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Structural Properties of Feasible Bookings in the European Entry-Exit Gas Market System

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Abstract. In this work, we analyze the structural properties of the set of feasible bookings in the European entry-exit gas market system. We present formal definitions of feasible bookings and then analyze properties that are important if one wants to optimize over them. Thus, we study whether the sets of feasible nominations and bookings are bounded, convex, connected, conic, and star-shaped. The results depend on the specific model of gas flow in a network. Here, we discuss a simple linear flow model with arc capacities as well as nonlinear and mixed-integer nonlinear models of passive and active networks, respectively. It turns out that the set of feasible bookings has some unintuitive properties. For instance, we show that the set is nonconvex even though only a simple linear flow model is used.

1. Introduction

Mathematical optimization of gas transport networks has been a highly active field of research during the last decades. For an overview of the field see, e.g., the recent book [35] and the recent survey article [44] as well as the references therein. Most of the research in this field so far dealt with the case of a single pattern of supplies and withdrawals (which we will call a nomination throughout the paper) that need to be transported through a given network. In this setting, two major tasks for mathematical optimization arise:

Feasibility: Given a nomination, check whether it is feasible w.r.t. physical and technical laws and rules.

Optimization: Given a feasible nomination, determine the cheapest way of transporting the nomination.

Both questions have been addressed very comprehensively in the literature of the last 50 years; see, e.g., [2–5, 10, 11, 15, 22, 23, 29–31, 37–39, 41, 43, 47, 54] to name only a few publications that stem from different fields of mathematical optimization like mixed-integer linear optimization, nonlinear optimization, robust optimization, optimization with complementarity constraints, or optimization with partial differential equations.

However, since the gas market liberalization that started in the 1990s, also other mathematical questions came up. In Europe, the gas market liberalization lead to the entry-exit system; see, e.g., [13, 17, 18]. At the core of this system, the interplay of so-called bookings and nominations has been established. A booking is a mid- to long-term contract between a gas trader and a gas transport company in which a capacity right is granted to the trader. This means, that for a booking $b \geq 0$, the trader has the right to nominate (for the day ahead) every amount below, i.e., every amount ℓ with $0 \leq \ell \leq b$, if it is in balance with all other nominations. By signing such a booking contract, the gas transport company guarantees that every

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balanced set of nominations that is in compliance with the corresponding bookings can actually be transported.

In contrast to the rich literature on nominations, there is much less literature on the mathematics of bookings. Among the first mathematical treatments of bookings are the two PhD theses [32, 53] and the technical report [20]. Moreover, a rather detailed discussion is given in Part III of the book [35]. From a mathematical point of view, the feasibility of a booking can be seen as some special case of robust feasibility in the sense of robust optimization [6]. In this context, an efficient test for checking the feasibility of a booking in a passive tree-structured network is given in [45] and the feasibility of bookings as well as complexity results for checking this feasibility is studied in [36]. Other related problems like the computation of maximum possible bookings (the so-called technical capacity) are introduced in [40]. Lastly, we also want to mention studies like they can be found in [1, 7], where entry-exit tariffs are discussed but where the authors do not study feasibility or optimization questions based on bookings.

Since the concept of bookings is at the interface of gas transport and trading, it needs to be studied both from a technical and physical as well as from an economic viewpoint. The latter is, e.g., given in [27], where a mathematical multilevel model of the European entry-exit system is presented that also includes the trading of booking contracts. Other modeling approaches for economic questions in gas markets have been discussed recently in [26]. In mathematical economics, the general hardness of multilevel or equilibrium models often requires to abstract from certain physical details of gas flow models. This is why linear flow models are often discussed in this area; see, e.g., [8, 9, 12, 14, 21, 42, 52] and the references therein. As a consequence, the consideration of bookings also needs to be done on different levels of physical and technical modeling of gas transport networks—ranging from very simple linear models to highly sophisticated nonlinear ones [50].

The above discussion is the main motivation for this paper. We analyze structural properties of the set of feasible nominations and bookings. As already discussed above, the structural properties of the set of feasible nominations has already been discussed in the literature. Thus, we focus on the set of feasible bookings. Moreover, and in contrast to the existing literature, we do not consider the set of feasible nominations structurally "only" for the case in passive but also in gas transport networks that include active elements such as compressors. The analysis of the set of feasible bookings and nominations is an important prerequisite for deciding feasibility of bookings or for optimizing over these sets. Our contribution is the following. We formalize the concept of bookings and study the set of feasible nominations and bookings for three important and different classes of gas flow models:

(a) a linear flow model with arc capacities;
(b) a nonlinear flow model for passive networks, i.e., for networks without controllable elements like compressors or valves;
(c) a nonlinear flow model for active networks, i.e., for networks with controllable elements.

For these models, we prove or present counterexamples for the boundedness, convexity, and connectivity of the set of feasible nominations and bookings and additionally study whether it is conic or star-shaped. By this, we pave the way for further studies like, e.g., computing the largest possible bookings, which is of practical importance for gas transport companies. Our results show that one needs to be very careful when considering bookings because rather unintuitive properties can be observed—for instance that we obtain a nonconvex set of feasible bookings even for linear flow models. This is mainly based on the definition of a feasible booking b, which requires that *every* balanced and component-wise smaller nomination

$0 \leq \ell \leq b$ is feasible. At this point, let us note that the rather counter-intuitive definition considered in this paper is chosen as it is discussed in the respective legislative texts on the European entry-exit gas market system; see [17, 18]. The motivation of the notion of bookings in particular and the European entry-exit system in general is the decoupling of economic trading and the technical transport through the network. Since a feasible booking guarantees the feasibility of every balanced and component-wise smaller nomination, no further technical restrictions for trading booking-compliant nominations exist. Thus, economic trading and the technical transport of gas are decoupled.

We remark that all models studied in this paper describe stationary flows. Since unexpected effects w.r.t. bookings already appear in these simplified gas transport models, it is likely to assume that this is also the case in more detailed gas transport models—most probably, it will even be more pronounced. The analysis of transient flow models as well as even more complicated stationary flow models is out of scope of this paper and part of our future research.

The remainder of the paper is structured as follows. In Section 2 we introduce the basic definitions and discuss a preliminary example of a very basic gas flow model in order to illustrate the concepts that we afterward analyze for more complicated models. The Sections 3–5 then analyze the properties of the set of feasible bookings for a linear flow model with arc capacities (Section 3), a nonlinear flow model (Section 4), and a mixed-integer nonlinear flow model (Section 5) in which controllable elements are modeled using binary variables. Finally, we conclude in Section 6 and pose some questions for future research.

