A meta-measure of performance related to both investors and investments characteristics

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Abstract

We introduce hereafter a new flexible *meta*-measurement of portfolio performance, called the Generalized Utility-based N-moment measure, relying both on a characterization of the whole return distribution and on the set of preferences of the investor, which is adapted to analyze the performance of hedge funds. It could also serve as the basis of a Fraudulent Behavior Index aiming to detect fraudulent funds.

Keywords Performance Measurement · Hedge Funds · Higher-moments · Statistical Expansion.

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1 Introduction

Fund performance measurement is a significant subject for both academics and practitioners in finance, first founded on the pioneer works by Markowitz (1952) on mean-variance efficient portfolios[4] and by Sharpe reward-to-volatility ratio (1966). Since published rankings can have an important impact on inflows and outflows (*Cf.* Hendricks *et al.*, 1993; Powell *et al.*, 2002), they have also consequences on selection and allocation decisions by fund managers[5], who can be tempted to manipulate the measures (Ingersoll *et al.*, 2007; Ornelas *et al.*, 2008). However, numerous measures have been proposed since the seminal works on this topic (*e.g.*, Caporin *et al.*, 2014, for a survey[6]), and some new performances measures still continuously appear from time to time (see Cheng *et al.*, 2017; Gzyl and Rios, 2018; Ardia and Boudt, 2018; Ardia *et al.*, 2018; Fays *et al.*, 2018; Peters and Seydel, 2018; Fischer and Lundtofte, 2019; Bi *et al.*, 2019).

And there are still debates around properties of some measures[7]. The choice between different performance measures, of course, depends first upon the preferences of investors, but also, secondly, on the characteristics of underlying return distributions. From a theoretical point of view, the Sharpe (1966) ratio is a meaningful portfolio performance measure when risk can be adequately measured by standard deviation. Although this ratio remains a reference indicator for assessing the accuracy of investment strategies, its use is doubtful in the presence of non-null skewness or/and excess *kurtosis*.

Accordingly, we first propose hereafter to express the expected utility of an investor from the moments of a return distribution relying on a classical fourth-order Taylor expansion. In this context, the new proposed measure of performance, called the Generalized Utility-based N-moment measure of performance (GUN in short), is an extension of the mean-variance framework which aims to better characterize both the shape of return distributions to be

[4] See also Markowitz (2014) for a complete survey.
[5] See Kolm *et al.* (2014) for the main trends on applications in operational research for portfolio optimizations.
[6] See also, for instance, Cherny and Madan (2009), Capocci (2009), Darolles *et al.* (2009), Jha *et al.* (2009), Jiang and Zhu (2009), Stavetski (2009), Zakamouline and Koekebakker (2009), Darolles and Gouriéroux (2010), Glawischnig and Sommersguter-Reichmannn (2010), Billio *et al.* (2012, 2013 and 2015), Joenväärä *et al.* (2013), Cremers *et al.* (2013), Smetters and Zhang (2014), Kadan and Liu (2014), Ferson and Lin (2014), Brown *et al.* (2015). To our best knowledge, the most comprehensive works dedicated to performance assessment, published in the two last decades, are those of Knight and Satchell (2002), Amenc and Lesourd (2003), Aftalion and Poncet (2003), Le Sourd (2007), Bacon (2008a and 2008b), Cogneau and Hübner (2009a and 2009b), Fischer and Wermers (2012), and Caporin *et al.* (2014).
[7] See, for instance, the controversy about the consistency of the Sharpe and *Omega* ratio and the second-order stochastic dominance criterion (*Cf.* Hodges, 1998; Fong, 2016; Balder and Schweizer, 2017; Klar and Müeller, 2017; Caporin *et al.*, 2018; Bi *et al.*, 2019).

evaluated and the preferences of investors. As a direct generalization of the Sharpe (1966) ratio, it happens that it nicely competes with the Morningstar (2002) measure and the Ingersoll-Spiegel-Goetzmann-Welch (2007) Manipulation-Proof Performance Measure (MPPM in short). It is thus theoretically founded on an extension of the mean-variance analysis in a first four moment framework. More precisely, this approach is based on numerous articles in the literature in finance on multi-moment analyses (*e.g.*, Jean, 1971 and 1973; Rubinstein, 1973), and is grounded on a linear combination of sensitivities to the first four moments that represents greediness, risk aversion, prudence and temperance[8]. Our measure is flexible enough so it can adapt to various investors utilities when varying their sensitivities to the mean, the variance, the skewness and the *kurtosis* of the underlying return density. In other words, our measure is a general one, that encompasses most classical measures in a sound way based on a utility setting. Actually, we show that each traditional performance measure goes with an (hidden) implicit choice of a set of preferences, that the GUN measure can, in fact, reproduce and reveal. In particular, it happens in our empirical study that the MPPM ranking places a lot of emphasis on the average return, and does not fully take into account all the features of return densities when considering several types of investors.

Our article contributes to the literature on performance measurement in four ways. First, we underline some weaknesses of traditional and more recent performance measures, in link with their intrinsic structure. Secondly, we show that most common performance measures are clearly linked to an implicit set of individual preferences. In this sense, our proposal is to write and make readers aware that main performance measures, indeed, do, in fact, depend on moments, at least implicitly, even if they do not precisely appear in the definition of some *formulae* of the measures. It happens, for instance, that Sharpe ratio corresponds to an almost 2 *versus* 1 trade-off between performance and risk, whilst the MPPM largely privileges performance against risk.[9] Thirdly, we propose a new flexible *meta*-measure of performance. One objective of our proposal is to show that we can recover implicit parameters (which are functions in our setting of mixtures of utility derivatives and moments) and that we can replicate rankings of funds obtained with main measures with a set of these parameters (whatever the utility function chosen among the classical ones). Fourthly, we briefly propose an ensemble of applications of our measures, from the original building of an index of potential fraud (called

[8] See Kimball (1990, 1992 and 1993) and Scott and Horvath (1980).

[9] And when we also know that the Keating-Shadwick (2002) *Omega* measure, in some sense, disregards risk – see Caporin *et al.* (2018).

Fraudulent Behavior Index hereafter) to a methodology for revealing preferences of investors from their fund ranking, or for designing fund characteristics in order to target investor preferences.

The rest of the article is structured as follows. Section 2 introduces the GUN measure and presents our setting. In Section 3, we attempt to directly compare the GUN measure to some of the main performance metrics. Section 4 further empirically compares some of the main performance measures to our measure when using some realistic simulations and real market data. Section 5 resumes the article findings, and introduces several potential applications of the GUN measure for future researches.

2 The Generalized Utility-based N-moment Measure of Performance

In this section, we present the GUN measure that is based on a whole return distribution characterization. More precisely, this measure of performance takes into account the first four moments of the return distribution and the associated sensitivities of the studied agent, reflecting his preferences and risk profile.

2.1 Utility Functions with Higher-order Moments

We first propose, hereafter, to express the expected utility of an investor from the moments of a return distribution through a fourth-order Taylor expansion. Secondly, we introduce our measure that is written as a linear combination of the first four moments of the investor's return distribution.

In economics, agents' behaviors are indeed represented by utility functions which describe their preferences and risk profiles. The main objective of any agent (in the mainstream classical theoretical approach) is supposed to be the maximization of his expected utility, which can be represented by an indirect function that is strictly concave and decreasing with even moments, and strictly concave and increasing with odd moments. Traditionally, only the first two moments, namely the mean and the variance, are considered to describe the preferences and risk profiles of an investor in terms of asset allocation in a risky environment. We can establish, however, a link between the expected utility of an agent and higher-order moments of a return distribution through an expansion of Taylor to an infinite-order (Tsiang, 1972; Loistl, 1976; Lhabitant, 1997; Dávila, 2011).

More generally, there is a long strand of literature using various statistical expansions[10] when pricing options[11] to compete with the Black and Scholes (1973) benchmark, and when pricing and optimally allocate assets[12] extending the canonical CAPM by Sharpe (1964) or the mean-variance model by Markowitz (1952). Statistical expansion is nowadays a well-known technique, from its origins (see Cramer, 1972; Hald, 1981 and 2000), limitations and potential drawbacks[13] (see, *e.g.*, Hamburger, 1920; Barton and Dennis, 1952; Tsiang, 1972; Scott and Horvath, 1980; Hlawitschka, 1994; Stoyanov, 2000; Jondeau and Rockinger, 2001; Schlögl, 2013)[14].

More specifically in our performance measure context, the utility of an investor i, denoted $U_i(.)$, can be formulated *via* a utility function that is arbitrarily continuous and differentiable in D with $\forall D \subset \mathbb{R}$. It represents the N-th order Taylor expansion, evaluated at the expected return on the investment, $\forall r_p \in D$, as:

$$U_i(r_p) = \sum_{n=0}^{N} (n!)^{-1} U_i^{(n)}[E(r_p)] \times [r_p - E(r_p)]^n + \tilde{\varepsilon}_{N+1}(r_p), \tag{1}$$

where r_p are the returns of the portfolio p, $n!$ is the n-factorial, $U_i^{(n)}(\cdot)$ is the n-th derivative of the utility function of the agent i and $\tilde{\varepsilon}_{N+1}(\cdot)$ is the Lagrange remainder.

Let us notice here that the latter tends to 0 when N goes to infinity. The Lagrange remainder can be decomposed as (with the previous notations):

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10 called Gram-Charlier type A, Edgeworth, Mac Laurin or Taylor expansion, depending on the field, context, ways of regrouping terms, and reference function or distribution.

11 See, *e.g.*, Jarrow and Rudd (1982), Corrado and Su (1996), Bakshi *et al.* (1997), Bakshi and Madan (2000), Jondeau and Rockinger (2001), Jurczenko *et al.* (2004), Martin *et al.* (2005), Lim *et al.* (2006), Corrado (2007), León *et al.* (2009), Andreou *et al.* (2010), Jha and Kalimipalli (2010), Tanaka *et al.* (2010), Chateau and Dufresne (2012), Del Brio and Perote (2012), Schlögl (2013), Chateau (2014), Lin *et al.* (2015).

12 See, *e.g.*, Arrow (1964), Feldstein (1969), Samuelson (1970), Jean (1971 and 1973), Arditti and Levy (1972), Rubinstein (1973), Borch (1974), Ingersoll (1975), Loistl (1976), Simkowitz and Beedles (1978), Levy and Markowitz (1979), Scott and Horvath (1980), Pulley (1983), Kroll *et al.* (1984), Dittmar (2002), Briec *et al.* (2007), Martellini and Ziemann (2010). See also Jurczenko and Maillet (2006) for further references.

13 We refere here to the survey by Jurczenko and Maillet (2006) on the theoretical foundations of a mean-variance-skewness-*kurtosis* decision criterion, discussing the conditions of convergence and the potential drawbacks of expansions.

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$$\tilde{\varepsilon}_{N+1}(r_p) = [(N-1)!]^{-1} U_i{}^{(N+1)}(\xi)\left[r_p - E(r_p)\right]^{N+1}, \quad (2)$$

where $N \in \mathbb{N}^*$ and ξ is a positive constant corresponding to the radius of convergence of the Taylor series expansion of $U_i(\cdot)$ around $E(r_p)$ as:

$$\xi = \lim_{N \to +\infty} \left| \frac{(N+1)!\, U_i^{(N)}[E(r_p)]}{N!\, U_i^{(N+1)}[E(r_p)]} \right|,$$

where $\xi \in \left]r_p, E(r_p)\right[$ if $r_p < E(r_p)$, or $\xi \in \left]E(r_p), r_p\right[$ otherwise.

In this analysis framework, we use a Taylor expansion in order to define the expected utility of any agent (see Jondeau and Rockinger, 2006; Jurczenko and Maillet, 2006), respecting accurate conditions (see Garlappi and Skoulakis, 2011) for the development to be exact (or approximative). To be valid, this approach requires that the Taylor approximation of the agent's utility function $U_i(.)$ at the N-th order around $E_i(.)$ absolutely converges towards $U_i(.)$. Moreover, since the summand and integral operators are commutative, we assume that conventional moments for all orders exist and are unique to characterize the return distribution. Then, we can take, under some regularity conditions, the limit of N towards infinity and the expected value on both sides in Equation (2), that leads us to (with the previous notations):

$$E\left[U_i(r_p)\right] = E\left\{\lim_{N\to\infty}\left\{\sum_{n=0}^{N}(n!)^{-1}U_i^{(n)}\left[E(r_p)\right]\left[r_p - E(r_p)\right]^n + \tilde{\varepsilon}_{N+1}(r_p)\right\}\right\}. \quad (3)$$

Through Equation (4), it is thus possible to express the expected utility of an economic agent from the first four moments of a return distribution. Considering an exact (or accurate) approximate Taylor expansion at the fourth-order of a general utility function[15] (see, for details, Jurczenko and Maillet, 2006), we have (with the previous notations):

$$\begin{aligned} E\left[U_i(r_p)\right] = {} & U_i\left[m_{1,p}(r_p)\right] + (2!)^{-1}U_i^{(2)}\left[m_{1,p}(r_p)\right]m_{2,p}(r_p) \\ & + (3!)^{-1}U_i^{(3)}\left[m_{1,p}(r_p)\right]m_{3,p}(r_p) + (4!)^{-1}U_i^{(4)}\left[m_{1,p}(r_p)\right]m_{4,p}(r_p) + \tilde{\epsilon}_5(r_p), \end{aligned} \quad (4)$$

[15] Please see our Web Appendix C (available on demand to the authors) for a list of the main utility functions expressed with the first four moments of returns and a table of elements' decomposition. Note here that $U_i^{(0)}(.) = U_i\ (.)$ *per* convention, and that the term in $U_i^{(1)}(.)$ disappears since $E\,[r_p - m_{1,p}(r_p)] = 0$ *per* definition. Pre-multiplying the first term (in $U_i\ (.)$) by $m_{1,p}(r_p)^{-1}$(for a non null mean), leads to the artificial but compact form of the following Equation (5) with *i*) a sum of terms with $n = 1$ to N , whilst *ii*) the sum of terms goes from $n = 0$ to N in Equation (1) and *iii*) the Jensen inequality residual term in Equation (10) below such as:

$$\tilde{\xi}_p = \sum_{n=2}^{N}(n!)^{-1}U_i^{(n)}\left[E(r_p)\right]\left[r_p - E(r_p)\right]^n + \tilde{\varepsilon}_{N+1}(r_p),$$

with a sum of terms going from $n = 2$ to N.

where $\tilde{\epsilon}_5(\cdot)$ is the Lagrange remainder and $m_{n,p}(\cdot)$ corresponds to the n-th moment with $n = [1, \dots, 4]$ that we can define more generally in the following way (with the previous notations, for $n > 1$):

$$m_{n,p}(r_p) = \int_{-\infty}^{+\infty} f(r_p)\left[r_p - m_{1,p}(r_p)\right]^n dr_p,$$

with:

$$m_{1,p}(r_p) = \int_{-\infty}^{+\infty} f(r_p) r_p dr_p,$$

where $f(\cdot)$ is the probability density function of returns r_p.

