# $N$-qubit Quantum Teleportation, Information Splitting and Superdense Coding through the composite GHZ-Bell Channel 

Debashis Saha ${ }^{\dagger}$ and Prasanta K. Panigrahi*<br>Indian Institute of Science Education and Research (IISER) Kolkata, Mohanpur Campus<br>BCKV Campus Main Office, Mohanpur - 741252, Nadia, West Bengal, India


#### Abstract

We introduce a general odd qubit entangled system composed of GHZ and Bell pairs and explicate its usefulness for quantum teleportation, information splitting and superdense coding. After demonstrating the superdense coding protocol on the five qubit system, we prove that ' $2 N+1$ ' classical bits can be sent by sending ' $N+1$ ' quantum bits using this channel. It is found that the five-qubit system is also ideal for arbitrary one qubit and two qubit teleportation and quantum information splitting (QIS). For the single qubit QIS, three different protocols are feasible, whereas for the two qubit QIS, only one protocol exists. Protocols for the arbitrary N -qubit state teleportation and quantum information splitting are then illustrated.


## I. INTRODUCTION

Entanglement is the fascinating aspect of quantum mechanics, due to which non-intuitive quantum correlations can exist between two or more particles [1]. Using entanglement, one can perform different types of quantum tasks like teleportation [2], superdense coding [3], secret sharing [4,5], quantum cryptography, one-way quantum computation [6] etc. Teleportation is a unique quantum procedure that allows one party to send a quantum state to another, without the state being physically transmitted. Teleportation of arbitrary single qubit, through an entangled channel of Einstein-Podolsky-Rosen [EPR] pair between the sender and receiver, was first demonstrated by Bennett et al., [2]. Experimentally it has been achieved using different quantum systems, inside and outside laboratory conditions [7-11]. Another related protocol is superdense coding, which allows one to send two classical bits of information encoded in a single qubit using a bipartite entangled channel [3]. 'Quantum information splitting' is a quantum state sharing protocol, where sharing of quantum information can be done between a group of parties, such that none of them can reconstruct the unknown information, without knowing the others' measurement outcomes. This was first demonstrated for a single qubit state, using three qubit Greenberger-Horne-Zeilinger (GHZ) state, with three agents having one qubit each [12]. Experimental realization of QIS has been achieved using several states [13-15] and single photon sources [14].

A number of multipartite entangled states like the GHZ states [16], modified W states [17,18], magnon state [19], mirror state [20] and the cluster states [21,22] have been introduced and exploited for carrying out several quantum tasks. Recently, a genuinely entangled five-qubit state [23] and a six-qubit state [24] have been used for successfully carrying out many quantum communication protocols [25,26]. Superdense coding has been done using combined Bell states, where ' $2 N$ ' bits of classical information is transmitted via ' $N$ ' qubits [27]. $N$-qubit teleportation has been discussed in several cases using different entangled quantum channels [28,29]. Also, a number of QIS protocols have been recently implemented [30-32].

In this paper, we demonstrate the utility of a quantum channel, composed of GHZ and Bell pairs for various quantum communication protocols. It has been shown that, some of the Bell states are decoherence free under certain environment [33]. Being the superposition of two terms, the GHZ state can be less prone to decoherence. This fact gives our primary motivation for studying such state. Before explicating those protocols, we investigate the nature of entanglement between the subparties of the five-qubit composite GHZ-Bell state:

$$
\begin{equation*}
|\zeta\rangle=\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle) \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)=\frac{1}{2}(|00000\rangle+|00011\rangle+|11100\rangle+|11111\rangle) \tag{1}
\end{equation*}
$$

* electronic address : pprasanta@iiserkol.ac.in
$\dagger$ electronic address: debashis7112@iiserkol.ac.in

