Applied Categorical Structures manuscript No.
(will be inserted by the editor)

Relative cohomology of algebraic theories

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Abstract We construct relative abelian categories in the sense of MacLane for models of algebraic systems in (co)complete abelian categories. As an example, we consider an analogue of Hochschild-Mitchell cohomology for the functor of Yoneda embedding.

Keywords algebraic theories · abelian categories · functor categories · Hochschild-Mitchell cohomology

Mathematics Subject Classification (2010) 18C10 · 18E10 · 18A25

1 Introduction

Recently, cohomology of algebraic systems was studied using the methods of algebraic topology [17], André-Quillen cohomology [12,4], Baues-Wirsching cohomology of small categories [12], cotriple cohomology associated with a neglecting funtor [11] and ordinary homological algebra [11,12]. Also in [12, 11] some of these approaches were compared. In the present paper we propose the relative homological algebra approach to the cohomology of multi-sorted algbraic systems. We identify the category of models of an algebraic system in a given abelian category $\mathcal{A}$ as the category of functors from an appropriate small category to $\mathcal{A}$. Then we construct a relative abelian category in the sense of [14] starting from the same neglecting functor as in [11], and we study some properties of the associated cohomology theory. Also we establish a connection between this relative cohomology and the "absolute" one, thus generalizing results of [11].

After some categorical preliminaries in Sec. 2, we consider resolvents in relative pair of abelian categories in the sense of [14] for models of algebraic

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theories in abelian categories in Sec. 3. As an example, in Sec. 4 we construct an analogue of Hochschild-Mitchell resolvent for small categoies [16,15,7]. This reduces the cohomology of algebraic theories to the one of locally finite (dimensional) categories considered by many authors [18,10,6]. For other approaches to cohomology of algebraic theories see [17,12,4].

2 Categorical preliminaries

We fix some notations from category theory (see [13,2] for details). We denote objects of $\mathcal{C}$ by $c, c', d, \ldots$, morphisms of $\mathcal{C}$ by $f, g, \ldots$, functors from $\mathcal{C}$ to $\mathcal{A}$ by $K, T, S, \ldots$ and natural transformations between these functors by $\varphi, \varkappa, \ldots$. The last term means that for any object c of $\mathcal{C}$ there exists a morphism φ_c in $\mathcal{A}$ such that the following diagram

$$\begin{tikzcd} T(c) \arrow[r, "\varphi_c"] \arrow[d, "T(f)"'] & S(c) \arrow[d, "S(f)"] \\ T(d) \arrow[r, "\varphi_d"'] & S(d) \end{tikzcd} \tag{1}$$

commutes for any morphism $f : \ c \to d$ in $\mathcal{C}$. We write $\varphi : \ T \rightharpoonup S$ for the above natural transformation, and morphisms φ_c are called components of φ.

We write $\begin{tikzcd} \bullet \arrow[r, "f"'] \arrow[rr, bend left, "fg"] & \bullet \arrow[r, "g"'] & \bullet \end{tikzcd}$ for composition of two morphisms in $\mathcal{C}$.

We say that "a" diagram of product of two objects exists in $\mathcal{C}$ if (a) the following diagram

$$c_1 \xleftarrow{\mathrm{pr}_1} c_1 \times c_2 \xrightarrow{\mathrm{pr}_2} c_2$$

exists in $\mathcal{C}$ and (b) this diagram is universal. We say that the category has binary products if the above diagram exists for any pair of objects. It is well known that $\times$ is a bifunctor, so for any pair (f_1, f_2) of morphisms in $\mathcal{C}$ there exist "a" morphism $f_1 \times f_2$ in $\mathcal{C}$ such that the diagram

$$\begin{tikzcd} c_1 \arrow[r, "f_1"] & d_1 \\ c_1 \times c_2 \arrow[u, "\mathrm{pr}_1"] \arrow[r, "f_1 \times f_2"] \arrow[d, "\mathrm{pr}_2"] & d_1 \times d_2 \arrow[u, "\mathrm{pr}_1"'] \arrow[d, "\mathrm{pr}_1"] \\ c_2 \arrow[r, "f_2"] & d_2 \end{tikzcd} \tag{2}$$

commutes.