2. Basic Definitions

We model a gas network as a directed graph $G = (V, A)$ with node set V and arc set A. The set of nodes is partitioned into the set V_+ of entry nodes, at which gas is supplied, the set V_- of exit nodes, where gas is withdrawn, and the set V_0 of the remaining inner nodes. We abbreviate the set $V_+ \cup V_-$ by V_b. The orientation of the arcs is artificial and thus negative flow along an arc can occur. In real-world gas networks, the arc set is typically partitioned into different types of arcs that correspond to different elements of the network; e.g., pipes, compressors, etc. We introduce these sets when we consider them for the first time. Finally, we always assume in the following that the undirected graph underlying G is connected. We now introduce basic definitions that we use in the following.

Definition 2.1 (Load flow)**.** A *load flow* is a vector

$$\ell = (\ell_u)_{u \in V_\mathrm{b}} \in \mathbb{R}^{V_\mathrm{b}}_{\geq 0}.$$

The set of load flow vectors is denoted by L.

A load flow thus corresponds to an actual situation at a single point in time by specifying the amount of gas that is supplied (ℓ_u for $u \in V_+$) or withdrawn (ℓ_u for $u \in V_-$). Since we only consider stationary flows, we need to impose that the supplied amount of gas equals the withdrawn amount, which leads to the definition of a nomination.

Definition 2.2 (Nomination)**.** A *nomination* is a balanced load flow ℓ, i.e., $\sigma^\top \ell = 0$ with $\sigma \in \{\pm 1\}^{V_\mathrm{b}}, \sigma_u = 1$ for all $u \in V_+$, $\sigma_u = -1$ for all $u \in V_-$. The set of nominations is called N, i.e.,

$$N := \{\ell \in L : \sigma^\top \ell = 0\} \subseteq L.$$

Definition 2.3 (Booking)**.** A *booking* is a vector $b = (b_u)_{u \in V_\mathrm{b}} \in \mathbb{R}^{V_\mathrm{b}}_{\geq 0}$. The set of bookings is denoted by B.

Nominations and bookings are connected by the following definition.

Definition 2.4 (Booking-compliant nomination)**.** A nomination ℓ is called *booking-compliant* with respect to the booking b if $\ell \leq b$ holds, where "$\leq$" is meant componentwise. The set of booking-compliant (or b-compliant) nominations is given by

$$N(b) := \{\ell \in N : \ell \leq b\}.$$

Obviously, $N(b) \subseteq N \subseteq L$ holds for finite b.

We now define *feasible nominations* and *feasible bookings*, where "feasible" is meant with respect to technical, physical, and legal constraints of gas transport. To this end, let $c_{\mathcal{E}}(x, s; \ell) = 0$ and $c_{\mathcal{I}}(x, s; \ell) \geq 0$ be the possibly nonlinear, nonconvex, and nonsmooth constraints that model the full problem of gas transport possibly including models of nodes, pipes, compressors, etc. Moreover, let $z := (x, s) \in \mathbb{R}^{n_x} \times \mathbb{Z}^{n_s}$ be the discrete-continuous variable vector that is required to state this model.

Definition 2.5 (Feasible nomination)**.** A nomination $\ell \in N$ is *feasible* if a vector $z := (x, s) \in \mathbb{R}^{n_x} \times \mathbb{Z}^{n_s}$ exists such that

$$c_{\mathcal{E}}(x, s; \ell) = 0, \quad c_{\mathcal{I}}(x, s; \ell) \geq 0 \tag{1}$$

holds. The set of feasible nominations is denoted by F_N.

We note that the set of feasible nominations F_N depends on the chosen model of gas transport. The only constraint that we need in all formulations is mass conservation at each node of the network that is modeled by Kirchhoff's first law, i.e.,

$$\sum_{a \in \delta^{\mathrm{out}}(u)} q_a - \sum_{a \in \delta^{\mathrm{in}}(u)} q_a = q_u \quad \text{for all } u \in V, \tag{2}$$

where $q_u \geq 0$ for entries, $q_u \leq 0$ for exits, and $q_u = 0$ for inner nodes.

The dependency of the feasible set defined by (1) on the nomination is given by fixing the entry and exit flows according to the nomination ℓ, i.e.,

$$q_u = \ell_u \quad \text{for all } u \in V_+, \quad q_u = -\ell_u \quad \text{for all } u \in V_-.$$

These constraints are part of $c_{\mathcal{E}}(x, s; \ell) = 0$ in (1).

We note that to check whether a given nomination is feasible may lead to a mixed-integer nonlinear and nonconvex problem depending on the constraints $c_{\mathcal{E}}$ and $c_{\mathcal{I}}$. For more information on this problem, see [35] or [43] and the references therein.

Definition 2.6 (Feasible booking)**.** We say that a booking b is *feasible* if all booking-compliant nominations $\ell \in N(b)$ are feasible. The set of feasible bookings is denoted by F_B.

The definition of a feasible booking is very strict and may appear counter-intuitive at a first glance. However, this definition directly follows from the legislative texts about the European entry-exit gas market system, which aims at decoupling the economic trading and the technical transport of the gas.

For later reference, we also state definitions of cones and star-shaped sets; see [33] for more details.

Definition 2.7 (Cone)**.** The set $K \subseteq \mathbb{R}^n$ is a *cone*, if $\lambda x \in K$ holds for any $x \in K$ and $\lambda \geq 0$.

Definition 2.8 (Star-shaped set)**.** The set $K \subseteq \mathbb{R}^n$ is *star-shaped* w.r.t. $x_0 \in K$ if for every element $x \in K$ and $\lambda \in [0, 1]$ the relation

$$\lambda x + (1 - \lambda)x_0 \in K$$

holds.

TABLE 1. Properties of F_N and F_B w.r.t. the simple gas transport model (3)

Properties	Gas transport constraints (3)
Bounded F_N	×
Bounded F_B	×
Convex F_N	✓
Convex F_B	✓
Connected F_N	✓
Connected F_B	✓
Star-shaped F_N	✓
Star-shaped F_B	✓
Conic F_N	✓
Conic F_B	✓

From the definition of a star-shaped set K w.r.t. x_0 it directly follows that for each element $x \in K$ the line segment $[x_0, x]$ is contained in K.

