

Robust product design and process planning in using process capability analysis

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Abstract In the past, the process capability index (PCI) was the only method used in on-line quality management. Recently however, attempts have been made to extend the on-line application to off-line applications, such as product design or process planning. Because the conventional PCI index, C_{pm} , does not truly represent the measurement score, alternatives cannot be differentiated during off-line applications. Hence, a new process capability index, C_{pmc} , was developed to reflect the differences among alternatives for easy decision making at the product design and process planning stages; however, the deterministic approach in using this new process capability index has the disadvantage of dealing with uncertainties during the product design and process planning activities. Quality engineering often employs an effective way of ensuring that high product quality and low production cost result from robust design, particularly in terms of its application in an uncertain environment. The new PCI was the score mainly used for off-line applications; thus, there is motivation for using new PCI values, C_{pmc} , as the observed levels in the course of robust design implementation. The associated statistical method, response surface methodology, will be adopted for robust design in this study. Then, for robustness, the mean and tolerance values can be determined appropriately, as well as a measurement score for reasonable comparison and selection among candidates. Consequently, an economical and quality product design and process planning can be achieved statistically for the off-line applications.

Keywords Process capability index (PCI) · Mean · Tolerance · Off-line application · Robustness · RSM

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Introduction

Since the manufacturing industry is currently facing intensive competition, both cost and quality have become important concerns of management. Thus, producers strive to provide economical processes which are also capable of meeting customers' quality requirement. In recent years, as the concept of concurrent engineering has become widely accepted, design engineers have hoped to achieve simultaneous product design and process planning, side by side, at an early stage of product development (Carter and Baker 1992; Bare and Cox 2008; Marshall and Jordan 2008; Michel et al. 2010; Mohsen et al. 2010). The goals are to shorten the time span required for introducing the new product onto the market, while attaining the lowest production cost coupled with premium product quality. Hence, what is needed is a way to measure the degree to which the producer's process capability satisfies customers' quality requirement. More importantly, a growing number of producers include this measurement value in their purchase contracts with customers, as a documentation requirement. One such measurement is the process capability index (PCI).

The PCI is a value that reflects real-time quality status. The PCI acts as a reference for real-time monitoring; it enables process controllers to acquire a better grasp of the quality of on site processes (Kotz and Johnson 1993; Kotz and Lovelace 1998). Although the PCI is one of the quality measurements employed during on-line quality management, several authors have pointed out that the PCI should be addressed at the beginning of the design stage rather than at the production stage, where process capability analysis is typically done. Thus, Jeang recently developed a PCI expression, C_{pmc} , extended from C_{pm} for an off-line application (Jeang and Chung 2009; Jeang 2009).

The statistical approach combines robust design with experimental design methods (Hussain et al. 2011). The purpose of the statistical approach is to determine which components significantly affect the quality, functionality and cost of a product or manufacturing process. One of the experimental design methods, RSM, is adopted for analyzing the measurement scores, the process capability index (C_{pk}) (Lee et al. 2010). To continue examining robust design with process capability index, C_{pmc} values are used for process planning and product design in this study. RSM is an effective method of exploring the relationship between controllable variables and response values, and indicates the critical controllable variables, while mathematically searching for the optimal values and statistically analyzing problems (Montgomery 1991). Another important feature of RSM is that it requires fewer experimental runs than the factorial design of experiments (DOEs) as attested in the literature. In this study, a Box-Behnken Design (BBD) was used because it allows the efficient estimation of the first and second order coefficients of regression functions.

This paper is organized as follows: Sect. 1 presents the introduction, Sects. 2 and 3 contain information relevant to the background. Section 2 discusses why the statistical design via statistical method is necessary for uncertain environments. Section 3 introduces the statistical method, RSM, which is adopted in this research. Section 4 introduces the new PCI which will be used for statistical analysis. Section 5 provides an application showing the proposed approach. Finally, a summary is given in Sect. 6.

Robust design for uncertainty

There are a few shortcomings in a design that uses the deterministic approach. Models that represent the interaction between the variables and their relationship to the end product's performance are virtually unknown. Conventionally, the approach in determining quality values (design values) of controllable variables according to the expected product performance is often an iterative process. Namely, it is possible to go back and assign new component design values to produce different product performance until a satisfactory yield has been achieved. The trial used in assigning new quality value (design value) of the controllable variable is inefficient because of possible dependence among various controllable variables. In addition, the approach cannot suggest which component is important and what the exact variable quality values should be, particularly when dependence is present. Hence, moving trials forward and backward becomes a very time consuming and tedious task; it slows down the pace of product development. Moreover, designers prefer to have as many feasible designs as possible in order to allow for changes when they encounter complexities. This

tendency makes the above discussions even more problematic. For design activities, besides optimal solutions, a statistical view of the analysis of the design problem is also necessary, particularly under uncertain design conditions (Jeang and Chang 2002). Quality engineering often employs the statistical method for robust design; it improves product or process quality by reducing the effects of variation in order to facilitate product functionality (Phadke 1989; Taguchi and Wu 1985; Du et al. 2012; Sibaliija and Majstorovic 2012). The effectiveness of robust design can be achieved by using a statistical method such as DOE (Montgomery 1991). Usually, product or process engineers utilize DOE to reduce the effects of variation by suggesting a proper direction for design or process improvements. The purpose of the DOE is to determine which of the design variables significantly affect the response values. This feature, in determining critical design variables, is very important, particularly when new products or processes are developed. The statistical method, response surface methodology (RSM), will be adopted in this study (Montgomery 1991).

Experimental designs with RSM

Usually, the relationship between the dependent variable and the independent variable is extremely complex or unknown; however, RSM provides a procedure that solves this problem (Montgomery 1991). Assume that the designer is concerned with a system involving some response values R , which depend on input variables X_i . For convenience, these input variables are also called design variables in later discussion. It is also assumed that X_i is continuous and controllable. The functional relationship between the response and the levels of n inputs can be written as:

$$R = f(X_1, X_2, \dots, X_n) \quad (1)$$

This determines whether a mechanistic model for such a relationship exists. Thus, the first step in RSM is to find a suitable approximation for $f(\cdot)$ using a low-order polynomial in some region of the independent variables. If the approximated function has linear variables, a first-order polynomial can be used and written in terms of the design variables:

$$R = a_0 + a_1 X_1 + a_2 X_2 + \dots + a_n X_n \quad (2)$$

Otherwise, a second-order polynomial can be used:

$$R = a_0 + \sum_{i=1}^n a_i X_i + \sum_{i=1}^n b_i X_i^2 + \sum_{i=1}^n \sum_{j=1}^n c_{ij} X_i X_j \quad (3)$$

The frequent use of second-order polynomial models is justified by the fact that they reflect the nonlinear behavior of the system. Experimental designs for fitting a second-order response surface must involve at least three levels of each

variable so that the coefficients in the model can be estimated. A rotability property is desirable for response surface models because the orientation of the design with respect to its surface is unknown. This study uses a Box-Behnken design because it allows efficient estimation of the first and second order coefficients. Using this experimental design, the levels of each input factor are assumed to be equally spaced. A least squares estimate is used to estimate the coefficients in approximating the polynomials. The response surface analysis then proceeds in terms of the fitted surface. If the fitted surface is an adequate estimation of the true functional relationship, then the analysis of the fitted response will be nearly equivalent to the analysis of the studied problem.

