), Tarhan et al. (2009), Colvin and Maravelias (2010), Gupta and Grossmann (2011), Mercier and Van Hentenryck (2011), Tarhan et al. (2013), Apap and Grossmann (2016).

The contribution of this paper is as follows. First, we introduce a novel node-formulation for multistage stochastic programs where decisions (at all stages) influence probability distributions at future stages. A node formulation automatically ensures non-anticipative decisions and thus avoids writing explicit nonanticipativity constraints (NACs) that are necessary when the uncertainty is represented via scenarios. NACs have typically been addressed as a bottleneck of available models for MSPEU, and motivated recent research to find and discard redundant NACs, see Apap and Grossmann (2016), Hooshmand and MirHassani (2016) and Hooshmand and MirHassani (2018). The new formulation is based on a novel scenario tree structure which incorporates the possibility that several (finitely many) different distributions for later stages can emanate based on the decisions made at a certain decision stage. The new scenario tree structure represents the second contribution of this paper. Third, we propose a general-purpose efficient algorithm for a special class of MSPEUs. We demonstrate the use of the new formulation, scenario tree structure and solution algorithm on instances of the *Football Team Composition Problem*. The instances are made available online for the benefit of future research at https://github.com/GioPan/instancesFTCPwithEndogenousUncertainty.

The remainder of this paper is organized as follows. In Section 2 we provide a model formulation and new scenario tree structure for MSPEUs. In Section 3 we describe a solution algorithm for a special case of MSPEUs. In Section 4 we present a case study in which we formulate the Football Team Composition Problem as a MSPEU and solve it using our specialized algorithm. Finally, we provide concluding remarks in Section 5.

2 Multistage stochastic programs with decision-dependent uncertainty

The decision maker is concerned with a sequence of decisions $(x_t)_{t=1}^T$ at decision stages $t = 1, \ldots, T$, conditional on the partial resolution of a random process $(\xi_t)_{t=1}^T$. At decision stage t, decisions are non-anticipative, meaning that they are based only on the realization of the random process up to, and including, ξ_t . The realization of the remaining random process ξ_{t+1}, \ldots, ξ_T is still uncertain. For $t = 2, \ldots, T$, the probability distribution of ξ_t, \ldots, ξ_T is dependent on past decisions x_1, \ldots, x_{t-1} . The resulting multistage stochastic program is thus characterized by endogenously defined uncertainty.

Consistently with Jonsbråten et al. (1998) we assume that the set of potential probability distributions enforced by decisions is finite and countable. Furthermore, we assume that the probability distributions are discrete (possibly after a scenario generation phase). The latter assumption is rather standard and required in order to solve real-life stochastic programs (except perhaps for a number of specific applications). The resulting decision-dependent discrete stochastic process can be depicted by means of the scenario tree structure illustrated in Figure 1. In what follows we refer to this scenario tree structure as a *multi-distribution*



Figure 1: Multidistribution scenario tree.

scenario tree (MDST) (see, e.g., also how Kaut et al. (2014) modified the classical structure of scenario trees in order to account for multiple time resolutions).

In an MDST the root node, arbitrarily named 0, represents the current state of the world, when first-stage decisions are made. First-stage decisions will enforce one out of a finite number of distributions represented by the set \mathcal{D}_0 . As an example, in Figure 1, decisions might determine, among other, distributions $d \in \mathcal{D}_0$ or $d+1 \in \mathcal{D}_0$. Distribution d is characterized by realizations, represented by nodes, that include l and m, while distribution d+1 is characterized by realizations that include q and o. Similarly, at stage t = 2, assuming realization m of distribution d occurs at stage t = 1, the actions of the decision maker will determine one out of a number of different distributions including d and d+1 from the set \mathcal{D}_m . This process continues in a similar manner until stage T-1. Given a realization n at stage T-1 the actions of the decision maker will determine one at stage T and d + 1. Finally, at stage T all the uncertainty is resolved and the decision maker makes final decisions.

Given an MDST, let us introduce the notation necessary for formulating the MSPEU. Let \mathcal{N} be the set of nodes in the scenario tree, \mathcal{N}_t be the set of nodes at stage t, 0 the root node, $t(n) \in \{1, \ldots, T\}$ the stage of node n, and $a(n) \in \mathcal{N}$ the parent node of node n except the root node. Let \mathcal{D}_n be the set of possible



Figure 2: Example multidistribution scenario tree with three stages, two possible distributions for each node, and two possible realizations for each distribution.

distributions which can be enforced by decision made at node $n \in \mathcal{N}$, and \mathcal{N}_{nd} be the child nodes of node n if distribution $d \in \mathcal{D}_n$ is enforced. Let π_n be the probability of node n with $\pi_0 = 1$, and $\sum_{m \in \mathcal{N}_{dn}} \pi_m = \pi_n$ for all $n \in \mathcal{N}$, $d \in \mathcal{D}_n$. An example of this notation is provided in Figure 2 for a three-stage scenario tree, with two possible distributions emanating from each node and each distribution being characterized by two possible realizations.

Let decision variables $x_n \in \mathbb{R}^{N_{t(n)}}$, $n \in \mathcal{N}$ represent decisions made at node n. These decisions may be integer or fractional and represent ordinary decision made in the decision process. Let δ_{nd} be a binary variable which captures the probability distribution enforced by the decisions made at node n. It takes value 1 if the decisions at node n enforce probability distribution d at the child nodes of n, 0 otherwise. Finally, let $\theta_n \in \mathbb{R}^1$ be a decision variable which holds the expected value of the decisions made at the nodes descending from n. For the reader's convenience, the notation is also reported in Appendix A in a tabular format. An MSPEU is thus

$$\max \ r_0^T x_0 + \sum_{d \in \mathcal{D}_0} q_{0d} \delta_{0d} + \theta_0 \tag{1a}$$

s.t.
$$\sum_{d \in \mathcal{D}_{-}} \delta_{nd} = 1$$
 $n \in \mathcal{N},$ (1b)

$$A_n x_n + \sum_{d \in \mathcal{D}_n} B_{nd} \delta_{nd} + C_{a(n)} x_{a(n)} + \sum_{d \in \mathcal{D}_{a(n)}} D_{a(n),d} \delta_{a(n),d} = h_n \qquad n \in \mathcal{N},$$
(1c)

$$\theta_n = \sum_{d \in \mathcal{D}_n} \delta_{nd} \bigg(\sum_{m \in \mathcal{N}_{nd}} \pi_m (r_m^T x_m + \sum_{d \in \mathcal{D}_m} q_{md} \delta_{md} + \theta_m) \bigg) \qquad \qquad n \in \mathcal{N} \setminus \mathcal{N}_T,$$
(1d)

$$\theta_n = \Theta_n \qquad \qquad n \in \mathcal{N}_T, \qquad (1e)$$

$$x_n \in X_{t(n)} \qquad \qquad n \in \mathcal{N}, \qquad (1f)$$

$$b_{nd} \in \{0,1\} \qquad \qquad n \in \mathcal{N}, d \in \mathcal{D}_n, \qquad (1g)$$

$$D_n \in \mathbb{R}$$
 $n \in \mathcal{N}$. (1h)

where, for each $n \in \mathcal{N}$, $r_n \in \mathbb{R}^{N_{t(n)}}$ and $q_{nd} \in \mathbb{R}^1$ represent the rewards of decisions x_n and δ_{nd} , respectively, $A_n \in \mathbb{R}^{M_{t(n)} \times N_{t(n)}}$, $B_{nd} \in \mathbb{R}^{M_{t(n)} \times 1}$, $C_{a(n)} \in \mathbb{R}^{M_{t(n)} \times N_{t(a(n))}}$, and $D_{a(n),d} \in \mathbb{R}^{M_{t(n)} \times 1}$ are matrices of coefficients and $h_n \in \mathbb{R}^{M_{t(n)}}$ a right-hand-side vector, with the assumption that $C_{a(0),d} \coloneqq \mathbf{0}$ and $D_{a(0),d} \coloneqq \mathbf{0}$. Finally, $\Theta_n \in \mathbb{R}^1$ represents the future expected value at leaf node n, and $X_{t(n)} \subseteq \mathbb{R}^{N_{t(n)}}$ the domain of the x_n variables. Objective function (1a) represents the sum of the profit for the decisions made a the root node (n = 0) and expected profit of future decisions. Constraints (1b) ensure that the decisions made at each node determine exactly one probability distribution. That is, the stochastic phenomena following the decisions at node n, are described by exactly one probability distribution (among the available ones). enforced by the decisions made. Constraints (1c) describe the dependency between decision variables x and δ at each node, and between them and the corresponding decision variables at the parent node. That is, the choice of a probability distribution at node n, δ_{nd} , depends on the decisions x_n made at the same node as well as on the decisions $x_{a(n)}$ made at the parent node and on the consequent choice of a probability distribution, $\delta_{a(n),d}$. Constraints (1d) ensure that decision variables θ_n hold the expected value of future decisions calculated according to the probability chosen. Consider a generic node n other than a leaf node, and note that, according to (1b), at this node there will be exactly one δ_{nd} equal to one, that is exactly one distribution be chosen. Thus, the right-hand-side of constraints (1d) will be equal to the term of the outer summation corresponding to the index d whose δ_{nd} is set to one. The remaining terms are zero. Correspondingly, θ_n will hold the expected value calculated according to the chosen probability distribution d. Constraints (1d) can be linearized using standard techniques. A linear reformulation is provided in Appendix B and a general procedure to determine the necessary big-M values is described in Appendix C. Constraints (1e) set the future expectation at the leaf nodes. Finally, constraints (1f) to (1h) set the domain of the decision variables. Particularly, $X_{t(n)}$ represents the domain of the x_n variables and may impose integrality restrictions on some/all variables.