In the remainder of this paper, we always consider the point of view of a transmission system operator (TSO) and we thus focus on technical and physical restrictions of the gas network. As a preliminary example let us first consider the case that except of Kirchhoff's first law (2) no further restrictions for gas transport exist. Hence, for a nomination $\ell \in N$ the Constraints (1) are given by

$$\sum_{a \in \delta^{\text{out}}(u)} q_a - \sum_{a \in \delta^{\text{in}}(u)} q_a = \sigma_u \ell_u \quad \text{for all } u \in V, \tag{3}$$

i.e.,

$$c_{\mathcal{E}}(x, s; \ell) = c_{\mathcal{E}}(x; \ell) = (c_{\mathcal{E},u}(q; \ell))_{u \in V}$$

with

$$c_{\mathcal{E},u}(q; \ell) = \sum_{a \in \delta^{\text{out}}(u)} q_a - \sum_{a \in \delta^{\text{in}}(u)} q_a - \sigma_u \ell_u$$

and $c_{\mathcal{I}}(x, s; \ell)$ is empty. Moreover, we set $\ell_u = \sigma_u = 0$ for all inner nodes $u \in V_0$ in Constraint (3).

We now analyze the feasibility of nominations and bookings w.r.t. (3). Each nomination ℓ of the set of nominations N is feasible, which can be shown by a direct proof or by Theorem 7.1 in [34]. The feasibility of each nomination implies that the set of feasible nominations F_N is unbounded, convex, connected, conic, and star-shaped. Furthermore, it follows that each booking is feasible and that the set of feasible bookings F_B has the same properties as the set of feasible nominations. We summarized the results w.r.t. linear flow without arc capacities in Table 1.

3. A Capacitated Linear Flow Model

In addition to the model used in the last section, we now further assume lower and upper flow bounds $q_a^- \leq q_a^+$ to be given for every arc $a \in A$. That means, we consider a standard capacitated linear flow model. Consequently, for a nomination $\ell \in N$ the Constraints (1) are given by (3) and the flow bounds

$$q_a^- \leq q_a \leq q_a^+ \quad \text{for all } a \in A, \tag{4}$$

i.e.,

$$c_{\mathcal{I}}(x, s; \ell) = c_{\mathcal{I}}(x; \ell) = (c_{\mathcal{I},a}(q; \ell))_{a \in A}$$

with

$$c_{\mathcal{I},a}(q; \ell) = (q_a - q_a^-, q_a^+ - q_a)$$

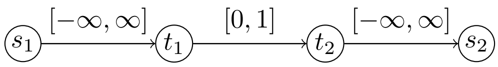

FIGURE 1. The graph G of Example 3.1

and $c_{\mathcal{E}}$ stays the same as in Section 2, i.e.,

$$c_{\mathcal{E}}(x, s; \ell) = c_{\mathcal{E}}(x; \ell) = (c_{\mathcal{E},u}(q; \ell))_{u \in V}$$

with

$$c_{\mathcal{E},u}(q; \ell) = \sum_{a \in \delta^{\text{out}}(u)} q_a - \sum_{a \in \delta^{\text{in}}(u)} q_a - \sigma_u \ell_u.$$

Note that checking the feasibility of a nomination $\ell \in N$ w.r.t. Conditions (1) of this section is a standard ℓ-transshipment problem; see Chapter 11 of the book [51].

In contrast to Section 2, we will see at the end of this section that a nomination may be infeasible w.r.t. given flow bounds. Hence, infeasible bookings may also exist. Additionally, for a feasible nomination there may exist a component-wise smaller nomination which is infeasible. To make this more concrete, we consider the following example.

Example 3.1. Let $G = (V, A)$ be a directed graph with nodes $V = \{s_1, s_2, t_1, t_2\}$ and arcs $A = \{(s_1, t_1), (t_1, t_2), (t_2, s_2)\}$. The nodes s_1, s_2 are entry nodes and t_1, t_2 are exit nodes. Furthermore, we set the flow bounds of the arcs (s_1, t_1) and (t_2, s_2) to $[-\infty, \infty]$ and for the remaining arc (t_1, t_2) to $[0, 1]$.

Then, one can show that the nomination

$$\ell_1 := (s_1, t_1, t_2, s_2)^\top = (5, 5, 5, 5)^\top$$

is feasible, i.e., $\ell \in F_N$. But the component-wise smaller nomination

$$\ell_2 := (s_1, t_1, t_2, s_2)^\top = (4, 0, 4, 0)^\top$$

is infeasible.

Thus, Example 3.1 leads to the following lemma.

Lemma 3.2. *The feasibility of a nomination $\ell \in F_N$ does not guarantee, in general, the feasibility of every component-wise smaller nomination $\tilde{\ell} \le \ell$.*

Despite the previous result, the set of feasible nominations is still convex.

Lemma 3.3. *The set of feasible nominations F_N is convex.*

Proof. Let ℓ and $\tilde{\ell}$ be feasible nominations, i.e., $\ell, \tilde{\ell} \in F_N$ with corresponding flows q and $\tilde{q}$ that satisfy Conditions (3) and (4). Additionally, let λ be in $[0, 1]$. Then, $\lambda\ell + (1 - \lambda)\tilde{\ell} \in N$ is valid because $\lambda\ell, (1 - \lambda)\tilde{\ell} \in \mathbb{R}^{V_b}_{\ge 0}$ and

$$\sigma^\top(\lambda\ell + (1 - \lambda)\tilde{\ell}) = \lambda\sigma^\top\ell + (1 - \lambda)\sigma^\top\tilde{\ell} = \lambda 0 + (1 - \lambda)0 = 0$$

holds. Furthermore,

$$\begin{aligned}
&\lambda\sigma_u\ell_u + (1 - \lambda)\sigma_u\tilde{\ell}_u \\
&= \left(\sum_{a \in \delta^{\text{out}}(u)} \lambda q_a - \sum_{a \in \delta^{\text{in}}(u)} \lambda q_a\right) + \left(\sum_{a \in \delta^{\text{out}}(u)} (1 - \lambda)\tilde{q}_a - \sum_{a \in \delta^{\text{in}}(u)} (1 - \lambda)\tilde{q}_a\right) \\
&= \sum_{a \in \delta^{\text{out}}(u)} (\lambda q_a + (1 - \lambda)\tilde{q}_a) - \sum_{a \in \delta^{\text{in}}(u)} (\lambda q_a + (1 - \lambda)\tilde{q}_a)
\end{aligned}$$

holds for all $u \in V$ and thus $\lambda q + (1 - \lambda)\tilde{q}$ satisfies Constraint (3). Moreover, the relation $q_a^- \le \lambda q + (1 - \lambda)\tilde{q} \le q_a^+$ is valid because q and $\tilde{q}$ satisfy the flow bounds (4). Consequently, $\lambda\ell + (1 - \lambda)\tilde{\ell}$ is a feasible nomination. $\square$

Lemma 3.3 implies the following corollary.