Process capability index as response values for statistical analysis

The PCI expression in Eq. (4), C_{pm} , is able to simultaneously reflect the influences of process deviation and process variance (Boyle 1991; Chan et al. 1989). This is only legitimate at the post-production stage due to the fact that U and t are realized values which are not controllable for design:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{K[\sigma^2 + (U - T)^2]}} \tag{4}$$

USL and LSL are the upper and lower limits, respectively. However, when C_{pm} is used as a measurement scale in the pre-production stage under the assumption that σ is t/P , U and t become controllable variables. Then, it is possible that various combinations of U and t will result in the same C_{pm} value. Thus, it is difficult to make a distinction among alternatives in seeking to make a correct choice from among them. For example, the curve shown in Fig. 1 represents all possible combinations of U and t which have C_{pm} as 1.2 without differentiation.

Additionally, when C_{pm} is used as a measurement scale in the pre-production stage, the designers would most likely establish the process mean U as close as possible to the design target T , within the process feasibility range, and attempt to decrease the process variance as much as possible, within the process capability limits, in order to attain a higher PCI value. It is known that cutting down process variance with small process tolerance t normally results in high tolerance cost (production cost). In other words, with the exclusive use of the process mean and process variance as the determinants of conventional PCI, C_{pm} , regardless of the cost impact on customer and production, there is a tendency for designers to position the process mean as close to the target value T as possible, and cut down the process tolerance to lower capability limit in order to increase the PCI value. Apparently, the found PCI value is erroneous. This unhelpful result will

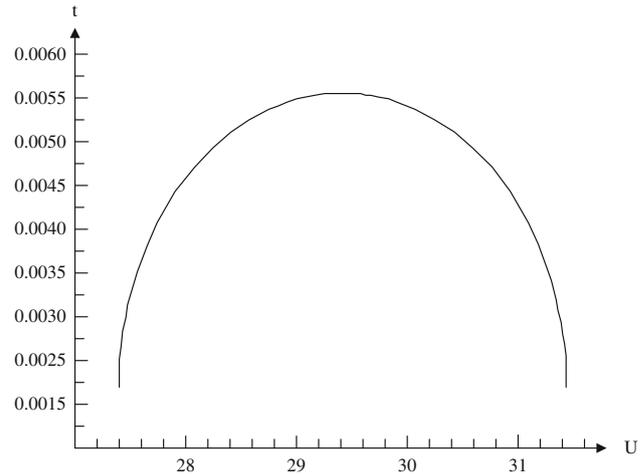


Fig. 1 Feasible range for various combination of U and t with $T = 30$, $LSL = 29.95$, $USL = 30.05$, $\sigma = t/P$, $P = 3$, when $C_{pm} = 1.2$

prevent the conventional C_{pm} from being a suitable index for possible alternatives during product design or process planning.

To overcome the above weaknesses, the lack of considering cost influence on deviation and variability should be avoided. It is known that the cost in representing the magnitude of deviation and variability is formed as a function of controllable variables, which contain process mean U and process tolerance t . In this regard, a new PCI expression is developed by having the non-cost expression $E(X)$, like Eq. (5), replaced with the quality and production related cost expression $C(X)$ as Eq. (6):

$$E(X) = \sigma^2 + (U - T)^2 \tag{5}$$

$$C(X) = K[\sigma^2 + (U - T)^2] + C_M(t) \tag{6}$$

The first term of Eq. (6) is “quality related cost” which is also referred to as “loss function” (Phadke 1989; Taguchi and Wu 1985). The second term of Eq. (6) is “production related cost” which is also called “tolerance cost function” (Chase et al. 1990). These two terms are needed to balance economical and quality considerations. They are depicted in Figs. 2 and 3 correspondingly. A narrower σ^2 can result in small quality related cost; however, the narrower one may lead to a greater production cost because of contracted tolerance t . Thus, we usually try to locate process mean U as close to design target T as possible before involving competitive process variance σ^2 and process tolerance t .

The sum of two cost terms is the total cost representing the quality level of a product or process (Jeang 1994). Consequently, the cost effectiveness and quality achievement PCI score becomes capable of evaluating and distinguishing alternative product designs or process planning. This PCI expression, C_{pmc} , is shown as follows:

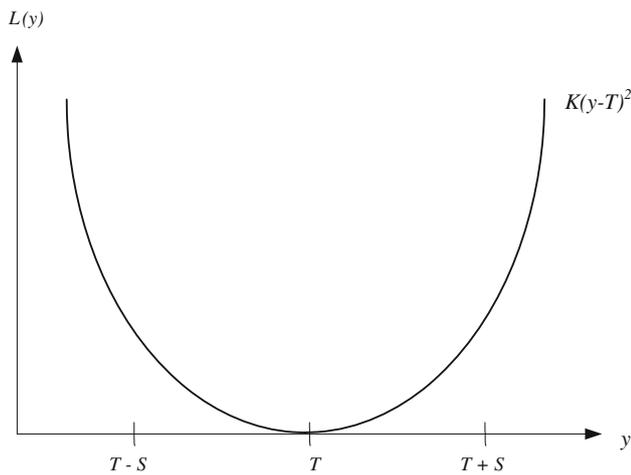


Fig. 2 Quality loss function

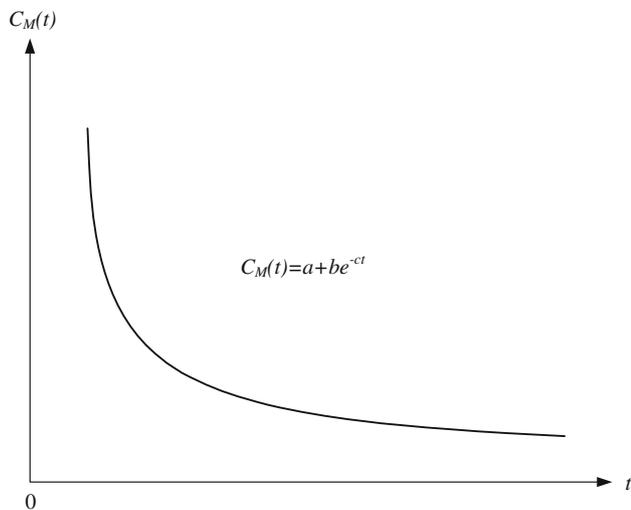


Fig. 3 Tolerance cost function

$$C_{pmc} = \frac{USL - LSL}{6(\sqrt{K[\sigma^2 + (U - T)^2]} + C_M(t_i))} \quad (7)$$