A node formulation implicitly includes non-anticipativity, that is, automatically ensures that the decisions made at a given stage are only based on available information. On the other hand, a classical scenario formulation requires non-anticipativity explicitly enforced by means of constraints, and in general, generates a much larger problem. This can be illustrated by the following example. Consider Figure 3 which provides the scenario representation of the example MDST in Figure 2. The number of scenarios is 15, and is the same as the number of leaf nodes in Figure 2. Assuming the decision at each stage are represented by N decision variables and have to satisfy M constraints, the corresponding scenario formulation would include

- $N \times 15 \times 3$ decision variables, where 3 is the number of stages,
- $M \times 15 \times 3$ constraints, and
- approximately $N \times (15 + 8)$ non-anticipativity constraints (approximately 15 for the first stage and 8 for the second stage, though more efficient specifications may be possible).

The node formulation would include

- $N \times 21$ variables (where 20 is the number of nodes in the scenario tree in Figure 2), and
- $M \times 21$ constraints.

Clearly, node formulations generate, in general, much smaller problems with non-anticipativity constraints playing an important role in scaling up the dimension of a scenario formulation, see e.g., Apap and Grossmann (2016), Hooshmand and MirHassani (2016) and Hooshmand and MirHassani (2018) for how to reduce the number of non-anticipativity constraints.



Figure 3: Scenarios in the example multidistribution scenario tree in Figure 2. Plain lines connect the nodes belonging to the same scenario. Dashed lines represent non-anticipativity constraints.

3 A solution algorithm for a special case

In this section we introduce an algorithm for solving the special case of problem (1) with $C_{a(n)} = \mathbf{0}$ for all n. The algorithm builds on the fact that, when $C_{a(n)} = \mathbf{0}$, the link between decisions at subsequent stages is created by the δ variables, i.e., those that enforce a probability distribution for the next stage. In this case, since we have finitely many distributions, we are allowed to enumerate the expected values obtainable at each node in the scenario tree. This procedure would become impractical when $C_{a(n)}$ is different from $\mathbf{0}$, unless further assumptions on x_n are made.

Let us thus consider the following equivalent formulation of problem (1).

$$z = \max \ r_0^T x_0 + \sum_{d \in \mathcal{D}_0} q_{0d} \delta_{0d} + \theta_0$$
(2a)

s.t.
$$\sum_{d \in \mathcal{D}_n} \delta_{nd} = 1$$
 $n \in \mathcal{N},$ (2b)

$$A_n x_n + \sum_{d \in \mathcal{D}_n} B_{nd} \delta_{nd} + \sum_{d \in \mathcal{D}_{a(n)}} D_{a(n),d} \delta_{a(n),d} = h_n \qquad n \in \mathcal{N}, \qquad (2c)$$

$$\theta_n \le \phi_{nd} + M_{nd}(1 - \delta_{nd}) \qquad \qquad n \in \mathcal{N} \setminus \mathcal{N}_T, d \in \mathcal{D}_n \qquad (2d)$$

$$\phi_{nd} = \sum_{m \in \mathcal{N}_{nd}} \pi_m (r_m^T x_m + \sum_{d' \in \mathcal{D}_m} q_{md'} \delta_{md} + \theta_m) \qquad n \in \mathcal{N} \setminus \mathcal{N}_T, d \in \mathcal{D}_n$$
(2e)

$$\begin{aligned} \theta_n &= \Theta_n & n \in \mathcal{N}_T, \quad (2f) \\ r_n &\in \mathcal{N}_{t(n)} & n \in \mathcal{N} \quad (2g) \end{aligned}$$

$$\delta_{nd} \in \{0,1\} \qquad \qquad n \in \mathcal{N}, d \in \mathcal{D}_n, \qquad (2b)$$

$$\theta_n \in \mathbb{R}$$
 $n \in \mathcal{N},$ (2i)

$$\phi_{nd} \in \mathbb{R} \qquad \qquad n \in \mathcal{N} \setminus \mathcal{N}_T, d \in \mathcal{D}_n.$$
 (2j)

Problem (2) modifies problem (1) in two elements. First, constraints (2c) take into account that $C_{a(n)} = \mathbf{0}$. Second, constraints (1d) have been linearized using constants M_{nd} and auxiliary decision variables ϕ_{nd} yielding constraints (2d) and (2e) (see e.g., Appendix C for a general purpose procedure to determine these big-M values).

Problem (2) can be solved in a dynamic programming fashion using the following backward procedure. The procedure starts by calculating the optimal last-stage expectation for each node at the second-last stage and for each possible probability distribution. For each $m \in \mathcal{N}_{T-1}$ and probability distribution $k \in \mathcal{D}_m$, the optimal expectation at the last stage is obtained by solving the following problem.

$$\Phi_{mk} = \max \sum_{n \in \mathcal{N}_{mk}} \pi_n (r_n^T x_n + \sum_{d \in \mathcal{D}_n} q_{nd} \delta_{nd} + \Theta_n)$$
(3a)

s.t.
$$\sum_{d \in \mathcal{D}_n} \delta_{nd} = 1,$$
 $n \in \mathcal{N}_{mk},$ (3b)

$$A_n x_n + \sum_{d \in \mathcal{D}_n} B_{nd} \delta_{nd} = h_n - D_{mk} \qquad \qquad n \in \mathcal{N}_{mk}, \tag{3c}$$

$$x_n \in X_T \qquad \qquad n \in \mathcal{N}_{mk}, \tag{3d}$$

$$\delta_{nd} \in \{0, 1\} \qquad \qquad n \in \mathcal{N}_{mk}, d \in \mathcal{D}_n.$$
(3e)

Problem (3) provides the last-stage expectation for each node $m \in \mathcal{N}_{T-1}$ at the second-last stage, and for each possible selection of a probability distribution from \mathcal{D}_m . Notice in (3) that Θ_n is input data and that \mathcal{D}_{mk} is moved to the right-hand-side to stress that it does not multiply a decision variable as in (2c), since the choice of a probability distribution at the parent node m has been fixed to k.

Then, for stage t = T - 2, ..., 1, for node $m \in \mathcal{N}_t$, and for distribution $k \in \mathcal{D}_m$ we calculate Φ_{km} by solving problem (4).

$$\Phi_{mk} = \max \sum_{n \in \mathcal{N}_{mk}} \pi_n (r_n^T x_n + \sum_{d \in \mathcal{D}_n} q_{nd} \delta_{nd} + \theta_n)$$
(4a)

s.t.
$$\sum_{d \in \mathcal{D}_n} \delta_{nd} = 1,$$
 $n \in \mathcal{N}_{mk},$ (4b)

$$A_n x_n + \sum_{d \in \mathcal{D}_r} B_{nd} \delta_{nd} = h_n - D_{mk} \qquad \qquad n \in \mathcal{N}_{mk}, \tag{4c}$$

$$\theta_n \le \Phi_{nd} + M_{nd}(1 - \delta_{nd}), \qquad d \in \mathcal{D}_n, n \in \mathcal{N}_{mk},$$
(4d)

$$x_n \in X_{t(n)} \qquad \qquad n \in \mathcal{N}_{mk}, \qquad (4e)$$

$$\delta_{-} \in [0, 1] \qquad \qquad n \in \mathcal{N}_{-} \quad d \in \mathcal{D} \qquad (4f)$$

$$\theta_n \in \mathbb{R} \qquad \qquad n \in \mathcal{N}_{mk}, a \in \mathcal{D}_n, \qquad (41)$$
$$\theta_n \in \mathbb{R} \qquad \qquad n \in \mathcal{N}_{mk}. \qquad (4g)$$

Notice in problem (4) that Φ_{nd} is input data and has been calculated in the previous steps of the algorithm. Finally, we can solve the following problem for the root node.

$$z = \max r_0^T x_0 + \sum_{d \in \mathcal{D}_0} q_{0d} \delta_{0d} + \theta_0 \tag{5a}$$

s.t.
$$\sum_{d \in \mathcal{D}_0} \delta_{0d} = 1,$$
 (5b)

$$A_0 x_0 + \sum_{d \in \mathcal{D}_0} B_{0d} \delta_{0d} = h_0,$$
 (5c)

$$\theta_0 \le \Phi_{0d} + M_{0d}(1 - \delta_{0d}), \qquad \qquad d \in \mathcal{D}_0, \tag{5d}$$

$$\theta_0 \in \mathbb{R}.$$
 (5g)

Letting $D = \max_{n \in \mathcal{N}} |\mathcal{D}_n|$, the algorithm entails solving $\mathcal{O}(D|\mathcal{N}|)$ mixed-integer programs of size significantly smaller than (1).

4 Case study

In this section we present a case study based on the *Football Team Composition Problem* (FTCP – Pantuso (2017); Pantuso and Hvattum (2020)) which we extend in order to account for decision-dependent uncertainty. The problem consists of selecting players for a football team while their future market value is uncertain and influenced by the team for which they play. The scope of the computational study is to compare the performance of the algorithm to that of a state-of-the-art commercial solver. We remark, however, that formulation (1) and the solution algorithm proposed are general and applicable beyond the context of the FTCP, which we use solely as an example. Decision problems under endogenous uncertainty may, in fact, arise in several business context, some of which are mentioned in Section 1. To provide another practical example, consider an agribusiness making periodic production planning decisions for a number of different crops while demand is uncertain. Product substitution is a common phenomenon in the production of crops. In fact, a customer, say a farmer, may view multiple crops as suitable for their farming needs, see, e.g., Bansal and Dyer (2020). Thus, the uncertain demand for a crop may depend in part on the portfolio of crops an agribusiness offers for sale, yielding a decision problem under endogenous uncertainty.