From Equation (4), we can express most of the utility functions as a linear combination of moments associated with the investor's sensitivities according to the following form[16] (with the previous notations):

$$E\left[U_i(r_p)\right] = \sum_{n=1}^{N} \lambda_{n,i,p} \times m_{n,p}(r_p) + \tilde{\epsilon}_{N+1}(r_p), \tag{5}$$

where $\tilde{\epsilon}_{N+1}(.)$ is the Lagrange remainder and $\lambda_{n,i,p}$ is the sensitivity of a representative investor i regarding the n-th moment defined as:

$$\lambda_{n,i,p} = (-1)^{n+1}(n!)^{-1}\omega_{n,i} \times g_{n,i}\left[m_{1,p}(r_p)\right]^{\tau_n},$$

where $n = [1, \dots, N]$, $n!$ is the n-factorial, $\omega_{n,i}$ is a weight, $g_{n,i}(\cdot)$ is a function of the first moment $m_{1,p}(\cdot)$ of the distribution of underlying returns r_p and the τ_n are some constants.

The HARA class of utility functions encompasses most of the popular functional forms of utility used in finance and economics, including the Constant Absolute Risk Aversion (CARA) and the Constant Relative Risk Aversion (CRRA) classes, as well as some functions belonging to the subclass of the quartic utility. The HARA power utility function is, for instance, written as:

$$U_i(r_p) = \frac{a}{1-a} \times \left[c + \frac{b}{a} \times r_p\right]^{(1-a)} \tag{6}$$

with:

$$\begin{cases} c + b/a \times r_p > 0 \\ 1/a > -1/2, \end{cases}$$

[16] In our Web Appendix K (available on demand to the authors), we have checked the accuracy of this expression with the main utility functions, using the first four moments of returns and a coefficient of risk aversion (a) equal to 3.

where $a > 0$ is the risk aversion coefficient, $c \geq 0$ and $b > 0$ are some constants, and r_p are the returns on portfolio p.

It is straightforward to verify that the HARA power utility function satisfies the fourth-order stochastic dominance requirements since we have (with the previous notations):

$$\begin{cases} U_i^{(1)}(r_p) = b\left[c + b/a \times r_p\right]^{-a} > 0, \\ U_i^{(2)}(r_p) = -b^2\left[c + b/a \times r_p\right]^{-(a+1)} < 0, \\ U_i^{(3)}(r_p) = b^3[(a+1)/a]\left[c + b/a \times r_p\right]^{-(a+2)} > 0, \\ U_i^{(4)}(r_p) = -b^4[(a+1)(a+2)/a^2]\left[c + b/a \times r_p\right]^{-(a+3)} < 0. \end{cases} \tag{7}$$

Focusing on terms up to the fourth-order, the expected utility of an agent i with preferences characterized by a HARA power utility function can then be approximated by a Taylor expansion. From Equation (6), the corresponding four-moment function reads[17] (with the previous notations):

$$\begin{aligned} E\left[U_i(r_p)\right] \cong \frac{a\left[m_{1,p}(r_p)\right]^{-1}}{(1-a)} \times \left\{c + \frac{b}{a} \times \left[m_{1,p}(r_p)\right]\right\}^{(1-a)} \times m_{1,p}(r_p) \\ - \frac{b^2}{2!} \times \left\{c + \frac{b}{a} \times \left[m_{1,p}(r_p)\right]\right\}^{-(a+1)} \times m_{2,p}(r_p) \\ + \frac{b^3}{3!}\frac{(a+1)}{a} \times \left\{c + \frac{b}{a} \times \left[m_{1,p}(r_p)\right]\right\}^{-(a+2)} \times m_{3,p}(r_p) \\ - \frac{b^4}{4!}\frac{(a+1)(a+2)}{a} \times \left\{c + \frac{b}{a}\left[m_{1,p}(r_p)\right]\right\}^{-(a+3)} \times m_{4,p}(r_p), \end{aligned} \tag{8}$$

with:

$$\begin{cases} \omega_{1,i} = a/(1-a) \\ \omega_{2,i} = b^2 \\ \omega_{3,i} = b^3(a+1)/a \\ \omega_{4,i} = b^4(a+1)(a+2)/a^{-2} \\ g_{n,i}\left[m_{1,p}(r_p)\right] = c + \frac{b}{a} \times m_{1,p}(r_p). \end{cases}$$

In the following, we discuss the main properties of the traditional utility functions and we make a link with the generalized formulation of the expected utility.

[17] See our Web Appendix C (available on demand to the authors) for a decomposition of special cases of the HARA class, such as utility functions belonging to the CARA and CRRA classes.

2.2 A Generalized Expression of Traditional Utility Functions reflecting Investor's Preferences

The main characteristics of the previous development allow us to differentiate several investors according to their preferences and risk profiles. It is indeed possible to define these characteristics, assimilated to sensitivities, to study moments of the return distribution (usually limited to the order four). The sensitivity to the first moment governs the so-called "greediness" of the investor, the sensitivity to the second moment represents his "risk aversion" , whilst the third[18] and the fourth terms characterize respectively the "prudence" (see Kimball, 1990; Lajeri-Chaherli, 2004) and the "temperance" (see Kimball, 1992, 1993; Eeckhoudt *et al.*, 1995; Menezes and Wang, 2005)[19]. We know that investors' preference functions determine what is the optimal combination between risky assets and the risk free rate an investor will hold, and how much this investor will consume and invest. The main restrictions existing on rational utility functions, *i.e.* corresponding to the supposed rationality of investors, are: non satiation, absolute and relative risk aversion, prudence and temperance. In other words, among all possible combinations of sensitivities of moments, only a restricted set is rational. Caballé and Pomansky (1996) analyze general utility functions exhibiting all derivatives of alternate signs and then propose an additional constraint on such utility functions, namely the Mixed Risk Aversion restriction. In particular, they formulate the property of Mixed Risk Aversion such as (with the previous notations):

$$\frac{U_i^{(n+1)}(r_p)}{U_i^{(n)}(r_p)} \leq \frac{U_i^{(n+2)}(r_p)}{U_i^{(n+1)}(r_p)}, \tag{9}$$

18 Some interesting related works, however, also show that the ratio $\left[U_i^{(3)}(.)/U_i^{(1)}(.)\right]$ is also linked to a risk aversion characteristic of a rational agent, who makes an arbitrage between the first and the third moments (*Cf.* Crainich and Eeckhoudt, 2008).

19 Lajeri-Chaherli (2004) proposes an expansion to the order five, mentioning the fifth-order risk as being the "edginess", whilst Caballé and Pomansky (1996) refine even further the expansion to the *N*-th order, referring to the "risk aversion of order *N*", as an analogue to the traditional classical Absolute Risk Aversion (see also Eeckhoudt and Schlesinger, 2006).

where $U_i^{(n)}(\cdot)$, for $n = [1, \dots, N]$, corresponds to the n-th derivative of the utility function of an individual i with respect to the return denoted r_p.

As an illustration, Figure 1 below displays a typical mixed risk aversion utility function (Panel A) and its first three (signed) derivatives (Panel B). that mainly show that, under the Mixed Risk Aversion hypothesis: 1) the impact of increases in returns are lower on the higher moment derivatives and thus the importance of the various derivatives should be ranked by decreasing order (which is moreover amplified by the $(n!)^{-1}$ factors in the Taylor expansion) and 2) the fact that the utility function is concave implies, under the Mixed Risk Aversion hypothesis, this precise ordering (with a decreasing influence of changes in returns on the different characteristics of the investor). In other words, rational individuals mainly focus first on expected performance, secondly on risk, then only thirdly and fourthly on superior higher-order moments.

The concept of Mixed Risk Aversion can also be linked to the other concepts in risk theory, namely the Proper Risk Aversion, the Standard Risk Aversion and the Risk Vulnerability. The first one corresponds to utility functions for which successive derivatives alternate in sign, the first being positive. The second concept reflects that both Absolute Risk Aversion and Absolute Prudence are decreasing, whereas the last one implies that Absolute Risk Aversion is decreasing and convex. A graphical observation of this condition is to say that the absolute values of the first, second, third and fourth derivatives of such utility functions, increasing and concave in the returns, all tend to a horizontal line when the returns are increasing.

It is obvious that Mixed Risk Aversion implies Standardness, Properness and Risk Vulnerability (see Caballé and Pomansky, 1996). The advantage with the concept of Mixed Risk Aversion is that it allows us to deal with higher moments, while Risk Aversion is restricted to the second-order moment and Standard Risk Aversion is directly related to the second-order and third-order moments.

Finally, most of the traditional utility functions[20] that respect the property of Mixed Risk Aversion can be expressed according to a generalized form such as (with the previous notations):

$$\begin{aligned} E[U_i(r_p)] &= U_i[E(r_p)] + \tilde{\xi}_p \\ &= \sum_{n=1}^{N} \lambda_{n,i,p} \times m_{n,p}(r_p) + \tilde{\epsilon}_{N+1}(r_p), \end{aligned} \tag{10}$$

[20] Please refer to our Web Appendix C (available on demand to the authors) for a decomposition to the fourth moment of the most common utility functions.

where $\tilde{\xi}_p$ is the Jensen inequality residual, and $\lambda_{n,i,p}$ is the sensitivity of investor *i* regarding the *n*-th moment of the portfolio *p* returns, such as:

$$\lambda_{n,i,p} = (-1)^{n+1}(n!)^{-1}\omega_{n,i} \times g_{n,i}\left[m_{1,p}(r_p)\right]^{\tau_n},$$

where $n = [1, \dots, N]$, $n!$ is the n-factorial, $\omega_{n,i}$ is a weight, $g_{n,i}(\cdot)$ is a function of the first moment $m_{1,p}(\cdot)$ of the distribution of underlying returns r_p, and the τ_n are some constants.

Dropping here the remainder and hypothesizing that the sensitivities are fixed[21] (*i.e.* $\lambda_{n,i,p} = \lambda_{n,i}$ for all Funds *p* considered), ultimately leads to our new measure that reflects the utility of a rational investor under some mild conditions[22], and which reads (with the previous notations):

$$\underset{[1\times1]}{GUN_{N,i,p}} \coloneqq \underset{[1\times N]}{\boldsymbol{\lambda}'_{N,i}} \times \underset{[N\times1]}{\mathbf{C}_{N,p}} \tag{11}$$
$$\cong E\left[U_i(r_p)\right],$$

where the $GUN_{N,i,p}$ statistic, summarizing the performance of a portfolio *p* held by an investor *i*, is expressed as a linear combination of the Conventional moments of a return distribution, where $\boldsymbol{\lambda}_{N,i}$ is a column vector composed of the $N \times 1$ sensitivities of an investor *i* to the *n*-th moment for $n = [1,..., N]$ and $i = [1,..., I]$, and $\mathbf{C}_{N,p}$ is a column vector composed of the *N* moments of the studied return distribution[23].

From the expression of $\lambda_{n,i}$ in $\boldsymbol{\lambda}_{N,i}$, which are general indeed, we see here that whatever the underlying implicit utility function (logarithmic, power…, rational or not, respecting the mixed-risk aversion conditions or not…), there is a link between our measure, a function of the derivatives and the moments in $\boldsymbol{\lambda}_{N,i}$, and the moments alone in $\mathbf{C}_{N,p}$.[24]

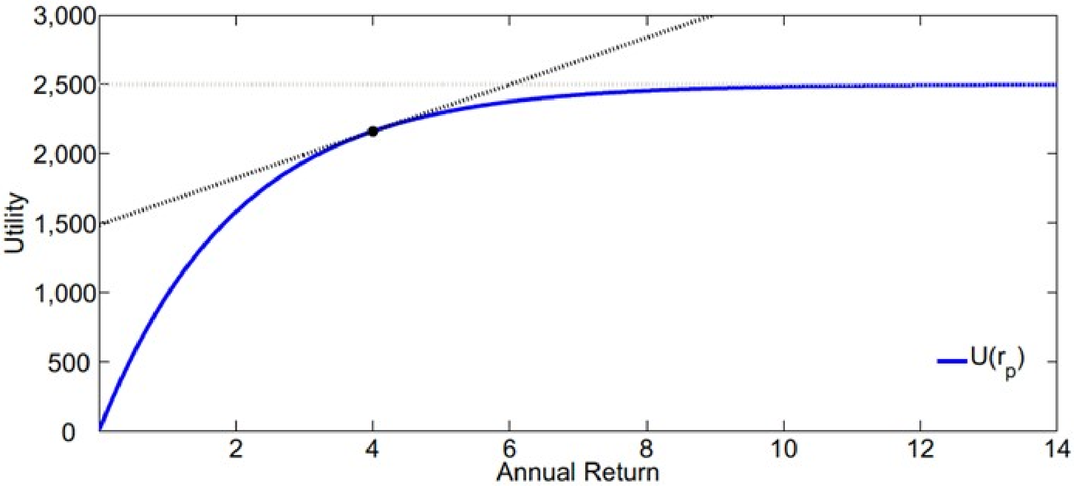
21 Because the measure is merely applied both to homogeneous investments and for similar individuals.

22 There also exists a link between the new Generalized Utility-based N-moment measure of performance and the Cumulative Prospect Theory when considering the investor's sensitivities as modified (subjective) probabilities associated with the distribution of non-distorted returns.

23 Our measure, following Billio *et al.* (2013), shares some similarities with the (*N*-th order) General Ranking Measure (see Smetters and Zhang, 2014, page 9).

