# The role of Surface Plasmon modes in the Casimir Effect

F. Intravaia\*

Universität Potsdam - Institut für Physik, Am Neuen Palais 10, 14469 Potsdam - Germany & E-mail: francesco.intravaia@quantum.physik.uni-potsdam.de

#### A. Lambrecht

Laboratoire Kastler-Brossel ENS, UPMC, CNRS, Université Pierre et Marie Curie, 4, place Jussieu, Case 74, F75252 Paris Cedex 05 & E-mail: lambrecht@spectro.jussieu.fr

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Abstract. In this paper we study the role of surface plasmon modes in the Casimir effect. First we write the Casimir energy as a sum over the modes of a real cavity. We may identify two sorts of modes, two evanescent surface plasmon modes and propagative modes. As one of the surface plasmon modes becomes propagative for some choice of parameters we adopt an adiabatic mode definition where we follow this mode into the propagative sector and count it together with the surface plasmon contribution, calling this contribution "plasmonic". The remaining modes are propagative cavity modes, which we call "photonic". The Casimir energy contains two main contributions, one coming from the plasmonic, the other from the photonic modes. Surprisingly we find that the plasmonic contribution to the Casimir energy becomes repulsive for intermediate and large mirror separations. Alternatively, we discuss the common surface plasmon definition, which includes only evanescent waves, where this effect is not found. We show that, in contrast to an intuitive expectation, for both definitions the Casimir energy is the sum of two very large contributions which nearly cancel each other. The contribution of surface plasmons to the Casimir energy plays a fundamental role not only at short but also at large distances.

## 1. Introduction

An important prediction of quantum theory is the existence of irreducible fluctuations of electromagnetic fields even in vacuum, that is in the thermodynamical equilibrium state with a zero temperature. These fluctuations have a number of observable consequences in microscopic physics for example in atomic physics the Van der Waals force between atoms in vacuum.

Vacuum fluctuations also have observable mechanical effects in macroscopic physics and the archetype of these effects is the Casimir force between two mirrors at rest in vacuum. This force was predicted by H. Casimir in 1948 [1] who considered two plane parallel perfect reflectors as shown in Figure 1 and found an interaction energy  $E_{\text{Cas}}$  depending only on geometrical parameters, the mirrors

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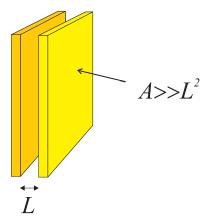


Figure 1: The configuration studied by H. Casimir

distance L and surface  $A \gg L^2$ , and two fundamental constants, the speed of light c and Planck constant  $\hbar$ 

$$E_{\rm Cas} = -\frac{\hbar c \pi^2 A}{720 L^3}.\tag{1}$$

The signs have been chosen to fit the thermodynamical convention with the minus sign of the energy  $E_{\text{Cas}}$  corresponding to a binding energy. The Casimir energy for perfect mirrors is usually obtained by summing the zero-point energies  $\frac{\hbar\omega}{2}$  of the cavity eigenmodes, substracting the result for finite and infinite separation, and extracting the regular expression (1) by inserting a formal high-energy cutoff and using the Euler-McLaurin formula [2].

The Casimir force was soon observed in different experiments which confirmed its existence [3, 4, 5]. Recent experiments have reached a good precision, in the %range, which makes possible an accurate comparison between theoretical predictions and experimental observations [6, 7].

Casimir considered an ideal configuration with two perfectly reflecting mirrors in vacuum. But the experiments are performed with real reflectors, for example metallic mirrors which have a perfect reflection only at frequencies below a plasma frequency  $\omega_{\rm P}$  or alternatively for mirror separations much larger than the plasma wavelength  $\lambda_{\rm P}$  characetristic for the metal. Accounting for this imperfect reflection and its frequency dependence is thus essential for obtaining a reliable theoretical expectation of the Casimir force in a real situation.

The consideration of real mirrors is important not only for the analysis of experiments but also from a conceptual point of view. Real mirrors are certainly transparent at the limit of high frequencies and this allows one to dispose of the divergences associated with the infiniteness of vacuum energy. This point was already alluded to in Casimir's papers and an important step in this direction was the Lifshitz theory of the Casimir force between two dielectric bulks [8, 9]. However this idea was fully implemented in theoretical derivations after a quite long period.