where σ is t/P and $C_M(t)$ is given value or $a + be^{-ct}$ (Jeang 1994, 1995), K is C_A/S^2 .

Because the process capability is a production process’s capability of producing a product according to the desired expectation of customers, the advantage in adopting C_{pmc} as the observed response for process design is that the designed parameter and tolerance values must represent the capability of producing a quality and cost-effective product. Then, by following the designed values, the production processes have the capability of producing products meeting customers’ expectations. Thus, instead of other responses used in previous works, the C_{pmc} value is considered as observed responses in the present research.

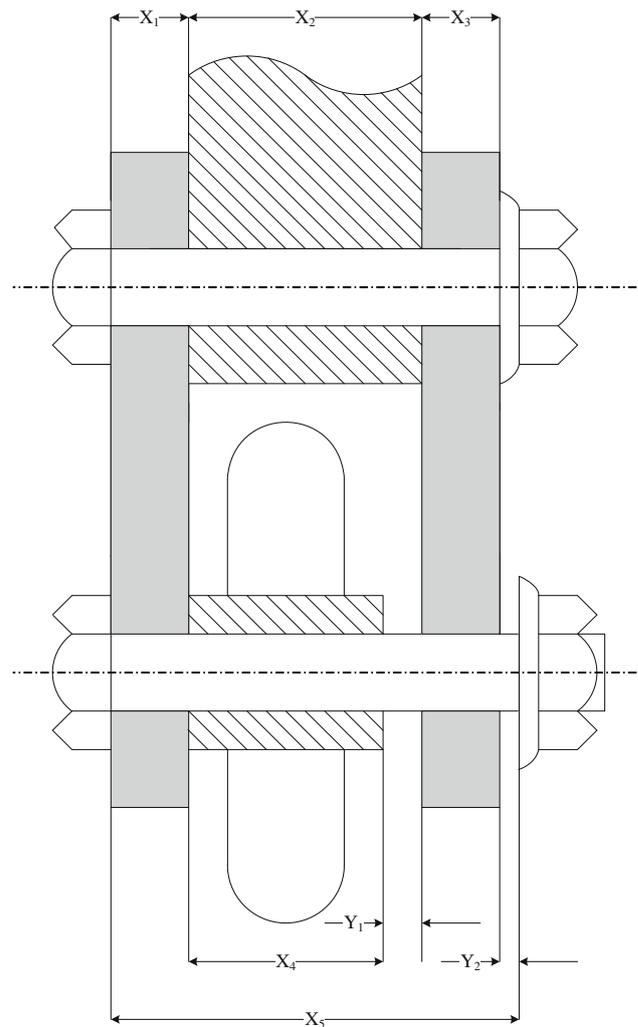


Fig. 4 The wheel mounting assembly drawing

The application for product design and process planning

Example 1 Assembly is the process by which the various parts and subassemblies are brought together to form a completed assembly or product which is designed to fulfill a certain mechanical function. Since assembly in the manufacturing process consists of putting together all the component parts and subassemblies of a given product, a proper allocation and analysis of tolerances among the assembly components is important to ensure that the functionality and quality of the design requirement are met. However, from the preceding discussion, in addition to tolerance design, the element of component dimensions (parameter values) should also be considered in an assembly design.

Figure 4 is a wheel mounting assembly, which consists of components X_1, X_2, X_3, X_4 and X_5 . They are linked with

Table 1 Three levels, U_j and t_j , for each component and tolerance cost $C_M(t_j)$

Component j	Lower level	Middle level	Upper level
U_1	4.9654 mm	5.0000 mm	5.0346 mm
U_2	8.4740 mm	8.5000 mm	8.5260 mm
U_3	3.9654 mm	4.0000 mm	4.0346 mm
U_4	8.3311 mm	8.3600 mm	8.3889 mm
U_5	17.6740 mm	17.7000 mm	17.7260 mm
t_1	\$10.7702 (0.03 mm)	\$10.7653 (0.06 mm)	\$10.7543 (0.09 mm)
t_2	\$12.7705 (0.02 mm)	\$12.7618 (0.04 mm)	\$12.7437 (0.06 mm)
t_3	\$10.7702 (0.03 mm)	\$10.7653 (0.06 mm)	\$10.7543 (0.09 mm)
t_4	\$5.7009 (0.04 mm)	\$5.6971 (0.06 mm)	\$5.6879 (0.08 mm)
t_5	\$7.1243 (0.03 mm)	\$7.1227 (0.05 mm)	\$7.1185 (0.07 mm)

two interrelated tolerance and dimension chains. The assembly functions for representing these two dimension chains are:

$$Y_1 = X_2 - X_4 \tag{8}$$

$$Y_2 = -X_1 - X_2 - X_3 + X_5 \tag{9}$$

The associated component dimensions and tolerances, U_1, U_2, U_3, U_4 and U_5, t_1, t_2, t_3, t_4 and t_5 , must be determined simultaneously (Jeang and Chang 2002). The target values T_1 and T_2 are 0.14 and 0.20 mm, respectively. The matching quality loss coefficients K_1 and K_2 are 250 and 350. The design tolerances (specifications) S_1 and S_2 are 0.12 and 0.16 mm, respectively. Obviously, $USL-LSL$ for Y_1 and Y_2 is $2 S_1$ and $2 S_2$ correspondingly. For the purpose of demonstration in an off-line application, the PCI expression C_{pmc} will be used as an index in measuring quality characteristics Y_1 and Y_2 . The high, middle, low tolerance levels and the associated tolerance cost are illustrated in Table 3. The feasible ranges for parameter U_i and the process capability limits for t_i are the extreme values connected with high and low levels in Table 1. Certainly, rooted in the discussion in Sect. 5, the formulation must contain decision variables, which are component parameters, U_1, U_2, U_3, U_4 and U_5 , and tolerances, t_1, t_2, t_3, t_4 and t_5 . They must be determined simultaneously to ensure C_{pmc} is maximized (Jeang and Chang 2002). The Monte Carlo Simulation is performed with the various level combinations of U_i and t_i , as suggested in ‘‘Appendix A’’. The normality assumption, $X_i \sim N(U_i, \sigma(t_i))$, is used to generate random values X_1, X_2, X_3, X_4 and X_5 in e^{th} experimental run where $\sigma(t_i)$ is defined in Eq. (10).