24 See Eq. (8) for the precise expressions of the $\lambda_{n,i}$ in the case of a HARA-type of utility function (and to our Web Appendix C - available on demand to the authors, for a decomposition to the fourth moment of the most common utility functions.

Panel A: An Illustration of a Mixed-Risk Aversion (Exponential) Utility Function



Panel B: Decomposition of the First Three Derivatives of an Exponential Utility Function

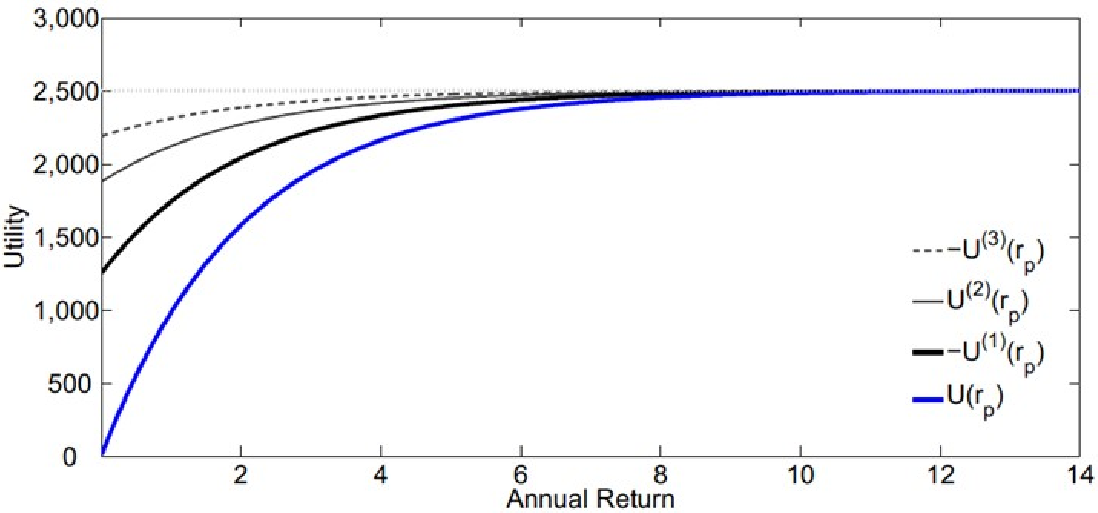


Fig.1. A Simple Illustration of a Mixed Risk Aversion Utility Function. Source: Simulations by the authors. Panel A displays a mixed risk aversion exponential utility function, such as: $U_i(rp) = 2,500 \times [1 - \exp(-a \times r_p \times 100)]$, with a coefficient $a = .50$ (*Cf.* our Web Appendix C available on demand to the authors). Panel B displays the utility function and its first three derivatives. The x-axis corresponds to annual returns and the y-axis represents the utility levels.

We propose in the next section to show how some of the main performance measures can be linked to linear combinations of moments adjusted to the agents' sensitivities.

3. A Comparison with the Main Performance Measures

We present in this section an interpretation of the GUN measure of performance and we explain how it can be seen as a generalization of some main performance measures.

First, we show that it is possible to express some of the main performance measures, namely the Sharpe (1966) ratio, the Morningstar (2002) RAR and the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM, as a linear combination of distorted moments in the case of a log transformation. Secondly, using the simulation scheme presented in Ingersoll *et al.* (2007), we show how the GUN can identify good and bad performances by comparing four declinations of it, each characterizing a specific investor's profile. Furthermore, we will show that our measure can almost perfectly replicate some rankings that come from the main performance measures[25]. Finally, using a set of hedge funds, we also show that the same results can be obtained when using, this time, market data with non-normal densities.

3.1 A Generalization of the Main Performance Measures

The new measure of performance is based on the study of the first four moments of return distributions. The main innovation of this is to consider the whole probability distribution. However, in a simplified framework, we show below that it is possible to exhibit some similar properties that share some main performance measures and a simple linear function of distorted moments. Our goal hereafter is both to show 1) how flexible our GUN measure of performance is and 2) that main performance measures are indeed very specific to a set of investors, *i.e.* that using a specific measure implies, in fact, to only consider some special (trade-offs of) preferences.

The Sharpe (1966) ratio is defined as a "Risk *Premium*" over the risk level of the studied portfolio as (with the previous notations):

[25] Note here that these measures belong to one of the four main families of performance measures identified in the survey of Caporin *et al.* (2014).

$$S_p = \left[E(r_p) - r_f\right] \times (\sigma_p)^{-1}, \tag{12}$$

where S_p is the Sharpe (1966) ratio of the portfolio p, r_f is the risk-free rate and σ_p is the standard deviation of returns of the portfolio p.

The numerator can be interpreted as the expectation of an individual in terms of returns and the denominator as the risk level of the agent's portfolio.

The Morningstar Risk-Adjusted Return, MRAR (*Cf.* Morningstar, 2002), is derived from a power-utility function and is defined as the expected value of the certainty equivalent annualized geometric return. It is thus defined as such:

$$MRAR_p = \begin{cases} E\left\{\left[(1+r_p)(1+r_f)^{-1}\right]^{-a}\right\}^{-\left(\frac{12}{a}\right)} - 1 \ with\ a > -1, a \neq 0 \\ \exp\left\{E\left[\ln\left[(1+r_p)(1+r_f)^{-1}\right]\right]\right\} - 1\ when\ a = 0, \end{cases} \tag{13}$$

where a is the risk aversion coefficient (set to 2 by Morningstar).

Finally, the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM indicators are written (depending on the pertinent risk aversion coefficient denoted a, with the previous notations) as such:

$$\Theta_p = [(1-a)\Delta t]^{-1} \ln\left\{E\left\{\left[(1+r_p)(1+r_f)^{-1}\right]^{1-a}\right\}\right\} \tag{14}$$

where the Θ_p statistic is the portfolio's *premium* return after adjusting for risk, Δt is the frequency of observations expressed on a yearly basis, r_p is the portfolio's (unannualized) rate of return of the portfolio p, r_f is the risk-free rate and a is the considered risk aversion coefficient.

If we now apply the logarithmic function to the Sharpe (1966) ratio, the Morningstar (2002) RAR and the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM indicators, we obtain (with the previous notations):

$$\begin{cases} \ln(S_p) = \ln[E(r_x)] - \ln(\sigma_p) \\ \ln(MRAR_p) = \varphi \ln\{E[(r^*)^{-a}]\} \\ \ln(\Theta_p) = \ln\{\ln\{E[(r^*)^{1-a}]\}\} - \ln(\nu), \end{cases} \tag{15}$$

where σ_p is the standard deviation of returns of the portfolio p, $E(r_x)$ is the difference between the expected return of the portfolio p and the risk-free rate r_f , $\varphi = -12/a$ is a constant, $r^* = (1 + r_p)/(1 + r_f)$ is the gross return, $\nu = (1 - a)\Delta t$ is a constant, where Δt is the frequency of observations of the agent's portfolio on a yearly basis.

3.2 A Direct Comparison of Moments Sensitivities of the Main Performance Measures

If we first restrict our GUN measure to the first two moments (normalizing $\lambda_{1,p}$ to unity without any loss of generality), we have (with the previous notations):

$$GUN_{2,i,p} = m_{1,p}(r_p) + \lambda_{2,i} \times m_{2,p}(r_p), \tag{16}$$

where $\lambda_{2,i}$ corresponds to the sensitivity of an individual i for the second moment, and with moments denoted by $m_{n,p}(.)$, with $n = [1, 2]$ that represents the first two moments of the return distribution of the portfolio p.

We clearly see in the above log-formulation of the classical performance measures that all are positively linked to some rescaled (excess) mean return, and negatively impacted by some notions of smoothness of the underlying return series, directly through a measure of the second moment of returns or, indirectly, *via* the use of a function which, more or less, undermines the dispersion of the series (up to some constants).

Even if a simple and clear analytical transformation of the measures cannot be easily provided, we can show through a simple illustration the relationship between the first two moments and four performance measures: the Sharpe (1966) ratio, the Morningstar (2002) Risk-Adjusted Return, the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM (for a risk-aversion coefficient of 3), and the GUN measure of performance (for a standard individual who has a sensitivity to the first and second moment equal to, respectively, 1.00 and -.50).

In Figure 2, we thus draw in Panel A the rescaled values of the four measures, when, based on the simulation scheme of Ingersoll *et al.* (2007), we artificially increase the annual mean return of the underlying return distribution, for a constant volatility level. In the same vein, Panel B displays results when we increase the annual volatility for a given annual mean return.

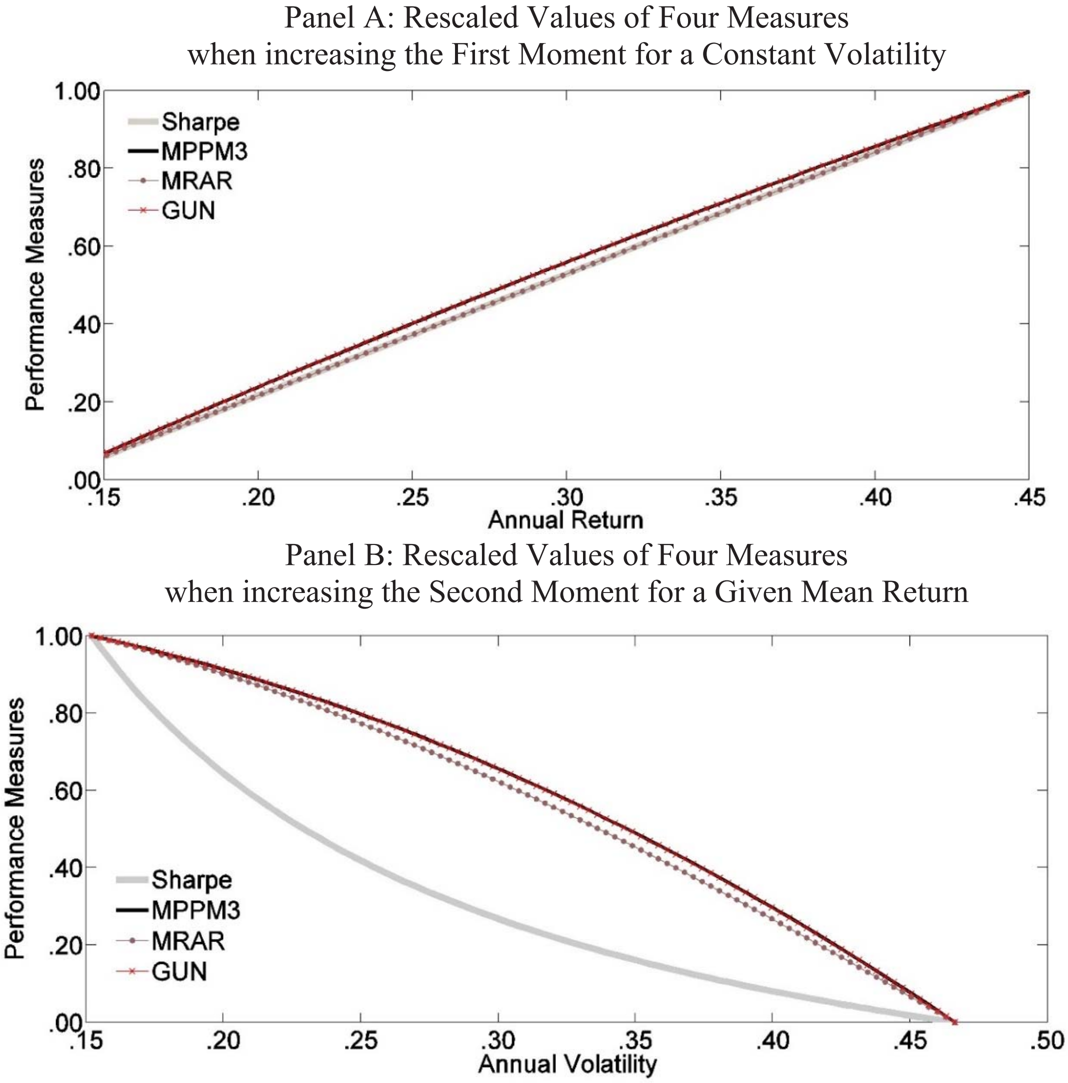


Fig 2. Impact of the First Two Moments on some Main Performance Measures. Source: Simulations by the authors. These figures draw on the impact of a positive variation of the first moment (Panel A) and of the second moment (Panel B) on the values of the GUN measure and three main performance measures: the Sharpe (1966) ratio, the Morningstar (2002) Risk-Adjusted Return and the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM. For Panel A, we draw one time-series from the simulation scheme presented in Ingersoll *et al.* (2007) and we slowly increase its mean return without modifying its volatility. The x-axis corresponds to the annual mean return and the y-axis represents the level of performance measures. For Panel B, in the same vein, we simulate returns of one portfolio and we gently increase its volatility without changing (by rescaling) its mean return. The x-axis is the annual volatility and the y-axis displays the rescaled values of the studied measures. The sensitivities to the first two moments are respectively equal to 1.00 and .50 for the computation of the GUN measure. Each measure is rescaled from 0% to 100%.

We note that all measures evolve in a similar way: they increase when the mean return goes up (for a constant standard deviation of returns) and when the volatility goes down (*ceteris paribus*). More precisely, we observe that the impact of the first (Panel A in Figure 2) and second moment (Panel B) on the GUN measure (thin line with cross markers), characterizing an agent with standard preferences, and on the MPPM3 (bold black line) are quite identical in rescaled values.

If we now compare the two other measures, we see that the Sharpe ratio (bold gray line) decreases faster than the Morningstar ratio (thin line with circle markers) when the volatility increases. Then, if we adjust the sensitivities of the GUN measure to the category of individuals considered, we should be able to get a similar ranking with our measure than the ones obtained with the Sharpe (1966) ratio, the Morningstar (2002) RAR and the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM. To sum up, our main idea is to show that the GUN measure is 1) flexible (because we can think about various sensitivity values in its definition), 2) more complete (since we can extend the analysis to higher-order moments for a better description of return densities) and 3) shares identical properties with other main classes of performance measures (in terms of preferences for the moments).