In the limit of small separations  $L \ll \lambda_{\rm P}$ , the Casimir effect has another interpretation establishing a bridge between quantum field theory of vacuum fluctuations and condensed matter theory of forces between two metallic bulks. It can indeed be understood as resulting from the Coulomb interaction between surface plasmons, that is the collective electron excitations propagating on the interface between each bulk and the intracavity vacuum [10, 11]. The corresponding field modes are evanescent waves and have an imaginary longitudinal wavevector. At short distances, surface plasmon modes are known to dominate the interaction and the Casimir energy reduces to [12, 13]

$$E \approx E_{\rm pl} = \frac{3}{2} \alpha \frac{L}{\lambda_{\rm P}}$$
 with  $\alpha = 1.193...$  (2)

Surface plasmons play an important role in many fields of physics. Let us only mention as recent examples the surface plasmon assisted enhancement of the transmission of light through metallic structures [14, 15, 16] or in the context of biomolecular physics, the importance of plasmon fluctuations of a 2-dimensional Wigner-like crystal where they can generate an attractive component to the dispersion forces between parallel surfaces [17].

Here, we derive and expand in more detail the results previously given in Ref. [18], investigating more closely the influence of surface plasmons on the Casimir energy, not only at short but at arbitrary distances.

### 2. Casimir energy for real mirrors

We restrict our attention to the situation of two infinitely large plane mirrors at zero temperature so that the only modification of the Casimir formula (1) is due to the metals finite conductivity. This modification is calculated by evaluating the radiation pressure of vacuum fields upon the two mirrors [19]

$$E = -\sum_{\epsilon} \sum_{\mathbf{k}} \sum_{\omega} \frac{i\hbar}{2} \ln(1 - r_{\mathbf{k}}^{p}[\omega]^{2} e^{2ik_{z}L}) + c.c.$$
(3)  
$$\sum_{\mathbf{k}} \equiv A \int \frac{\mathrm{d}^{2}\mathbf{k}}{4\pi^{2}} \quad , \quad \sum_{\omega} \equiv \int_{0}^{\infty} \frac{\mathrm{d}\omega}{2\pi}$$

The energy E is obtained by summing over polarization p=(TE,TM), transverse wavevector  $\mathbf{k} \equiv (k_x, k_y)$  (with z the longitudinal axis of the cavity) and frequency  $\omega$ ;  $k_z$  is the longitudinal wavevector associated with the mode.  $r_{\mathbf{k}}^p$  are the reflection amplitudes here supposed to be the same for the two mirrors.

Imperfectly reflecting mirrors will be described by scattering amplitudes which depend on the frequency, wavevector and polarization while obeying general properties of stability, high-frequency transparency and causality. The two mirrors

form a Fabry-Perot cavity with the consequences well-known in classical or quantum optics : the energy density of the intracavity field is increased for the resonant frequency components whereas it is decreased for the non resonant ones. The Casimir force is but the result of the balance between the radiation pressure of the resonant and non resonant modes which push the mirrors respectively towards the outer and inner sides of the cavity [20]. This balance includes not only the contributions of ordinary waves propagating freely outside the cavity but also that of evanescent waves. These two sectors of ordinary and evanescent waves are directly connected by analyticity properties of the scattering amplitudes.

Expression (3) holds for dissipative mirrors as well as for non dissipative ones [19]. It tends towards the ideal Casimir formula (1) as soon as the mirrors are nearly perfect for the modes contributing to the integral.

The reduction of the Casimir energy (3) with respect to the ideal formula (1) due to the imperfect reflection of mirrors is described by a factor

$$\eta_{\rm E} = \frac{E}{E_{\rm Cas}} \tag{4}$$

This factor plays an important role in the discussion of the most precise recent experiments.

## 3. Mode decomposition of the Casimir energy

We now recalculate the Casimir energy as a sum over the cavity modes using the plasma model for the mirrors dielectric function.

$$\varepsilon(\omega) = 1 - \frac{\omega_{\rm P}^2}{\omega^2} , \qquad \lambda_{\rm P} = \frac{2\pi c}{\omega_{\rm P}}$$
(5)

with  $\omega_{\rm P}$  the plasma frequency and  $\lambda_{\rm P}$  the plasma wavelength.