Suppose a category has binary product and "nullary" product (i.e. terminal object), then we say that it has finite products. Suppose both $\mathcal{C}$ and $\mathcal{A}$ have finite products, then we say that a functor $F \in \mathrm{Funct}(\mathcal{C}, \mathcal{A})$ is product preserving if it takes diagrams of finite products in $\mathcal{C}$ to those in $\mathcal{A}$. It is easily seen that product preserving functors and natural transformations between them form a full subcategory $\mathrm{FPFunct}(\mathcal{C},\mathcal{A})$ in $\mathrm{Funct}(\mathcal{C},\mathcal{A})$. Let $\varphi: T \rightharpoonup S$ be a natural transformation between two product-preserving functors, then comparing (1) and (2) we obtain

$$\varphi_{c_1 \times c_2} \simeq \varphi_{c_1} \times \varphi_{c_2}. \tag{3}$$

We call a category preadditive if it is enriched in **Ab** (some authors use the term "additive" in this case). We say that a functor is preadditive if it is an **Ab**-functor in the sense of enriched category theory, and we retain the notations $\mathrm{Funct}(\mathcal{C},\mathcal{A})$ and $\mathrm{FPFunct}(\mathcal{C},\mathcal{A})$ for categories of preadditive and preadditive product-preserving functors respectively. If a preadditive category has a terminal object, then it is also a zero one, and we denote the category of normalized (i.e. zero object-preserving) functors by $\mathrm{Funct}_{\mathrm{N}}(\mathcal{C},\mathcal{A})$.

Let $\mathcal{A}$ be a preadditive category, then a diagram of the form

$$\begin{tikzcd} a_1 \arrow[r, shift left, "\iota_1"] & a_1 \oplus a_2 \arrow[l, shift left, "\pi_1"] \arrow[r, shift right, "\pi_2"'] & a_2 \arrow[l, shift right, "\iota_2"'] \end{tikzcd} \tag{4}$$

is called diagram of direct sum if the equalities

$$\pi_1\iota_1 + \pi_2\iota_2 = 1_{c_1 \oplus c_2}, \qquad \iota_k\pi_l = \begin{cases} 1_{c_k}, & k = l \\ 0, & k \neq l \end{cases}$$

are satisfied. It is well known that a diagram of direct sum is universal whenever it exists, so it defines a diagram of products in $\mathcal{A}$. Conversely, any diagram of product in preadditive category defines the one of direct sum, and preadditive category which has direct sums of any pair objects and has "a" zero object is called additive category, so an additive category always has finite products.

Let $\mathcal{C}$ be category with finite products and let $\mathcal{A}$ be an additive category. Define objectwise direct sum $F \oplus G$ of two product preserving functors F, G from $\mathcal{C}$ to $\mathcal{A}$, then considering an iterated direct sum in $\mathcal{A}$ we see that $F \oplus G$ is in turn a product-preserving functor, so $\mathrm{FPFunct}(\mathcal{C},\mathcal{A})$ is an additive category.

Let $\mathcal{C}$ be a category, denote by $\mathbb{Z}\mathcal{C}$ a preadditive category with $Ob\,\mathbb{Z}\mathcal{C} = Ob\,\mathcal{C}$ and $Mor\,\mathbb{Z}\mathcal{C} = \mathbb{Z}\,Mor\,\mathcal{C}$ where $\mathbb{Z}$ is the functor of free abelian group, then we have an identification

$$\mathrm{Funct}(\mathcal{C},\mathcal{A}) \cong \mathrm{Funct}(\mathbb{Z}\mathcal{C},\mathcal{A}) \tag{5}$$

provided $\mathcal{A}$ is a preadditive category, so we can consider $\mathrm{FPFunct}(\mathcal{C},\mathcal{A})$ as a subcategory of $\mathrm{Funct}(\mathcal{C},\mathcal{A})$ provided $\mathcal{C}$ has finite products and $\mathcal{A}$ is additive, so we can use the theory of preadditive functors developed in [15].