4. A Comparison with the MPPM and other Performance Measures with Simulated and Real Data

In this section, we propose three complementary studies. The first ones are related to some empirical results based on simulated data, generated thanks to the simulation scheme described in Ingersoll *et al.* (2007). We start by illustrating the *Iso*-MPPM curves that display the amount of over-volatility, for a given level of over-performance, required to reverse the MPPM ranking between two funds. Next, we reproduce and complete two tables obtained by Ingersoll *et al.* (2007) in which we show that the MPPM corresponds to a greedy investor, that is, an investor that cares a lot about mean returns and less on the other moments of the return distribution. Then, we exhibit the flexibility of the GUN to replicate the rankings of some main performance measures when underlying return distributions result from the realistic simulation scheme by Ingersoll *et al.* (2007). Finally, the last study presented in this section is dedicated to some empirical results based on real market data, relying on a hedge fund sample as in Darolles *et al.* (2009). Our main objective here is to confirm that the GUN measure is flexible enough to also replicate the ordering of some main performance measures with market data and when return distributions are not normal.

4.1 Empirical Results with Simulated Data

In this sub-section, we first show how the GUN identifies good and bad performances by comparing four variants of the measure, each characterizing a specific investor's profile, to some traditional measures, namely the Sharpe (1966) ratio, the Jensen (1968) alpha, the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM, the Henriksson-Merton (1981) and the Treynor-Mazuy (1966) measures. Secondly, we show that our measure can replicate the rankings of performance measures previously mentioned, and, also, of the Darolles-Gouriéroux-Jasiak (2009) L-performance, the Keating-Shadwick (2002) *Omega* and the Morningstar (2002) Risk-Adjusted Return measures. Using the simulation scheme presented in Ingersoll *et al.* (2007), we built *Iso*-MPPM curves in order to estimate the amount of *extra* unit of performance, for a given amount of over-volatility, that is required to reverse the MPPM ranking between the two funds (see Figure 3 below).

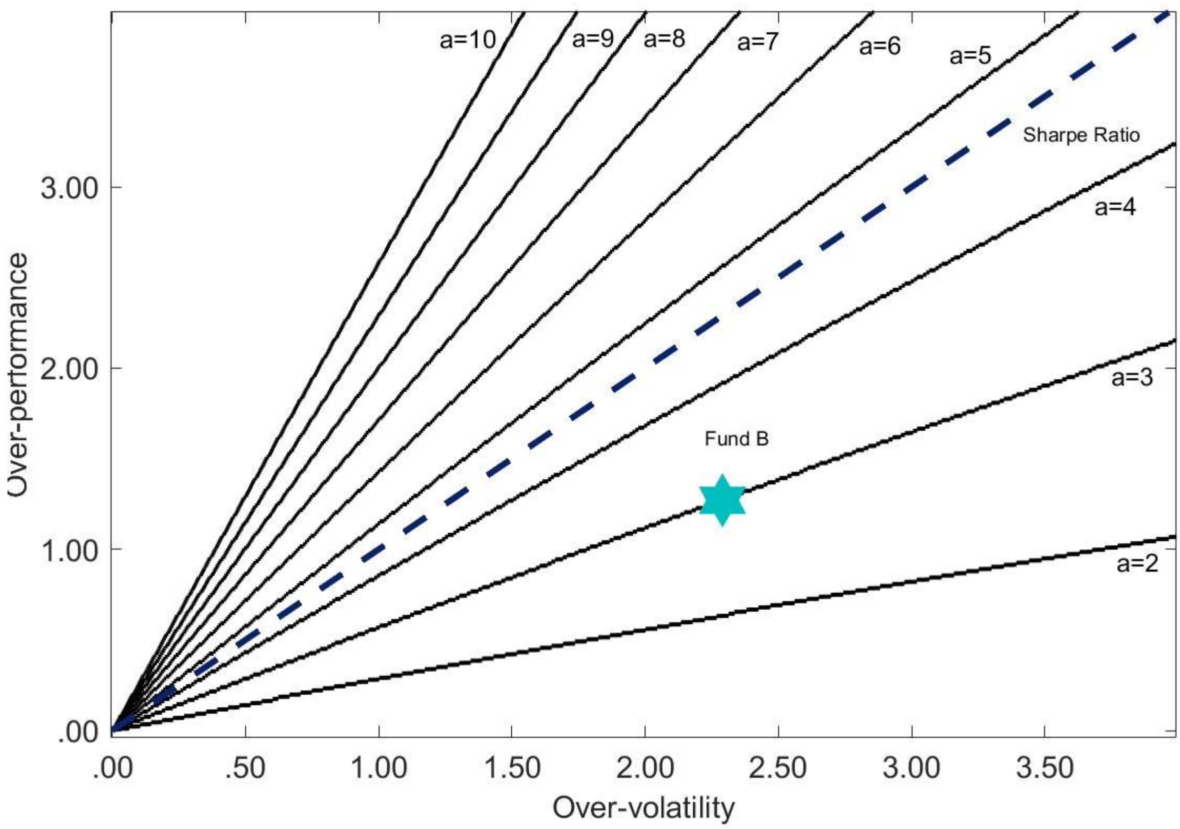


Fig. 3. *Iso*-MPPM Curves displaying the Quantity of Over-volatility required for a Given Over-performance for reversing the MPPM Ranking - in a Pure Simulation Case. Source: Simulations by the authors. Illustration of *Iso*-MPPM curves, *i.e.* over-performances (y-axis) *versus* over-volatilities (x-axis) yielding the same ranking according to the MPPM; both are expressed in percentage. We represent by a blue star the amount of over-performance of Fund *B* compared to Fund *A*, for a given level of over-volatility. The ranking curves (solid lines) are computed for several Manipulation-Proof Performance Measures (*Cf.* Ingersoll *et al.*, 2007) when varying the risk aversion coefficient, denoted *a*, from 2 to 10 here (*a* is equal to 3 in the original paper). Each *Iso*-curve represents the amount of over-performance of a portfolio compared to another one, for a given over-volatility, required to reverse the MPPM ranking between these two funds. The dashed bold line is the *Iso*-Sharpe (1966) ratio curve assuming that a unit of *extra* over-volatility - for a unit of a given over-performance - is required to inverse the ranking between fund *A* and fund *B*.

These curves are realized by comparing 10,000 pairs of portfolios obtained from the following algorithm[26]. We start by randomly choosing one time-series of returns among thousands simulated according to the scheme used in Ingersoll *et al.* (2007). This (initial) portfolio is composed by 250 returns distributed according to a Gaussian law characterized by an annual mean return equal to 17% and an annual volatility set to 20% as in Ingersoll *et al.* (2007). Next, we distort the return distribution of this initial portfolio in two different directions. First, we gradually increase its mean return, while keeping (by rescaling) its annual volatility equal to 20%, until we obtain 10,000 new portfolios. Secondly, we gradually decrease the volatility of the initial portfolio, without modifying (by rescaling) the mean return equal to 17% until we obtain 10,000 other portfolios. Finally, for a given risk aversion coefficient, we compute the MPPM for the two sets of 10,000 simulated portfolios in order to estimate the amount of *extra* units of performance that is required, for a given amount of over-volatility, to reverse the MPPM ranking between two funds.

For instance, when considering in MPPM a risk aversion coefficient equal to 3 (respectively 5), it happens that a 50 basis point *extra* performance may compensate a supplementary limit over-volatility of .91% (respectively .45%), whilst (in a quasi-linear manner in this region) a 100 basis point of *extra* performance may offset a *surplus* of volatility as large as 1.80% (respectively .90%), which has to be compared to the (theoretical) one-to-one performance-volatility relation in the case of the definition of the Sharpe ratio[27]. Therefore, the relationship approximately equals to 1 over 2, *versus* a relationship of 1 out of 1 for the Sharpe ratio. This former 1-for-2 trade-off might be seen as quite aggressive and it may illustrate the fact that, even for reasonable value of the risk aversion (equal here to 2, 3 or 5), the MPPM would only correspond to some rather greedy investors.

Let us now to turn to a comparison between MPPM and GUN, still in the same framework of simulations used in Ingersoll *et al.* (2007). Obviously, we would like that our measure, just as the MPPM, recognizes good performances as well as penalizes bad performances when they occur. The following Table 1 and Table 2 display the performances of portfolio managers, who exhibit, respectively, stock selection and market timing abilities. Next, we show that the GUN is flexible enough to replicate the rankings of most of the traditional performance measures, as shown in Table 3 and Table 4, which present illustrations of the GUN

[26] See our Web Appendix B (available on demand to the authors) for a full sketch of the algorithm.
[27] Results for an illustration of *Iso*-MPPM curves when considering over-skewness and over-performance are available upon request.

ranking equivalence on a sample of 30 (randomly chosen) simulated managed portfolios according to several measures of performance.

More precisely, Table 1 reports[28] the average excess, standard deviation and frequencies of the difference between the managed and market portfolios according to the annualized Sharpe (1966) ratio, the Jensen (1968) *alpha*, the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM and four variants of the GUN (on the various rows), each characterizing a specific investor's profile[29], for both an informed (left columns) and an uninformed manager (right columns). The former informed is supposed to generate an annual *extra*-performance superior to 1% compared to that of the latter uninformed manager. The two traders hold similar under-diversified portfolios, but the uninformed does not engage in any manipulation. For both managers, we distinguish three panels in Table 1, corresponding to different annual residual risk levels equal, respectively, to 20.00% (Panel A), 2.00% (Panel B) and .20% (Panel C). As mentioned by Ingersoll *et al.* (2007), these specific risk levels could reflect the level of diversification of portfolios composed by a few, hundreds and thousands of component stocks.

When we look at the global results, the Jensen (1968) *alpha*[30] is on average, as expected, equal to 1.00% and 0.00% for, respectively, the informed and uninformed managers in the three panels (see second columns, second lines in the three panels for the two blocks corresponding to the informed and the uninformed traders). Moreover, the uninformed investors' portfolios are significantly positive or negative, relative to the market, just about the predicted 5.00% of the time. Indeed, the Jensen (1968) *alpha* does not penalize for under-diversification.

For Panel C (that corresponds to a large number of stock portfolios with a small idiosyncratic risk), all the measures show that 1) the informed managers' portfolios are better than the market and 2) that the uninformed managers' portfolio are essentially identical to the market. For Panel B (that is associated with portfolios, composed of hundreds of stocks, with a reasonable specific risk), the Sharpe, MPPM and GUN measures give similar results. For Panel A (that reflects portfolios, composed of few component stocks, with a high specific volatility),

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28 Table 1 is an exact replication of Table 5 (on page 1,534) in Ingersoll *et al.* (2007), using their simulation scheme. See also our Web Appendix E for more details (available on demand to the authors).

29 We first start by defining a "neutral" agent for whom the scalar products, respectively, between the first four sensitivities and the first four average moments of the studied sample are strictly identical. Secondly, we specify four different categories of investors characterized by a high sensitivity to only one of the four moments (*ceteris paribus*). More precisely, we have a greedy investor, denoted $GUN_{4,G,p}$, who is focused on the mean, a risk-averse agent, named $GUN_{4,RA,p}$, with a high sensitivity to the variance, a prudent one, called $GUN_{4,P,p}$, characterized by a significant preference to the third moment and a very temperate investor, *alias* $GUN_{4,T,p}$, who severely dislikes the fourth moment (*Cf.* Caporin *et al.*, 2014, for the definition of the different performance measures used in the following tables).

30 Using the same hypothesis defined in Ingersoll *et al.* (2007), we compute the Jensen (1968) *alpha* assuming a systematic risk sensitivity of informed and uninformed managers' portfolios equal to 1.

the MPPM as well as the four variants of the GUN do substantially better in showing that the 1% *extra* -performance does not properly compensate for the lack of diversification. We also observe that the GUN measure characterizing a risk-averse investor, is more penalized when the portfolio residual specific risk is high compared to the other agents' profiles. Results obtained with our four variants of GUN lead us to think that the MPPM would correspond, once again, to a greedy investor.

In the main, we can here write that the GUN measure always coherently leads us to prefer the informed manager, whatever the quality of the signal, just as the MPPM does. Similarly, the more precise the signal, the better the performance (as shown when we compare the results in Panel A, B and C), for all the profiles of investors considered. However, we also show that the final impact on measures deeply depends on the preferences of investors.

Also grounded on the simulation parameters defined in Ingersoll *et al.* (2007), Table 2 reports[31] the average, standard deviation and frequencies of timing coefficients and total contributed values for the Henriksson-Merton (1981) and Treynor-Mazuy (1966) measures, of the differences between the managed and market portfolios according to the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM and the four variants of the GUN, previously defined in Table 1, for both an informed (left columns) and a random market timer (right columns).

The timing measures of Treynor and Mazuy (1966) and Henriksson and Merton (1981) come from regressions based on an *extra* market factor to capture managers' timing abilities defined as:

$$\tilde{r}_p - r_f = \alpha_p + \left(\tilde{r}_m - r_f\right) \times \beta_{1,p} + \tilde{\omega}_m \times \beta_{2,p} + \tilde{\varepsilon}_p, \tag{17}$$

where $\tilde{r}_p$ are the managed portfolio returns, r_f is the risk-free rate, α_p is the Jensen (1968) *alpha*, $\beta_{1,p}$ is the systematic risk sensitivity of Portfolio p to the market portfolio m, $\tilde{r}_m$ are the market portfolio returns, $\beta_{2,p}$ is the market timing coefficient, w_m is equal, respectively, to $\tilde{\omega}_m = max\left(\tilde{r}_m - r_f, 0\right)$ in Henriksson and Merton (1981) and $\tilde{\omega}_m = \left(\tilde{r}_m - r_f\right)^2$ in Treynor and Mazuy (1966), and $\tilde{\varepsilon}_p$ is the residual.

As shown in the setting of Ingersoll *et al.* (2007), the total contribution values corresponding to the money manager's contribution to timing and selectivity are, respectively, written as such (with the previous notations):

[31] Table 2 is also an exact replication of Table 6 (on page 1,535) in Ingersoll *et al.* (2007), using their simulation scheme. See also our Web Appendix F (available on demand to the authors) for more details.

$$\begin{cases} HM_\beta = \beta_{2,p} \text{ with } \widetilde{\omega}_m = \max\left(r_f - \tilde{r}_m \,, 0\right) \\ TM_\beta = \beta_{2,p} \text{ with } \widetilde{\omega}_m = \left(\tilde{r}_m - r_f\right)^2 \,, \end{cases} \tag{18}$$

where $\beta_{2,p}$ is the market timing coefficient, w_m is equal, respectively, to $\widetilde{\omega}_m = max(\tilde{r}_m - r_f, 0)$ in Henriksson and Merton (1981) and $\widetilde{\omega}_m = \left(\tilde{r}_m - r_f\right)^2$ in Treynor and Mazuy (1966), and $\tilde{\varepsilon}_p$ is the residual.