Consider $|\zeta\rangle$ as $\frac{1}{2}\left(\left|0_{A} 0_{A} 0_{B}\right\rangle+\left|1_{A} 1_{A} 1_{B}\right\rangle\right)\left(\left|0_{A} 0_{B}\right\rangle+\left|1_{A} 1_{B}\right\rangle\right)$, where Alice has three qubits and Bob has two qubits. The reduced density matrix obtained by partial tracing over the sub-system ' A ' is given by,

$$
\rho_{B}=\operatorname{Tr}_{A}|\zeta\rangle\langle\zeta|=\frac{1}{4}\left(|00\rangle_{B}\left\langle\left. 00\right|_{B}+\mid 01\right\rangle_{B}\left\langle\left. 01\right|_{B}+\mid 10\right\rangle_{B}\left\langle\left. 10\right|_{B}+\mid 11\right\rangle_{B}\left\langle\left. 11\right|_{B}\right)=\frac{1}{4}\left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right)\right. \text { which is maxi- }
$$

mally mixed. For another inequivalent distribution, $|\zeta\rangle=\frac{1}{2}\left(\left|0_{A} 0_{A} 0_{A}\right\rangle+\left|1_{A} 1_{A} 1_{A}\right\rangle\right)\left(\left|0_{A} 0_{B}\right\rangle+\left|1_{A} 1_{B}\right\rangle\right)$, the reduced density matrix is given by, $\rho_{B}=\operatorname{Tr}_{A}|\zeta\rangle\langle\zeta|=\frac{1}{4}\left(|0\rangle_{B}\left\langle\left. 0\right|_{B}+\mid 1\right\rangle_{B}\left\langle\left. 1\right|_{B}\right)=\frac{1}{4}\left(\begin{array}{cc}1 & 0 \\ 0 & 1\end{array}\right)\right.$. Hence, the state $|\zeta\rangle$ has the maximum possible entanglement between two subsystem satisfying the general condition for teleportation [34]. In the following section, we first show the superdense coding of five bits of information using this state and subsequently generalize it to ' $2 N+1$ ' bits of information. In the section III, we explicate arbitrary one, two and $N$-qubit state teleportation using composite GHZ-Bell measurement. The next section deals with QIS of one and two-qubit states using different protocols and show that this channel can also be used for $N$-qubit QIS. We then conclude in the last section, with several directions for future work. We now procced to explicate various protocols starting with superdense coding.

## II. SUPERDENSE CODING

We start with the protocol of superdense coding of five classical bit, through the above mentioned GHZ-Bell channel: $|\zeta\rangle=\frac{1}{2}\left(\left|0_{A} 0_{A} 0_{B}\right\rangle+\left|1_{A} 1_{A} 1_{B}\right\rangle\right)\left(\left|0_{A} 0_{B}\right\rangle+\left|1_{A} 1_{B}\right\rangle\right)$. Alice and Bob know the correspondence of quantum state and classical bit string. Alice can transform the state $|\zeta\rangle$ into 32 entangled orthonormal states by applying Paulli matrices for encoding classical bits. Depending on which bit string Alice wants to send to Bob, she has to perform an appropriate unitary operation on her qubits. Alice then sends her qubits to Bob and by making a measurement using the orthonormal states given in Table I, Bob successfully gets the five bits of classical information. In Table I, we have shown that all the orthonormal states can be written as product of the following states:
$\left|\xi^{ \pm}\right\rangle=\frac{1}{\sqrt{2}}(|000\rangle \pm|111\rangle),\left|\chi^{ \pm}\right\rangle=\frac{1}{\sqrt{2}}(|011\rangle \pm|100\rangle),\left|\vartheta^{ \pm}\right\rangle=\frac{1}{\sqrt{2}}(|010\rangle \pm|101\rangle),\left|\theta^{ \pm}\right\rangle=\frac{1}{\sqrt{2}}(|001\rangle \pm|110\rangle)$,
$\left|\psi^{ \pm}\right\rangle=\frac{1}{\sqrt{2}}(|00\rangle \pm|11\rangle),\left|\phi^{ \pm}\right\rangle=\frac{1}{\sqrt{2}}(|01\rangle \pm|10\rangle)$.
Hereforth, we use $I$ as the identity matrix, $X$ as $\sigma_{x}, Z$ as $\sigma_{z}$ and $U_{i}$ denotes the operator, where $U$ acts on the $i$-th qubit from the left hand side.