In Section 4.1 we describe the FTCP in more details and formulate it as an MSPEU. In Section 4.2 we illustrate an efficient procedure to obtain big-M values. In Section 4.3 we introduce the problem instances and finally in Section 4.4 we present and discuss the results. The data of the problem instances is available online at https://github.com/GioPan/instancesFTCPwithEndogenousUncertainty for the benefit of future research.

4.1 The Football Team Composition Problem

The FTCP is the problem of composing a football team by purchasing and selling professional football players. A complete description of the problem can be found in Pantuso (2017). In what follows, we report

the basic elements necessary for this case study. The decision problem can be described as follows. At every stage, i.e., *transfer market window* (TMW), professional football clubs can renew their teams by purchasing players from other clubs or selling available players to other clubs. In order to participate in national and international competitions, professional clubs must compose a team made of a fixed number of players. In addition, the coach of the team requires players with different roles (e.g., defenders or mid-fielders) and skills.

Club managers are often given a budget to spend in the transfer market and are typically allowed to reinvest the revenue from the sale of players. The current market value of football players is known, while the future value is stochastic as it depends on a number of random events such as injuries, fitness, motivation and ultimately luck. Therefore, at every TMW, football clubs make decisions in conditions of uncertainty with the scope of maximizing the expected value of the team. Furthermore, the market values of the players in the same team are strongly correlated. This generates a multistage stochastic program with endogenous uncertainty, since the decision of hiring or selling a player will change the correlations of the joint value distribution of the players considered. That is, hiring a football player will make their value strongly correlated with the value of the other players in the team, while selling or not hiring a player will make their value uncorrelated with the value of the players in the team.

In order to model the FTCP we assume the club is evaluating a finite number of alternative *team* compositions. Every team composition consists of the required number of players and contains the necessary mix of skills. Let \mathcal{I} be the set of possible team compositions, \mathcal{N} the set of nodes in the MDST describing the underlying uncertainty, \mathcal{N}_{in} the set of child nodes of node $n \in \mathcal{N}$ if team composition $i \in \mathcal{I}$ is chosen at node n. Note in fact that the selection of a team composition determines the correlations between the values of the players in the instance, and thus the probability distribution. Let \mathcal{N}^L be the set of leaf nodes, i.e., the nodes at the last decision stage, and $t(n) \in \{1, \ldots, T\}$ the stage corresponding to node n.

Let V_{in} be the value of team composition $i \in \mathcal{I}$ at node n (i.e., the sum of the values of the players in team composition i), let C_{in}^S be the cost of the salary of the players in team composition i at node n, C_{jn}^T the cost of transitioning from team composition i to team composition j at node n, i.e., the cost of the transfer fees for the players bought minus the revenue for the transfer fees of the players sold. Furthermore, let B be the budget available to the club for the transfer market at node n and E_n a stochastic extra-budget conditional on events such as sport successes. Let X_i be equal to one if team composition $i \in \mathcal{I}$ is the initial team composition, M_{in} a suitably large constant for each i and n (see Section 4.2), and ρ a discount rate. Let δ_{in} be a binary decision variable which is equal 1 if team composition $i \in \mathcal{I}$ is chosen at node $n \in \mathcal{N}$, x_{ijn} a binary decision variable which is equal 1 if the club transitions from team composition $i \in \mathcal{I}$ to team composition $j \in \mathcal{J}$ at node $n \in \mathcal{N}$ and, ϕ_{in} the expected net value of the team at the child nodes of node $n \in \mathcal{N}$. The FTCP with endogenous uncertainty is hence:

$$\max \sum_{i \in \mathcal{I}} (V_{i0}\delta_{i0} - C_{i0}^S\delta_{i0} - \sum_{j \in \mathcal{I}} C_{ij0}^T x_{ij0}) + \theta_0$$
(6a)

s.t.
$$\sum_{i \in \mathcal{I}} \delta_{in} = 1$$
 $n \in \mathcal{N},$

$$\delta_{i0} - X_i + \sum_{j \in \mathcal{I}: j \neq i} (x_{ij0} - x_{ji0}) = 0 \qquad i \in \mathcal{I},$$

(6b)

(6c)

(6d)

(6e)

$$\delta_{in} - \delta_{i,a(n)} + \sum_{j \in \mathcal{I}: j \neq i} (x_{ijn} - x_{jin}) = 0 \qquad n \in \mathcal{N} \setminus \{0\}, i \in \mathcal{I},$$

$$\sum_{i \in \mathcal{I}} \sum_{j \neq i \in \mathcal{I}} x_{ijn} \le 1 \qquad \qquad n \in \mathcal{N},$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I}} C_{ijn}^T x_{ijn} \le B + E_n \qquad \qquad n \in \mathcal{N},$$

$$\theta_n \le \phi_{in} + M_{in}(1 - \delta_{in})$$
 $i \in \mathcal{I}, n \in \mathcal{N},$

(6f)

(6g)

(6i)

(6k)

(61)

$$\phi_{in} = \sum_{m \in \mathcal{N}_{in}} \frac{1}{1 + \rho^{t(m)}} \pi_m \left\{ \theta_m + \sum_{j \in \mathcal{I}} \left(V_{jm} \delta_{jm} - C_{jm}^S \delta_{jm} - \sum_{k \in \mathcal{I}} C_{jkm}^T x_{jkm} \right) \right\} \quad i \in \mathcal{I}, n \in \mathcal{N} \setminus \mathcal{N}^L,$$
(6h)

$$\theta_n = \frac{1}{1 + \rho^{t(n)+1}} \sum_{i \in \mathcal{I}} V_{in} \delta_{in} \qquad \qquad n \in \mathcal{N}^L,$$

$$\delta_{in} \in \{0, 1\} \qquad \qquad i \in \mathcal{I}, n \in \mathcal{N},$$
(6j)

$$x_{ijn} \in \{0,1\} \qquad \qquad i,j \in \mathcal{I}, n \in \mathcal{N},$$

$$\theta_n \in \mathbb{R}$$

Objective function (6a) represents the sum of the value of the team composition chosen here-and-now, minus the expenses for salaries and, if any, the transition cost from the initial team composition. In addition it takes into account the expected value of the team at future nodes. Constraints (6b) ensure that only one team composition is chosen at each decision node, while constraints (6c)-(6d) state that either the club holds the same team composition as in the previous season or a new team composition is chosen at node 0 and at the rest of the nodes, respectively. Constraints (6e) state that at most one team change can be done at each node. Constraints (6f) ensure that the net expenses for transitioning from a team composition to another (i.e., the money spent in the transfer market) do not exceed the available budget. Constraints (6g) state that the future net expected value of the team depends on the team composition chosen. Constraints (6h) sets the expected value at the children of node n if composition i is chosen, while constraints (6i) set the final value of θ_n to the value of the team chosen at the leaf nodes. This corresponds to a sunset value which accounts the termination of an infinite horizon problem. Notice that the final value of the team is discounted as a future value. Finally, constraints (6j)-(6l) define the domain for the decision variables.

4.2 Finding big-M values for the FTCP

We illustrate a fast method to set the M_{in} values in (6). The method proposed resulted, on this special case, significantly faster than the general method described in Appendix C.

We start by observing that, for all $i \in \mathcal{I}$ and $n \in \mathcal{N}$ we are looking for a value M_{in} such that:

$$\theta_n - \phi_{in} \le M_{in}$$

and that, based on constraints (6g), we have

$$\theta_n \le \max_{i \in \mathcal{I}} \phi_{in}$$

consequently, we need to set

$$M_{in} \ge \max_{i \in \mathcal{I}} \phi_{in} - \phi_{in}$$

thus, under the very mild assumption that $\phi_{in} \geq 0$ (i.e., that the team does not spend in transfers and salaries more than the value of the entire team), for every $i \in \mathcal{I}$ a valid M_{in} is

$$M_{in} = M_n = \max_{i \in \mathcal{I}} \phi_{in}$$

By expanding ϕ_{in} according to constraints (6h) and (6i) we can set

• $M_{in} = \max_{i \in \mathcal{I}} \frac{1}{1+\rho^{t(n)+1}} V_{in}$ for all $i \in \mathcal{I}$ if $n \in \mathcal{N}^L$

•
$$M_{in} = \max_{i \in \mathcal{I}} \left\{ \sum_{m \in \mathcal{N}_{in}} \frac{1}{1 + \rho^{t(m)}} \pi_m \left(M_m + \max_{j \in \mathcal{I}} \left\{ V_{jm} \right\} \right) \right\}$$
 for all $i \in \mathcal{I}$ if $n \in \mathcal{N} \setminus \mathcal{N}^L$.

4.3 **Problem instances**

Instances are generated from the case studies on the FTCP presented by Pantuso (2017) and are available online at https://github.com/GioPan/instancesFTCPwithEndogenousUncertainty. The instances consist of the 20 teams competing in the English Premier League 2013/2014 and dealing with the transfer market of summer 2014. Each team is characterized by a budget, a list of players (made of the current and target players), and a number of randomly generated team compositions \mathcal{I} complying with the regulations of the league and with the coach's specifics. In turn, each player is characterized by age, role, current market value, salary, selling and purchase price. The value and cost of salaries of a team composition are calculated as the sum of values and salaries, respectively, of the players contained. Similarly, transition costs are calculated as the revenue for the players sold, minus the cost of the players bought.