3 Relative abelian categories

An additive category $\mathcal{A}$ is called abelian if it obeys the following three axioms:

(Abel-1) $\ker f$ and $\operatorname{coker} f$ are nonempty for any morphism f of $\mathcal{A}$.
(Abel-2) Suppose f is a monomorphism and g is an epimorphism, then $f \in \ker g$ iff $g \in \operatorname{coker} f$.
(Abel-3) We can decompose any morphism in $\mathcal{A}$ into the epimorphism followed by monomorphism.

It is well known that the decomposition provided by (Abel-3) is functorial.

Let T, S be product preserving functors from $\mathcal{C}$ to $\mathcal{A}$ and let $\varphi :\ T \rightharpoonup S$ be a natural transformation. Like the case of ordinary functors [9,14], for any object c of $\mathcal{C}$ we can choose a morphism $\varkappa_c :\ K(c) \to C(c)$ in $\mathcal{A}$ such that $\varkappa_c \in \ker \varphi_c$. Like the case of ordinary functors, it follows that an exact sequence

$$0 \longrightarrow K(c) \xrightarrow{\varkappa_c} T(c) \xrightarrow{\varphi_c} S(c)$$

is functorial, so K is a functor and $\varkappa_c$ is a component of natural transformation $\varkappa :\ K \rightharpoonup T$. Then the diagram of direct sum (4) yields a diagram in $\mathcal{A}$

$$\begin{tikzcd}
0 \arrow[r] & K(c_2) \arrow[r, "\varkappa_{c_2}"] \arrow[d, shift left, "K(\iota_2)"] & T(c_2) \arrow[r, "\varphi_{c_2}"] \arrow[d, shift left, "T(\iota_2)"] & S(c_2) \arrow[d, shift left, "S(\iota_2)"] \\
0 \arrow[r] & K(c_1) \oplus K(c_2) \arrow[r, "\varkappa_{c_1 \times c_2}"] \arrow[u, shift left, "K(\pi_2)"] \arrow[d, shift right, "K(\pi_1)"'] & T(c_1) \oplus T(c_2) \arrow[r, "\varphi_{c_1 \times c_2}"] \arrow[u, shift left, "T(\pi_2)"] \arrow[d, shift right, "T(\pi_1)"'] & S(c_1) \oplus S(c_2) \arrow[u, shift left, "S(\pi_2)"] \arrow[d, shift right, "S(\pi_1)"'] \\
0 \arrow[r] & K(c_1) \arrow[r, "\varkappa_{c_1}"] \arrow[u, shift right, "K(\iota_1)"'] & T(c_1) \arrow[r, "\varphi_{c_1}"] \arrow[u, shift right, "T(\iota_1)"'] & S(c_1) \arrow[u, shift right, "S(\iota_1)"']
\end{tikzcd}$$

with exact rows and obvious commutativity properties. Since $\varkappa_c$ is a monomorphism, we see that left column defines a diagram of direct sum, so K is again a product preserving functor.

Let λ be a natural transformation such that $\lambda\varphi = 0$, then for any object c of $\mathcal{C}$ there exists a morphism λ'_c in $\mathcal{A}$ such that $\lambda_c = \lambda'_c \varkappa_c$ with $\varkappa$ constructed above. Since $\varkappa_c$ is a monomorphism, it can be easily proved that the above decomposition of λ_c is functorial like the case of ordinary functors, so λ'_c are components of a natural transformation. In other words, $\varkappa \in \ker \varphi$; dual statement may be proved analogously. This verifies axioms (Abel-1) and (Abel-2) in $\mathrm{FPFunct}(\mathcal{C}, \mathcal{A})$. Axiom (Abel-3) may be verified along the same lines as (Abel-1) was, so we have proved the following theorem.

