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CORRECTION

Correction to: convergence rates for Kaczmarz-type algorithms

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1 Comments and notations

We made corrections only on Theorem 7 from Section "4.2 Extended Kaczmarz single - projection algorithm" of the original paper. We will refer to the equations, results, and references from the original paper by adding the sign (*). Else, they are related to this Erratum.

2 Erratum to Theorem *7

Theorem 1 *The algorithm MREK has linear convergence.*

Proof Let $(x^k)_{k\geq 0}$ be the sequence generated with the MREK algorithm. According to the selection procedure (*44) of the projection index i_k and (*9), we successively obtain (see also Section 1 of the paper [*1])

$$m|\langle A_{i_k}, x^{k-1}\rangle - b^k_{i_k}|^2 \geq \sum_{1\leq i\leq m} |\langle A_i, x^{k-1}\rangle - b^k_i|^2 = \| Ax^{k-1} - b^k\|^2$$
$$= \| (Ax^{k-1} - b) + (r - y^k)\|^2. \tag{1}$$

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We have the following elementary inequality.

Lemma 1 *Let* α, β *be real numbers such that*

$$\alpha \in [0, 1], \ \beta \geq -1 \text{ and } \beta - \alpha = \alpha \beta. \tag{2}$$

Then

$$(r_1 + r_2)^2 \geq \alpha r_1^2 - \beta r_2^2, \ \forall r_1, r_2 \in \mathbb{R}. \tag{3}$$

This gives us the following result.

Corollary 1 *Let* α, β *be as in* (2). *Then*

$$\| x + y \|^2 \geq \alpha \| x \|^2 - \beta \| y \|^2, \ \forall x, y \in \mathbb{R}^n. \tag{4}$$

Proof Indeed, we observe that, in the hypothesis (2), we have

$$\| x + y \|^2 - \alpha \| x \|^2 + \beta \| y \|^2 = \| \sqrt{1-\alpha}x - \sqrt{1+\beta}y \|^2 \geq 0. \qquad \square$$

Therefore, from (1) and (4), we obtain

$$-|\langle A_{i_k}, x^{k-1}\rangle - b_{i_k}^k|^2 \leq -\frac{\alpha}{m} \| Ax^{k-1} - b \|^2 + \frac{\beta}{m} \| r - y^k \|^2 . \tag{5}$$

In [*19], Proposition 1, Eq. (59) (for $\omega = 1$) it is proved the equality

$$\| x^k - x \|^2 = \| x^{k-1} - x \|^2 - \frac{\left(\langle A_{i_k}, x^{k-1}\rangle - b_{i_k}\right)^2}{\|A_{i_k}\|^2} + \| \gamma_{i_k} \|^2, \tag{6}$$

where

$$\gamma_{i_k} = \frac{r_{i_k} - y_{i_k}^k}{\| A_{i_k} \|^2} A_{i_k}, \tag{7}$$

and $x \in LSS(A; b)$ is such that $P_{\mathcal{N}(A)}(x) = P_{\mathcal{N}(A)}(x^0)$. If δ is the smallest nonzero singular value of A (therefore also of A^T) and because $P_{\mathcal{N}(A)}(x^k) = P_{\mathcal{N}(A)}(x^0)$, $\forall k \geq 0$ it holds that $x^k - x \in \mathcal{R}(A^T)$ (see also [*1]), hence

$$\| Ax^{k-1} - b \|^2 \geq \delta^2 \| x^{k-1} - x \|^2 . \tag{8}$$

Then, from (1), (6), and (5), the obvious inequality

$$\| \gamma_{i_k} \|^2 \leq \frac{\| r - y^k \|^2}{\| A_{i_k} \|^2},$$

and (8) we get

$$\begin{aligned} \| x^k - x \|^2 &\leq \| x^{k-1} - x \|^2 - \frac{\alpha}{m} \frac{\| Ax^{k-1} - b \|^2}{\| A_{i_k} \|^2} + \frac{\beta}{m} \frac{\| r - y^k \|^2}{\| A_{i_k} \|^2} + \frac{\| r - y^k \|^2}{\| A_{i_k} \|^2} \\ &\leq \left(1 - \frac{\alpha\delta^2}{m \cdot M}\right) \| x^{k-1} - x \|^2 + \frac{1}{\mu}\left(1 + \frac{\beta}{m}\right) \| y^k - r \|^2, \end{aligned} \tag{9}$$

where

$$M = \max_{1 \leq i \leq m} \| A_i \|^2, \ \ \mu = \min_{1 \leq i \leq m} \| A_i \|^2 . \tag{10}$$

In [*19], Lemma 2 it is proved that

$$\parallel y^k - r \parallel^2 \le \left(1 - \frac{\delta^2}{n}\right)^k \parallel y^0 - r \parallel^2, \ \forall k \ge 0. \tag{11}$$