$$\sigma(t_i) = \left(\frac{t_i}{3}\right) \tag{10}$$

These random values are plugged into the above Eqs. (8) and (9) to have one set of Y_1 , and Y_2 correspondingly. Each level combination of U_i and t_i will be repeated 30 times in the e^{th} experimental run to ensure the accuracy of the

normality assumption. Then, the outputs, U_{Yr} and σ_{Yr} , can be obtained in the e^{th} experimental run, where r is 1 and 2. The response values, C_{pmc1} and C_{pmc2} are found from Eq. (7). Then, having C_{pmc} as the response value, the RSM optimization technique is used for problem analysis. C_{pmc} is defined in the following equation:

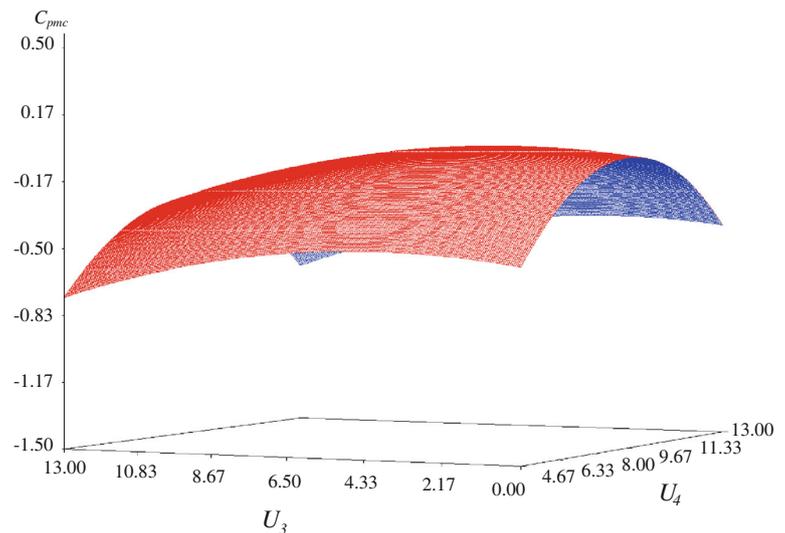
$$C_{pmc} = \frac{C_{pmc1} + C_{pmc2}}{2} \tag{11}$$

The optimal solutions are $t_1^* = 0.0439, t_2^* = 0.0600, t_3^* = 0.0439, t_4^* = 0.0759, t_5^* = 0.0446, U_1^* = 4.9654, U_2^* = 8.5260, U_3^* = 3.9825, U_4^* = 8.3860, U_5^* = 17.6740$ and $C_{pmc}^* = 0.0069$. A second-order model C_{pmc} predicting equation is: $0.4739 - 0.0083 U_1 - 0.0095 U_2 - 0.0124 U_3 - 0.0201 U_4 - 0.0336 U_5 - 0.0101 U_1^2 - 0.0215 U_1 U_2 - 0.0371 U_2^2 - 0.0217 U_1 U_3 - 0.0219 U_2 U_3 - 0.0101 U_3^2 + 0.0551 U_2 U_4 - 0.0268 U_4^2 + 0.02144 U_1 U_5 + 0.0212 U_2 U_5 + 0.0219 U_3 U_5 - 0.0097 U_5^2 + 0.0002 U_1 t_1 + 0.0002 U_3 t_1 - 0.0001 U_5 t_1 - 0.0005 t_1^2 - 0.0049 t_2^2 - 0.0048 t_3^2 - 0.0082 t_3 t_4 - 0.0117 t_4^2 + 0.0082 t_3 t_5 + 0.0186 t_4 t_5 - 0.0092 t_5^2$. Table 2 shows that the R^2 value is 0.9979. The fact that the R^2 value is greater than 0.95 indicates that the second order model provides an excellent fit. For statistical analysis, as completed by the computer programs, SAS (SAS Institute 2001), the contour and surface of the response of C_{pmc} is plotted in Figs. 5 and 6 to illustrate its 3D shape. Each contour represents a specific response value for combinations of the levels of factors. Various level combinations will be examined so that design feasibility and restrictions are satisfied. In addition, the F-ratio and F-ordering for all inputs U_i and t_i are listed in Table 3. F-ratio and F-ordering indicate that U_2, U_3 and U_1 should be closely controlled. If design improvement is needed, component 2 is to be given first priority.

For the purpose of comparison with other optimization methods, Example 1 is formulated with the mathematical programming as given in ‘‘Appendix B’’. The optimal solutions found from ‘‘Appendix B’’ are very close to the values obtained in the proposed approach. The solution closeness

Table 2 Statistical analysis for C_{pmc} of Example 1

Response surface for variable C_{pmc}					
Response mean					0.006908
Root MSE					0.000000181
R-square					0.9979
Coefficient of variation					0.4648
Regression	DF	S S	R-Square	F value	Pr > F
Linear	10	4.8437E-9	0.0424	14865.9	<0.0001
Quadratic	10	3.1978E-8	0.2797	98144.2	<0.0001
Crossproduct	45	7.74886E-8	0.6779	52848.7	<0.0001
Total Model	65	0.000000114	1.0000	53973.7	<0.0001
Factor	Degrees of freedom	Sum of squares	Mean square	Prob > F	
U_1	11	2.6443291E-8	2.4039356E-9	<0.0001	
U_2	11	4.3891891E-8	3.9901719E-9	<0.0001	
U_3	11	3.2048017E-8	2.913456E-9	<0.0001	
U_4	11	2.8242178E-8	2.5674708E-9	<0.0001	
U_5	11	2.3253698E-8	2.1139726E-9	<0.0001	
t_1	11	1.170191E-10	1.06381E-11	<0.0001	
t_2	11	3.1594869E-9	2.872261E-10	<0.0001	
t_3	11	1.233381E-10	1.121255E-11	<0.0001	
t_4	11	7.152263E-10	6.502057E-11	<0.0001	
t_5	11	2.986217E-11	2.714743E-12	<0.0001	