Theorem 1 *Let $\mathcal{C}$ be a small additive category and let $\mathcal{A}$ be an abelian category, then the category $\mathrm{FPFunct}(\mathcal{C}, \mathcal{A})$ of product preserving functors from $\mathcal{C}$ to $\mathcal{A}$ is an abelian category with kernels and cokernels being defined objectwise.*

Let S be a set and let S^* be "the" set of words generated by alphabet S (including an empty word), then S^* is a category with obvious finite products. Let $\mathbf{T}$ be an equationally defined algebraic theory of signature S and let $\mathrm{Alg}(\mathbf{T},\mathcal{A})$ be the category of models of $\mathbf{T}$ in a category $\mathcal{A}$ with finite products. Then it is well known that there exist a small locally finite category $\mathcal{T}$ with finite products such that $\mathrm{Alg}(\mathbf{T},\mathcal{A})$ is equivalent to $\mathcal{M} = \mathrm{FPFunct}(\mathcal{T},\mathcal{A})$ and we have a product preserving functor $T: \ S^* \to \mathcal{T}$ which is identity on objects [3,1,8].

Let I be a discrete subcategory of S^*, then $\mathrm{Funct}(I,\mathcal{A})$ reduces to the category $\mathcal{A}^I$ of functions from I to $\mathcal{A}$ with pointwise structure of abelian category inherited from $\mathcal{A}$. For any object F of $\mathcal{M}$ define an object $\Box_I F$ of $\mathcal{A}^I$ as a composition of F with the inclusion $U_I: \ I \hookrightarrow S^* \xrightarrow{T} \mathcal{T}$, and natural transformations go to maps of functions. This defines an exact functor $\Box_I$ from $\mathcal{M}$ to $\mathcal{A}^I$, and we obtain the following proposition as a consequence of (3), cf. [3], Corollary 1.2.2.2.

Proposition 1 *Suppose $S \subset I$, then $\Box_I$ is faithful.*

Suppose now that $\mathcal{A}$ is complete, this is true e.g. for $\mathcal{A} = \mathrm{Alg}(\mathbf{T},\mathcal{A}')$ provided $\mathcal{A}'$ is. Let $\boxtimes$ be tensor product of preadditive categories. Then it was shown in [15] that for a preadditive category $\mathcal{I}$ and a complete abelian category $\mathcal{A}$ there exists a preadditive bifunctor

$$\hom_{\mathcal{I}}(-,-): \ (\mathrm{Funct}(\mathcal{I},\mathbf{Ab}))^{\mathrm{op}} \boxtimes \mathrm{Funct}(\mathcal{I},\mathcal{A}) \to \mathcal{A}$$

limit preserving in the first variable, and it may be extended to preadditive functor $\mathrm{Funct}(\mathcal{C},\mathrm{Funct}(\mathcal{I}^{\mathrm{op}},\mathbf{Ab})) \boxtimes \mathrm{Funct}(\mathcal{I},\mathcal{A}) \to \mathrm{Funct}(\mathcal{C},\mathcal{A})$ also denoted by $\hom_{\mathcal{I}}(-,-)$ (this is called "picking up the operators" in [15], pp.14-15) in such a way that $\hom_{\mathcal{I}}(F,G)$ is additive provided the first argument is additive considered as a functor from $\mathcal{C}$ to $\mathrm{Funct}(\mathcal{I}^{\mathrm{op}},\mathcal{A})$.