Future player values are stochastic and modeled by means of the regression equation available in Pantuso (2017). The joint probability distribution of the market values for all the players forms a multivariate normal distribution. In addition, we set a 0.8 correlation between the players belonging to the focal team and no correlation between the players belonging to the focal team and the remaining players. Therefore, the correlations change with the decisions of the team to buy or sell players creating an MSPEU. Scenario trees are obtained by sampling realizations from the underlying multivariate normal distributions. Finally, for each node in the scenario tree, the random extra-budget E_n is calculated as a percentage of the deterministic budget B. The percentage is the same as percentage increase of team value from the parent node, that is max $\{0, 100 * (V_{in} - V_{i,a(n)})/V_{i,a(n)}\}$. That is, an increase in the team value yields an increase in the spending capability of the team.

The instances represent a four-stage horizon and are identified by (i) the focal team, where Team \in { ARS, ASV, CAR, CHE, CRP, EVE, FUL, HUL, LIV, MAC, MAU, NEC, NOR, SOU, STO, SUN, SWA, TOT, WBA, WHU }, (ii) the number of team composition $|\mathcal{I}| \in \{3, 4, 5\}$, and (iii) the number of samples in each distribution $S \in \{4, 5\}$, yielding 120 numerically different instances with up to approximately 400 thousand binary variables.

4.4 Results

We implemented our algorithm in Python 2.7 using Cplex 12.8 for solving the subproblems. Cplex 12.8 has also been used to solve the full problems. All experiments have been run on a server with 64 double AMD Opteron 6380 processors and 251 GB RAM. Tables 1 to 3 report the results for different cardinalities $|\mathcal{I}|$ and different number of samples S for all the focal teams. The computation times do not include the time required to find big-M values as illustrated in Section 4.2. An account of these times is reported in Appendix D. Note that big-M values are required both when solving the full problem and when using our algorithm and in both cases are calculated beforehand. Thus, their computation time does not have an impact on the comparison between the two solution methods. Since the algorithm entails iteratively building and solving mixed-integer programs, the times reported include the time required for building the models.

Table 1 reports the results with $|\mathcal{I}| = 3$, generating instances with up to approximately 32000 binary variables. For these instances it can be noticed that Cplex performs much better than our method on the smaller test cases (with S = 4 samples) while our algorithm is more competitive on the instances with S = 5. Altogether, Cplex solves the problems approximately 25% faster than our method.

Table 1: Results with $|\mathcal{I}| = 3$. S indicates the number of realizations describing each distribution. #Var, #Bin and #Con indicate the total number of variables, the number of binary variables, and the number of constraints, respectively. τ (Cpx) indicate the elapsed time when using the algorithm (Cplex). Obj. (Cpx) indicates the objective value obtained using the algorithm (Cplex). $\Delta \tau$ is calculated as $100(\tau - \tau \text{ Cpx})/\tau$ Cpx.

m	a	11 3 7	// D'	110		01:0	r 1	01:	A [07]
Team	S	#Var	#Bin	#Con	τ Cpx [sec]	Obj. Cpx	τ [sec]	Obj.	$\Delta \tau$ [%]
ARS	4	24505	16965	22620	86.924	1487.985	178.413	1487.985	105.252
ASV	4	24505	16965	22620	107.462	858.189	228.801	858.189	112.914
CAR	4	24505	16965	22620	112.806	642.253	282.521	642.253	150.448
CHE	4	24505	16965	22620	144.798	2604.929	297.918	2604.929	105.748
CRP	4	24505	16965	22620	105.784	471.838	232.649	471.838	119.929
EVE	4	24505	16965	22620	198.908	1216.299	189.362	1216.299	-4.799
FUL	4	24505	16965	22620	116.068	694.942	187.200	694.942	61.284
HUL	4	24505	16965	22620	174.406	571.997	215.792	571.997	23.729
LIV	4	24505	16965	22620	457.376	1371.466	196.851	1371.466	-56.961
MAC	4	24505	16965	22620	165.375	1734.123	166.470	1734.123	0.662
MAU	4	24505	16965	22620	2036.260	2474.198	231.169	2474.198	-88.647
NEC	4	24505	16965	22620	95.849	974.510	213.350	974.510	122.589
NOR	4	24505	16965	22620	69.662	486.204	161.434	486.204	131.739
SOU	4	24505	16965	22620	153.804	640.762	143.529	640.762	-6.681
STO	4	24505	16965	22620	79.237	674.964	165.104	674.964	108.366
SUN	4	24505	16965	22620	229.976	837.582	202.287	837.582	-12.040
SWA	4	24505	16965	22620	90.615	659.840	228.751	659.840	152.442
TOT	4	24505	16965	22620	193.830	1383.202	171.743	1383.202	-11.395
WBA	4	24505	16965	22620	223.691	570.527	179.085	570.527	-19.941
WHU	4	24505	16965	22620	259.587	785.027	202.723	785.027	-21.905
ARS	5	47008	32544	43392	189.159	1508.956	403.177	1508.956	113.142
ASV	5	47008	32544	43392	486.235	801.662	400.098	801.662	-17.715
CAR	5	47008	32544	43392	223.908	661.053	389.787	661.053	74.083
CHE	5	47008	32544	43392	593.065	2667.682	628.685	2667.682	6.006
CRP	5	47008	32544	43392	222.443	478.977	483.007	478.977	117.138
EVE	5	47008	32544	43392	726.302	1134.583	387.368	1134.583	-46.666
FUL	5	47008	32544	43392	275.205	717.025	377.999	717.025	37.352
HUL	5	47008	32544	43392	2785.695	494.981	446.281	494.981	-83.980
LIV	5	47008	32544	43392	7386.953	1403.374	391.775	1403.374	-94.696
MAC	5	47008	32544	43392	349.382	1769.971	306.545	1769.971	-12.261
MAU	5	47008	32544	43392	7755.757	2260.280	488.009	2260.280	-93.708
NEC	5	47008	32544	43392	481.964	1014.031	440.037	1014.031	-8.699
NOR	5	47008	32544	43392	356.411	499.601	317.165	499.601	-11.012
SOU	5	47008	32544	43392	148.499	656.613	322.197	656.613	116.968
STO	5	47008	32544	43392	391.803	689.878	359.498	689.878	-8.245
SUN	5	47008	32544	43392	481.092	854.091	406.024	854.091	-15.604
SWA	5	47008	32544	43392	444.930	681.336	386.791	681.336	-13.067
TOT	5	47008	32544	43392	418.714	1409.204	400.087	1409.204	-4.449
WBA	5	47008	32544	43392	299.717	522.849	367.367	522.849	22.571
WHU	5	47008	32544	43392	654.066	678.902	426.684	678.902	-34.764
Avg					744.342		305.093		25.378

By increasing the number of team compositions we generate larger MDSTs, and the corresponding MSPEUs also become larger. Table 2 reports the results with $|\mathcal{I}| = 4$ generating a MDST with four possible distributions at each decision node and problems with up to approximately 135 thousand binary variables. In this case, our algorithm performs significantly better that Cplex on almost all instances, solving the

problems 32.8% faster.

Table 2: Results with $|\mathcal{I}| = 4$. S indicates the number of realizations describing each distribution. #Var, #Bin and #Con indicate the total number of variables, the number of binary variables, and the number of constraints, respectively. τ (Cpx) indicate the elapsed time when using the algorithm (Cplex). Obj. (Cpx) indicates the objective value obtained using the algorithm (Cplex). $\Delta \tau$ is calculated as $100(\tau - \tau \text{ Cpx})/\tau$ Cpx.

Case	S	#Var	#Bin	#Con	τ Cpx [sec]	Obj. Cpx	τ [sec]	Obj.	$\Delta \tau ~[\%]$
ARS	4	91749	69904	65535	469.440	1409.074	585.920	1409.074	24.812
ASV	4	91749	69904	65535	560.788	916.248	453.329	916.248	-19.162
CAR	4	91749	69904	65535	551.781	574.859	524.636	574.859	-4.920
CHE	4	91749	69904	65535	756.516	2476.158	729.832	2476.158	-3.527
CRP	4	91749	69904	65535	558.236	431.065	632.984	431.065	13.390
EVE	4	91749	69904	65535	7656.373	1282.401	535.382	1282.401	-93.007
FUL	4	91749	69904	65535	932.283	711.560	488.729	711.560	-47.577
HUL	4	91749	69904	65535	7746.679	559.913	608.927	559.913	-92.140
LIV	4	91749	69904	65535	7627.856	1452.961	545.440	1452.961	-92.849
MAC	4	91749	69904	65535	888.410	1657.326	371.674	1657.326	-58.164
MAU	4	91749	69904	65535	7672.755	2489.203	701.230	2489.203	-90.861
NEC	4	91749	69904	65535	545.592	908.271	518.564	908.271	-4.954
NOR	4	91749	69904	65535	393.393	443.482	371.138	443.482	-5.657
SOU	4	91749	69904	65535	368.125	603.976	342.766	603.976	-6.889
STO	4	91749	69904	65535	453.101	633.148	396.288	633.148	-12.539
SUN	4	91749	69904	65535	529.445	785.688	513.954	785.688	-2.926
SWA	4	91749	69904	65535	469.695	607.176	440.718	607.176	-6.169
TOT	4	91749	69904	65535	639.796	1295.453	421.155	1295.453	-34.174
WBA	4	91749	69904	65535	3945.793	633.948	473.058	633.948	-88.011
WHU	4	91749	69904	65535	8498.951	828.664	601.910	828.664	-92.918
ARS	5	176841	134736	126315	1210.558	1440.502	1340.235	1440.502	10.712
ASV	5	176841	134736	126315	1946.890	760.043	1381.227	760.043	-29.055
CAR	5	176841	134736	126315	1357.787	597.870	1329.530	597.870	-2.081
CHE	5	176841	134736	126315	1706.027	2602.808	1896.954	2602.808	11.191
CRP	5	176841	134736	126315	1365.008	467.102	1489.021	467.102	9.085
EVE	5	176841	134736	126315	8505.074	1232.924	1260.209	1232.924	-85.183
FUL	5	176841	134736	126315	7808.254	699.597	1388.999	699.597	-82.211
HUL	5	176841	134736	126315	8642.826	553.807	1300.262	553.807	-84.956
LIV	5	176841	134736	126315	8326.633	1460.389	1234.644	1460.389	-85.172
MAC	5	176841	134736	126315	1020.989	1678.841	1149.133	1678.841	12.551
MAU	5	176841	134736	126315	8397.024	2285.317	1405.108	2285.317	-83.267
NEC	5	176841	134736	126315	1623.349	1062.903	1427.509	1062.903	-12.064
NOR	5	176841	134736	126315	1041.199	480.087	1085.893	480.087	4.293
SOU	5	176841	134736	126315	1000.712	616.957	1020.687	616.957	1.996
STO	5	176841	134736	126315	1111.253	663.486	1059.614	663.486	-4.647
SUN	5	176841	134736	126315	1480.353	891.809	1609.855	891.809	8.748
SWA	5	176841	134736	126315	1197.202	623.004	1596.017	623.004	33.312
TOT	5	176841	134736	126315	2560.338	1441.642	1264.625	1441.642	-50.607
WBA	5	176841	134736	126315	8304.973	601.984	1152.420	601.984	-86.124
WHU	5	176841	134736	126315	8432.614	755.708	1416.893	755.708	-83.197
Avg					3207.601		926.661		-32.873