Fig. 5 Response surface

may be because of this particular example. However, there are still some advantages with the proposed approach. They are: (1) Other than determining the optimal solutions, the proposed approach also provided the important ranking listed in Table 3 as a reference for design improvement. (2) Because “Appendix B” shows a deterministic approach, design functions, Eqs. (8–9), tolerance cost functions, $a + b \cdot e^{-c \cdot t}$, need to be well defined in advance. However, sometimes design

functions are unknown, and tolerance cost $C_M(t)$ from tolerance function $a + b \cdot e^{-c \cdot t}$ and the quality loss coefficient K from C_A/S^2 are stochastic in nature. These facets render the deterministic approach in “Appendix B” infeasible. Conversely, the proposed approach is well adapted for design in an uncertain environment. When the constraints of Eqs. (33–42) on decision variables appear in nonlinear forms, the proposed approach becomes infeasible.

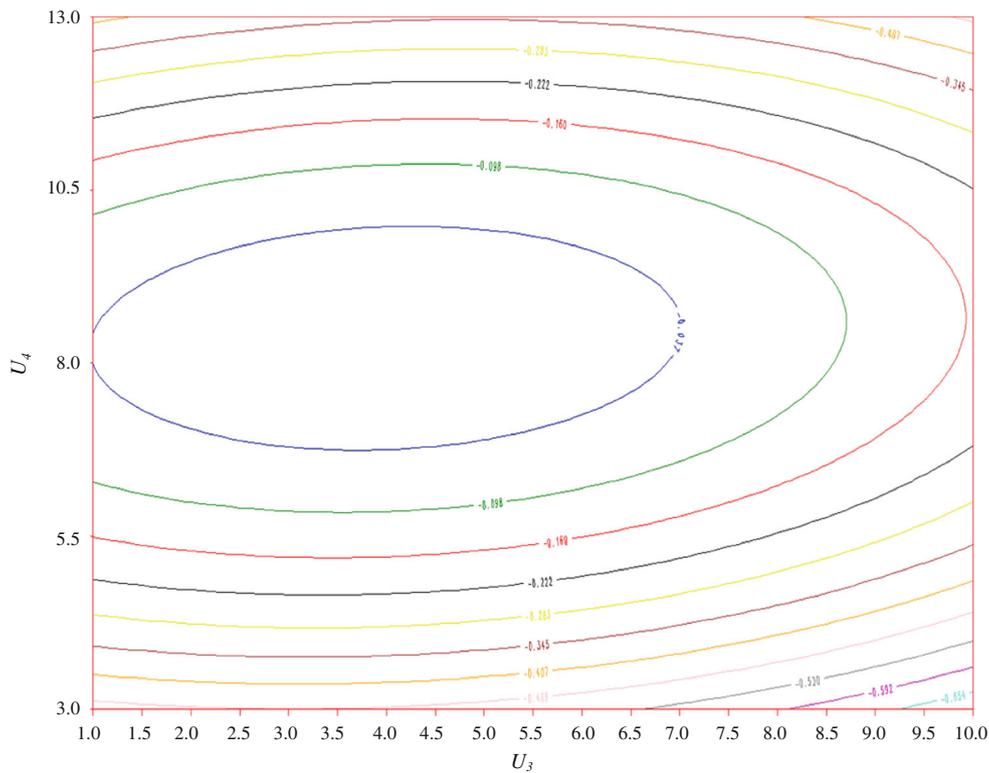


Fig. 6 Contour plot of C_{pmc} for process mean U_3 and U_4

Table 3 F-ratio and F-ranking

Factors	F-ratio	F-ranking
U_1	73778.80	3
U_2	122462.00	1
U_3	89416.40	2
U_4	78797.80	4
U_5	64879.60	5
t_1	326.49	9
t_2	8815.21	6
t_3	344.12	8
t_4	1995.54	7
t_5	83.32	10

Example 2 A practical example of manufacturing process planning is introduced to illustrate how the presented approach is applied to machining operations. Figure 7 is a drawing of the work piece. The various combinations of component dimension values X_1, X_2, X_3, X_4 and X_5 from each cutting operation result in five B/P dimension values, Y_1, Y_2, Y_3, Y_4 and Y_5 . Table 4 shows the procedures for the manufacturing process, which include the working machine, the reference surface, the processed surface, and the process capability range allowed in the manufacturing process. Table 5 is the related information concerning the

B/P dimensions. It covers the DC, DC vector, design tolerance S_r , design target T_r and quality loss coefficient K_r , where r is 1, 2, 3 . . . 5. As mentioned in the preceding discussion, the associated process means and process tolerances are U_1, U_2, U_3, U_4 and U_5, t_1, t_2, t_3, t_4 and t_5 . These values must be determined simultaneously so that all B/P dimension values fall within their own specification limits. By referring to dimension chain DC vector, A_{re} , in Table 5, the design functions are formed as follows:

$$Y_1 = X_5 + X_4 \tag{12}$$

$$Y_2 = X_3 - X_1 \tag{13}$$

$$Y_3 = X_3 - X_2 \tag{14}$$

$$Y_4 = X_4 - X_3 \tag{15}$$

$$Y_5 = X_5 \tag{16}$$

If we let the optimal solutions, U^* and t^* , found from C_{pmc} as an objective function, be substituted into the C_{pm} expression to have C_{pm}^l , the values $C_{pm}^l, 1.2756$, still fall above the acceptable limit for general application. Accordingly, the optimal solutions U^* and t^* not only maximize the C_{pmc} expression, but also ensure that the C_{pm} value is at a satisfactory level.