## Generalized superdense coding of ' $2 N+1$ ' bits of information

This superdense coding can be generalized to ( $2 N+1$ )-qubit channel. For transmiting ' $2 N+1$ ' bits of information, Alice and Bob share a multipartite entangled state, which is the product of one $\left|\xi^{+}\right\rangle$and $(N-1)$ number of $\left|\psi^{+}\right\rangle$ states such that Alice possesses ' $N+1$ ' qubits and Bob possesses ' $N$ ' qubits:

$$
\begin{equation*}
\left|\zeta_{0}\right\rangle=\left|\xi^{+}\right\rangle_{A B}\left|\psi^{+}\right\rangle_{A B} \ldots \ldots \ldots\left|\psi^{+}\right\rangle_{A B} \tag{3}
\end{equation*}
$$

where $\quad\left|\xi^{+}\right\rangle_{A B}=\frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{A} 0_{B}\right\rangle+\left|1_{A} 1_{A} 1_{B}\right\rangle\right)$ and $\left|\psi^{+}\right\rangle_{A B}=\frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{B}\right\rangle+\left|1_{A} 1_{B}\right\rangle\right)$. The capacity of superdense coding of this channel is $2 N+1$, satisfying 'Holevo bound' [35]. Alice wants to send a classical bit string, which is $a_{2 N+1} a_{2 N} \ldots \ldots . a_{2} a_{1}\left(a_{i}=0\right.$ or 1$)$ and the corresponding quantum state is $\left|\zeta_{j}\right\rangle$, where ' $j$ ' is the decimal representation of $a_{2 N+1} a_{2 N} \ldots \ldots . a_{2} a_{1}$. Here, ' $j$ ' can take value from 0 to $2^{2 N+1}-1$. The unitary operations performed by Alice to obtain the quantum state $\left|\zeta_{j}\right\rangle$ is given by,
$\left|\zeta_{j}\right\rangle=\otimes_{k=1}^{N}\left(Z_{2 k}\right)^{a_{k}} \otimes_{l=1}^{N+1}\left(X_{|2 l-2|}\right)^{a_{l+N}}\left|\zeta_{0}\right\rangle$.
It is noted that, $I$ is referred as $Z^{0}$ or $X^{0}$. All $\left|\zeta_{j}\right\rangle$ 's are mutually orthonormal and can be written as products of $\left|\xi^{ \pm}\right\rangle_{A B},\left|\chi^{ \pm}\right\rangle_{A B},\left|\vartheta^{ \pm}\right\rangle_{A B},\left|\theta^{ \pm}\right\rangle_{A B},\left|\psi^{ \pm}\right\rangle_{A B},\left|\phi^{ \pm}\right\rangle_{A B}$ states. The subscript $A B$ denotes that the last particle of that state belongs to Bob and others belong to Alice. After applying the unitary operation, Alice has to send her ' $N+1$ ' qubits to Bob and he will be able to get ' $2 N+1$ ' bits of information by making a measurement in the $\left\{\left|\zeta_{j}\right\rangle\right\}$ basis, which satisfy the superdense protocol perfectly.

TABLE I: The unitary operations performed by Alice on her qubits and the resulting 32 orthonormal states $\left|\zeta_{j}\right\rangle$ ( $j=0,1, . ., 31$ ) with their corresponding classical bit strings.