This pattern illustrates that our algorithm scales significantly better than Cplex, and is confirmed also by our results on the instances with $|\mathcal{I}| = 5$, the largest we tested. These instances generate problems with up

to approximately 400 thousand binary variables and the corresponding results are provided in Table 3. It is possible to notice that our algorithm outperforms Cplex on almost all instances. Altogether, our algorithm is able to solve the same instances 61.1% faster than Cplex. Only on two instances (CAR and CHE, with S = 4) Cplex performs better than our algorithm. Similar cases are to be expected since our algorithm entails solving several subproblems which are mixed-integer programs. Therefore, it is possible that some numerically difficult subproblems are found that slow down the entire algorithm (in our case we do not solve problems in parallel). However, despite that, the average performance of our algorithm on the largest instance is by far better.

These results illustrate that MSPEU easily generate very large optimization problems. The size of the problems, in our instances, increases by approximately three to five times by increasing the number of possible distributions. However, the algorithm we provide scales better than the solver Cplex. Particularly, when using Cplex, the average solution time increases by approximately 330% when increasing $|\mathcal{I}|$ from 3 to 4, and by approximately 175% when increasing $|\mathcal{I}|$ from 4 to 5. With our algorithm the average solution time increasing $|\mathcal{I}|$ from 3 to 4, and by approximately 203% when increasing $|\mathcal{I}|$ from 3 to 4, and by approximately 158% when increasing $|\mathcal{I}|$ from 4 to 5.

Table 3: Results with $|\mathcal{I}| = 5$. S indicates the number of realizations describing each distribution. #Var, #Bin and #Con indicate the total number of variables, the number of binary variables, and the number of constraints, respectively. τ (Cpx) indicate the elapsed time when using the algorithm (Cplex). Obj. (Cpx) indicates the objective value obtained using the algorithm (Cplex). $\Delta \tau$ is calculated as $100(\tau - \tau \text{ Cpx})/\tau$ Cpx.

Case	S	#Var	#Bin	#Con	τ Cpx [sec]	Obj. Cpx	τ [sec]	Obj.	$\Delta\tau~[\%]$
ARS	4	261051	210525	151578	4148.394	1412.322	2487.630	1412.322	-40.034
ASV	4	261051	210525	151578	9128.747	888.394	2360.369	888.394	-74.144
CAR	4	261051	210525	151578	2324.632	559.909	2951.872	559.909	26.982
CHE	4	261051	210525	151578	2941.786	2351.709	4074.808	2351.709	38.515
CRP	4	261051	210525	151578	4873.336	403.008	2757.925	403.008	-43.408
EVE	4	261051	210525	151578	9240.398	1318.359	2367.225	1328.380	-74.382
FUL	4	261051	210525	151578	9385.279	676.308	2775.651	676.308	-70.425
HUL	4	261051	210525	151578	9616.179	496.264	2430.993	600.252	-74.720
LIV	4	261051	210525	151578	9892.436	1584.601	1522.159	1584.601	-84.613
MAC	4	261051	210525	151578	1748.831	1589.133	1070.475	1589.133	-38.789
MAU	4	261051	210525	151578	9294.710	2529.355	1935.253	2533.948	-79.179
NEC	4	261051	210525	151578	6984.354	1202.156	1502.823	1202.156	-78.483
NOR	4	261051	210525	151578	3516.805	420.312	1038.191	420.312	-70.479
SOU	4	261051	210525	151578	3593.291	599.703	947.766	599.703	-73.624
STO	4	261051	210525	151578	1966.818	632.875	1182.410	632.875	-39.882
SUN	4	261051	210525	151578	4767.951	731.767	1460.145	731.767	-69.376
SWA	4	261051	210525	151578	4397.444	576.314	1280.492	576.314	-70.881
TOT	4	261051	210525	151578	3894.520	1392.754	1169.367	1392.754	-69.974
WBA	4	261051	210525	151578	11192.050	590.701	1298.701	661.566	-88.396
WHU	4	261051	210525	151578	9314.423	600.960	1755.932	768.724	-81.148
ARS	5	504556	406900	292968	5985.321	1453.229	3231.294	1453.229	-46.013
ASV	5	504556	406900	292968	13100.017	969.935	2557.610	969.935	-80.476
CAR	5	504556	406900	292968	12990.803	612.171	3115.356	612.171	-76.019
CHE	5	504556	406900	292968	15815.020	2608.034	4221.096	2608.034	-73.310
CRP	5	504556	406900	292968	13160.004	476.955	3668.813	476.955	-72.121
EVE	5	504556	406900	292968	18368.885	1289.379	2913.376	1452.827	-84.140
FUL	5	504556	406900	292968	13385.821	746.554	2811.966	755.759	-78.993
HUL	5	504556	406900	292968	12995.828	516.157	3335.790	655.447	-74.332
LIV	5	504556	406900	292968	12808.767	1473.064	2854.447	1706.759	-77.715
MAC	5	504556	406900	292968	5341.258	1681.019	2050.793	1681.019	-61.605
MAU	5	504556	406900	292968	13282.279	2690.920	3722.067	2712.864	-71.977

NEC	5	504556	406900	292968	13368.708	1365.929	2971.268	1365.929	-77.774
NOR	5	504556	406900	292968	5352.415	462.292	2111.632	462.292	-60.548
SOU	5	504556	406900	292968	5347.118	624.205	1897.896	624.205	-64.506
STO	5	504556	406900	292968	5679.705	660.999	2308.348	660.999	-59.358
SUN	5	504556	406900	292968	13483.620	927.387	2961.979	927.387	-78.033
SWA	5	504556	406900	292968	11568.065	631.495	2500.653	631.495	-78.383
TOT	5	504556	406900	292968	12955.368	1490.643	2323.134	1490.643	-82.068
WBA	5	504556	406900	292968	12852.950	709.452	2696.335	718.245	-79.022
WHU	5	504556	406900	292968	13408.572	843.851	3371.308	844.027	-74.857
Avg					8836.822		2399.883		-65.192

5 Conclusions

This paper introduced 1) a novel scenario tree structure and 2) a node formulation for multistage stochastic programs with endogenous uncertainty, as well as 3) a solution algorithm for a special case. A computational study shows that while, as expected, problems with endogenous uncertainty tend to generate large optimization problems, all our instances where solvable by Cplex to optimality in at most approximately 5 hours. Furthermore, our algorithm outperformed Cplex on the medium and large instances and showed that it scales well with the size of the problem.

Despite the encouraging results obtained in our study, solving multistage stochastic programs with endogenous uncertainty remains, in general, a challenging task. Our algorithm requires an explicit scenario tree structure, and solves a number of problems which grows linearly with the number of nodes. However, the number of nodes in a scenario tree grows exponentially with the number of stages and the treatment of cases with more than a handful of stages may soon become prohibitive. New approaches in the spirit of Pereira and Pinto (1991); Zou et al. (2019), based on progressive approximations of future stages, may be proven more scalable. Furthermore, our models employ so called big-M constants. Poorly chosen big-M values, e.g., by trial-and-error, may become problematic. It is well known that they may create numerical difficulties when solving mixed-integer programs. In addition, as discussed in Pineda and Morales (2019), they may lead to highly sub-optimal solutions. The authors use a simple bilevel programming problem (which can be reformulated as a mixed-integer program that uses big-Ms) to show how a poorly designed trial-and-error procedure may generate the false belief that the solution to the reformulation is indeed optimal for the original bilevel problem. Based on these evidences we also advocate caution and the use of more sophisticated procedures for setting big-M values. The procedure in Appendix C goes in this direction. Finally, cases more general than the special one treated Section 3 remain to be addressed.