	Informed Timer (δ_p = .10%)					Random Timer				
Residual Risk	Avg. Excess	Std. Dev.	Freq. Won	Freq. Signif. +	Freq. Signif. -	Avg. Excess	Std. Dev.	Freq. Won	Freq. Signif. +	Freq. Signif. -
Panel A: Annual Logarithmic Residual Standard Deviation = 20.00%										
Sharpe	-.140	.344	34.22%	1.99%	10.74%	-.176	.344	30.47%	1.56%	12.75%
Jensen	1.01%	8.99%	54.50%	6.14%	3.90%	.01%	8.99%	50.11%	4.93%	4.95%
MPPM	-4.99%	8.99%	29.04%	1.37%	13.76%	-5.99%	8.99%	25.31%	1.03%	16.30%
$GUN_{4,G,p}$	-6.99%	17.97%	34.93%	2.06%	10.41%	-8.49%	17.97%	31.88%	1.69%	12.01%
$GUN_{4,RA,p}$	-17.00%	9.03%	2.99%	.02%	59.08%	17.00%	9.03%	2.98%	.02%	59.14%
$GUN_{4,P,p}$	-7.94%	8.99%	18.88%	.57%	22.20%	-8.44%	8.99%	17.37%	.48%	23.90%
$GUN_{4,T,p}$	-8.05%	8.99%	18.55%	.55%	22.55%	-8.54%	8.99%	17.07%	.47%	24.25%
Panel B: Annual Logarithmic Residual Standard Deviation = 2.00%										
Sharpe	.047	.045	85.15%	26.29%	.35%	-.003	.045	47.45%	4.28%	5.66%
Jensen	1.00%	.90%	86.65%	29.67%	.29%	-.00%	.90%	50.13%	4.93%	4.95%
MPPM	.94%	.90%	85.17%	27.37%	.34%	-.06%	.90%	47.45%	4.29%	5.66%
$GUN_{4,G,p}$	1.82%	1.80%	84.39%	26.31%	.38%	-.17%	1.80%	46.36%	4.05%	5.97%
$GUN_{4,RA,p}$	.64%	.92%	75.78%	17.11%	.93%	-.34%	.91%	35.78%	2.18%	10.02%
$GUN_{4,P,p}$	.82%	.90%	81.85%	23.06%	.52%	-.17%	.90%	42.77%	3.32%	7.13%
$GUN_{4,T,p}$	.82%	.90%	81.80%	22.97%	.52%	-.17%	.90%	42.66%	3.29%	7.17%
Panel C: Annual Logarithmic Residual Standard Deviation = .20%										
Sharpe	.050	.005	100.00%	100.00%	.00%	.000	.005	49.88%	4.84%	4.99%
Jensen	1.00%	.09%	100.00%	100.00%	.00%	.00%	.09%	50.13%	4.93%	4.99%
MPPM	1.00%	.09%	100.00%	100.00%	.00%	.00%	.09%	49.88%	4.85%	5.02%
$GUN_{4,G,p}$	2.00%	.18%	100.00%	100.00%	.00%	.00%	.18%	49.76%	4.82%	5.04%
$GUN_{4,RA,p}$	.99%	.09%	100.00%	100.00%	.00%	.00%	.09%	48.63%	4.53%	5.33%
$GUN_{4,P,p}$	1.00%	.09%	100.00%	100.00%	.00%	.00%	.09%	49.36%	4.72%	5.13%
$GUN_{4,T,p}$	1.00%	.09%	100.00%	100.00%	.00%	.00%	.09%	49.37%	4.72%	5.13%

Tab.1. The GUN Measure of Performance: Informed *versus* Uninformed Traders. This table shows the effect of a variation of the portfolio residual risk according to four variants of the Generalized Utility-based N-moment measure of performance and three other measures (Sharpe, 1966; Jensen, 1968; Ingersoll *et al.*, 2007) for an (un-)informed trader who holds a portfolio with a positive *extra* -performance by taking on various levels of increased unsystematic risk. The Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM is defined as:

$$\Theta_p = [(1-a)\Delta t]^{-1} \ln\left\{E\left\{\left[(1+r_p)(1+r_f)^{-1}\right]^{1-a}\right\}\right\}$$

and the Generalized Utility-based N-moment measure of performance – in short GUN – for an individual i as follows:

$$GUN_{4,i,p} = \lambda_{1,i,p} \times m_{1,p}(r_p) + \lambda_{2,i,p} \times m_{2,p}(r_p) + \lambda_{3,i,p} \times m_{3,p}(r_p) + \lambda_{4,i,p} \times m_{4,p}(r_p).$$

The latter is declined according to four general investor's profiles, namely $GUN_{4,G,p}$, $GUN_{4,RA,p}$, $GUN_{4,P,p}$ and $GUN_{4,T,p}$, which respectively refer to an investor strongly greedy, risk averse, prudent and temperate. The frequencies with which the investors' portfolio beats the market portfolio according to each measure are given along with the approximate frequencies with which the portfolio significantly (5.00%) outperforms or underperforms the market. These numbers are estimated as the frequency with which the performance measure was more than 1.65 standard deviations positive or negative. The computation is based on 350,000 simulated managed portfolios with a 5-year return history, respecting the following market hypotheses (with a four-digit accuracy): risk free rate 5.00% *per* year, market *premium* 12.00%, market standard deviation 20.00% and an investor's degree of risk aversion a set to 3. Simulations by the authors.

The second set of the Merton-Henriksson (1981) and Treynor-Mazui (1968) measures ultimately write (with previous notations):

$$\begin{cases} HM_V = \alpha_p e^{-r_f \Delta t} + \beta_{2,p} P\left(1, \Delta t, e^{-r_f \Delta t}\right) \text{ with } \tilde{\omega}_m = \max\left(r_f - \tilde{r}_m, 0\right) \\ TM_V = \alpha_p e^{-r_f \Delta t} + \beta_{2,p} e^{r_f \Delta t}\left(e^{\sigma_m^2 \Delta t} - 1\right) \text{ with } \tilde{\omega}_m = \left(\tilde{r}_m - r_f\right)^2, \end{cases} \tag{19}$$

whith $P(1, \tau, K)$ the value of a τ-period put option with a strike price equal to K.

	Informed Timer (δ_p = .10%)					Random Timer				
Residual Risk	Avg. Excess	Std. Dev.	Freq. Won	Freq. Signif. +	Freq. Signif. -	Avg. Excess	Std. Dev.	Freq. Won	Freq. Signif. +	Freq. Signif. -
HMβ	.012	.043	60.42%	8.60%	2.64%	.000	.044	49.29%	4.78%	5.06%
HM*V*	.10%	.35%	60.68%	8.66%	2.62%	.00%	.36%	49.22%	4.74%	5.02%
TMβ	.211	.940	58.58%	7.72%	3.18%	-.001	.955	49.94%	4.64%	4.66%
TM*V*	.01%	.02%	62.02%	9.38%	2.58%	.00%	.02%	49.62%	4.92%	5.02%
MPPM	.42%	2.37%	57.22%	6.80%	3.52%	-.50%	2.46%	42.02%	3.14%	7.38%
$GUN_{4,G,p}$	.51%	4.76%	54.28%	5.92%	3.94%	-1.28%	4.95%	39.42%	2.90%	8.20%
$GUN_{4,RA,p}$	-1.45%	3.28%	33.18%	1.62%	11.42%	-2.33%	2.56%	18.16%	.32%	22.52%
$GUN_{4,P,p}$	-.27%	2.56%	46.30%	3.62%	6.22%	-1.17%	2.55%	31.88%	1.92%	11.38%
$GUN_{4,T,p}$	-.33%	2.53%	45.44%	3.36%	6.50%	-1.22%	2.51%	30.76%	1.74%	12.08%

	Informed Timer (δ_p = 10%)					Random Timer				
Residual Risk	Avg. Excess	Std. Dev.	Freq. Won	Freq. Signif. +	Freq. Signif. -	Avg. Excess	Std. Dev.	Freq. Won	Freq. Signif. +	Freq. Signif. -
HMβ	.114	.138	80.12%	20.14%	.82%	.000	.138	50.16%	5.22%	4.44%
HM*V*	.95%	1.14%	80.42%	20.38%	.74%	.00%	1.14%	50.24%	5.20%	4.50%
TMβ	2.057	3.014	75.68%	16.14%	1.10%	.006	2.99	49.70%	5.26%	4.94%
TM*V*	.07%	.07%	82.64%	23.16%	.52%	.05%	.07%	50.38%	5.14%	4.46%
MPPM	4.09%	7.50%	70.34%	13.38%	1.16%	-4.16%	7.67%	29.04%	1.62%	13.48%
$GUN_{4,G,p}$	6.09%	14.99%	65.52%	10.72%	1.90%	-6.57%	15.32%	33.18%	1.94%	11.22%
$GUN_{4,RA,p}$	-12.55%	8.14%	6.46%	.06%	45.66%	-15.23%	7.81%	2.50%	.02%	62.04%
$GUN_{4,P,p}$	-2.06%	7.62%	38.94%	2.92%	8.30%	-7.20%	7.76%	17.40%	.58%	22.96%
$GUN_{4,T,p}$	-2.24%	7.58%	38.14%	2.74%	8.76%	-7.33%	7.71%	16.72%	.50%	23.58%

Tab.2. The GUN Measure of Performance: Informed *versus* Random Market Timers. This table compares the timing coefficient and the contributed value, denoted HM_β and HM_V when using the Henriksson-Merton (1981) parametric model TM_β and TM_V for the Treynor-Mazuy (1966) market timing model, along with the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM and four variants of the GUN, for an informed market timer whose information about a changing mean explains .10% and 1.00% of the market's variance, and a random market timer who varies leverage randomly to the same degree. The Ingersoll *et al.* (2007) Manipulation-Proof Performance Measure – say MPPM – is defined as:

$$\Theta_p = [(1-a)\Delta t]^{-1} \ln\left\{E\left\{\left[\left(1+r_p\right)\left(1+r_f\right)^{-1}\right]^{1-a}\right\}\right\}$$

and the Generalized Utility-based N-moment measure of performance – in short GUN – for an individual i as follows:

$$GUN_{4,i,p} = \lambda_{1,i,p} \times m_{1,p}(r_p) + \lambda_{2,i} \times m_{2,p}(r_p) + \lambda_{3,i} \times m_{3,p}(r_p) + \lambda_{4,i} \times m_{4,p}(r_p).$$

The latter is declined according to four general investor's profiles, namely $GUN_{4,G,p}$, $GUN_{4,RA,p}$, $GUN_{4,P,p}$ and $GUN_{4,T,p}$, which respectively refer to an investor strongly greedy, risk averse, prudent and temperate. The frequencies with which the investors' portfolio beats the market portfolio according to each measure are given along with the approximate frequencies with which the portfolio significantly (5.00%) outperforms or underperforms the market. These numbers are estimated as the frequency with which the performance measure was more than 1.65 standard deviations positive or negative. The computation is based on 350,000 simulated managed portfolios with a 5-year return history, respecting the following market hypotheses (with a four-digit accuracy): risk free rate 5.00% *per* year, market *premium* 12.00%, market standard deviation 20.00% and an investor's degree of risk aversion a set to 3. Simulations by the authors.

The total contribution is thus here the amount by which the value of the protective put exceeds its average "cost" measured by the lowered present value of the *extra* average return. The informed market timer optimally adjusts the systematic risk sensitivity of his portfolio according to his information that explains .10% (top part) or 1.00% (bottom part) of the market portfolio's variation. The uninformed trader wrongly believes that he has the same quality information and adjusts his leverage randomly to the same degree.

The Henriksson-Merton (1981) and Treynor-Mazuy (1966) measures are built to only identify informed traders, but not to penalize the uninformed ones. Thus, these two models show consistent results with the null hypothesis for the uninformed timer, just as they would for a manager not trying to time at all. Regarding the MPPM, we observe that the uninformed market timer, who incorrectly thinks he has better information, has definitely a very low portfolio performance relative to the market. The Henriksson-Merton (1981) and Treynor-Mazuy (1966) models frequently recognize the informed traders because, unlike the MPPM, they do not penalize the portfolio's performance for the induced lack of inter-temporal diversification. However, when we look at the four GUN measures, only the greedy investor prefers the very informed trader compared to the three other specific profiles. Consequently, these results lead us to think, once again, that the MPPM may be biased towards the mean.

Let us show now that the GUN measure is flexible enough to be able to be aligned with main performance measures when pertinently choosing the right set of sensitivities. Panel A in Table 3 reports performance measure scores and rankings of an illustrative sample of 30 portfolios[32] according to the Sharpe (1966) ratio, the Jensen (1968) *alpha* and four variants of the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM when varying the risk aversion level (from 2 to 5). Panel B (first line) displays the vector of the first two implied sensitivities, corresponding to coefficients used in the computation of our GUN (denoted Av. Sensi. in the table). that allows us to exactly replicate the portfolio rankings obtained with each measure of the six studied (with, below, Spearman, Kendall, and Goodman-Kruskal correlation coefficients equal to 1 on the studied sample as indicated in Panel B). Panel C and D present a statistics summary of Spearman, Kendall, and Goodman-Kruskal correlation coefficients computed for 1,000 samples of 30 funds drawn from a large sample of simulated portfolios, when varying the first two sensitivities (with the average on the first line of Panel C - denoted Av. Sensi.) and all the sensitivities (with the average on the first line on Panel D).

32 The 30 ranked portfolios correspond, as a mere illustration, to a random sample of 15 informed and 15 uninformed managers whose portfolio returns respect the simulation scheme defined in Table 1 (*Cf.* Ingersoll *et al.*, 2007). For each measure of performance in this table, the value and rank of funds are reported in the various columns of Panel A. Funds are sorted according to their Sharpe ratios (first two columns). Then, for the other measures (following columns), related relative ranks are presented. For instance, the highest Sharpe ratio fund is also the best, according to the Jensen, MPPM2 and MPPM3 measures, whilst it is the third fund in terms of Sharpe, which the best according to the MPPM4 measure, and the 20th fund for the MPPM5. In this sample also, the 30th fund is the worst fund, whatever the measure.