Consider now $\mathcal{C}$ as a functor from $\mathcal{C}^{\mathrm{op}} \otimes \mathcal{C}$ to $\mathbf{Ab}$. This gives the functor of Yoneda embedding $Y_{\mathcal{C}}(-): \ \mathcal{C}^{\mathrm{op}} \to \mathrm{Funct}(\mathcal{C},\mathbf{Ab})$, $c \mapsto \mathrm{Hom}_{\mathcal{C}}(c,-)$. Let $U_{\mathcal{I}}: \mathcal{I} \to \mathcal{C}$ be a preadditive functor and let $U_{\mathcal{I}}^*$ be the corresponding neglecting functor from $\mathrm{Funct}(\mathcal{C},\mathbf{Ab})$ to $\mathrm{Funct}(\mathcal{I},\mathbf{Ab})$. Then the right adjoint to the neclecting functor $\Box_{\mathcal{I}}$ from $\mathrm{Funct}(\mathcal{C},\mathcal{A})$ to $\mathrm{Funct}(\mathcal{I},\mathcal{A})$ reads [15]

$$\mathrm{Ran}_{U_{\mathcal{I}}}(-) = \hom_{\mathcal{I}}((Y_{\mathcal{C}})^{\mathrm{op}} U_{\mathcal{I}}^*,-).$$

Dually, suppose that $\mathcal{A}$ is cocomplete, then there exist the bifunctor

$$-\otimes_{\mathcal{I}} -: \ \mathrm{Funct}(\mathcal{I},\mathcal{A}) \boxtimes \mathrm{Funct}(\mathcal{C},\mathrm{Funct}(\mathcal{I}^{\mathrm{op}},\mathbf{Ab})) \to \mathrm{Funct}(\mathcal{C},\mathcal{A})$$

colimit preserving in the second variable and the left adjoint to $\Box_{\mathcal{I}}$ has the form

$$\mathrm{Lan}_{U_{\mathcal{I}}}(-) = - \otimes_{\mathcal{I}} Y_{\mathcal{C}}^{\mathrm{op}} (U_{\mathcal{I}}^*)^{\mathrm{op}},$$

where $Y_{\mathcal{C}}^{\mathrm{op}}$ is the product-preserving "second Yoneda embedding" $\mathcal{C} \to \mathrm{Funct}(\mathcal{C}^{\mathrm{op}}, \mathbf{Ab})$ given by $c \mapsto \mathrm{Hom}_{\mathcal{C}}(-, c)$ (cf. [2], Prop. 5.3.18). Observe that $Y_{\mathcal{C}}$ preserves zero object, then putting $\mathcal{I} = I$ and using the identification (5) we obtain the following proposition.

Proposition 2 *Suppose $\mathcal{A}$ is cocomplete, then the functor $\Box_I : \mathcal{M} \to \mathcal{A}^I$ has left adjoint.*

4 Cohomology

Combining Prop. 1 and Prop. 2, we can construct a resolvent pair of abelian categories in the sense of ([14], Ch. IX §§5,6) for any $I \supset S$ using the pair of adjoint functors $L_{\Box_I} \dashv \Box_I$, so we can define relative extension functor $\mathrm{Ext}^*_{\Box_I}(-,-)$ from $\mathcal{M}^{\mathrm{op}} \times \mathcal{M}$ to $\mathbf{Ab}$ w.r.t. the proper class of $\Box_I$-split short exact sequences along the lines of ([14], Ch. XII §§4,5), and we obtain the following proposition.

Proposition 3 *Suppose $\mathcal{A}$ is complete, then an isomorphism*

$$H^n_{\Box_I}(F, G) := \mathrm{Ext}^n_{\Box_I}(F, G) \cong H^n(\mathrm{Hom}_{\mathcal{M}}(\beta^{\Box_I}(F), G)$$

there exist, where $\beta^{\Box_I}(F)$ is a relatively projective resolvent of F in $\mathcal{M}$ constructed using the pair of adjoint functors $\mathrm{Lan}_{\Box_I} \dashv \Box_I$.

Then proceeding along the lines of [5], Ch. XVI §1 and [14], Ch.XII §§9,10 we obtain the following proposition which relates this cohomology to "absolute" one, thus extending Theorem C of [11].