The low, middle and high levels for input factors U_i , and t_i and associated tolerance costs are shown in Table 6. The Monte Carlo Simulation is performed with the various level

Fig. 7 The Part for the presented machining process

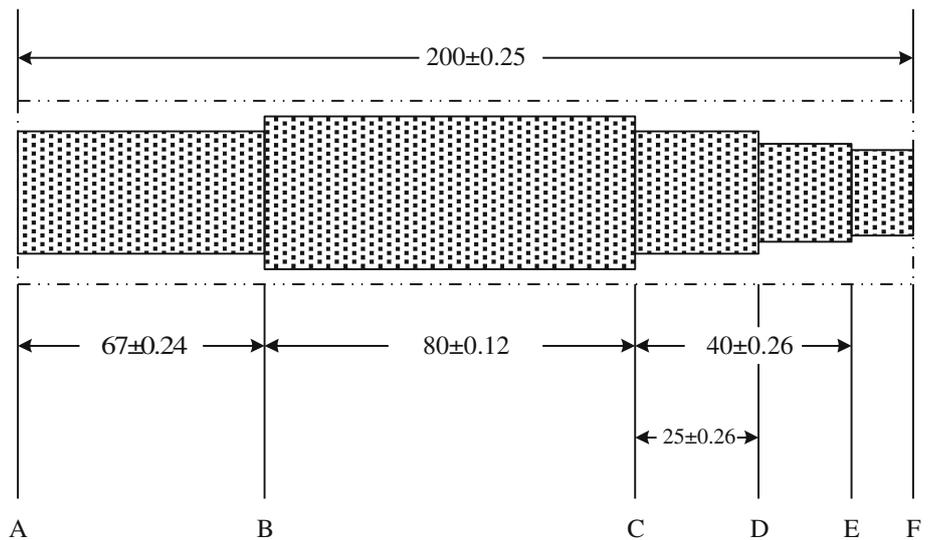


Table 4 The operations involved in the manufacturing process

Operation number i	Working machine	Reference surface	Processed surface	Lower process capability t_{Li} (mm)	Upper process capability t_{Ui} (mm)
1	Turret lathe	F	E	0.04	0.14
2	Turret lathe	F	D	0.04	0.14
3	Turret lathe	F	C	0.05	0.13
4	Turret lathe	F	B	0.05	0.13
5	Turret lathe	B	A	0.05	0.13

Table 5 Related information on the work piece diagram

r	Blueprint dimensions	Dimension chain r	Dimensions chain vector A_{re}	Design tolerance S_r	Target value T_r	Quality loss coef. K_r
1	A–F	5–4	[1,1]	0.25	200	6,000
2	C–E	3–1	[1,-1]	0.26	41	10,500
3	C–D	3–2	[1,-1]	0.26	25	9,000
4	B–C	4–3	[1,-1]	0.12	79	1,500
5	A–B	5	[1]	0.24	42	10,500

Table 6 Three levels, U_i and t_i , for each operation and tolerance cost $C_M(t_i)$

Operation i	Lower level	Middle level	Upper level
U_1	38 mm	39 mm	40 mm
U_2	55 mm	56 mm	57 mm
U_3	79 mm	81 mm	82 mm
U_4	157 mm	160 mm	163 mm
U_5	42 mm	43 mm	44 mm
t_1	\$93.0028 (0.04) mm	\$71.1156 (0.09) mm	\$63.0939 (0.14) mm
t_2	\$93.7241 (0.04) mm	\$71.3709 (0.09) mm	\$63.0717 (0.14) mm
t_3	\$78.5737 (0.05) mm	\$56.9872 (0.09) mm	\$50.5266 (0.13) mm
t_4	\$79.4912 (0.05) mm	\$57.2412 (0.09) mm	\$50.5153 (0.13) mm
t_5	\$79.4991 (0.05) mm	\$57.2688 (0.09) mm	\$50.5201 (0.13) mm

Table 7 Statistical analysis for C_{pmc}

Response surface for variable C_{pmc}					
Response mean					0.007992
Root MSE					0.001249
R^2					0.9624
Coefficient of variation					15.6241
Regression	DF	S S	R-square	F value	Pr > F
Linear	10	0.000584	0.1356	37.47	<0.0001
Quadratic	10	0.003526	0.8183	226.14	<0.0001
Crossproduct	45	0.0000363	0.0084	0.52	0.9927
Total Model	65	0.004147	0.9624	40.91	<0.0001
Factors	Degrees of freedom	Sum of squares	Mean square	Prob > F	
U_1	11	0.000010044	0.000000913	0.8368	
U_2	11	0.000015449	0.000001404	0.5425	
U_3	11	0.000127	0.000011517	<0.0001	
U_4	11	0.000276	0.000025111	<0.0001	
U_5	11	0.000010136	0.000000921	0.8325	
t_1	11	0.000001496	0.000000136	1.0000	
t_2	11	0.000002247	0.000000204	0.9996	
t_3	11	0.000013530	0.000001230	0.6506	
t_4	11	0.000042951	0.000003905	0.0078	
t_5	11	0.000094262	0.000008569	<0.0001	

combinations of U_i and t_i , as suggested in “Appendix A. The normality assumption, $X_{i-N}(U_i, \sigma(t_i))$, is used to generate experimental values X_1, X_2, X_3, X_4 and X_5 randomly in e^{th} experimental run. These experimental values are plugged into the above Eqs. (12)–(16) to have one set of Y_1, Y_2, Y_3, Y_4 and Y_5 , correspondingly. Each level combination of U_i and t_i will be repeated 30 times in e^{th} experimental run to ensure the accuracy of the normality assumption. Then, U_{Y_r} and σ_{Y_s} are obtained from 30 sets of Y_1, Y_2, Y_3, Y_4 and Y_5 , where r is 1, 2, 3, 4 and 5. The values, $C_{pmc1}, C_{pmc2}, C_{pmc3}, C_{pmc4}$ and C_{pmc5} , can be found from Eq. (7) with the e^{th} experimental outputs, U_{Y_r} and σ_{Y_s} . Then, the response value is the sum of $C_{pmc1}, C_{pmc2}, C_{pmc3}, C_{pmc4}$ and C_{pmc5} , which is considered as the response value for RSM analysis. A second-order model C_{pmc} predicting equation is: $-24.8924 + 0.4539U_3 + 0.0923U_4 + 0.0005U_2U_3 - 0.0031U_3^2 - 0.0003U_4^2 + 0.0002U_4U_5 - 3.5787t_3^2 - 4.0503t_4^2$. Table 7 indicates that the coefficient of determination R^2 is 0.9624, and that a good fit exists. The optimal solutions are $t_1^* = 0.0908, t_2^* = 0.0898, t_3^* = 0.0904, t_4^* = 0.0892, t_5^* = 0.0891, U_1^* = 38.9456, U_2^* = 56.0464, U_3^* = 80.5716, U_4^* = 158.8937, U_5^* = 42.8089, C_{pmc}^* = 0.01712$ and $C_{pm}^* = 1.3972$. For statistical analysis as completed by the computer programs, SAS (SAS Institute 2001), the contour and surface of the response of C_{pmc} is plotted in Figs. 8 and 9 to illustrate its 3D shape.

In addition, the F-ratio and F-ordering for all inputs U_i and t_i are listed in Table 8. F-ratio and F-ordering indicate that U_3, U_4, t_4 and t_5 should be closely controlled. If process improvement is needed, operation 4 is to be given first priority.