Results in this illustration show that the rankings of the 30 simulated portfolios obtained according to the Sharpe (1966) ratio, the Jensen (1968) *alpha* and the four variants of the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM (varying the risk aversion coefficients from 2 to 5 by 1) can, most of the time, be exactly replicated – as shown by the Spearman, Kendall and Goodman-Kruskal correlation coefficients equal (or close) to 1.00, for each of the six measures – when adjusting the sensitivities applied to the first two (or first four) moments in the computation of the GUN measures. These results lead us to conclude that, first, all traditional measures indeed correspond to an equivalent implicit choice regarding the preferences, with a mean risk sensitivity varying from -.67 for the Jensen *alpha* to -2.50 for MPPM5 when two moments are under study (Panel C) and from -.59 for the Jensen *alpha* to -2.57 for MPPM5 when all moments are implied (Panel D), whilst the temperance and prudence representative sensitivities are small (equal to some percents in Panel D). Secondly, as illustrated by the comparison between MPPM for various risk aversion coefficients, the higher is the agent's risk aversion coefficient and the lower his sensitivity to the second moment. In other words, choosing one measure or the other is in fact a choice in terms of preferences. Thirdly, we can also write that our measure is flexible enough to replicate, with a fine accuracy in most cases, the rankings coming from the main traditional performance measures (since for the vast majority of the rankings - see Panel C and D, correlation coefficients are close to 1, with minimum coefficients that are high).

Table 4 (Panel A, B, C and D) reports the same results as the previous ones in Table 3, but this time 1) when related to the scheme of simulations for both informed and random market timer (see Table 2) and 2) according here to the Henriksson-Merton (1981) and the Treynor-Mazuy (1966) measures, and four variants of the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM when varying the risk aversion level. More precisely, Panel A reports performance measure scores and rankings of a sample of 30 portfolios[33] according to the various measures. Panel B (first line) displays the vector of the first two implied sensitivities, corresponding to coefficients used in the computation of our GUN measures that allow us to exactly replicate the portfolio rankings obtained with each measure of the six studied.

[33] The 30 ranked portfolios correspond to 15 informed and 15 uninformed managers whose portfolio returns respect the simulation scheme defined in Table 1 (*Cf.* Ingersoll *et al.*, 2007).

Panel A: An illustration of Iso-GUN Rankings on a Sample of 30 Portfolios

Sharpe		Jensen		MPPM2		MPPM3		MPPM4		MPPM5	
Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank
.40	1	8.23%	1	3.28%	1	-.75%	1	-4.16%	5	-6.15%	5
.35	2	6.50%	2	1.81%	6	-2.17%	5	-4.50%	6	-6.48%	6
.29	3	4.70%	3	.19%	3	-2.21%	2	-4.80%	1	-6.87%	7
.26	4	3.82%	4	-.18%	5	-2.52%	6	-4.87%	7	-7.01%	8
.19	5	2.15%	5	-.54%	4	-2.86%	7	-5.04%	8	-7.05%	9
.17	6	1.76%	6	-.54%	2	-3.07%	8	-5.08%	9	-7.14%	11
.16	7	1.43%	7	-.86%	7	-3.11%	9	-5.17%	11	-7.21%	12
.14	8	1.15%	8	-1.10%	8	-3.20%	11	-5.24%	12	-7.26%	10
.14	9	1.11%	9	-1.14%	9	-3.26%	10	-5.26%	10	-7.28%	13
.14	10	1.01%	10	-1.23%	20	-3.27%	12	-5.31%	13	-7.39%	14
.14	11	1.00%	11	-1.26%	11	-3.34%	13	-5.42%	14	-7.81%	15
.13	12	.94%	12	-1.30%	12	-3.44%	14	-5.81%	15	-7.95%	16
.13	13	.87%	13	-1.37%	13	-3.81%	15	-5.98%	16	-8.01%	17
.12	14	.78%	14	-1.47%	14	-3.88%	3	-6.04%	17	-8.03%	18
.11	15	.42%	15	-1.81%	19	-4.01%	16	-6.06%	18	-8.07%	20
.10	16	.16%	16	-2.04%	16	-4.07%	17	-6.10%	20	-8.11%	21
.09	17	.09%	17	-2.10%	15	-4.09%	18	-6.15%	21	-8.16%	19
.09	18	.08%	18	-2.12%	18	-4.13%	20	-6.17%	19	-8.43%	22
.09	19	.04%	19	-2.16%	17	-4.18%	19	-6.23%	2	-8.48%	23
.09	20	.03%	20	-2.19%	10	-4.18%	21	-6.43%	22	-8.85%	1
.09	21	-.02%	21	-2.21%	21	-4.33%	4	-6.48%	23	-8.92%	24
.08	22	-.23%	22	-2.43%	22	-4.43%	22	-6.93%	24	-10.24%	2
.08	23	-.28%	23	-2.48%	26	-4.48%	23	-7.96%	3	-11.93%	4
.05	24	-.77%	24	-2.95%	24	-4.94%	24	-8.13%	4	-12.04%	3
.04	25	-2.81%	25	-7.00%	25	-11.14%	25	-15.30%	25	-19.46%	25
-.01	26	-4.06%	26	-8.19%	28	-12.18%	26	-16.18%	26	-20.17%	26
-.24	27	-10.54%	27	-14.91%	27	-18.92%	27	-22.92%	27	-26.91%	27
-.30	28	-12.15%	28	-16.36%	29	-20.28%	28	-24.90%	28	-28.12%	28
-.35	29	-13.52%	29	-17.94%	23	-21.94%	29	-25.94%	29	-29.94%	29
-.57	30	-19.45%	30	-24.90%	30	-29.09%	30	-33.94%	30	-37.49%	30

Panel B: Statistics Summary of Ranking Correlations on 1,000 Samples of 30 Randomly Chosen Funds when only varying the First Two Sensitivities

	Sharpe			Jensen			MPPM2			MPPM3			MPPM4			MPPM5		
Av. Sens.	(1.00, -.42)			(1.00, -.29)			(1.00, -1.00)			(1.00, -1.50)			(1.00, -2.00)			(1.00, -2.50)		
Correlation*	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ
The Sample	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Panel C: Statistics Summary of Ranking Correlations on 1,000 Samples of 30 Randomly Chosen Funds when varying the First Two Sensitivities

Av. Sens.	Sharpe (1.00, -.78)			Jensen (1.00, -.67)			MPPM2 (1.00, -.91)			MPPM3 (1.00, -1.31)			MPPM4 (1.00, -1.83)			MPPM5 (1.00, -2.44)		
Correlation*	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ
Minimum	.90	.85	.80	.84	.80	.78	1.00	.99	.73	.94	.89	.87	.86	.83	.60	.80	.69	.63
First Quartile	1.00	.99	.92	.98	.95	.87	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.97	1.00	1.00	1.00
Median	1.00	1.00	1.00	1.00	1.00	.87	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Thrid Quartile	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Maximum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Panel D: Statistics Summary of Ranking Correlations on 1,000 Samples of 30 Randomly Chosen Funds when only varying All Four Sensitivities

Av. Sens.	Sharpe (1.00, -.88, .03, -.02)			Jensen (1.00, -.59, .05, -.02)			MPPM2 (1.00, -.95, .04, -.04)			MPPM3 (1.00, -1.36, .03, -.03)			MPPM4 (1.00, -1.71, .05, -.04)			MPPM5 (1.00, -2.57, .06, -.04)		
Correlation*	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ
Minimum	.90	.85	.80	.84	.80	.78	1.00	.99	.73	.94	.89	.87	.86	.83	.60	.80	.69	.63
First Quartile	1.00	.99	.92	.98	.95	.87	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.97	1.00	1.00	1.00
Median	1.00	1.00	1.00	1.00	1.00	.87	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Thrid Quartile	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Maximum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Tab. 3. GUN Ranking Equivalence These tables compare the ranking of simulated funds obtained according to several performance measures (respectively, Sharpe, 1966; Jensen, 1968; Ingersoll *et al.*, 2007). Panel A displays the values and the ranking, for each studied measure, for a sample of 30 simulated portfolios. Panel B presents the sensitivities of the first two moments, used for the computation of the GUN measure, to replicate the ranking of the six other performance measures as well as the corresponding correlation coefficients (*ρ: Spearman; τ: Kendall; γ: Goodman and Kruskal). Panel C and Panel D present, respectively, the average first two and the average first four sensitivities associated with moments, and a statistics summary of ranking correlations (minimum, first quartile, median, third quartile, maximum) on 1,000 samples of 30 randomly chosen funds.
Similar to Table 1 and to Ingersoll *et al.* (2007), managed portfolio return distributions are defined as:

$$\tilde{r}_p = exp\{[\mu_m + \psi_p - .50(\sigma_m^2 + \upsilon_m^2)]\Delta t + (\sigma_m\tilde{\vartheta} + \upsilon_p\tilde{\eta})\sqrt{\Delta t}\} - 1$$

and market portfolio returns as:

$$\tilde{r}_m = exp\{[\mu_m - .50(\sigma_m^2)]\Delta t + (\sigma_m\tilde{\vartheta})\sqrt{\Delta t}\} - 1$$

where $\tilde{r}_p$ the are the managed portofolio returns, $\tilde{r}_m$ are the market ones, μ_m is the mean market rate of return, ψ_p is the extra -performance of managed portfolios, σ_m is the standard deviation of market returns, υ_p is the residual standard deviation of returns of managed portfolios, $\tilde{\vartheta}$ and $\tilde{\eta}$ are Gaussian random variables, and Δt is the considered time increment. Each sample of 30 portfolios corresponds to 15 informed traders and 15 uninformed managers when setting the annual residual risk to .20%, 2.00% and 20.00%. The computation is based on 1,250 random series, equivalent to a 5-year return history or so, respecting the following market hypotheses: risk free rate 5.00% per year, market premium 12.00%, market standard deviation 20.00%. Simulations by the authors.

Panel C and D present a statistics summary of Spearman, Kendall, and Goodman-Kruskal correlation coefficients computed for 1,000 samples of 30 funds drawn from a large sample of simulated portfolios, when varying the first two sensitivities (with the average on the first line of Panel C - noted Av. Sensi.) and all four sensitivities (with the average on the first line of Panel D). Such implied sensitivities in Panel C and Panel D (displayed in the first lines), corresponding to coefficients used in the computation of our GUN measures, allow us to fairly replicate the portfolio rankings obtained with the six studied performance measures. We also present a statistics summary of the Spearman, Kendall, and Goodman-Kruskal rank correlation coefficients computed from a large sample of portfolios when only varying the first two sensitivities (Panel C) and all four sensitivities (Panel D).

Panel A: An illustration of *Iso* -GUN Rankings on a Sample of 30 Portfolios

HMβ		TMβ		MPPM2		MPPM3		MPPM4		MPPM5	
Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank
23.81%	1	15.90%	4	38.37%	2	32.72%	2	27.08%	2	21.51%	17
84.00%	2	13.86%	1	32.41%	17	28.77%	17	25.14%	17	21.43%	2
16.52%	3	11.85%	14	32.29%	14	28.42%	14	24.54%	14	20.66%	14
15.80%	4	7.74%	29	26.89%	1	22.13%	1	17.63%	10	13.99%	10
8.93%	5	6.57%	3	26.24%	3	21.27%	10	17.39%	1	12.73%	5
8.38%	6	5.92%	26	24.92%	10	20.74%	3	16.60%	5	12.66%	1
8.01%	7	4.68%	8	24.36%	5	20.48%	5	15.24%	3	11.00%	19
5.24%	8	4.21%	9	21.86%	19	18.23%	19	14.61%	19	10.40%	9
5.15%	9	3.76%	11	21.40%	6	17.28%	9	13.84%	9	9.73%	3
4.43%	10	3.24%	7	20.72%	9	16.60%	6	12.82%	16	9.32%	16
4.06%	11	3.19%	25	19.84%	16	16.33%	16	11.81%	6	7.04%	6
3.27%	12	2.76%	27	17.13%	30	12.55%	27	8.70%	27	4.85%	27
2.69%	13	2.52%	5	16.40%	27	12.35%	30	7.55%	30	2.75%	30
2.32%	14	2.20%	13	15.19%	12	10.40%	12	5.60%	12	.78%	12
2.19%	15	1.32%	10	14.36%	7	9.57%	7	4.77%	7	-.03%	7
1.81%	16	.92%	18	10.83%	15	7.16%	15	3.50%	15	-.16%	15
1.54%	17	.39%	21	9.91%	4	4.83%	8	1.53%	8	-1.77%	8
.69%	18	.16%	16	8.59%	25	4.81%	21	1.18%	21	-2.45%	21
.31%	19	-.36%	15	8.45%	21	4.52%	4	-.86%	4	-5.18%	22
.23%	20	-.45%	23	8.13%	8	3.43%	25	-1.43%	18	-5.19%	18
-.07%	21	-.81%	22	8.10%	29	3.01%	29	-1.59%	22	-6.24%	4
-.09%	22	-1.39%	24	7.34%	26	2.34%	18	-1.74%	25	-6.52%	13
-.45%	23	-1.79%	28	6.10%	18	2.28%	26	-2.10%	29	-6.92%	25
-1.37%	24	-1.82%	20	5.59%	22	2.00%	22	-2.56%	13	-7.23%	29
-2.04%	25	-2.89%	19	5.37%	13	1.41%	13	-2.80%	26	-7.89%	26
-2.37%	26	-4.51%	12	2.02%	20	-2.60%	20	-7.21%	20	-11.83%	20
-2.38%	27	-7.63%	30	.64%	11	-4.25%	11	-9.14%	11	-14.03%	11
-3.79%	28	-11.05%	6	-3.14%	28	-8.54%	28	-13.98%	28	-17.57%	23
-4.14%	29	-12.60%	2	-7.64%	23	-10.94%	23	-14.25%	23	-19.47%	28
-9.84%	30	-18.00%	17	-10.44%	24	-14.65%	24	-18.87%	24	-23.10%	24