Corollary 3.4. The set of feasible nominations F_N is connected and star-shaped w.r.t. every point $x_0 \in F_N$.

Despite the set of feasible nominations is convex, connected, and star-shaped, it is in general not conic.

Lemma 3.5. *Let the flow bounds $q_a^- \leq q_a^+, a \in A$, be finite and assume that a feasible nonzero nomination $\ell \neq 0 \in F_N$ exists. Then, the set of feasible nominations F_N is not conic.*

Proof. Let $\ell \in F_N$ be a feasible nonzero nomination. Then, the corresponding flows satisfying Constraints (3) and (4) contain at least one nonzero arc flow. We can assume w.l.o.g. that the considered flows are nonnegative. Thus, scaling the nonzero nomination ℓ by a parameter $\lambda > 1$, increases at least one arc flow. Due to this and the finite flow bounds, we can scale ℓ by $\tilde{\lambda} \in \mathbb{R}_{>0}$ such that $\tilde{\lambda}\ell$ is feasible and for each $\varepsilon > 0$, the nomination $(\tilde{\lambda} + \varepsilon)\ell$ is infeasible. Hence, the set F_N of feasible nominations is not conic. $\square$

Moreover, the set of feasible nominations is bounded if we consider finite flow bounds.

Lemma 3.6. *If the flow bounds $q_a^- \leq q_a^+$ are finite for all $a \in A$, then the set of feasible nominations F_N is bounded.*

Proof. We assume that the set of feasible nominations is unbounded and consequently, a feasible nomination $\ell \in F_N$ with $\sum_{u \in V_+} \ell_u > \sum_{a \in A} q_a^+$ exists. We assume w.l.o.g. that the corresponding arc flows are nonnegative. Hence, from Constraint (3) it follows that at least one arc flow violates its upper flow bound because in the considered nomination the injected flow is larger than the aggregated upper arc flow bounds. This is a contradiction to the feasibility of ℓ. $\square$

After analyzing the feasibility of nominations, we now turn to the feasibility of bookings. In contrast to nominations, a feasible booking implies the feasibility of each component-wise smaller booking.

Lemma 3.7. *Let $b \in F_B$ be a feasible booking. Then, each booking $\tilde{b} \leq b$ is feasible. Furthermore, the set of feasible bookings F_B is star-shaped w.r.t. the zero booking.*

Proof. Let b be a feasible booking and $\tilde{b} \in B$ a booking with $\tilde{b} \leq b$. Consequently, $N(\tilde{b}) = \{\ell \in N : \ell \leq \tilde{b}\} \subseteq N(b)$ holds. Thus, the feasibility of the booking b implies the feasibility of $\tilde{b}$. From this it follows that the set F_B of feasible bookings is star-shaped w.r.t. the zero booking. $\square$

Furthermore, we know that the set of feasible bookings is connected due to Lemma 3.7.

Corollary 3.8. The set of feasible bookings F_B is connected.

In analogy to the case of nominations, we can show that the set of feasible bookings is in general not conic.

Corollary 3.9. Let the flow bounds $q_a^- \leq q_a^+$, $a \in A$, be finite. Assume further that a feasible booking b with a b-compliant nonzero nomination $\ell \in F_N$ exists. Then, the set of feasible booking F_B is not conic.

Proof. From the proof of Lemma 3.5 it follows that $\lambda > 0$ exists so that $\lambda\ell$ is infeasible. Consequently, the set of bookings B is not conic. $\square$

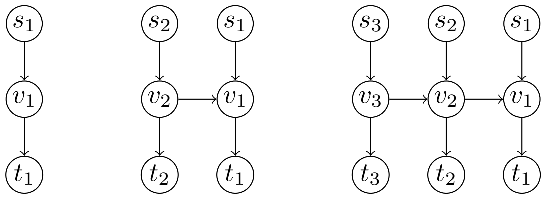

FIGURE 2. The H-networks H_1, H_2, H_3

With the help of an example, we now show that the set of all feasible bookings F_B is nonconvex in contrast to the set of feasible nominations F_N; see Lemma 3.3.

Definition 3.10 (H-networks)**.** The family of H-networks $H_n = (V_n, A_n)$ is defined by

$$H_1 := (V_1, A_1), \quad V_1 = \{s_1, v_1, t_1\}, \quad A_1 = \{(s_1, v_1), (v_1, t_1)\}$$

and $H_n := (V_n, A_n)$, $n \geq 2$, with

$$V_n = V_{n-1} \cup \{s_n, v_n, t_n\}, \quad A_n = A_{n-1} \cup \{(s_n, v_n), (v_n, t_n), (v_n, v_{n-1})\}.$$

See Figure 2 for the networks H_1, H_2, H_3. The nodes s_i are entry nodes, t_i are exit nodes, and v_i are inner nodes. The family of H-networks is also considered in [20] and in the chapter [25] of the book [35].

Example 3.11. We now consider the network H_2 and only impose a lower flow bound of zero of arc (v_2, v_1). All other flow bounds are formally set to $[-\infty, +\infty]$. One can show that

$$b_1 := (b_{s_2}, b_{s_1}, b_{t_2}, b_{t_1})^\top = (2, 2, 0, 2)^\top$$

and

$$b_2 := (b_{s_2}, b_{s_1}, b_{t_2}, b_{t_1})^\top = (2, 0, 2, 0)^\top$$

are feasible bookings, i.e., $b_1, b_2 \in F_B$. Consider now the convex combination b_3 with convex coefficient $\lambda = 1/2$, i.e.,

$$b_3 := (b_{s_2}, b_{s_1}, b_{t_2}, b_{t_1})^\top = (2, 1, 1, 1)^\top.$$

Obviously, the nomination

$$(b_{s_2}, b_{s_1}, b_{t_2}, b_{t_1})^\top = (0, 1, 1, 0)^\top$$

is b_3-compliant but *not* feasible.

Let us note at this point that a feasible booking, in contrast to a feasible nomination, does not need to be balanced.

Example 3.11 leads to the following theorem.

Theorem 3.12. *The feasible set of bookings F_B is, in general, nonconvex.*

Note this means that even a linear gas physics or engineering model may lead to a nonconvex set of feasible bookings.