References

- S. Ahmed. Strategic planning under uncertainty: Stochastic integer programming approaches. Phd, University of Illinois at Urbana-Champaign, 2000.
- R. M. Apap and I. E. Grossmann. Models and computational strategies for multistage stochastic programming under endogenous and exogenous uncertainties. *Computers & Chemical Engineering*, 103:233–274, 2016. ISSN 0098-1354.
- S. Bansal and J. S. Dyer. Planning for end-user substitution in agribusiness. Operations Research, 68(4): 1000–1019, 2020.
- J. R. Birge and F. Louveaux. Introduction to stochastic programming. Springer, New York, 1997.
- M. Colvin and C. T. Maravelias. A stochastic programming approach for clinical trial planning in new drug development. *Computers & Chemical Engineering*, 32(11):2626–2642, 2008.
- M. Colvin and C. T. Maravelias. Modeling methods and a branch and cut algorithm for pharmaceutical clinical trial planning using stochastic programming. *European Journal of Operational Research*, 203(1): 205–215, 2010.

- B. da Costa Flach. Stochastic Programming with Endogenous Uncertainty: An Application in Humanitarian Logistics. PhD thesis, PUC-Rio, 2010.
- L. F. Escudero, M. A. Garín, J. F. Monge, and A. Unzueta. On preparedness resource allocation planning for natural disaster relief under endogenous uncertainty with time-consistent risk-averse management. *Computers & Operations Research*, 98:84–102, 2018.
- V. Goel and I. E. Grossmann. A stochastic programming approach to planning of offshore gas field developments under uncertainty in reserves. *Computers & Chemical Engineering*, 28:1409–1429, 2004.
- V. Goel and I. E. Grossmann. A Class of stochastic programs with decision dependent uncertainty. *Mathe-matical Programming*, 108:355–394, 2006.
- V. Gupta and I. E. Grossmann. Solution strategies for multistage stochastic programming with endogenous uncertainties. *Computers & Chemical Engineering*, 35:2235–2247, 2011.
- H. Held and D. L. Woodruff. Heuristics for Multi-Stage Interdiction of Stochastic Networks. Journal of Heuristics, 11(5-6):483–500, 2005.
- L. Hellemo. Managing Uncertainty in Design and Operation of Natural Gas Infrastructure. Phd, Norwegian University of Science and Technology, 2016.
- L. Hellemo, P. I. Barton, and A. Tomasgard. Decision-dependent probabilities in stochastic programs with recourse. *Computational Management Science*, 15(3):369–395, 2018.
- F. Hooshmand and S. A. MirHassani. Efficient constraint reduction in multistage stochastic programming problems with endogenous uncertainty. Optimization Methods and Software, 31:359–376, 2016.
- F. Hooshmand and S. A. MirHassani. Reduction of nonanticipativity constraints in multistage stochastic programming problems with endogenous and exogenous uncertainty. *Mathematical Methods of Operations Research*, 87(1):1–18, 2018.
- T. W. Jonsbråten, R. J.-B. Wets, and D. L. Woodruff. A class of stochastic programs withdecision dependent random elements. Annals of Operations Research, 82:83–106, 1998.
- P. Kall and S. W. Wallace. Stochastic programming. John Wiley and Sons Ltd, Chichester, 1994.
- M. Kaut, K. T. Midthun, A. S. Werner, A. Tomasgard, L. Hellemo, and M. Fodstad. Multi-horizon stochastic programming. *Computational Management Science*, 11(1-2):179–193, 2014.
- M. Laumanns, S. Prestwich, and B. Kawas. Distribution shaping and scenario bundling for stochastic programs with endogenous uncertainty. In J. L. Higle, W. Römisch, and S. Sen, editors, *Stochastic Programming E-Print Series*, Stochastic Programming E-Print Series. Institut für Mathematik, 2014.
- L. Mercier and P. Van Hentenryck. An anytime multistep anticipatory algorithm for online stochastic combinatorial optimization. Annals of Operations Research, 184(1):233–271, 2011.
- G. Pantuso. The Football Team Composition Problem: A Stochastic Programming approach. Journal of Quantitative Analysis in Sports, 13(3):113–129, 2017.
- G. Pantuso and L. M. Hvattum. Maximizing performance with an eye on the finances: a chance-constrained model for football transfer market decisions. TOP, pages 1–29, 2020.
- S. Peeta, F. S. Salman, D. Gunnec, and K. Viswanath. Pre-disaster investment decisions for strengthening a highway network. *Computers & Operations Research*, 37:1708–1719, 2010.
- M. V. Pereira and L. M. Pinto. Multi-stage stochastic optimization applied to energy planning. Mathematical programming, 52(1-3):359–375, 1991.
- S. Pineda and J. M. Morales. Solving linear bilevel problems using big-ms: Not all that glitters is gold. IEEE Transactions on Power Systems, 34(3):2469–2471, 2019. doi: 10.1109/TPWRS.2019.2892607.

- B. Tarhan and I. E. Grossmann. A multistage stochastic programming approach with strategies for uncertainty reduction in the synthesis of process networks with uncertain yields. *Computers & Chemical Engineering*, 32:766–788, 2008.
- B. Tarhan, I. E. Grossmann, and V. Goel. Stochastic Programming Approach for the Planning of Offshore Oil or Gas Field Infrastructure under Decision-Dependent Uncertainty. *Industrial & Engineering Chemistry Research*, 48:3078–3097, 2009.
- B. Tarhan, I. E. Grossmann, and V. Goel. Computational strategies for non-convex multistage MINLP models with decision-dependent uncertainty and gradual uncertainty resolution. Annals of Operations Research, 203(1):141–166, 2013.
- K. Tong, Y. Feng, and G. Rong. Planning under Demand and Yield Uncertainties in an Oil Supply Chain. Industrial & Engineering Chemistry Research, 51(2):814–834, 2012.
- K. Viswanath, P. Srinivas, and S. F. Sibel. Investing in the Links of a Stochastic Network to Minimize Expected Shortest Path Length. 2004. URL https://www.krannert.purdue.edu/programs/phd/working-papers-series/2004/1167.pdf.
- J. Zou, S. Ahmed, and X. A. Sun. Stochastic dual dynamic integer programming. *Mathematical Programming*, 175(1-2):461–502, 2019.

A Notation table

	Sets
$\{1, \ldots, T\}$	Set of decision stages
$\tilde{\mathcal{N}}$	Set of nodes in the multi-distribution scenario tree
$\mathcal{N}_t \subseteq \mathcal{N}$	Set of nodes at stage t
${\mathcal D}_n$	Set of possible distributions applicable at node n
$\mathcal{N}_{nd}\subseteq\mathcal{N}$	Set of child nodes of node n if distribution $d \in \mathcal{D}_n$ is enforced
X_t	Domain of the decision variables at stage t
	Parameters
t(n)	Stage of node n
a(n)	Parent node of node n
π_n	Probability of node n
$r_n \in \mathbb{R}^{N_{t(n)}}$	Coefficients of decision variables x_n in the objective function
$q_{nd} \in \mathbb{R}^1$	Coefficient of decision variable δ_{nd} in the objective function
$A_n \in \mathbb{R}^{M_{t(n)} \times N_{t(n)}}$	Coefficients of variables x_n in the constraints that connect x_n and δ_{nd} decisions
$B_{nd} \in \mathbb{R}^{M_{t(n)} \times 1}$	Coefficients of variable δ_{nd} in the constraints that connect x_n and δ_{nd} decisions
$C_{a(n)} \in \mathbb{R}^{M_{t(n)} \times N_{t(a(n))}}$	Coefficients of variables $x_{a(n)}$ in the constraints that connect x_n and δ_{nd} decisions
$D_{a(n),d} \in \mathbb{R}^{M_{t(n)} \times 1}$	Coefficients of variable $\delta_{a(n),d}$ in the constraints that connect x_n and δ_{nd} decisions
$h_n \in \mathbb{R}^{M_{t(n)}}$	Right-hand-side coefficients of the constraints that connect x_n and δ_{nd} decisions
$\Theta_n \in \mathbb{R}^1$	Terminal value of the decisions following node $n \in \mathcal{N}_T$
	Variables
$x_n \in \mathbb{R}^{N_{t(n)}}$	Decisions made at node n
$\delta_{nd} \in \{0,1\}$	Decision on whether to apply probability distribution d at node n
$\theta_n \in \mathbb{R}^1$	Expected value of the decisions made at the nodes descending from n

Table 4: Notation of problem (1).