Then, from (*5) and (11), we obtain

$$\parallel x^k - x \parallel^2 \le \left(1 - \frac{\alpha\delta^2}{mM}\right) \parallel x^{k-1} - x \parallel^2 + \frac{1}{\mu}\left(1 + \frac{\beta}{m}\right)\left(1 - \frac{\delta^2}{n}\right)^k \parallel y^0 - r \parallel^2 . \tag{12}$$

If we introduce the notations

$$\tilde{\alpha} = 1 - \frac{\alpha\delta^2}{m \cdot M} \in [0, 1), \ \tilde{\beta} = 1 - \frac{\delta^2}{n} \in [0, 1), \ C = \frac{1}{\mu}\left(1 + \frac{\beta}{m}\right) \parallel y^0 - r \parallel^2 \tag{13}$$

from (9) – (10), we obtain

$$\parallel x^k - x \parallel^2 \le \tilde{\alpha} \parallel x^{k-1} - x \parallel^2 + \tilde{\beta}^k C, \forall k \ge 1. \tag{14}$$

From (14), a recursive argument gives us

$$\parallel x^k - x \parallel^2 \le \tilde{\alpha}^k \parallel x^0 - x \parallel^2 + \sum_{j=0}^{k-1} \tilde{\alpha}^j \tilde{\beta}^{k-j} C$$

or, for $\nu = \max\{\tilde{\alpha}, \tilde{\beta}\} \in [0, 1)$

$$\parallel x^k - x \parallel^2 \le \nu^k \left(\parallel x^0 - x \parallel^2 + Ck\right), \forall k \ge 1. \tag{15}$$

If we define $\epsilon_k = \nu^k \left(\parallel x^0 - x \parallel^2 + Ck\right), \forall k \ge 1$, we obtain that $\lim_{k\to\infty} \frac{\epsilon_{k+1}}{\epsilon_k} = \nu \in [0, 1)$, which gives us the linear convergence for MREK algorithm and completes the proof. □

Typos mistakes

1. On page 9, at the end of the proof of Corollary 1, replace the equation

$$\frac{\epsilon_{n\Gamma}}{\epsilon_{n\Gamma-1}} = \delta \in [0, 1), \forall n \ge 1.$$

by the equation

$$\frac{\epsilon_{n\Gamma}}{\epsilon_{(n-1)\Gamma}} = \delta \in [0, 1), \forall n \ge 1.$$

2. On page 11, in equation (45), instead of

$$\mathbb{E}\left[\|x^k - x_{LS}\|\right] \le \left(1 - \frac{1}{\hat{k}^2(A)}\right)^{\lfloor k/2 \rfloor} (1 + 2hatk^2(A))\|x_{LS}\|^2,$$

write

$$\mathbb{E}\left[\|x^k - x_{LS}\|\right] \le \left(1 - \frac{1}{\hat{k}^2(A)}\right)^{\lfloor k/2 \rfloor} (1 + 2\hat{k}^2(A))\|x_{LS}\|^2,$$

3. On page 10, after the equation (42), please write:
Note. We used formula (40) to update the vector y^{k-1}, instead of the formula

$$y^k = y^{k-1} - \frac{\langle y^{k-1}, A^{j_k} \rangle}{\| A^{j^k} \|^2} A^{j_k}$$

because we supposed that $\| A^j \| = 1, \forall\, j = 1, \ldots, n$. This can be achieved by a scalling of A of the form

$$A \implies A\,D, \text{ with } D = diag\left(\frac{1}{\| A^1 \|}, \frac{1}{\| A^2 \|}, \ldots, \frac{1}{\| A^n \|}\right),$$

which transforms the initial problem (10) into the equivalent one

$$\| (AD)(D^{-1}x) - \hat{b} \| = \min_{z \in \mathbb{R}^n} \| (AD)(D^{-1}z) - \hat{b} \| .$$

4. On page 14, second line from top, instead of the formula

$$x^{k+\Gamma-j} - x = P_{i_{k+\Gamma-j-1}}(x^{k+\Gamma-j-1} - x) + \gamma_{i_{k+\Gamma-j-1}}$$

the formula

$$x^{k+\Gamma-j} - x = P_{i_{k+\Gamma-j}}(x^{k+\Gamma-j-1} - x) + \gamma_{i_{k+\Gamma-j}}.$$

5. On page 14, the fourth line from top, instead of the formula

$$x^{k+\Gamma} = P_{k+\Gamma-1} \circ \cdots \circ P_{i_k}(x^k - x) + \sum_{j=1}^{\Gamma} \Pi_j \gamma_{ik+\Gamma-j}$$

please write

$$x^{k+\Gamma} = P_{i_k+\Gamma} \circ \cdots \circ P_{i_k}(x^k - x) + \sum_{j=1}^{\Gamma} \Pi_j \gamma_{i_k+\Gamma-j}.$$

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