Panel B: Statistics Summary of Ranking Correlations on 1,000 Samples of 30 Randomly Chosen Funds when only varying the First Two Sensitivities

Av. Sens.	HMβ (1.00, -.42)			TMβ (1.00, -.29)			MPPM2 (1.00, -1.00)			MPPM3 (1.00, -1.50)			MPPM4 (1.00, -2.00)			MPPM5 (1.00, -2.50)		
Correlation*	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ
The Sample	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Panel C: Statistics Summary of Ranking Correlations on 1,000 Samples of 30 Randomly Chosen Funds when varying the First Two Sensitivities

	HMβ			TMβ			MPPM2			MPPM3			MPPM4			MPPM5		
Av. Sens.	(1.00, -.18)			(1.00, -.32)			(1.00, -.98)			(1.00, -1.42)			(1.00, -2.08)			(1.00, -2.53)		
Correlation*	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ
Minimum	1.00	1.00	1.00	1.00	.85	.80	1.00	.99	.87	.97	.92	.83	.97	.88	.79	.93	.85	.72
First Quartile	1.00	1.00	1.00	1.00	1.00	.86	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.99	.86
Median	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Thrid Quartile	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Maximum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Panel D: Statistics Summary of Ranking Correlations on 1,000 Samples of 30 Randomly Chosen Funds when only varying All Four Sensitivities

	HMβ			TMβ			MPPM2			MPPM3			MPPM4			MPPM5		
Av. Sens.	(1.00, -.29, .14, -.02)			(1.00, -.30, .22, -.03)			(1.00, -.92, .04, -.03)			(1.00, -1.34, .03, -.03)			(1.00, -1.82, .05, -.04)			(1.00, -2.19, .05, -.04)		
Correlation*	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ
Minimum	1.00	1.00	1.00	1.00	.91	.89	1.00	1.00	.93	1.00	.93	.88	1.00	.92	.85	1.00	.98	.87
First Quartile	1.00	1.00	1.00	1.00	1.00	.91	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Median	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Thrid Quartile	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Maximum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Tab. 4. GUN Ranking Equivalence. These tables compare the ranking of simulated funds according to several performance measures (respectively, Henriksson and Merton, 1981; Treynor and Mazuy, 1966; Ingersoll *et al.,* 2007). Panel A displays the values and the ranking, for each studied measure, for a sample of 30 simulated portfolios. Panel B presents the sensitivities of the first two moments, used for the computation of the GUN measure, to replicate the ranking of the six other performance measures as well as the corresponding correlation coefficients (ρ: Spearman; τ : Kendall; γ: Goodman and Kruskal). Panel C and Panel D present, respectively, the average first two and the average first four sensitivities associated with moments, and a statistics summary of ranking correlations (minimum, first quartile, median, third quartile, maximum) on 1,000 samples of 30 randomly chosen funds. Similar to Table 2 and to Ingersoll *et al.* (2007), market portfolio return distributions are defined as:

$$\tilde{r}_m = exp\left\{(\mu_m + \tilde{s} - .50\sigma_m^2)\Delta t + (\sigma_m\tilde{\vartheta})\left[(1-\delta_p^2)\Delta t\right]^{1/2}\right\} - 1$$

where $\tilde{r}_m$ are the market returns, μ_m is the mean market rate of return, $\tilde{s}$ is the signal, σ_m is the standard deviation of market returns, δ_p is the information known by the market timer about the market variance, $\tilde{\vartheta}$ is a Gaussian random variable, and Δt is the considered time increment. The market's unconditional expected rate of return and logarithmic variance *per* unit time are μ_m and σ_m^2, respectively. The information about the changing mean is in the signal $\tilde{s}$, which is normally distributed with mean zero and variance $\delta_p^2\sigma_m^2\Delta t$, where δ_p is the fraction of variation known by the informed trader. In the *m* simulations, δ_p is set to .10% and 1.00%. The optimal market holding conditional on a signal $\tilde{s}$ is equal to the conditional risk *premium* divided by the relative risk aversion *a* (set to 3) times the conditional variance, such as:

$$(\mu_m + \tilde{s} - r)\left[a(1-\delta_p^2)\sigma_m^2\right]^{-1}.$$

Since the unconditional risk *premium* is equal to the relative risk aversion times the unconditional variance, the optimal leverage conditional on signal $\tilde{s}$ is:

$$[1+\tilde{s}(a)^{-1}](1-\delta_p^2)^{-1}.$$

Each sample of 30 portfolios corresponds to 15 informed market timers and 15 random managers as defined in Table 2. The computation is based on 1,250 random series, equivalent to a 5-year or so return history, respecting the following market hypotheses: risk free rate 5.00% per year, market premium 12.00%, market standard deviation 20.00%. Simulations by the authors.

Indeed, this second illustration of the flexibility of our measure also shows that: 1) it is possible to reproduce the rankings obtained according to the Henriksson-Merton (1981) and Treynor-Mazuy (1966) measures (for both versions), when varying the first two moment sensitivities, and 2) to reveal explicitly the preferences of the investors when they select some particular ranking.

In this section, we have shown, in a standard simulation framework, that it is possible to replicate the rankings of some main performance measure with the GUN. In the following, we show that we can obtain the same results when we use market series, *i.e.* the dataset of hedge funds partially described in Darolles *et al.* (2009), with return distributions that are not normal.

4.2. Empirical Results with Real Market Data

We hereafter compare the rankings of hedge funds according to several performance measures, computed on funds in the database presented in Darolles *et al.* (2009). NAV are expressed in US dollars on a monthly basis from June 2004 to July 2007. These hedge funds are self-declared in one or several style categories. These categories describe the type of assets ("Currencies", "Distressed Securities"), the type of management ("Global Macro", "Merger Arbitrage"), or both ("Fixed Income Arbitrage", "Equity Long/Short"). Most of the hedge funds belong to the following pure categories: "Equity Long/Short", "Fixed Income", "Global Macro", "Currency", "Futures", "Equity Long/Short Equally Weighted", "Fixed Income Arbitrage", "Merger Arbitrage" and "Distressed Securities". We have here in the sample 30 hedge funds that represent the main styles and report the information on the management companies, the strategies and the assets under management in the next Table 5.

On the model of previous tables, Table 6 reports performance measure scores and rankings of a sample of 30 hedge funds according to the Sharpe (1966) ratio, the Jensen (1968) *alpha* and four variants of the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM. Table 7 presents the same results according to the Henriksson-Merton (1981), the Treynor-Mazuy (1966), two variants of the Darolles-Gouriéroux-Jasiak (2009) L-performance, the Morningstar (2002) RAR and the Keating-Shadwick (2002) *Omega* measure.

As previously, implied sensitivities reported in these tables are coefficients that make equal (or the closest) the ranking obtained with the GUN and the six studied measures. Panels A and B in both tables provide results for the specific sample of hedge funds under study, whilst Panel C and D present outputs for 1,000 draws of samples of 30 funds randomly chosen in a larger HFR database containing 333 hedge funds.

As anticipated, the implied (normalized) sensitivities in Table 6 are in the same ranges for the second, third and fourth-order preferences (from -.57 for the Jensen *alpha* risk aversion coefficient to -2.24 for the MPPM5 - Panel C; from -.63 for the Jensen alpha to -2.42 for MPPM5 - Panel D), with, as previously, small coefficients corresponding to the third and fourth sensitivities. We also observe that, for the four variants of the MPPM measure, the higher the agent's risk aversion coefficient is, the lower his sensitivity to the second moment. Correlation coefficients are also very high in this Table.

Fund#	Fund Name	Style	Company Name
1	Exane Investors Gulliver Fund	Equity Hedge	Exane Structured Asset Management
2	Ibis Capital, LP	Equity Hedge	Ibis Management, LLC
3	Odey European Inc.	Equity Hedge	Odey Asset Management Limited
4	Platinum Fund Ltd.	Equity Hedge	Optima Fund Management
5	Permal U.S. Opportunities Ltd.	Equity Hedge	Permal Investment Management Services Ltd
6	RAB Europe Fund	Equity Hedge	RAB Capital PLC
7	Pioneer Long Short European Equity	Equity Hedge	Pioneer Asset Management
8	Emerging Value Opportunities Fund Ltd.	Equity Hedge	Value Line, Inc
9	Robbins Capital Partners, L.P.	Equity Hedge	T. Robbins Capital Management, LLC
10	Invesco QLS Equity	Equity Market Neutral	Invesco Structured Products Group
11	Thames River European Fund	Equity Non-Hedge	Thames River Capital LLP
12	Craigmillar Partners L.P.	Equity Non-Hedge	Craigmillar Ltd.
13	SSI Long/Short Equity Market Neutral L.P.	Long/Short	SSI Investment Management, Inc.
14	Friedberg Global Macro Hedge Fund Ltd.	Macro	Friedberg Mercantile Group Ltd.
15	Sunrise Capital Diversified, Ltd.	Macro	Sunrise Capital Partners
16	FX Concepts Global Currency Program	Macro	FX Concepts, Inc.
17	Haidar Jupiter International Ltd.	Macro	Haidar Capital Management, LLC
18	GLC Directional Fund, Ltd.	Macro	Glc Directional Fund, L.P.
19	R.G. Niederhoffer Diversified Fund II, Ltd.	Macro	R.G. Niederhoffer Diversified Fund II, Ltd.
20	QM Premier Fund USD Share Class	Macro	QM Premier Fund USD Share Class
21	Alternative Treasury Strategy, LLC	Macro	Alternative Treasury Strategy, LLC
22	Forest Multi Strategy Fund LLC	Relative Value Arbitrage	Forest LLC
23	Aristeia International, Ltd.	Convertible Arbitrage	Aristeia Capital LLC
24	Paulson International Ltd.	Merger Arbitrage	Paulson & Co., Inc.
25	Schultze Offshore Fund, Ltd.	Event-driven	Schultze Offshore Ltd.
26	York European Opportunities Fund, L.P.	Event-driven	York European Opportunities L.P
27	Lion Fund Limited	Event-driven	Lion Fund Limited
28	Fletcher Income Arbitrage Fund, Ltd.	Fixed Income Arbitrage	Fletcher Income Arbitrage Fund, Ltd.
29	Coast Arbitrage Fund II, Ltd.	Fixed Income Arbitrage	Coast Arbitrage Fund II, Ltd.
30	Global Distressed Fund	Distressed	Global Investment House

Tab. 5. Hedge Funds classified by Styles. Source: *HFR*; this table presents the fund names, the styles and the company names of several hedge funds. Among the 30 hedge funds reported, 18 are also present in the database used in Darolles *et al.* (2009). We complete our dataset in order to have the same style categories as those listed in Darolles *et al.* (2009).

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Panel A: An illustration of *Iso* -GUN Rankings on a Sample of 30 Hedge Funds

HMβ		TMβ		MPPM2		MPPM3		MPPM4		MPPM5	
Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank
1.90	1	37.85%	1	34.41%	1	31.08%	1	27.83%	1	24.63%	1
1.82	2	17.23%	6	19.90%	6	18.50%	6	17.13%	6	16.22%	3
1.69	3	17.16%	3	18.75%	3	17.88%	3	17.04%	3	15.80%	6
1.57	4	14.12%	2	17.17%	2	16.56%	2	15.96%	2	15.36%	2
1.55	5	12.74%	7	16.18%	7	15.27%	7	14.36%	7	13.46%	7
1.49	6	11.45%	4	14.39%	4	13.78%	4	13.16%	4	12.55%	4
1.47	7	10.69%	5	13.93%	5	13.34%	5	12.75%	5	12.16%	5
1.44	8	9.13%	10	11.36%	8	10.93%	8	10.52%	8	10.10%	8
1.36	9	8.09%	12	11.00%	12	10.41%	10	9.87%	10	9.34%	10
1.23	10	7.78%	8	10.95%	11	10.24%	11	9.52%	11	8.80%	11
1.12	11	6.30%	9	10.95%	10	10.23%	12	9.45%	12	8.66%	12
1.11	12	6.20%	11	9.81%	13	9.10%	13	8.43%	9	8.15%	9
1.03	13	5.71%	16	9.67%	14	9.00%	14	8.39%	13	7.69%	14
1.01	14	5.56%	14	9.63%	16	8.87%	16	8.34%	14	7.67%	13
.99	15	4.99%	13	9.00%	9	8.71%	9	8.12%	16	7.37%	16
.98	16	3.57%	15	6.63%	15	6.33%	15	6.02%	15	5.72%	15
.80	17	2.79%	20	5.98%	24	5.21%	20	4.79%	20	4.36%	20
.79	18	2.44%	25	5.91%	23	5.18%	23	4.61%	17	4.31%	17
.75	19	2.05%	24	5.81%	25	5.14%	24	4.45%	23	4.11%	18
.74	20	2.01%	22	5.63%	20	4.90%	17	4.37%	18	3.94%	19
.68	21	1.60%	17	5.20%	17	4.79%	25	4.31%	24	3.71%	23
.68	22	1.53%	18	4.90%	18	4.64%	18	4.22%	19	3.50%	22
.66	23	1.43%	23	4.78%	19	4.50%	19	3.81%	22	3.46%	24
.63	24	1.27%	21	4.42%	22	4.11%	22	3.76%	25	3.44%	21
.59	25	1.21%	19	4.31%	21	4.02%	21	3.73%	21	2.73%	25
.37	26	-.94%	29	2.57%	26	2.00%	26	1.43%	26	.85%	26
.16	27	-2.28%	26	.61%	27	.30%	27	.00%	27	-.31%	27
.09	28	-2.86%	27	-.78%	28	-1.78%	28	-2.77%	28	-3.76%	28
.03	29	-5.56%	28	-1.85%	29	-3.00%	29	-4.14%	29	-5.27%	29
-.07	30	-10.11%	30	-5.23%	30	-7.16%	30	-9.10%	30	-11.05%	30

Panel B: Statistics Summary of Ranking Correlations on 1,000 Samples of 30 Randomly Chosen Hedge Funds when only varying the First Two Sensitivities