Remark 3.13. For arbitrary $H_n = (V_n, A_n), n \geq 2$, we see that the following holds: If $b \in F_B$ is a feasible booking with $b_{t_k} > 0$ it follows $b_{s_j} = 0$ for all $j < k$. Thus, an exit node t_k, $k \geq 2$, can exclude every entry node s_j, $j < k$, from the market. Consider now for a moment the multicriteria optimization problem for which the feasible region is given by B. Moreover, the $|V_n|$ objective functions f_u, $u \in V_n$, are given by e_u, $u \in V_n$, with e_u being the uth unit vector; see, e.g., [16] for general multicriteria optimization. Then, from the above discussion and Example 3.11 it follows that the ideal point (in the sense of multicriteria optimization) is not bookable.

TABLE 2. Summary about properties of F_N and F_B w.r.t. gas transport model (3)–(4)

Properties	Gas transport constraints (3)–(4)
Bounded F_N	✓, see Lemma 3.6
Bounded F_B	×, see Lemma 3.14
Convex F_N	✓, see Lemma 3.3
Convex F_B	×, see Example 3.11
Connected F_N	✓, see Corollary 3.4
Connected F_B	✓, see Corollary 3.8
Star-shaped F_N	✓, see Corollary 3.4
Star-shaped F_B	✓, see Lemma 3.7
Conic F_N	×, see Lemma 3.5
Conic F_B	×, see Corollary 3.9

Furthermore, the set of feasible bookings is unbounded.

Lemma 3.14. *If the set of feasible bookings F_B is nonempty, then it is unbounded.*

Proof. The zero nomination is booking-compliant for every booking. Due to $F_B \neq \emptyset$, the zero nomination is feasible. Consequently, for each node $u \in V$ and nonnegative value M the booking b with $b_u = M$ and $b_v = 0, v \in V, v \neq u$, is feasible because the zero nomination is the only booking-compliant nomination for b. □

We note that for gas transport with flow bounds, the set of feasible bookings is unbounded in contrast to the set of feasible nominations; see Lemma 3.6 and 3.14. This is due to the definition of a feasible booking that only requires the feasibility of every balanced and component-wise smaller nomination. Consequently, bookings exist that only contain a single feasible booking-compliant nomination and scaling this booking does not change the set of balanced and booking-compliant nominations.

With the help of the following example, we show that even in the case of linear constraints of gas transport like (3) and (4) the set of feasible nominations and bookings can be empty. We consider the graph H_1. Additionally, we set the lower and upper flow bounds of arc (s_1, v_1) to 1 and of arc (v_1, t_1) to 2. For a given nomination $\ell \in N$ the flows q satisfying Conditions (3) are unique because G is a tree. Furthermore, the flow q_a on each arc $a \in A$ equals ℓ_{t_1}. Thus, no feasible nomination for G exists due to the chosen lower and upper arc flow bounds. Especially, the zero nomination, which is always b-compliant, is infeasible. Consequently, the set of feasible bookings is empty.

Finally, we summarize the results for a gas transport model using capacitated linear flows in Table 2.

4. A Nonlinear Flow Model for Passive Networks

We now extend the model of the last section to a more realistic model of gas physics by introducing a bounded pressure variable p_u for every node $u \in V$. Additionally, the pressure levels are coupled to arc flows. Hence, for a nomination $\ell \in N$ the Constraints (1) are given by (3), (4), and the classical Weymouth pressure drop conditions

$$p_v^2 = p_u^2 - \Lambda_a |q_a| q_a \quad \text{for all } a = (u, v) \in A, \tag{5}$$

where $\Lambda_a > 0$ is a constant for every arc $a \in A$. Furthermore, the pressures are bounded, i.e.,

$$0 < p_u^- \leq p_u \leq p_u^+ \quad \text{for all } u \in V. \tag{6}$$

Consequently, the Constraints (1) are represented by

$$c_{\mathcal{I}}(x, s; \ell) = c_{\mathcal{I}}(x; \ell) = \begin{pmatrix} (c_{\mathcal{I},u}(q; \ell))_{u \in V} \\ (c_{\mathcal{I},a}(q; \ell))_{a \in A} \end{pmatrix}$$

with

$$c_{\mathcal{I},u}(q; \ell) = (p_u - p_u^-, p_u^+ - p_u)$$

and the arc flow bounds

$$c_{\mathcal{I},a}(q; \ell) = (q_a - q_a^-, q_a^+ - q_a)$$

as well as

$$c_{\mathcal{E}}(x, s; \ell) = c_{\mathcal{E}}(x; \ell) = \begin{pmatrix} (c_{\mathcal{E},u}(q; \ell))_{u \in V} \\ (c_{\mathcal{E},a}(q; \ell))_{a \in A} \end{pmatrix}$$

with the flow conservation

$$c_{\mathcal{E},u}(q; \ell) = \sum_{a \in \delta^{\text{out}}(u)} q_a - \sum_{a \in \delta^{\text{in}}(u)} q_a - \sigma_u \ell_u$$

and

$$c_{\mathcal{E},a}(q; \ell) = p_u^2 - p_v^2 - \Lambda_a |q_a| \, q_a.$$

We now analyze the feasibility of nominations and bookings in this extended setting. In contrast to Sections 2 and 3, the set of feasible nominations F_N is now, in general, nonconvex. This follows from a small counterexample in Section 5.1 of [24].

Lemma 4.1. *The set of feasible nominations F_N is, in general, nonconvex.*

Despite the nonconvexity of the set of feasible nominations, we can guarantee that the set of feasible nominations is star-shaped w.r.t. the zero nomination under certain pressure and flow bound requirements. The main idea behind the proof is the following observation: When we fix the pressure at one node, then decreasing the flow to another node will also decrease the pressure drops along this path.