B A big-*M* reformulation

In this appendix a big-M reformulation that linearizes model (1) is introduced. In addition to the notation introduced in Section 2, let $M_{nd} \in \mathbb{R}^1$ be a suitably high constant. The linearized EUMSP is thus

$$\max \ r_0^T x_0 + \sum_{d \in \mathcal{D}_0} q_{0d} \delta_{0d} + \theta_0 \tag{7a}$$

s.t.
$$\sum_{d \in \mathcal{D}_n} \delta_{nd} = 1$$
 $n \in \mathcal{N},$ (7b)

$$A_n x_n + \sum_{d \in \mathcal{D}_n} B_{nd} \delta_{nd} + C_{a(n)} x_{a(n)} + \sum_{d \in \mathcal{D}_{a(n)}} D_{a(n),d} \delta_{a(n),d} = h_n \qquad n \in \mathcal{N},$$
(7c)

$$\theta_n \leq \sum_{m \in \mathcal{N}_{nd}} \pi_m (r_m^T x_m + \sum_{d \in \mathcal{D}_m} q_{md} \delta_{md} + \theta_m) + M_{nd} (1 - \delta_{nd}) \qquad n \in \mathcal{N} \setminus \mathcal{N}_T, d \in \mathcal{D}_n$$
(7d)

$$\theta_n = \Theta_n \qquad \qquad n \in \mathcal{N}_T, \qquad (7e)$$

$$\begin{aligned} x_n \in \mathcal{A}_{t(n)} & n \in \mathcal{N}, \quad (1) \\ \delta_{t,t} \in \{0, 1\} & n \in \mathcal{N}, \quad d \in \mathcal{D}, \quad (7\sigma) \end{aligned}$$

$$\theta_n \in \mathcal{R} \tag{18}$$

Note, particularly, that constraints (7d) are equivalent to (1d). Consider a given node n, other than a leaf node. Observe that only for one distribution d there will be a δ_{nd} which takes value one at n (see (7b)). For the same n and for the same d, the second term on the right-hand-side of (7d) will be zero (i.e., the big-M will not be enforced), and the resulting right-hand-side will be the most binding among the $|\mathcal{D}_n|$ constraints for node n. Since we are maximizing, at optimality θ_n will take value of the expectation according to the distribution d for which $\delta_{nd} = 1$, as it happens in model (1).

C Finding big-M values

An efficient implementation of model (7) requires tight big-M values. Observe that, for $n \in \mathcal{N} \setminus \mathcal{N}_T$ and $d \in \mathcal{D}_n$, constant M_{nd} must be a valid upper bound for constraints (7d), that is:

$$\theta_n - \sum_{m \in \mathcal{N}_{nd}} \pi_m (r_m^T x_m + \sum_{d \in \mathcal{D}_m} q_{md} \delta_{md} + \theta_m) \le M_{nd}$$

Let us introduce ϕ_{nd} to represent the expectation at the children of node n for distribution d, that is:

$$\phi_{nd} = \sum_{m \in \mathcal{N}_{nd}} \pi_m (r_m^T x_m + \sum_{d \in \mathcal{D}_m} q_{md} \delta_{md} + \theta_m)$$

Consider the numerical example shown in Table 5 for a given node n and three possible distributions d. The table reports the highest and lowest values the expectation at the following stage can take for each possible distribution, and the corresponding value of θ_n should a specific distribution be chosen. When choosing M_{n,d_1} , notice that the maximum value θ_n can reach for other distributions is 9, and that the least value ϕ_{n,d_1} can reach is 5. Therefore, we can set $M_{n,d_1} = 9 - 5$. In fact, if distribution d_2 is chosen, θ_n will be at most 9 and ϕ_{n,d_1} at least 5, thus adding 4 to ϕ_{n,d_1} will ensure that θ_n is correctly set to 9. Similarly, we choose $M_{n,d_2} = 6$ as the highest value θ_n can take if d_2 is not selected is 10, while the least value of ϕ_{n,d_2} is 4. Finally, with a similar reasoning we can set $M_{n,d_3} = 7$.

From the example in Table 5 we understand that finding values for M_{nd} amounts to finding highest values for ϕ_{nd} and differences $\theta_n - \phi_{nd}$. In what follows we illustrate how these values can be found for t = T - 1in Section C.1 and t = T - 2, ..., 1 in Section C.2.

C.1 Big-M values for stages t = T - 1

We start at the second-last stage, t = T - 1. Our task is that of finding, for each node $\bar{n} \in \mathcal{N}_{T-1}$ and for each distribution $\bar{d} \in \mathcal{D}_{\bar{n}}$, a constant $M_{\bar{n}\bar{d}}$ which is slightly higher than the highest difference $\theta_{\bar{n}} - \phi_{\bar{n}\bar{d}}$, where

Table 5: Numerical example for the calculation of constants ${\cal M}_{nd}$

		$ \phi_r$		
d	$ heta_n$	Max	Min	M_{nd}
d_1	10	10	5	9 - 5 = 4
d_2	9	9	4	10 - 4 = 6
d_3	8	8	3	10 - 3 = 7

again

$$\phi_{\bar{n}\bar{d}} = \sum_{m \in \mathcal{N}_{\bar{n}\bar{d}}} \pi_m (r_m^T x_m + \sum_{d \in \mathcal{D}_m} q_{md} \delta_{md} + \theta_m)$$

Now, the highest difference can be found solving the following optimization problem:

$$M_{\bar{n}\bar{d}}^* = \max \ \theta_{\bar{n}} - \sum_{m \in \mathcal{N}_{\bar{n}\bar{d}}} \pi_m (r_m^T x_m + \sum_{k \in \mathcal{D}_m} q_{mk} \delta_{mk} + \Theta_m)$$
(8a)

s.t.
$$\sum_{d \in \mathcal{D}_n} \delta_{nd} = 1$$
 (8b)

$$A_n x_n + \sum_{d \in \mathcal{D}_n} B_{nd} \delta_{nd} + C_{a(n)} x_{a(n)} + \sum_{d \in \mathcal{D}_{a(n)}} D_{a(n),d} \delta_{a(n),d} = h_n \qquad n \in \mathcal{N},$$
(8c)

$$\theta_{\bar{n}} \le \Theta^*_{\bar{n}\bar{d}},$$
(8d)

$$x_n \in X_{t(n)} \qquad \qquad n \in \mathcal{N}, \qquad (8e)$$

$$\delta_{nd} \in \{0, 1\} \qquad \qquad n \in \mathcal{N}, d \in D_n, \tag{81}$$
$$\delta_{\bar{n}\bar{d}} = 0 \tag{8g}$$

$$\bar{n}\bar{d} = 0 \tag{8g}$$

Problem (8) consists of finding the feasible solution to problem (7) which yields the highest value for the left-hand-side of constraint (7d) for \bar{n} and \bar{d} . The following two elements must be noted in (8). Constraints (7d) of the original problem, which determine the correct expectations at the stages before T-1, are not included as they are irrelevant for stage T-1. The second element to note is constraint (8d) which sets an upper bound $\theta_{\bar{n}}$. This upper bound represents the highest value $\theta_{\bar{n}}$ can take for the distributions other than \bar{d} . This value can, in turn, be obtained solving optimization problems. The highest expectation for stage T, given distribution $d' \in D_{\bar{n}}$ is the optimal value to problem (9):

$$\Phi_{\bar{n}d'}^* = \max \sum_{m \in \mathcal{N}_{\bar{n}d'}} \pi_m (r_m^T x_m + \sum_{k \in \mathcal{D}_m} q_{mk} \delta_{mk} + \Theta_m)$$
(9a)

s.t.
$$\sum_{d \in \mathcal{D}_n} \delta_{nd} = 1$$
 $n \in \mathcal{N},$ (9b)

$$A_n x_n + \sum_{d \in \mathcal{D}_n} B_{nd} \delta_{nd} + C_{a(n)} x_{a(n)} + \sum_{d \in \mathcal{D}_{a(n)}} D_{a(n),d} \delta_{a(n),d} = h_n \qquad n \in \mathcal{N},$$
(9c)

$$x_n \in X_{t(n)} \qquad \qquad n \in \mathcal{N}, \qquad (9d)$$

$$\delta_{-} \in \{0, 1\} \qquad \qquad n \in \mathcal{N}, \quad d \in \mathcal{D} \qquad (9c)$$

$$\delta_{nd} \in \{0, 1\} \qquad \qquad n \in \mathcal{N}, u \in \mathcal{D}_n, \qquad (9e)$$

$$\delta_{\bar{n}d'} = 1 \qquad \qquad (9f)$$

Therefore, when calculating $M_{\bar{n}\bar{d}}$, the upper bound $\Theta^*_{\bar{n}}$ in (8d) is given by:

$$\Theta_{\bar{n}\bar{d}}^* = \max_{d \in \mathcal{D}_{\bar{n}}: d \neq \bar{d}} \Phi_{\bar{n}d}^*$$

Clearly, solving problems (8) and (9) amounts to solving integer programs of size comparable with the original problem (7). However, the tightest $\Theta_{\bar{n}\bar{d}}^*$ and $M_{\bar{n}\bar{d}}$ are not necessary, and higher values would still provide correct results. A suitable value for $M_{\bar{n}\bar{d}}$ can be obtained by solving any relaxation of problem (8), yielding $M_{\bar{n}\bar{d}}^R$, and (9), yielding $\Phi_{\bar{n}\bar{d}}^R$ and in turn $\Theta_{\bar{n}\bar{d}}^R$. As an example, one might solve the linear programming relaxation of problems (8) and (9) or, if the size of the problems is excessively high, one might choose to relax constraints (8c) and (9c) for some stages. Finally, since the procedure outlined might return negative values for some M_{nd} , we set $M_{nd} = \max\{0, M_{nd}^R\}$ to reduce, when possible, high big-M absolute values. The procedure is summarized in Algorithm 1.

Algorithm 1 Algorithm for calculating M_{nd} for T-1

1: Input: $\mathcal{N}, \mathcal{D}_n$ for $n \in \mathcal{N}, \Theta_n$ for $n \in \mathcal{N}_T$ 2: for Node $\bar{n} \in \mathcal{N}_{T-1}$ do for Distribution $d \in \mathcal{D}_{\bar{n}}$ do 3: Calculate $\Phi_{\bar{n}d}^{LP}$ by solving the LP relaxation to problem (9) 4: 5: end for for Distribution $\bar{d} \in \mathcal{D}_{\bar{n}}$ do 6: In constraint (8d) set $\Theta_{\bar{n}\bar{d}}^* = \max_{d \in \mathcal{D}_{\bar{n}}: d \neq \bar{d}} \Phi_{\bar{n}d}^{LP}$ Calculate $M_{\bar{n}\bar{d}}^R$ by solving a suitable relaxation to problem (8) 7: 8: Set $M_{\bar{n}\bar{d}} = \max\{0, M^R_{\bar{n}\bar{d}}\}$ 9: end for 10: 11: end for 12: **return** M_{nd} for $n \in \mathcal{N}_{T-1}$ and $d \in \mathcal{D}_n$.