Summary

The present research uses the PCI measurement, C_{pmc} , via RSM for robustness in process capability analysis to ensure that, at the initial blueprint stage, a lower production cost and a high quality product can be achieved. A time frame is developed for the off-line applications during product and process design. Prior to production, design engineers can establish process mean and process tolerance based on the optimized mean and tolerance values, and make use of the listed ranking as a reference for possible design improvement. As expected, the produced quality values after the production process must be distributed with statistical values as the established process mean and process tolerance before the production process. As a result, an effective PCI for a product’s life cycle becomes actualized. Two examples: product design and process planning, are introduced for demonstrating the present approach. The results, in terms

Fig. 8 Response surface

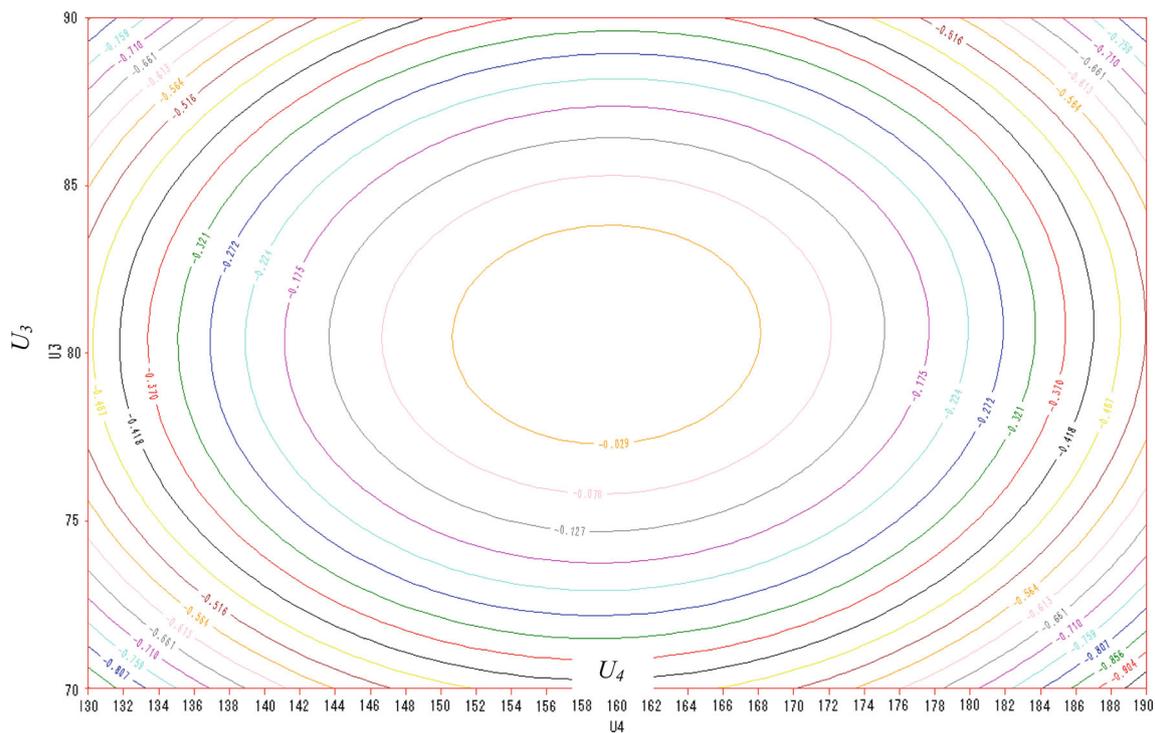
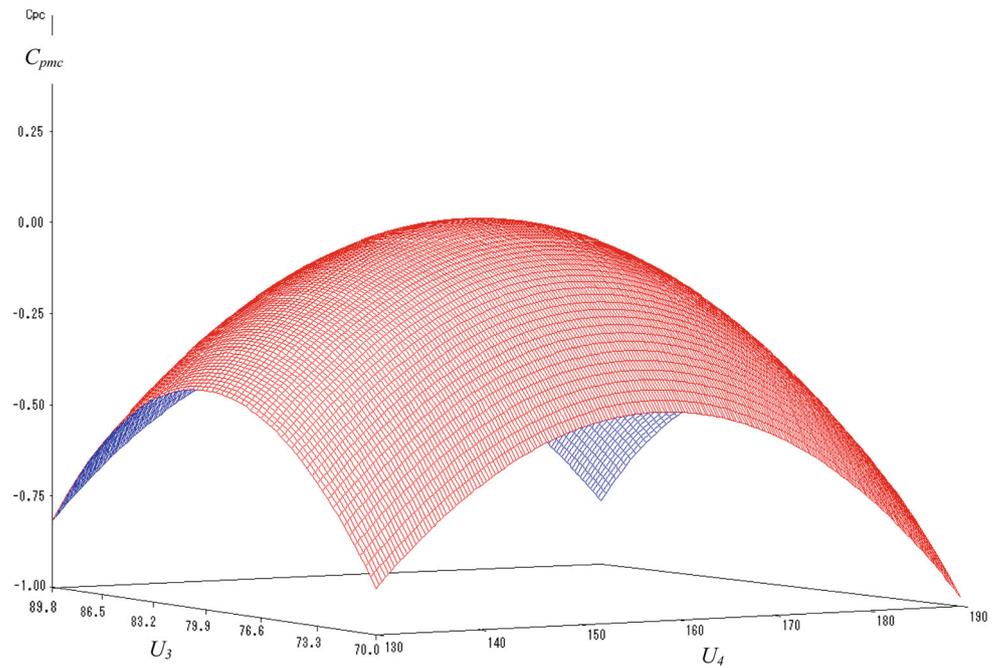


Fig. 9 Contour plot of C_{pmc} for process mean U_3 and U_4

of statistical sense, provide not only the optimal solutions but also the important ranking. Because previous works considered the tolerance costs and quality losses as deterministic values in product and process design, there is a need to relax the assumptions of deterministic values in future research. The present approach considers product specifications USL and LSL as given conditions; there is a need to

relax this given assumption in future studies, by considering USL and LSL as additional decision variables. Most of the time, during design and planning stages, design functions such as Eqs. (8–9) and (12–16) are unknown. Utilizing advanced computer software, such as Computer-Aided Engineering (CAE), can help engineers at the product design and process planning stage, to surmount design problems that

Table 8 F-ratio and F-ranking

Factors	F-ratio	F-ranking
U_1	0.59	7
U_2	0.90	5
U_3	7.39	2
U_4	16.10	1
U_5	0.59	8
t_1	0.09	10
t_2	0.13	9
t_3	0.79	6
t_4	2.50	4
t_5	5.50	3

have unknown design functions. As a result, an economical and quality process capability analysis for product and process design becomes possible at an earlier time in the design stage. The following itemizes and explains the steps involved:

Step 1: Provide the design functions (see Eqs. 8–9 and 12–16) and the design-related information (see Tables 4, 5). Offer LSL , USL , C_A , tolerance cost functions or tolerance costs.