	HMβ			TMβ			MPPM2			MPPM3			MPPM4			MPPM5		
Av. Sens.	(1.00, -.78)			(1.00, -.48)			(1.00, -1.00)			(1.00, -1.65)			(1.00, -2.26)			(1.00, -2.67)		
Correlation*	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ
The Sample	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Panel C: Statistics Summary of Ranking Correlations on 1,000 Samples of 30 Randomly Chosen Hedge Funds

when varying the First Two Sensitivities

Av. Sens.	HMβ (1.00, -.94)			TMβ (1.00, -.57)			MPPM2 (1.00, -.97)			MPPM3 (1.00, -1.48)			MPPM4 (1.00, -1.80)			MPPM5 (1.00, -2.24)		
Correlation*	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ
Minimum	.95	.81	.70	.94	.83	.75	1.00	.98	.87	1.00	.98	.80	.99	.94	.80	.99	.95	.87
First Quartile	.97	.87	.80	.97	.89	.82	1.00	.99	.87	1.00	.99	.87	1.00	.97	.87	1.00	1.00	1.00
Median	1.00	.95	.93	1.00	.95	.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Thrid Quartile	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Maximum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Panel D: Statistics Summary of Ranking Correlations on 1,000 Samples of 30 Randomly Chosen Funds

when only varying All Four Sensitivities

Av. Sens.	HMβ (1.00, -1.06, .03, -.03)			TMβ (1.00, -.63, .42, -.03)			MPPM2 (1.00, -.99, .04, -.04)			MPPM3 (1.00, -1.41, .05, -.03)			MPPM4 (1.00, -1.98, .17, -.05)			MPPM5 (1.00, -2.42, .05, -.11)		
Correlation*	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ
Minimum	.98	.89	.75	.97	.87	.80	1.00	1.00	.87	1.00	.99	.90	1.00	.98	.83	1.00	.98	.90
First Quartile	.98	.90	.80	.98	.89	.86	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.95	1.00	1.00	1.00
Median	1.00	1.00	.98	.98	.92	.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Thrid Quartile	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Maximum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Tab. 6. *Iso*-GUN Rankings on Hedge Funds. Source: HFR; monthly quotes in US dollars from June 2004 to July 2007. These tables compare the ranking of hedge funds obtained according to the Henriksson-Merton (1981) and Treynor-Mazuy (1966) (first) measures and four variations of the MPPM (Ingersoll *et al.*, 2007). Panel A displays the values and the ranking, for each studied measure, for a sample of 30 hedge funds. Panel B presents the sensitivities of the first two moments, used for the computation of the GUN measure, to replicate the ranking of the six other performance measures as well as the corresponding correlation coefficients (ρ: Spearman; τ: Kendall; γ: Goodman and Kruskal). Panel C and Panel D present, respectively, the average first two and the average first four sensitivities associated with moments, and a statistics summary of ranking correlations (minimum, first quartile, median, third quartile, maximum) on 1,000 samples of 30 randomly chosen portfolios among 333 hedge funds.

In Table 7, the results for the Henriksson-Merton (1981), the Treynor-Mazuy (1966), the Morningstar (2002) MRAR and the Keating-Shadwick (2002) *Omega*, are similar to the previous ones (same range for the sensitivities, similar small higher order sensitivities, same high correlations with a median correlation coefficient close to 1...), although the results in terms of correlations are slightly lower.

We also have to note that the Darolles *et al.* (2009) L_1 and L_3 measures give different results with comparatively high coefficients on the third and fourth moments (specifically for the L_3 measure), and lower global correlation coefficients (with an average between .80 and .88 for the different correlation coefficients - Panel D) which signal some differences between measures and the various rankings.

Nevertheless, as in the previous examples (*Cf.* Tables 3 and 4), Tables 6 and 7 also globally show that we can find very similar rankings based on traditional measures or on appropriate GUN measures, when using, this time, market data with asset returns that are non-Gaussian.

Panel A: An illustration of *Iso*-GUN Rankings on a Sample of 30 Hedge Funds

HMV		TMV		L1		L2		MRAR		*Omega*	
Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank
41.53%	1	30.17%	1	25.68%	18	33.50%	18	36.46%	2	3.55	18
20.84%	2	17.10%	2	25.43%	10	33.30%	10	20.32%	3	3.42	8
17.44%	3	13.71%	4	24.85%	8	32.83%	12	19.58%	4	3.21	10
13.70%	4	11.75%	3	23.64%	12	31.31%	8	18.01%	5	2.75	4
12.47%	5	10.33%	6	23.37%	13	31.01%	13	16.50%	6	2.67	13
11.32%	6	9.92%	5	22.17%	4	30.40%	30	14.77%	11	2.66	12
11.17%	7	9.88%	8	22.17%	2	30.30%	11	14.27%	7	2.64	2
9.16%	8	7.75%	7	21.79%	11	28.60%	19	11.56%	10	2.61	5
7.46%	9	6.67%	14	21.53%	19	27.76%	3	10.97%	8	2.59	11
6.33%	10	5.61%	9	20.51%	3	27.06%	20	10.78%	9	2.40	6
6.15%	11	5.45%	10	19.90%	5	26.93%	2	10.77%	12	2.27	3
5.94%	12	5.27%	13	19.44%	20	24.83%	4	9.53%	19	2.14	19
5.80%	13	5.19%	12	19.28%	17	24.81%	16	9.42%	13	2.05	7
5.80%	14	5.10%	15	18.85%	9	24.43%	9	9.28%	20	1.95	9
5.13%	15	4.56%	16	18.48%	16	24.13%	5	9.10%	15	1.91	17
4.76%	16	4.13%	18	18.37%	30	23.34%	17	6.53%	18	1.87	20
3.66%	17	3.42%	17	17.49%	6	22.91%	22	5.35%	21	1.72	15
2.87%	18	2.43%	11	17.11%	22	20.79%	24	5.32%	23	1.70	16
2.23%	19	1.83%	19	17.10%	7	19.70%	27	5.28%	24	1.66	23
1.94%	20	1.76%	20	16.61%	24	19.33%	7	5.02%	22	1.66	22
1.82%	21	1.69%	22	16.61%	15	18.47%	6	4.91%	14	1.57	21
1.37%	22	1.55%	23	14.90%	27	18.14%	25	4.75%	17	1.56	24
1.32%	23	1.33%	21	14.39%	25	17.27%	28	4.60%	16	1.56	1
1.11%	24	1.05%	28	13.21%	26	17.21%	26	4.20%	26	1.55	14
1.11%	25	.88%	30	12.15%	28	16.80%	29	4.11%	25	1.52	26
.89%	26	.47%	24	12.10%	29	15.38%	15	2.02%	29	1.27	30
-2.84%	27	-2.42%	25	9.89%	23	13.55%	14	.30%	30	1.12	29
-2.91%	28	-2.71%	29	3.33%	14	4.00%	23	-1.76%	27	1.06	25
-7.11%	29	-6.53%	26	.68%	21	.53%	21	-2.95%	28	1.02	27
-11.66%	30	-11.18%	27	-.19%	1	-1.04%	1	-6.91%	1	0.95	28

Panel B: Statistics Summary of Ranking Correlations on 1,000 Samples of 30 Randomly Chosen Hedge Funds when only varying the First Two Sensitivities

Av. Sens.	HMV (1.00, -.83)			TMV (1.00, -.95)			L1 (1.00, -1.32)			L2 (1.00, -.99)			MRAR (1.00, -1.38)			*Omega* (1.00, -.32)		
Correlation*	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ
The Sample	.91	.89	.81	.88	.83	.72	.81	.79	.73	.90	.84	.78	1.00	1.00	1.00	1.00	1.00	1.00

Panel C: Statistics Summary of Ranking Correlations on 1,000 Samples of 30 Randomly Chosen Hedge Funds when varying the First Two Sensitivities

	HMV			TMV			L1			L2			MRAR			*Omega*		
Av. Sens.	(1,.00 -.84)			(1.00, -.91)			(1.00, -1.23)			(1.00, -.94)			(1.00, -1.45)			(1.00, -.28)		
Correlation*	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ
Minimum	.83	.70	.65	.83	.70	.67	.70	.68	.65	.71	.69	.65	.99	.95	.76	.88	.80	.76
First Quartile	.89	.75	.71	.88	.73	.71	.78	.69	.71	.74	.71	.68	1.00	.99	.84	1.00	.99	.84
Median	.91	.81	.75	.91	.84	.75	.80	.73	.75	.81	.78	.76	1.00	1.00	1.00	1.00	1.00	1.00
Thrid Quartile	.94	.92	.82	.93	.79	.82	.83	.79	.80	.88	.81	.79	1.00	1.00	1.00	1.00	1.00	1.00
Maximum	.98	.94	.87	.97	.90	.85	.93	.85	.82	.94	.90	.88	1.00	1.00	1.00	1.00	1.00	1.00

Panel D: Statistics Summary of Ranking Correlations on 1,000 Samples of 30 Randomly Chosen Funds when only varying All Four Sensitivities

	HMV			TMV			L1			L2			MRAR			*Omega*		
Av. Sens.	(1.00, -1.28, .36, -.03)			(1.00, -.75, .56, -.01)			(1.00, -.84, 1.72, -2.10)			(1.00, -3.52, 10.00, -6.39)			(1.00, -1.40, .01, -.03)			(1.00, -.19, .02, -.08)		
Correlation*	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ	ρ	τ	γ
Minimum	.88	.84	.75	.93	.80	.75	.77	.73	.70	.73	.70	.67	1.00	.98	.71	.90	.88	.82
First Quartile	.96	.87	.80	.94	.83	.78	.80	.75	.71	.82	.80	.78	1.00	1.00	.92	1.00	1.00	.99
Median	1.00	1.00	1.00	.96	.87	.80	.85	.82	.80	.88	.86	.80	1.00	1.00	1.00	1.00	1.00	1.00
Thrid Quartile	1.00	1.00	1.00	.97	.89	.84	.88	.85	.84	.92	.88	.84	1.00	1.00	1.00	1.00	1.00	1.00
Maximum	1.00	1.00	1.00	1.00	.95	.90	.95	.90	.89	.98	.94	.90	1.00	1.00	1.00	1.00	1.00	1.00

Tab. 7. *Iso*-GUN Rankings on Hedge Funds (*bis*). Source: HFR; monthly quotes in US dollars from June 2004 to July 2007. These tables compare the ranking of hedge funds obtained according to the Henriksson-Merton (1981) and Treynor-Mazuy (1966) (first) measures, two variants of the L-performance (Darolles *et al.*, 2009), the MRAR (Morningstar, 2002) and the *Omega (*Keating and Shadwick, 2002). Panel A displays the values and the ranking, for each studied measure, for a sample of 30 hedge funds. Panel B presents the sensitivities of the first two moments, used for the computation of the GUN measure, to replicate the ranking of the six other performance measures as well as the corresponding correlation coefficients (ρ: Spearman; τ: Kendall; γ: Goodman and Kruskal). Panel C and Panel D present, respectively, the average first two and the average first four sensitivities associated with moments, and a statistics summary of ranking correlations (minimum, first quartile, median, third quartile, maximum) on 1,000 samples of 30 randomly chosen portfolios among 333 hedge funds.

5. Conclusion, Discussion and Future Researches

Portfolio performance measurement is a topic of interest within both academic and practitioner communities, as well as financial authorities. Funds are generally ranked according to different *criteria* by investment banks and financial advisors. Such published rankings can have a significant impact on investors' flows and thus, finally, on allocation decisions of fund managers.

Numerous measures have been proposed to evaluate the performance of an active management. However, a reasonable concern among those who use a particular measure is whether or not the manager being evaluated might react by attempting to manipulate it. If most of the performance measures can be manipulated, can we then find one that is hardly gameable by portfolio managers?

In this article, we propose the GUN measure of performance, built as a generalization of the Sharpe (1966) ratio. It is founded on an extension of the Mean-Variance analysis in a first four moment framework. This measure is flexible in the first four sensitivities in the utility function of the investor applied to the mean, the variance, the skewness and the *kurtosis*. The first main objective of this adjustment is to be able to adapt the proposed measure of performance for each category of individuals, by taking into account their preferences and risk profiles. For instance, the sensitivities applied to the first four moments of the GUN measure will be different for an individual who mainly wants to maximize the excess return, another one who prefers to minimize his portfolio risk, or the last one who essentially considers a constraint on the fourth moment adopting a “safety first” behavior. This is the main point for justifying the proposed measure of performance. Secondly, our measure, attempting to better characterize both the investor (*via* a set of sensitivities) and the entire distribution of returns (through the *n*-th moments), leaves indeed very little room for gaming the ranking. The key driver of this feature is that the GUN measure allows us to control for several dimensions of the returns on the portfolio and, therefore, it is difficult to game all of these dimensions contemporaneously.

Finally, we briefly introduce some further potential applications of the GUN measure for future research. The first one concerns the building of a Fraudulent Behavior Index for detecting potential fraudsters (or anomalies). The underlying idea is to compute several ranking with the GUN, when varying the sensitivities of investors in the *formula* of the measure and to point a set of best funds. It should be very unlikely that a given fund would be well ranked whatever the GUN measures, and thus this could serve to show the need to further investigate the behavior of a fund (see Bernard and Boyle, 2009, and, Bernard *et al.*, 2019, for more details). A second use might be in the information content of the investors’ fund rankings obtained with the GUN measure to characterize their investment behaviors, and thus, recover their preferences and individual attitudes towards risk. Revealed preference theory shows how to construct utility functions from price and choice observations (Samuelson, 1938 and 1948; Little, 1949; Houthakker, 1950; Afriat, 1967). A growing literature has made progress in developing empirical measures of individual risk attitudes, with the aim of capturing this crucial component of individual heterogeneity (see, *e.g.*, Bruhin *et al.*, 2010) as well as the time-varying property of risk attitudes (*e.g.*, Misina, 2003; Coudert and Gex, 2008). A third operational future work could be in the use of the GUN as a marketing tool. Based on a similar idea to the one leading the Fraudulent Behavioral Index for detecting frauds, we can also address the maximum of relevant clients, targeting with one specific product the group of investors for which the fund will be considered a good “choice” (in the top 10% of their preferred funds, for example). The GUN measure would thus aim to identify the specific category of agents that will perfectly fit, for instance, the objective of a new investment strategy (see De Palma and Prigent, 2008).

Finally and lastly, an intensive focus on measures taking into account some extreme risk measures (such as Value-at-Risk and Expected Shortfall) might be of interest in the post-COVID2019 period, marked by such very exceptional and dramatic systemic events.

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