In this case the zeros of the argument of the integrand in (3) lie on the real axis. In fact, they have to be pushed slightly below this axis by introducing a vanishing dissipation parameter in order to avoid any ambiguity in expression (3) [19]. We may then rewrite (3) as a sum over the solutions  $[\omega_{\mathbf{k}}^{p}]_{m}$  of the equation labeled by an integer index m

$$r^p_{\mathbf{k}}[\omega]^2 e^{2ik_z L} = 1. \tag{6}$$

Simple algebraic manipulations exploiting residues theorem and complex integration techniques [11] then lead to the Casimir energy expressed as sums over these modes

$$E = \sum_{p,\mathbf{k}} \left[ \sum_{m}^{\prime} \frac{\hbar \left[ \omega_{\mathbf{k}}^{p} \right]_{m}}{2} \right]_{L \to \infty}^{L}.$$
(7)

The prime in the sum over m signifies as usually that the term m = 0 has to be multiplied by 1/2. The sum over the modes is to be understood as a regularized quantity as it involves infinite quantities. The upper expression contains as limiting cases at large distances the Casimir expression with perfect mirrors and at short distances the expression in terms of surface plasmon resonances (2).

The ensemble of modes appearing in Eqn. (7) can be separated into two different ensembles. The TM polarization admits propagating cavity modes as well as evanescent modes, while the TE polarization allows only propagating cavity modes. The first ensemble contains two modes  $\omega_+$  and  $\omega_-$  which tend to the usual surface plasmon modes at short distances.  $\omega_{-}$  is always in the evanescent sector, while  $\omega_+$  lies either in the evanescent or in the propagating sector depending on its parameters as shown in Fig. 2. The second ensemble are propagating cavity modes which may have TE or TM polarization. In view of the particular behavior of the  $\omega_+$  mode, we adopt an adiabatic mode definition, where we follow it continuously from the evanescent into the propagative sector and attribute the whole mode to the surface plasmon modes, which we call "plasmonic modes" [18]. This denomination is chosen in order to avoid confusion with the (common) definition of surface plasmon modes which defines them as evanescent modes only and cuts the  $\omega_+$  mode into two pieces. Plasmonic modes are thus the cavity modes living in the evanescent sector at least for some particular value of their parameters. In the same line of reasoning we call all propagative modes minus the propagative part of the  $\omega_+$  mode "photonic" modes. Photonic modes are the cavity modes propagating for all cavity length. In the limit  $L \to \infty$ , all propagative modes tend asymptotically to the eigenmodes of the perfect cavity. The frequencies of the plasmonic modes  $\omega_+$  and  $\omega_-$  degenerate in the limit  $L \to \infty$  to the surface plasmon frequency  $\omega_0$  for a single interface [21]

$$\omega_{\pm} \xrightarrow{L \to \infty} \omega_0[\mathbf{k}] = \sqrt{\frac{\omega_P^2 + 2|\mathbf{k}|^2 - \sqrt{\omega_P^4 + 4|\mathbf{k}|^4}}{2}} \tag{8}$$

The Casimir energy can now be rewritten as

$$E = \sum_{p,\mathbf{k}} \left[ \sum_{n} \frac{\hbar \omega_{n}^{p}}{2} \right]_{L \to \infty}^{L} = \underbrace{\sum_{\mathbf{k}} \left[ \frac{\hbar \omega_{+}}{2} + \frac{\hbar \omega_{-}}{2} \right]_{L \to \infty}^{L}}_{\text{plasmonic Contribution } (E_{\text{pl}})} + \underbrace{\sum_{p,\mathbf{k}} \left[ \sum_{n} \frac{\hbar \omega_{n}^{p}}{2} \right]_{L \to \infty}^{L}}_{\text{photonic Contribution } (E_{\text{ph}})}$$
(9)

Note that both contributions have no physical meaning on their own, i.e. one cannot measure them separately. The only observable is the total Casimir energy which is the sum of both contributions. We may rewrite the plasmonic contribution to the Casimir energy in a more explicit way as follows

$$E_{\rm pl} = cA \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \frac{\hbar}{2} \left( \omega_+[\mathbf{k}, L] + \omega_-[\mathbf{k}, L] - 2\omega_0[\mathbf{k}] \right) \tag{10}$$



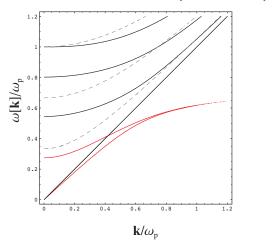


Figure 2: A representation of the dispersion relation for the TM-modes with the plasma model for  $L/\lambda_{\rm P} =$ 1.5. The dashed curves represent the perfect cavity modes. The solid grey curves represent the plasmonic modes and the black curves the photonic modes. The  $\omega_+$ mode changes its nature when crossing the light-dispersion curve.