C.2 Big-M values for stages t = T - 2, ..., 1

δ,

6

Once constants M_{nd} are available for every $n \in \mathcal{N}_{T-1}$ and $d \in \mathcal{D}_n$ we can proceed in a similar way to calculate big-Ms for stages $T-2, \ldots, 1$. Given a stage $\bar{t} \in \{T-2, \ldots, 1\}$, a node at that stage, $\bar{n} \in \mathcal{N}_{\bar{t}}$, and distribution available at that node $\bar{d} \in \mathcal{D}_{\bar{n}}$, the tightest value of constant $M_{\bar{n}\bar{d}}$, namely $M_{\bar{n}\bar{d}}^*$, is

$$M_{\bar{n}\bar{d}}^* = \max \ \theta_{\bar{n}} - \sum_{m \in \mathcal{N}_{\bar{n}\bar{d}}} \pi_m (r_m^T x_m + \sum_{k \in \mathcal{D}_m} q_{mk} \delta_{mk} + \theta_m)$$
(10a)

s.t.
$$\sum_{d \in \mathcal{D}_n} \delta_{nd} = 1$$
 $n \in \mathcal{N},$ (10b)

$$A_n x_n + \sum_{d \in \mathcal{D}_n} B_{nd} \delta_{nd} + C_{a(n)} x_{a(n)} + \sum_{d \in \mathcal{D}_{a(n)}, d} \delta_{a(n), d} = h_n \qquad n \in \mathcal{N}, \quad (10c)$$

$$\theta_n \ge \sum_{m \in \mathcal{N}_{nd}} \pi_m (r_m^T x_m + \sum_{k \in \mathcal{D}_m} q_{mk} \delta_{mk} + \theta_m) - M_{nd} (1 - \delta_{nd}) \qquad t = \bar{t} + 1, \dots, T - 1, n \in \mathcal{N}_t, d \in \mathcal{D}_n,$$
(10d)

$$\begin{aligned} \partial_n &= \Theta_n & n \in \mathcal{N}_T \quad (10e) \\ \partial_{\overline{z}} &\leq \Theta^* \, \overline{z} \end{aligned} \tag{10f}$$

$$x_n \in X_{t(n)} \qquad \qquad n \in \mathcal{N}, \quad (10g)$$

$$\delta_{nd} \in \{0,1\} \qquad \qquad n \in \mathcal{N}, d \in \mathcal{D}_n, \quad (10h)$$

$$\bar{n}\bar{d} = 0 \tag{10i}$$

Problem (10) consists of finding the feasible solution to problem (7) which yields the highest value of the left-hand-side of constraint (7d) for node \bar{n} and distribution \bar{d} . Notice that, unlike in problem (8), problem (10) includes constraints (10d) which are necessary to ensure that θ_n values are set to the lowest expectation for all stages between \bar{t} and T-1. Note that constant M_{nd} in constraints (10d) is also an upper bound to the quantity $\theta_n - \sum_{m \in \mathcal{N}_{nd}} \pi_m (r_m^T x_m + \sum_{k \in \mathcal{D}_m} q_{mk} \delta_{mk} + \theta_m)$ and can be thus set to the quantities determined at previous iterations. Also in this case, $\Theta_{\bar{n}}^*$ in (10f) represents the highest possible value $\theta_{\bar{n}}$ can take for distributions other than \bar{d} . The highest expectation for stage $\bar{t} + 1$, given distribution $d' \in D_{\bar{n}}$ is the optimal value to problem (11)

$$\Phi_{\bar{n}d'}^* = \max \sum_{m \in \mathcal{N}_{\bar{n}d'}} \pi_m (r_m^T x_m + \sum_{k \in \mathcal{D}_m} q_{mk} \delta_{mk} + \theta_m)$$
(11a)

s.t.
$$\sum_{d \in \mathcal{D}_n} \delta_{nd} = 1$$
 $n \in \mathcal{N},$ (11b)

$$A_n x_n + \sum_{d \in \mathcal{D}_n} B_{nd} \delta_{nd} + C_{a(n)} x_{a(n)} + \sum_{d \in \mathcal{D}_{a(n)}} D_{a(n),d} \delta_{a(n),d} = h_n \qquad n \in \mathcal{N}, \quad (11c)$$

$$\theta_n \le \sum_{m \in \mathcal{N}_{nd}} \pi_m (r_m^T x_m + \sum_{k \in \mathcal{D}_m} q_{mk} \delta_{mk} + \theta_m) + M_{nd} (1 - \delta_{nd}) \qquad t = \bar{t} + 1, \dots, T - 1, n \in \mathcal{N}_t, d \in \mathcal{D}_n,$$
(11d)

$$\theta_n = \Theta_n$$
 $n \in \mathcal{N}_T$
(11e)

 $m \in \mathcal{N}_T$
(11f)

$$\begin{aligned} & & n \in \mathcal{N}, \quad (\Pi) \\ & & & n \in \mathcal{N}, \quad d \in \mathcal{D}_n, \quad (\Pi) \end{aligned}$$

$$\bar{n}d' = 1 \tag{11h}$$

Therefore, the upper bound $\Theta^*_{\bar{n}\bar{d}}$ in (10f) is given by:

$$\Theta_{\bar{n}}^* = \max_{d \in \mathcal{D}_{\bar{n}}: d \neq \bar{d}} \Phi_{\bar{n}d}^*$$

Similarly to Section C.1, calculating the optimal $M_{\bar{n}\bar{d}}^*$ is cumbersome as well as not strictly necessary. Therefore, any computationally suitable relaxation to problems (10) and (11) can be adopted. The procedure for obtaining constants M_{nd} for stages $T - 2, \ldots, 1$ is sketched in Algorithm 2.

Algorithm 2 Algorithm for calculating M_{nd} for $\bar{t} = T - 2, ..., 1$

1: Input: $\mathcal{N}, \mathcal{D}_n$ for $n \in \mathcal{N}, \Theta_n$ for $n \in \mathcal{N}_T, M_{nd}$ for $n \in \mathcal{N}_t, t = \overline{t} + 1, \dots, T - 1, d \in D_n$. 2: for Node $\bar{n} \in \mathcal{N}_{\bar{t}}$ do for Distribution $d \in \mathcal{D}_{\bar{n}}$ do Calculate $\Phi_{\bar{n}d}^{LP}$ by solving the LP relaxation to problem (11) 3: 4: end for 5: for Distribution $\bar{d} \in \mathcal{D}_{\bar{n}}$ do 6: In constraint (10f) set $\Theta_{\bar{n}\bar{d}}^* = \max_{d \in \mathcal{D}_{\bar{n}}: d \neq \bar{d}} \Phi_{\bar{n}d}^{LP}$ Calculate $M_{\bar{n}\bar{d}}^R$ by solving a relaxation to problem (10) 7: 8: Set $M_{\bar{n}\bar{d}} = \max\{0, M^R_{\bar{n}\bar{d}}\}$ 9: end for 10: 11: end for 12: **return** M_{nd} for $n \in \mathcal{N}_{\bar{t}}$ and $d \in \mathcal{D}_n$.

D Computation time for finding big-M values for the FTCP

Table 6: Average elapsed time in seconds for the computations of big-M values using the procedure in Section 4.2. S indicates the number of realizations describing each distribution.

Team	S	$ \mathcal{I} = 3$	$ \mathcal{I} = 4$	$ \mathcal{I} = 5$
ARS	4	11.216	36.161	66.495
ASV	4	13.387	37.614	84.259
CAR	4	17.873	44.370	96.844
CHE	4	16.862	52.053	149.683
CRP	4	17.100	47.362	129.843
EVE	4	10.921	36.309	93.248
FUL	4	12.467	39.589	67.543
HUL	4	11.928	40.945	86.544
LIV	4	11.384	32.654	84.933
MAC	4	12.999	32.460	71.190
MAU	4	14.093	38.464	95.931
NEC	4	11.079	32.661	87.461
NOR	4	9.897	28.554	66.370

SOU	4	10.861	23.151	66.977
STO	4	11.009	37.650	68.867
SUN	4	18.335	36.425	85.658
SWA	4	12.770	32.966	99.143
TOT	4	11.475	28.322	69.286
WBA	4	10.644	28.465	85.157
WHU	4	17.390	45.385	78.934
ARS	5	23.028	59.372	165.883
ASV	5	21.531	62.066	167.843
CAR	5	24.472	74.075	241.720
CHE	5	28.144	84.103	301.168
CRP	5	21.557	74.754	257.036
EVE	5	21.673	72.323	183.927
FUL	5	19.659	50.441	142.312
HUL	5	23.749	51.421	176.955
LIV	5	18.195	40.295	150.142
MAC	5	20.799	47.659	117.553
MAU	5	31.487	64.327	155.962
NEC	5	35.180	64.231	175.313
NOR	5	26.893	51.132	146.938
SOU	5	18.282	43.805	120.134
STO	5	21.236	54.687	151.571
SUN	5	29.861	66.360	187.473
SWA	5	30.952	59.528	141.202
TOT	5	28.963	56.107	128.371
WBA	5	23.609	54.868	139.128
WHU	5	27.754	63.924	176.582
Avg		19.018	48.176	129.039