| Classical bit | Unitary operation | State obtained | Decomposition of the state |
| :---: | :---: | :---: | :---: |
| 00000 | $I$ | $\left\|\zeta_{0}\right\rangle=\frac{1}{2}(\|00000\rangle+\|00011\rangle+\|11100\rangle+\|11111\rangle)$ | $\left\|\xi^{+}\right\rangle_{A B}\left\|\psi^{+}\right\rangle_{A B}$ |
| 00001 | $Z_{2}$ | $\left\|\zeta_{1}\right\rangle=\frac{1}{2}(\|00000\rangle+\|00011\rangle-\|11100\rangle-\|11111\rangle)$ | $\left\|\xi^{-}\right\rangle_{A B}\left\|\psi^{+}\right\rangle_{A B}$ |
| 00010 | $Z_{3}$ | $\left\|\zeta_{2}\right\rangle=\frac{1}{2}(\|00000\rangle-\|00011\rangle+\|11100\rangle-\|11111\rangle)$ | $\left\|\xi^{+}\right\rangle_{A B}\left\|\psi^{-}\right\rangle_{A B}$ |
| 00011 | $Z_{2} Z_{3}$ | $\left\|\zeta_{3}\right\rangle=\frac{1}{2}(\|00000\rangle-\|00011\rangle-\|11100\rangle+\|11111\rangle)$ | $\left\|\xi^{-}\right\rangle_{A B}\left\|\psi^{-}\right\rangle_{A B}$ |
| 00100 | $X_{1}$ | $\left\|\zeta_{4}\right\rangle=\frac{1}{2}(\|01100\rangle+\|01111\rangle+\|10000\rangle+\|10011\rangle)$ | $\left\|\chi^{+}\right\rangle_{A B}\left\|\psi^{+}\right\rangle_{A B}$ |
| 00101 | $Z_{2} X_{1}$ | $\left\|\zeta_{5}\right\rangle=\frac{1}{2}(\|01100\rangle+\|01111\rangle-\|10000\rangle-\|10011\rangle)$ | $\left\|\chi^{-}\right\rangle_{A B}\left\|\psi^{+}\right\rangle_{A B}$ |
| 00110 | $Z_{3} X_{1}$ | $\left\|\zeta_{6}\right\rangle=\frac{1}{2}(\|01100\rangle-\|01111\rangle+\|10000\rangle-\|10011\rangle)$ | $\left\|\chi^{+}\right\rangle_{A B}\left\|\psi^{-}\right\rangle_{A B}$ |
| 00111 | $Z_{2} Z_{3} X_{1}$ | $\left\|\zeta_{7}\right\rangle=\frac{1}{2}(\|01100\rangle-\|01111\rangle-\|10000\rangle+\|10011\rangle)$ | $\left\|\chi^{-}\right\rangle_{A B}\left\|\psi^{-}\right\rangle_{A B}$ |
| 01000 | $X_{2}$ | $\left\|\zeta_{8}\right\rangle=\frac{1}{2}(\|01000\rangle+\|01011\rangle+\|10100\rangle+\|10111\rangle)$ | $\left\|\vartheta^{+}\right\rangle_{A B}\left\|\psi^{+}\right\rangle_{A B}$ |
| 01001 | $Z_{2} X_{2}$ | $\left\|\zeta_{9}\right\rangle=\frac{1}{2}(\|01000\rangle+\|01011\rangle-\|10100\rangle-\|10111\rangle)$ | $\left\|\vartheta^{-}\right\rangle_{A B}\left\|\psi^{+}\right\rangle_{A B}$ |
| 01010 | $Z_{3} X_{2}$ | $\left\|\zeta_{10}\right\rangle=\frac{1}{2}(\|01000\rangle-\|01011\rangle+\|10100\rangle-\|10111\rangle)$ | $\left\|\vartheta^{+}\right\rangle_{A B}\left\|\psi^{-}\right\rangle_{A B}$ |
| 01011 | $Z_{2} Z_{3} X_{2}$ | $\left\|\zeta_{11}\right\rangle=\frac{1}{2}(\|01000\rangle-\|01011\rangle-\|10100\rangle+\|10111\rangle)$ | $\left\|\vartheta^{-}\right\rangle_{A B}\left\|\psi^{-}\right\rangle_{A B}$ |
| 01100 | $X_{1} X_{2}$ | $\left\|\zeta_{12}\right\rangle=\frac{1}{2}(\|00100\rangle+\|00111\rangle+\|11000\rangle+\|11011\rangle)$ | $\left\|\theta^{+}\right\rangle_{A B}\left\|\psi^{+}\right\rangle_{A B}$ |
| 01101 | $Z_{2} X_{1} X_{2}$ | $\left\|\zeta_{13}\right\rangle=\frac{1}{2}(\|00100\rangle+\|00111\rangle-\|11000\rangle-\|11011\rangle)$ | $\left\|\theta^{-}\right\rangle_{A B}\left\|\psi^{+}\right\rangle_{A B}$ |
| 01110 | $Z_{3} X_{1} X_{2}$ | $\left\|\zeta_{14}\right\rangle=\frac{1}{2}(\|00100\rangle-\|00111\rangle+\|11000\rangle-\|11011\rangle)$ | $\left\|\theta^{+}\right\rangle_{A B}\left\|\psi^{-}\right\rangle_{A B}$ |
| 01111 | $Z_{2} Z_{3} X_{1} X_{2}$ | $\left\|\zeta_{15}\right\rangle=\frac{1}{2}(\|00100\rangle-\|00111\rangle-\|11000\rangle+\|11011\rangle)$ | $\left\|\theta^{-}\right\rangle_{A B}\left\|\psi^{-}\right\rangle_{A B}$ |
