



## Guest editorial: special issue on fair division

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This special issue is devoted to *fair division*, the study of how to divide resources among interested agents so that the agents receive their fair share. While fair division has long been an important topic of study among mathematicians and economists, the past decade or so has seen a considerable rise in interest in the area from computer scientists and in particular multi-agent system researchers. This is also witnessed by the fact that fair division papers now form an important part of the program at leading conferences such as the annual *International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*.

Following an open Call for Papers between May 2020 and March 2021, ten papers were submitted for consideration of inclusion in this special issue. After a rigorous reviewing process in accordance with the high standards of *Autonomous Agents and Multi-Agent Systems*, five papers were eventually accepted for publication. Each of these papers addresses a topic of current interest to the fair division community; together, these five papers offer a unified view of some of the latest trends and developments in this burgeoning field.

- Much of the fair division literature assumes that the items to be allocated are ‘goods’ which yield positive utility for the agents, with some work also investigating ‘chores’, i.e., items with negative utility. In their paper “Fair allocation of indivisible goods and chores”, Aziz, Caragiannis, Igarashi, and Walsh consider a general scenario in which an agent may have positive or negative utility for each item. Their framework captures settings such as fair task assignment, wherein agents can experience both positive and negative utility for each task. The authors present algorithms for computing fair allocations with respect to fairness notions such as envy-freeness up to one item (EF1) and proportionality up to one item (PROP1).
- Another key dichotomy in the fair division literature is the dichotomy between divisible and indivisible items. In their article “Maximin fairness with mixed divisible and indivis-

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ible goods”, Bei, Liu, Lu, and Wang investigate situations in which the set of goods to be allocated includes both types of goods. They focus on the notion of maximin share fairness (MMS) and show that the worst-case MMS approximation guarantee with mixed goods is no worse than that with indivisible goods only. On the other hand, they exhibit problem instances in which adding some divisible resources would strictly decrease the MMS approximation ratio. The authors also propose an algorithm that produces an approximate MMS allocation, where the approximation ratio is determined by how agents value the divisible goods relative to their MMS values.

- The article “Fair allocation of conflicting items” by Hummel and Hetland examines the fair allocation of indivisible items when some pairs of items are in conflict with each other and agents are not permitted to receive conflicting items. Constraints of this kind capture, for example, participation in events that overlap in time, or assignment of positions in an organization with conflicts of interest. The authors demonstrate that typical fairness concepts such as envy-freeness up to one item (EF1), maximin share fairness (MMS), and maximum Nash welfare (MNW) are useful under conflict constraints. Among other results, they show that a  $1/3$ -approximate MMS allocation always exists whenever the number of agents is larger than the maximum degree of the conflict graph. They also study the achievable fairness approximations in randomly generated instances.
- A classic problem in fair division is the problem of how to allocate a heterogeneous divisible resource. This problem is often referred to as ‘cake cutting’, with the cake serving as a playful metaphor for the resource. In his work entitled “Redividing the cake”, Segal-Halevi explores cake cutting in settings where there is already an initial allocation of the cake, and the goal is to achieve a fair outcome while ensuring that all agents retain a significant portion of the value that they initially own. The author devises protocols that attain various trade-off points between fairness and ownership rights, and investigates variants that incorporate geometric and connectivity constraints on the allocation. His protocols have implications on the price of fairness, which measures the loss in social welfare caused by fairness requirements.
- Finally, in their paper “Fair cake-cutting algorithms with real land-value data”, Shtechman, Gonen, and Segal-Halevi explore the problem of dividing land via tools from the cake-cutting literature. They present several heuristic adaptations of famous cake-cutting algorithms, and evaluate these heuristics using extensive simulations on real land-value data maps. Their simulations compare the performance of cake-cutting algorithms in this context to other common methods of dividing land, such as by hiring assessors or by selling and dividing the proceeds, with respect to metrics including utilitarian welfare, egalitarian welfare, Nash social welfare, envy, and geometric shape. The authors find that the cake-cutting algorithms perform better in most metrics, but that the performance of these algorithms is greatly influenced by technical implementation details.

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