Lemma 4.2. *Suppose that $q_a^- \leq 0 \leq q_a^+$, $a \in A$, and $\bigcap_{u \in V} [p_u^-, p_u^+] \neq \emptyset$ holds. Then, the set of feasible nominations F_N is star-shaped w.r.t. the zero nomination.*

Proof. Let $\ell \in F_N$ be a feasible nomination with corresponding flow q satisfying Conditions (3) and (4). Additionally, let $p_u \in [p_u^-, p_u^+]$, $u \in V$, be the corresponding pressure levels satisfying Condition (5) and let $\lambda \in [0, 1]$. We now show that F_N is star-shaped w.r.t. the zero nomination by proving that $\lambda\ell$ is a feasible nomination for any $\lambda \in [0, 1]$. To this end, we consider the nomination $\lambda\ell$ with corresponding flows λq. The latter satisfies $q_a^- \leq \lambda q_a \leq q_a^+$ for every arc $a \in A$, because q satisfies the flow bounds, the requirement $q_a^- \leq 0 \leq q_a^+$ for all $a \in A$ holds, and $\lambda \in [0, 1]$. We now have to find feasible pressure levels for the nomination $\lambda\ell$. From Theorem 7.1 in [34] it follows that we can compute an assignment $\tilde{p}$ of the pressure variables that satisfies Condition (5) with $\tilde{p}_u \leq p_u^+$ for all $u \in V$ and such that at least one node $u \in V$ satisfies $\tilde{p}_u = p_u^+$. We assume w.l.o.g. that the pressure level of the arbitrary node w is at its upper bound, i.e., $\tilde{p}_w = p_w^+$ holds.

We now contrarily assume that pressure levels $\tilde{p}$ do not satisfy the pressure bounds at each node. Due to the construction of $\tilde{p}$, the pressure level at every node satisfies its upper pressure bound. Due to this and the infeasibility of $\tilde{p}$, a node $v \in V$ with $\tilde{p}_v < p_v^-$ exists. We can further assume that $\lambda \in (0, 1]$ holds, because for $\lambda = 0$ we obtain the zero nomination, which is feasible with zero arc flows and a constant pressure level $p_v = p \in \bigcap_{u \in V} [p_u^-, p_u^+] \neq \emptyset$ for every node $v \in V$. The existence of p is guaranteed by the requirements.

Case 1: The relation $\tilde{p}_v > \tilde{p}_w$ holds. This together with $\bigcap_{u\in V}[p_u^-, p_u^+] \neq \emptyset$, the assumption w.r.t. $\tilde{p}$, and $\tilde{p}_w = p_w^+$ leads to the following relations

$$p_v^+ \geq p_v^- > \tilde{p}_v > \tilde{p}_w = p_w^+ \geq p_w^-.$$

From the latter relations follows that $[p_v^-, p_v^+] \cap [p_w^-, p_w^+] = \emptyset$, which is a contradiction to $\bigcap_{u\in V}[p_u^-, p_u^+] \neq \emptyset$.

Case 2: The relation $\tilde{p}_v \leq \tilde{p}_w$ holds. We assume w.l.o.g. that $\tilde{p}_v$ is nonnegative (otherwise we would consider $|\tilde{p}_v|$), which again satisfies Condition (5), and thus, we are either in Case 1 or again in Case 2. Additionally, we assume w.l.o.g. that $P(w,v)$ is a directed path from w to v. Furthermore, $\tilde{p}$ satisfies Condition (5) and, thus,

$$\tilde{p}_w^2 - \sum_{a\in P(w,v)} \Lambda_a \,|\lambda q_a|\, \lambda q_a = \tilde{p}_v^2 \tag{7}$$

holds. Due to the last equation, $\tilde{p}_w \geq \tilde{p}_v \geq 0$, and $\lambda \in (0,1]$, the relation

$$0 \leq \sum_{a\in P(w,v)} \Lambda_a \,|\lambda q_a|\, \lambda q_a \leq \sum_{a\in P(w,v)} \Lambda_a \,|q_a|\, q_a$$

is valid. This relation together with Equation (7), $\lambda \in (0,1]$, $\tilde{p}_v \geq 0$, Condition (5), $(p_w^+)^2 = (\tilde{p}_w)^2 \geq p_w^2$, and the assumption leads to

$$\begin{aligned}
(p_v^-)^2 > \tilde{p}_v^2 &= \tilde{p}_w^2 - \sum_{a\in P(w,v)} \Lambda_a \,|\lambda q_a|\, \lambda q_a \\
&\geq p_w^2 - \sum_{a\in P(w,v)} \Lambda_a \,|\lambda q_a|\, \lambda q_a \geq p_w^2 - \sum_{a\in P(w,v)} \Lambda_a \,|q_a|\, q_a \\
&= p_v^2,
\end{aligned}$$

This is a contradiction to the feasibility of ℓ with corresponding feasible pressure levels $p_u, u \in V$. □

Note that a related result is given in Theorem 3.7 of [28]. Lemma 4.2 directly implies that the set of feasible nominations F_N is connected.

Corollary 4.3. The set of feasible nominations F_N is connected.

If the pressure requirement $\bigcap_{u\in V}[p_u^-, p_u^+] \neq \emptyset$ is not valid, then the set of feasible bookings is empty.

Lemma 4.4. *If $\bigcap_{u\in V}[p_u^-, p_u^+] = \emptyset$ is satisfied, then $F_B = \emptyset$ holds.*

Proof. Due to the requirement $\bigcap_{u\in V}[p_u^-, p_u^+] = \emptyset$, the zero nomination is not feasible, which directly shows the claim. □

The model of gas transport of this section is more restrictive than the model of Section 2. Thus, we can transfer Lemmas 3.5 and 3.6 to the nonlinear flow model on passive networks. Consequently, we know that the set of feasible nominations is not conic and that it is bounded. We now turn to the analysis of the bookings.

From Lemma 3.12, it follows that the set of feasible bookings is, in general, nonconvex. Furthermore, the statements in 3.7–3.9 and 3.14 are valid for the nonlinear flow model on passive networks and can be shown in analogy to Section 3. Consequently, the set of feasible bookings is connected and star-shaped w.r.t. the zero booking. Furthermore, the set of feasible bookings is unbounded if it is nonempty and examples with an empty set of feasible bookings exist; see Lemma 4.4.

We summarize the results for gas transport w.r.t. the nonlinear flow model on passive networks in Table 3. The main difference about the structural properties of nominations and bookings between the linear flow model of Sections 2 and 3 and the

TABLE 3. Summary about properties of F_N and F_B w.r.t. gas transport model (3)–(6)

Properties	Gas transport constraints (3)–(6)
Bounded F_N	✓, see Lemma 3.6
Bounded F_B	×, see Lemma 3.14
Convex F_N	×, see Lemma 4.1
Convex F_B	×, see Example 3.11
Connected F_N	✓, see Corollary 4.3
Connected F_B	✓, see Corollary 3.8
Star-shaped F_N	✓, see Lemma 4.2
Star-shaped F_B	✓, see Lemma 3.7
Conic F_N	×, see Lemma 3.5
Conic F_B	×, see Corollary 3.9

nonlinear flow model on passive networks is that the set of feasible nominations is convex in case of the linear flow model, which is not valid anymore in the nonlinear flow model.