The basic idea of the explicit calculation of the plasmonic contribution to the Casimir energy resides in the fact that the frequencies functions  $\omega_i$ ,  $i = 0, \pm$  are solutions of simple equations. We rewrite Eq.(10) in terms of dimensionless variables

$$E_{\rm pl} = \eta_{\rm pl} E_{\rm Cas}, \quad \eta_{\rm pl} = -\frac{180}{\pi^3} \int_0^\infty \sum_i c_i k \ \Omega_i[k] d\mathbf{k}$$
(11)

$$\Omega = \omega L, \quad \Omega_{\rm P} = \omega_{\rm P} L, \quad |\mathbf{k}| \, L = k, \quad z = k^2 - \Omega^2 \tag{12}$$

with  $c_{+} = c_{-} = 1, c_{0} = -2$ .  $\Omega_{0}$  is the dimensionless surface plasmon frequency for a single mirror and we have introduced the corrective factor  $\eta_{\rm pl}$  for the plasmonic contribution to the Casimir energy. Note that

$$\eta_i = -\frac{180}{\pi^3} \int_0^\infty k \ \Omega_i[k] dk, \quad i = \pm, 0$$
(13)

is divergent despite the convergence of the whole expression given in Eq. (10). Without giving any details let us just mention that in order to perform the explicit calculation we need to introduce a regularizing factor. Such a modification is mathematically mathematical convenient and does not affect the final result.

It can be shown that the dimensionless frequencies  $\Omega_i[k]$  can be formally obtained as

$$k^{2} = f_{i}(z) \Rightarrow \Omega_{i}[k] = \sqrt{k^{2} - f_{i}^{-1}[k^{2}]}, \quad i = 0, \pm$$
 (14)

where

$$f_{+}(z) = z + \frac{\Omega_{p}^{2} \sqrt{z}}{\sqrt{z} + \sqrt{z + \Omega_{P}^{2}} \tanh[\frac{\sqrt{z}}{2}]} = z + g_{+}^{2}[z]$$
(15a)

$$f_{-}(z) = z + \frac{\Omega_{p}^{2} \sqrt{z}}{\sqrt{z} + \sqrt{z + \Omega_{P}^{2}} \coth[\frac{\sqrt{z}}{2}]} = z + g_{-}^{2}[z]$$
(15b)

$$f_0(z) = z + \frac{\Omega_p^2 \sqrt{z}}{\sqrt{z} + \sqrt{z + \Omega_P^2}} = z + g_0^2[z]$$
(15c)

Let us also define

$$y_i^2 = z_i^0 = -f_i^{-1}[0] = \Omega_i^2[0]$$
(16)

With the change of variable  $k^2 = z + g_i^2(z)$  and after some rearrangements Eqn. (11) can be rewritten as

$$\eta_{pl} = -\frac{180}{2\pi^3} \left[ \int_0^\infty \sum_i c_i g_i(z) dz + \int_{-z_+^0}^0 g_+(z) dz - \frac{2}{3} y_+^3 \right]$$
(17)

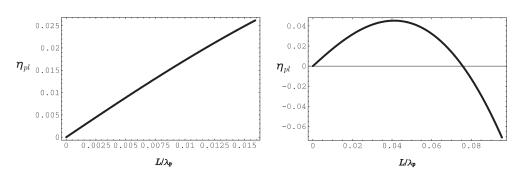
where we have exploited the fact that

$$z_{+}^{0} \neq z_{-}^{0} = z_{0}^{0} = 0, \quad f_{i}^{-1}[\infty] = \infty$$
$$f_{i}(-z_{i}^{0}) = 0 \Rightarrow g_{i}(-z_{i}^{0}) = \sqrt{z_{i}^{0}} \quad \text{and} \quad g_{i}(\infty) = \frac{\Omega_{\mathrm{P}}}{\sqrt{2}}$$

The corrective factor  $\eta_{\rm pl}$  has a well defined structure: it is indeed decomposed into an integral over the positive real z-axis plus an integral over an interval of the negative z-axis plus a constant depending only on  $\Omega_{\rm P}$ . Moreover only  $g_+$ , is involved in the last integral and in the constant. This particular structure can be traced back to the properties of the plasmonic modes. The positive zvalue domain coincides with the evanescent sector while the negative one describes the propagative sector. While the plasmonic mode  $\omega_-$  and the surface plasmon frequency for a single mirror  $\omega_0$  are totally contained in the evanescent sector, the plasmonic mode  $\omega_+$  lives in both sectors. Therefore  $g_0$  and  $g_-$  describing the properties of  $\omega_-$  and  $\omega_0$  are contained only in the first integral while  $g_+$  has to be evaluated in a wider range of z-values which includes an interval in the propagative part contribution of the plasmonic mode  $\omega_+$ .