| 10000 | $X_{3}$ | $\left\|\zeta_{16}\right\rangle=\frac{1}{2}(\|00001\rangle+\|00010\rangle+\|11101\rangle+\|11110\rangle)$ | $\left\|\xi^{+}\right\rangle_{A B}\left\|\phi^{+}\right\rangle_{A B}$ |
| 10001 | $Z_{2} X_{3}$ | $\left\|\zeta_{17}\right\rangle=\frac{1}{2}(\|00001\rangle+\|00010\rangle-\|11101\rangle-\|11110\rangle)$ | $\left\|\xi^{-}\right\rangle_{A B}\left\|\phi^{+}\right\rangle_{A B}$ |
| 10010 | $Z_{3} X_{3}$ | $\left\|\zeta_{18}\right\rangle=\frac{1}{2}(\|00001\rangle-\|00010\rangle+\|11101\rangle-\|11110\rangle)$ | $\left\|\xi^{+}\right\rangle_{A B}\left\|\phi^{-}\right\rangle_{A B}$ |
| 10011 | $Z_{2} Z_{3} X_{3}$ | $\left\|\zeta_{19}\right\rangle=\frac{1}{2}(\|00001\rangle-\|00010\rangle-\|11101\rangle+\|11110\rangle)$ | $\left\|\xi^{-}\right\rangle_{A B}\left\|\phi^{-}\right\rangle_{A B}$ |
| 10100 | $X_{1} X_{3}$ | $\left\|\zeta_{20}\right\rangle=\frac{1}{2}(\|01101\rangle+\|01110\rangle+\|10001\rangle+\|10010\rangle)$ | $\left\|\chi^{+}\right\rangle_{A B}\left\|\phi^{+}\right\rangle_{A B}$ |
| 10101 | $Z_{2} X_{1} X_{3}$ | $\left\|\zeta_{21}\right\rangle=\frac{1}{2}(\|01101\rangle+\|01110\rangle-\|10001\rangle-\|10010\rangle)$ | $\left\|\chi^{-}\right\rangle_{A B}\left\|\phi^{+}\right\rangle_{A B}$ |
| 10110 | $Z_{3} X_{1} X_{3}$ | $\left\|\zeta_{22}\right\rangle=\frac{1}{2}(\|01101\rangle-\|01110\rangle+\|10001\rangle-\|10010\rangle)$ | $\left\|\chi^{+}\right\rangle_{A B}\left\|\phi^{-}\right\rangle_{A B}$ |
| 10111 | $Z_{2} Z_{3} X_{1} X_{3}$ | $\left\|\zeta_{23}\right\rangle=\frac{1}{2}(\|01101\rangle-\|01110\rangle-\|10001\rangle+\|10010\rangle)$ | $\left\|\chi^{-}\right\rangle_{A B}\left\|\phi^{-}\right\rangle_{A B}$ |
| 11000 | $X_{2} X_{3}$ | $\left\|\zeta_{24}\right\rangle=\frac{1}{2}(\|01001\rangle+\|01010\rangle+\|10101\rangle+\|10110\rangle)$ | $\left\|\vartheta^{+}\right\rangle_{A B}\left\|\phi^{+}\right\rangle_{A B}$ |
| 11001 | $Z_{2} X_{2} X_{3}$ | $\left\|\zeta_{25}\right\rangle=\frac{1}{2}(\|01001\rangle+\|01010\rangle-\|10101\rangle-\|10110\rangle)$ | $\left\|\vartheta^{-}\right\rangle_{A B}\left\|\phi^{+}\right\rangle_{A B}$ |
| 11010 | $Z_{3} X_{2} X_{3}$ | $\left\|\zeta_{26}\right\rangle=\frac{1}{2}(\|01001\rangle-\|01010\rangle+\|10101\rangle-\|10110\rangle)$ | $\left\|\vartheta^{+}\right\rangle_{A B}\left\|\phi^{-}\right\rangle_{A B}$ |
| 11011 | $Z_{2} Z_{3} X_{2} X_{3}$ | $\left\|\zeta_{27}\right\rangle=\frac{1}{2}(\|01001\rangle-\|01010\rangle-\|10101\rangle+\|10110\rangle)$ | $\left\|\vartheta^{-}\right\rangle_{A B}\left\|\phi^{-}\right\rangle_{A B}$ |
| 11100 | $X_{1} X_{2} X_{3}$ | $\left\|\zeta_{28}\right\rangle=\frac{1}{2}(\|00101\rangle+\|00110\rangle+\|11001\rangle+\|11010\rangle)$ | $\left\|\theta^{+}\right\rangle_{A B}\left\|\phi^{+}\right\rangle_{A B}$ |
| 11101 | $Z_{2} X_{1} X_{2} X_{3}$ | $\left\|\zeta_{29}\right\rangle=\frac{1}{2}(\|00101\rangle+\|00110\rangle-\|11001\rangle-\|11010\rangle)$ | $\left\|\theta^{-}\right\rangle_{A B}\left\|\phi^{+}\right\rangle_{A B}$ |
| 11110 | $Z_{3} X_{1} X_{2} X_{3}$ | $\left\|\zeta_{30}\right\rangle=\frac{1}{2}(\|00101\rangle-\|00110\rangle+\|11001\rangle-\|11010\rangle)$ | $\left\|\theta^{+}\right\rangle_{A B}\left\|\phi^{-}\right\rangle_{A B}$ |
| 11111 | $Z_{2} Z_{3} X_{1} X_{2} X_{3}$ | $\left\|\zeta_{31}\right\rangle=\frac{1}{2}(\|00101\rangle-\|00110\rangle-\|11001\rangle+\|11010\rangle)$ | $\left\|\theta^{-}\right\rangle_{A B}\left\|\phi^{-}\right\rangle_{A B}$ |