5. A Mixed-Integer Nonlinear Flow Model for Active Networks

Besides nonlinear models of gas flow in pipes, real-world gas transport networks also comprise so-called active elements that can be controlled by the dispatcher. Examples for such devices are valves or compressors. Detailed descriptions of these active elements are, e.g., given in the chapters [19, 46] of the book [35]. For our nonlinear gas transport model, we focus on compressors as an example for active elements.

A compressor is represented by an arc $a = (u, v) \in A_{\text{cs}} \subseteq A$ and we use the following simplified model (for more complicated models see, e.g., [48–50]). A compressor can be in bypass mode or active. In bypass mode, the in- and outflow pressures of the compressor are the same ($p_v = p_u$) and the flow through the compressor is arbitrary (within certain arc-specific bounds). If the compressor is active, it can compress the gas, i.e., the compressor increases the pressure. This capability is limited by lower and upper bounds on the obtained compression ratio, i.e.,

$$\frac{p_v}{p_u} \in [\varepsilon_a^-, \varepsilon_a^+] \quad \text{for all } a = (u, v) \in A_{\text{cs}}$$

with $1 \leq \varepsilon_a^- \leq \varepsilon_a^+$. Both states of a compressor can be modeled by the constraints

$$\frac{p_v}{p_u} s_a \geq \varepsilon_a^- s_a + (1 - s_a)(p_u - p_v) \quad \text{for all } a = (u, v) \in A_{\text{cs}}, \tag{8}$$

$$\frac{p_v}{p_u} s_a \leq \varepsilon_a^+ s_a + (1 - s_a)(p_u - p_v) \quad \text{for all } a = (u, v) \in A_{\text{cs}}, \tag{9}$$

where the binary variable $s_a \in \{0, 1\}$, $a \in A_{\text{cs}}$, equals 1 if only if the compressor is active. Otherwise the compressor is in bypass mode. In addition, the compressor has a nonnegative lower arc flow bound $\hat{q}^-$ in the active state for which we assume that $\hat{q}_a^- > q_a^-$ holds. (Otherwise we can neglect this lower arc flow bound of the compressor because of its standard arc flow bound.) We model this tightened lower flow bound of the compressor by modifying the lower arc flow bound constraint in (4) as follows:

$$q_a \geq (1 - s_a) q_a^- + s_a \hat{q}_a^-. \tag{10}$$

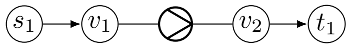

FIGURE 3. The graph of Example 5.1

Consequently, the Constraints (1) are given by

$$c_{\mathcal{I}}(x, s; \ell) = \begin{pmatrix} (c_{\mathcal{I},u}(q;\ell))_{u\in V} \\ (c_{\mathcal{I},a}(q;\ell))_{a\in A} \end{pmatrix}$$

with

$$c_{\mathcal{I},u}(q;\ell) = (p_u - p_u^-, p_u^+ - p_u)$$

and

$$c_{\mathcal{I},a}(s;q;\ell) = \begin{pmatrix} (s_a(p_v/p_u - \varepsilon_a^-) - (1-s_a)(p_u - p_v))_{a=(u,v)\in A_{\mathrm{cs}}} \\ (s_a(\varepsilon_a^+ - p_v/p_u) + (1-s_a)(p_u - p_v))_{a=(u,v)\in A_{\mathrm{cs}}} \\ (q_a - (1-s_a)q_a^- - s_a\hat{q}^-)_{a\in A_{\mathrm{cs}}} \\ (q_a - q_a^-)_{a\in A\setminus A_{\mathrm{cs}}} \\ (q_a^+ - q_a)_{a\in A\setminus A_{\mathrm{cs}}} \end{pmatrix}.$$

Additionally, we have the constraints

$$c_{\mathcal{E}}(x, s; \ell) = c_{\mathcal{E}}(x; \ell) = \begin{pmatrix} (c_{\mathcal{E},u}(q;\ell))_{u\in V} \\ (c_{\mathcal{E},a}(q;\ell))_{a\in A\setminus A_{\mathrm{cs}}} \end{pmatrix}$$

with the flow conservation equations

$$c_{\mathcal{E},u}(q;\ell) = \sum_{a\in\delta^{\mathrm{out}}(u)} q_a - \sum_{a\in\delta^{\mathrm{in}}(u)} q_a - \sigma_u \ell_u$$

and the pressure drop conditions

$$c_{\mathcal{E},a}(q;\ell) = p_u^2 - p_v^2 - \Lambda_a |q_a| q_a.$$

We again analyze the set of feasible nominations and bookings in this extended setting. In contrast to Sections 2–4, the set of feasible nominations F_N is, in general, not connected anymore, which we will show by the following example.

Example 5.1. We consider the graph $G = (V, A)$ with nodes $V = \{s_1, v_1, v_2, t_1\}$, where s_1 is an entry, v_1, v_2 are inner nodes, and t_1 is an exit. Additionally, the graph contains three arcs (s_1, v_1), (v_1, v_2), and (v_2, t_1), where the arc (v_1, v_2) represents a compressor. A graphical representation is given in Figure 3. We set the pressure bounds to $[2, 2]$ for node s_1 and to $[1, 2]$ for the remaining nodes. Furthermore, the lower and upper bounds for the compression ratio are given by $[\varepsilon^-, \varepsilon^+] = [2, 3]$ and the pressure drop coefficient Λ_a equals 0.5 for every arc $a \in A \setminus A_{\mathrm{cs}}$. We neglect flow bounds in this example. The graph G is a tree with one entry and one exit. Due to this and Definition 2.2, a nomination in G equals $(\ell, \ell), \ell \in \mathbb{R}_{\geq 0}$, with the arc flows $q_a = \ell$, $a \in A$, where ℓ is the amount of gas which is injected at the entry and withdrawn at the exit of the network. If the compressor is in bypass mode, then each feasible nomination has to satisfy the pressure Constraints (5) and (6), i.e.,

$$p_{s_1} = 2, \quad p_{v_1} = 2 - 0.5\ell^2 = p_{v_2} \geq 1, \quad p_{t_1} = p_{v_2} - 0.5\ell^2 = 2 - \ell^2 \geq 1.$$

Thus, the set of feasible nominations is $\{(\ell, \ell) : \ell \in [0, 1]\}$ in this case. If the compressor is active, then each feasible nomination $(\ell, \ell), \ell \in \mathbb{R}_{\geq 0}$, has to satisfy

$$p_{s_1} = 2,\ p_{v_1} = 2 - 0.5\ell^2 \geq 1,\ 2 \leq \frac{p_{v_2}}{p_{v_1}} \leq 3, 1 \leq p_{v_2} \leq 2,\ p_{t_1} = p_{v_2} - 0.5\ell^2 \geq 1.$$

and consequently, only the nomination $(\sqrt{2}, \sqrt{2})$ is feasible. Hence, the set of feasible nominations is $F_N = \{(\ell, \ell) : \ell \in [0, 1]\} \cup \left\{(\sqrt{2}, \sqrt{2})\right\}$, which is not connected.