Fig. 3 shows the numerical evaluation of Eqn. (17) for  $\eta_{\rm pl}$  as function of  $L/\lambda_{\rm P}$  for two different distance intervals. The left graphic illustrates the short distance behavior In the limit  $\Omega_{\rm P} \ll 1$  the corrective factor  $\eta_{\rm pl}$  can be approximated by the first integral of Eq. (17). leading to





**Figure 3:** The normalized plasmonic mode contribution to the Casimir force as function of  $L/\lambda_{\rm P}$  for two different distance intervals.

$$\eta_{\rm pl} \approx -\frac{180}{2\pi^3} \frac{\Omega_{\rm P}}{\sqrt{2}} \int_0^\infty \left(\sqrt{1 + e^{-\sqrt{z}}} + \sqrt{1 - e^{-\sqrt{z}}} - 2\right) dz = \frac{3}{2} \alpha \frac{L}{\lambda_{\rm P}} \tag{18}$$

We recover here the result of Eqn. (2). The right graphic in Fig. 3 shows the plasmonic mode contribution  $\eta_{\rm pl}$  at large distances. Surprisingly, it changes its sign for  $\frac{L}{\lambda_{\rm P}} \sim 0.08$  and its slope and diverges for  $L \gg \lambda_{\rm P}$  [18]. In the large distance limit  $\Omega_{\rm P} \gg 1$  we find the following asymptotic behavior

$$\eta_{\rm pl} \approx -\Gamma \sqrt{\Omega_{\rm P}}, \qquad \Gamma = 29.752$$
(19)

The contribution of the plasmonic modes to the Casimir energy becomes thus repulsive for intermediate and large distances. This result is based on an adiabatic definition of the surface plasmon modes, where we follow the  $\omega_+$  mode even when it crosses the barrier  $\omega = ck$  and becomes propagative.

Let us now compare this result to the common definition of surface plasmon modes which includes only evanescent waves and cuts the  $\omega_+$  mode at  $\omega = ck$  as for example done by Bordag recently [22]. This leads to

$$\eta_{\rm ev} = -\frac{180}{\pi^3} \int_{k_{\rm P}}^{\infty} k \left(\Omega_+[k] - \Omega_0[k]\right) dk - \frac{180}{\pi^3} \int_0^{\infty} k \left(\Omega_-[k] - \Omega_0[k]\right) dk \tag{20}$$

where

$$k_{\rm P} = g_+(0) = \Omega_{\rm P} / \sqrt{1 + \Omega_{\rm P} / 2}$$
 (21)

is associated with the value of  $|\mathbf{k}|$  for which this modes crosses the light cone.  $\eta_{ev}$  can be evaluated numerically [22], or using the same method developed here

$$\eta_{\rm ev} = -\frac{180}{2\pi^3} \left( \int_0^\infty \sum_i c_i g_i(z) dz - \int_{-z_0^P}^0 g_0(z) dz - \frac{2}{3} \left( k_{\rm P}^3 - \Omega_0^3[k_{\rm P}] \right) \right)$$
(22)

where we have exploited the following relations

$$-z_0^P = f_0^{-1}[k_P] = k_P^2 - \Omega_0^2[k_P] \Rightarrow g_0(-z_0^P) = \Omega_0[k_P]$$
(23)

Eq. (22) shows the short distance asymptotic behavior of the total Casimir energy given in (2) as it was reported in [12, 13]. It does not change its sign and, in agreement with [22], at long distances goes as

$$\eta_{\rm ev} = \beta_{\rm ev} \sqrt{\Omega_{\rm P}} \quad \text{with} \quad \beta_{\rm ev} = 1.62399...$$
 (24)

With this definition we therefore naturally find the result that the contribution of the evanescent modes to the Casimir energy is always attractive and reproduces well the short distance behavior of the Casimir energy.

### 4. Conclusion

As a concluding remark we would like to stress that, no matter how we attribute the propagative part of the  $\omega_+$  mode, whether to the surface plasmon modes in an adiabatic definition (plasmonic modes) or to the propagating modes, the influence of surface plasmons is very important at all distances. The Casimir energy is the detailed balance of two very large contributions of opposite sign which nearly cancel each other, but not quite, the difference being a small Casimir energy, much smaller than each of both contributions.

It might be interesting to investigate if a change in the photon-plasmon coupling could somehow influence this detailed balance and therefore the value or even the sign of the Casimir force. A different coupling could be obtained by using for example nanostructured surfaces. For such an analysis the adiabatic mode definition should be well suited, because for a different coupling the mode will change *as a whole* and in following continuously the mode one could trace back the changes introduced through the structured surface.

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