## III. TELEPORTATION

We now proceed to the implementation of teleportation protocols, starting with the teleportation of single and two qubit states. Subsequently, we illustrate the protocol of arbitrary $N$-qubit teleportation.

## Teleportation of single qubit state

For the purpose of single qubit teleportation, Alice and Bob are assigned to be in the state: $|\zeta\rangle=\frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{A} 0_{A}\right\rangle+\right.$ $\left.\left|1_{A} 1_{A} 1_{A}\right\rangle\right) \frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{B}\right\rangle+\left|1_{A} 1_{B}\right\rangle\right)$. For teleporting an arbitrary single qubit state $(\alpha|0\rangle+\beta|1\rangle)$, where $|\alpha|^{2}+|\beta|^{2}=1$, the combined six-qubit state obtained by Alice can be written as,

$$
\begin{align*}
& (\alpha|0\rangle+\beta|1\rangle) \frac{1}{2}(|00000\rangle+|00011\rangle+|11100\rangle+|11111\rangle) \\
& =\frac{1}{2}(|00000\rangle+|01110\rangle+|10001\rangle+|11111\rangle)(\alpha|0\rangle+\beta|1\rangle)+\frac{1}{2}(|00000\rangle+|01110\rangle-|10001\rangle-|11111\rangle)(\alpha|0\rangle-\beta|1\rangle)+ \\
& \frac{1}{2}(|00001\rangle+|01111\rangle+|10000\rangle+|11110\rangle)(\alpha|1\rangle+\beta|0\rangle)+\frac{1}{2}(|00001\rangle+|01111\rangle-|10000\rangle-|11110\rangle)(\alpha|1\rangle-\beta|0\rangle) . \tag{5}
\end{align*}
$$

Making a von Neumann five-particle measurement using the orthonormal states given in the first column of Table II, Alice sends her outcome to Bob via two classical bits. Subsequently, Bob applies suitable unitary operations to recover the original state $(\alpha|0\rangle+\beta|1\rangle)$. Hence, the teleportation protocol is deterministically implemented.