Proposition 4 *Suppose $\mathcal{A}$ is both complete and cocomplete, then an isomorphism*

$$H^n_{\Box_I}(F, G) \cong H^n(\mathrm{Hom}_{\mathrm{Funct}(\mathcal{T},\mathcal{A})}(F, \beta_{\Box_I}(G)) \qquad (*)$$

there exist, where $\beta_{\Box_I}$ is a relatively injective coresolvent of G in $\mathrm{Funct}(\mathcal{T}, \mathcal{A})$ constructed using the pair of adjoint functors $\Box_I \dashv \mathrm{Ran}_{\Box_I}$. Since $\mathrm{Funct}(\mathcal{T}, \mathcal{A})$ has enough "absolute" injectives, then we can take "absolute" injective resolvent of G in (*) instead of relative one.

Example 1 Let $\mathcal{A} = \mathrm{Funct}(\mathcal{C}^{\mathrm{op}}, \mathbf{Ab})$ and consider resolvent of the functor $Y_{\mathcal{C}}^{\mathrm{op}}$. Then like the case of rings and modules ([14], Ch.X, §2) we obtain that $\beta^{\Box_I}_{n+1}(Y_{\mathcal{C}}^{\mathrm{op}})$ is the functor from $\mathcal{C}$ to $\mathcal{A}$ defined by

$$c \mapsto \bigoplus_{i_1,\dots,i_n \in I} \mathrm{Hom}_{\mathcal{C}}(-, i_1) \otimes_{\mathbf{Z}} \mathrm{Hom}_{\mathcal{C}}(i_1, i_2) \otimes_{\mathbf{Z}} \dots \otimes_{\mathbf{Z}} \mathrm{Hom}_{\mathcal{C}}(i_n, c),$$

where explicit formula for tensor product of functors ([15],p.26) was used. This resolvent coincides with the ordinary Hochschild-Mitchell one ([15], p.70) except for the range of indices $i_1, \dots i_n$ which run I but not over the whole $\mathrm{Ob}\,\mathcal{C}$.

Let F be an element of $\mathcal{M} = \mathrm{FPFunct}(\mathcal{C}, \mathcal{A})$ and let $\phi(f, f_1, \ldots, f_n, f')$ be $\mathbf{Z}$-linear function from

$$\prod_{i, i_1, \ldots, i_n, i'} \mathrm{Hom}_{\mathcal{C}}(i, i_1) \times \mathrm{Hom}_{\mathcal{C}}(i_1, i_2) \times \ldots \times \mathrm{Hom}_{\mathcal{C}}(i_n, i')$$

to $\mathrm{Hom}_{\mathbf{Ab}}(\mathrm{Hom}_{\mathcal{C}}(i, i'), F(i, i'))$ with differential

$$\begin{aligned} \delta\phi(f, f_1, \ldots, f_n, f') = F(1 \boxtimes f)\phi(f_1, \ldots, f_n, f') + \\ + \sum_{p=1}^{n} (-1)^p \phi(f, f_1, \ldots, f_p f_{p+1}, \ldots, f_n, f') + \\ + \phi(f, f_1, \ldots, f_n) F(f' \boxtimes 1), \end{aligned} \quad (6)$$

where F is considered as a functor from $\mathcal{C} \boxtimes \mathcal{C}^{\mathrm{op}}$ to $\mathbf{Ab}$. Denote of this subcomplex with $I = S$ by $H^n_{\mathrm{red}}(Y^{\mathrm{op}}_{\mathcal{C}}, F)$, then using (3) we obtain the following decomposition of full cohomology groups

$$H^n(Y^{\mathrm{op}}_{\mathcal{C}}, F) = \bigoplus_{S^* \times S^*} H^n_{\mathrm{red}}(Y^{\mathrm{op}}_{\mathcal{C}}, F).$$

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