Step 2: Choose an appropriate design experimental matrix (see “Appendix A”) for the various levels of inputs, U and t (see Tables 1, 6).

Step 3: Use the combined levels of inputs, U and t , as the arrangement of an experimental design matrix to find: **a)** $K[\sigma^2 + (U - T)^2]$, $\sigma^2 = (t/P)^2$, normally $P = 3$, K is C_A/S^2 , **b)** $C_M(t)$ is $a + be^{-ct}$ or tolerance cost with given tolerance level (see Table 1 or Table 6). Then have the results from items a and b fed into Eq. (7) to find C_{pmc} .

Step 4: Perform RSM with SAS software to obtain the best values of U and t for the maximization of C_{pmc} and to find C_{pmc} prediction functions by regression analysis. See second-order model C_{pmc} predicting equation in Examples 1 and 2. Then, employ ANOVA to rank the important parameters (see Tables 3, 8).

Step 5: Employ ANOVA to rank the important parameters (see Tables 3, 8).

Step 6: If improvement is needed, then the above steps can be repeated based on the suggestions made in Step 5.

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Appendix A: Table of Box-Behnken Design (Factors = 10, blocks = 2, runs = 170)

$$\begin{pmatrix} 0 & \pm 1 & 0 & 0 & 0 & \pm 1 & \pm 1 & 0 & 0 & \pm 1 \\ \pm 1 & \pm 1 & 0 & 0 & \pm 1 & 0 & 0 & 0 & 0 & \pm 1 \\ 0 & \pm 1 & \pm 1 & 0 & 0 & 0 & \pm 1 & \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 & \pm 1 & 0 & \pm 1 & 0 & 0 & \pm 1 & 0 \\ \pm 1 & 0 & 0 & 0 & 0 & 0 & 0 & \pm 1 & \pm 1 & \pm 1 \\ 0 & 0 & \pm 1 & \pm 1 & \pm 1 & 0 & 0 & 0 & 0 & \pm 1 \\ \pm 1 & 0 & 0 & \pm 1 & 0 & 0 & \pm 1 & \pm 1 & 0 & 0 \\ 0 & 0 & \pm 1 & 0 \\ \pm 1 & 0 & \pm 1 & 0 & 0 & \pm 1 & 0 & 0 & \pm 1 & 0 \\ 0 & 0 & 0 & \pm 1 & \pm 1 & \pm 1 & 0 & \pm 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Appendix B: Presented problem formulated with mathematical programming

$$Max \quad C_{pmc} = \frac{C_{pmc1} + C_{pmc2}}{2} \tag{17}$$

s.t.

$$t_{Y1} = t_2 + t_4 \tag{18}$$

$$t_{Y2} = t_1 + t_2 + t_3 + t_5 \tag{19}$$

$$\sigma_{Y1} = \sqrt{\left(\frac{t_2}{P}\right)^2 + \left(\frac{t_4}{P}\right)^2} \tag{20}$$

$$\sigma_{Y2} = \sqrt{\left(\frac{t_1}{P}\right)^2 + \left(\frac{t_2}{P}\right)^2 + \left(\frac{t_3}{P}\right)^2 + \left(\frac{t_5}{P}\right)^2} \tag{21}$$

$$U_{Y1} = U_2 - U_4 \tag{22}$$

$$U_{Y2} = -U_1 - U_2 - U_3 + U_5 \tag{23}$$

$$C_M(t_{Y1}) = a_2 + b_2 \cdot e^{-c_2 \cdot t_2} + a_4 + b_4 \cdot e^{-c_4 \cdot t_4} \tag{24}$$

$$C_M(t_{Y2}) = a_1 + b_1 \cdot e^{-c_1 \cdot t_1} + a_2 + b_2 \cdot e^{-c_2 \cdot t_2} + a_3 + b_3 \cdot e^{-c_3 \cdot t_3} + a_5 + b_5 \cdot e^{-c_5 \cdot t_5} \tag{25}$$

$$C_{pmc1} = \frac{USL_1 - LSL_1}{6\sqrt{K_1 [(U_{Y1} - T_1)^2 + \sigma_{Y1}^2]} + C_M(t_{Y1})} \tag{26}$$

$$C_{pmc2} = \frac{USL_2 - LSL_2}{6\sqrt{K_2 [(U_{Y2} - T_2)^2 + \sigma_{Y2}^2]} + C_M(t_{Y2})} \tag{27}$$

$$C_{pmc1} = \frac{USL_1 - LSL_1}{6\sqrt{(U_{Y1} - T_1)^2 + \sigma_{Y1}^2}} \tag{28}$$

$$C_{pmc2} = \frac{USL_2 - LSL_2}{6\sqrt{(U_{Y2} - T_2)^2 + \sigma_{Y2}^2}} \tag{29}$$

$$C_{pmc} = \frac{C_{pmc1} + C_{pmc2}}{2} \quad (30)$$

$$T_1 - S_1 + t_{Y1} \leq U_{Y1} \leq T_1 + S_1 - t_{Y1} \quad (31)$$

$$T_2 - S_2 + t_{Y2} \leq U_{Y2} \leq T_2 + S_2 - t_{Y2} \quad (32)$$

$$4.9654 \leq U_1 \leq 5.0346 \quad (33)$$

$$8.4740 \leq U_2 \leq 8.5260 \quad (34)$$

$$3.9654 \leq U_3 \leq 4.0346 \quad (35)$$

$$8.3311 \leq U_4 \leq 8.3889 \quad (36)$$

$$17.6740 \leq U_5 \leq 17.7260 \quad (37)$$

$$0.03 \leq t_1 \leq 0.09 \quad (38)$$

$$0.02 \leq t_2 \leq 0.06 \quad (39)$$

$$0.03 \leq t_3 \leq 0.09 \quad (40)$$

$$0.04 \leq t_4 \leq 0.08 \quad (41)$$

$$0.03 \leq t_5 \leq 0.07 \quad (42)$$

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