Example 5.1 leads to the following lemma.

Lemma 5.2. *The set of feasible nominations F_N is, in general, not connected—even if no arc flow bounds are considered.*

We now present another example, which also proves Lemma 5.2. While in Example 5.1 the set of feasible nominations is not connected due to bounds on the compression ratio, in the next example the feasible set of nominations is disconnected due to the presence of a lower arc flow bound of the compressor.

Example 5.3. We again consider the network $G = (V, A)$ of the last example. We modify the lower and upper pressure bounds to $[0.875, 2]$ for v_1 and to $[1, 3]$ for v_2 and t_1. Furthermore, the lower and upper bounds for the compression ratio are set to $[1, 3]$. The set of feasible nominations is $F_N = \{(\ell, \ell) : \ell \in [0, 1.5]\}$ because each feasible nomination $(\ell, \ell), \ell \in \mathbb{R}_{\geq 0}$, has to satisfy the following pressure constraints

$$p_{s_1} = 2, \quad p_{v_1} = 2 - 0.5\ell^2 \geq 0.875,$$
$$1 \leq \frac{p_{v_2}}{p_{v_1}} \leq 3, \quad 1 \leq p_{v_2} \leq 3, \quad p_{t_1} = p_{v_2} - 0.5\ell^2 \geq 1.$$

We note that the set of feasible nominations is connected and that $p_{v_2}/p_{v_1} = 1$ corresponds to the bypass mode. If the compressor is inactive, the nominations $\{(\ell, \ell) : \ell \in [0, 1]\}$ are feasible and with the help of the compressor the remaining nominations of F_N are feasible. We now add the lower flow bound $\hat{q}^- = 1.25$ for the compressor, which comes into play if the compressor is active. Consequently, the set of feasible nominations is $F_N = \{(\ell, \ell) : \ell \in [0, 1] \cup [1.25, 1.5]\}$, which is not connected. Hence, we see that the lower flow bound of the compressor may also lead to a disconnected set of feasible nominations.

From Lemma 5.2 it follows that the set of feasible nominations is, in general, neither conic, star-shaped, nor convex.

Corollary 5.4. The set of feasible nominations F_N is, in general, neither conic, star-shaped, nor convex.

Furthermore, from Lemma 3.6 it follows that the set F_N of feasible nominations is bounded.

We now turn to the analysis of the bookings. The nonlinear flow model on passive networks is a special case of the considered nonlinear flow model on active networks. Hence, we can conclude from Section 4 that the set of feasible bookings is, in general, nonconvex and that examples with an empty set of feasible bookings exist. Additionally, we can prove in analogy to Section 4 that the set of feasible bookings is, in general, connected and star-shaped w.r.t. the zero booking. Moreover, the set of feasible bookings is unbounded if it is nonempty.

We summarize the results for gas transport w.r.t. the nonlinear flow model on active networks in Table 4. The main difference about the structural properties of nominations and bookings between the nonlinear flow model on passive and on active networks is that the set of feasible nominations is connected and star-shaped on passive networks, which is not the case anymore on active networks.

6. Conclusion

In this work, we analyzed the structural properties of the set of feasible nominations and feasible bookings in the European entry-exit gas market system. We presented a formal definition of (feasible) bookings and then studied whether this set is bounded, convex, connected, conic, and star-shaped—which are all important properties if one wants to optimize over this set. We carried out the analysis for different gas flow models on a network, ranging from a simple capacitated linear flow model to a mixed-integer nonlinear model of an active network. The results

TABLE 4. Summary about properties of F_N and F_B w.r.t. gas transport model (3)–(6), (8)– (10)

Properties	Gas transport constraints (3)–(6), (8)–(10)
Bounded F_N	✓, see Lemma 3.6
Bounded F_B	×, see Lemma 3.14
Convex F_N	×, see Corollary 5.4
Convex F_B	×, see Example 3.11
Connected F_N	×, see Example 5.1
Connected F_B	✓, see Corollary 3.8
Star-shaped F_N	×, see Corollary 5.4
Star-shaped F_B	✓, see Lemma 3.7
Conic F_N	×, see Corollary 5.4
Conic F_B	×, see Corollary 3.9

TABLE 5. Summary about properties of F_N and F_B w.r.t. different gas transport models

	Gas transport constraints			
Properties	(3)	(3), (4)	(3)–(6)	(3)–(6), (8)–(10)
Bounded F_N	×	✓	✓	✓
Bounded F_B	×	×	×	×
Convex F_N	✓	✓	×	×
Convex F_B	✓	×	×	×
Connected F_N	✓	✓	✓	×
Connected F_B	✓	✓	✓	✓
Star-shaped F_N	✓	✓	✓	×
Star-shaped F_B	✓	✓	✓	✓
Conic F_N	✓	×	×	×
Conic F_B	✓	×	×	×

are summarized in Table 5. It turns out that some of the results on the feasible set of bookings are rather counter-intuitive. For instance, one can see that all models (except for the very simplified linear one without arc capacities) lead to nonconvex sets of feasible bookings. This is remarkable because this already happens for the model that only uses a very simple linear flow model with arc capacities. It also indicates that it can be expected that optimizing over these sets is hard. This is in line with results from the literature like in [32], where it is shown that checking the feasibility of a booking is a coNP-hard problem on general graphs—even for linear flow models. However, the results in [45] imply that the same problem is easy for nonlinear flow models on trees. One interesting question for future research thus is to exactly draw the line between the easy and the hard cases. The analysis of feasible bookings in the context of instationary gas flow models is also part of our future work. However, with respect to stationary models, it can be seen in Table 5 that the only desirable properties of bookings that still hold in the nonlinear flow model with active elements are purely based on the nature of feasible bookings and thus do not depend on the chosen physical and technical models of gas transport.

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Compliance with Ethical Standards

The authors declare that they have no conflict of interest.

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