Table II: The outcome of the measurement performed by Alice and the stats obtained by Bob.

| Outcome of the measurement | State obtained |
| :---: | :---: |
| $\frac{1}{2}(\|00000\rangle+\|01110\rangle+\|10001\rangle+\|11111\rangle)$ | $\alpha\|0\rangle+\beta\|1\rangle$ |
| $\frac{1}{2}(\|00000\rangle+\|01110\rangle-\|10001\rangle-\|11111\rangle)$ | $\alpha\|0\rangle-\beta\|1\rangle$ |
| $\frac{1}{2}(\|00001\rangle+\|01111\rangle+\|10000\rangle+\|11110\rangle)$ | $\alpha\|1\rangle+\beta\|0\rangle$ |
| $\frac{1}{2}(\|00001\rangle+\|01111\rangle-\|10000\rangle-\|11110\rangle)$ | $\alpha\|1\rangle-\beta\|0\rangle$ |

## Teleportation of arbritrary two-qubit state

For two qubit teleportation, Alice and Bob share the state $\frac{1}{2}\left(\left|0_{A} 0_{A} 0_{B}\right\rangle+\left|1_{A} 1_{A} 1_{B}\right\rangle\right)\left(\left|0_{A} 0_{B}\right\rangle+\left|1_{A} 1_{B}\right\rangle\right)$, where Alice possesses three qubits and Bob possesses two. The combined seven-qubit state containing an arbitrary state $(\alpha|00\rangle+\gamma|01\rangle+\mu|10\rangle+\beta|11\rangle)$ with $|\alpha|^{2}+|\beta|^{2}+|\gamma|^{2}+|\mu|^{2}=1$, can be written as the linear combination of sixteen states which are decomposed into Alice and Bob system:

$$
\begin{align*}
& (\alpha|00\rangle+\gamma|01\rangle+\mu|10\rangle+\beta|11\rangle) \frac{1}{2}(|00000\rangle+|00011\rangle+|11100\rangle+|11111\rangle) \\
& =\left|\Omega_{0}\right\rangle(\alpha|00\rangle+\gamma|01\rangle+\mu|10\rangle+\beta|11\rangle)+\left|\Omega_{1}\right\rangle(\alpha|00\rangle+\gamma|01\rangle-\mu|10\rangle-\beta|11\rangle)+\left|\Omega_{2}\right\rangle(\alpha|00\rangle-\gamma|01\rangle+\mu|10\rangle-\beta|11\rangle) \\
& +\left|\Omega_{3}\right\rangle(\alpha|00\rangle-\gamma|01\rangle-\mu|10\rangle+\beta|11\rangle)+\left|\Omega_{4}\right\rangle(\alpha|10\rangle+\gamma|11\rangle+\mu|00\rangle+\beta|01\rangle)+\left|\Omega_{5}\right\rangle(\alpha|10\rangle+\gamma|11\rangle-\mu|00\rangle-\beta|01\rangle) \\
& +\left|\Omega_{6}\right\rangle(\alpha|10\rangle-\gamma|11\rangle+\mu|00\rangle-\beta|01\rangle)+\left|\Omega_{7}\right\rangle(\alpha|10\rangle-\gamma|11\rangle-\mu|00\rangle+\beta|01\rangle)+\left|\Omega_{8}\right\rangle(\alpha|01\rangle+\gamma|00\rangle+\mu|11\rangle+\beta|10\rangle) \\
& +\left|\Omega_{9}\right\rangle(\alpha|01\rangle+\gamma|00\rangle-\mu|11\rangle-\beta|10\rangle)+\left|\Omega_{10}\right\rangle(\alpha|01\rangle-\gamma|00\rangle+\mu|11\rangle-\beta|10\rangle)+\left|\Omega_{11}\right\rangle(\alpha|01\rangle-\gamma|00\rangle-\mu|11\rangle+\beta|10\rangle) \\
& +\left|\Omega_{12}\right\rangle(\alpha|11\rangle+\gamma|10\rangle+\mu|01\rangle+\beta|00\rangle)+\left|\Omega_{13}\right\rangle(\alpha|11\rangle+\gamma|10\rangle-\mu|01\rangle-\beta|00\rangle)+\left|\Omega_{14}\right\rangle(\alpha|11\rangle-\gamma|10\rangle+\mu|01\rangle-\beta|00\rangle) \\
& +\left|\Omega_{15}\right\rangle(\alpha|11\rangle-\gamma|10\rangle-\mu|01\rangle+\beta|00\rangle) . \tag{6}
\end{align*}
$$

where $\left|\Omega_{i}\right\rangle^{\prime} s(i=0,1,2, \ldots, 15)$ are mutually orthonormal five particle states, belonging to Alice, as shown in Table III. Now Alice makes a five-particle measurement using those orthonormal states and sends the outcome of her measurement to Bob via four classical bits. After applying suitable unitary operations Bob will be able to reconstruct the original state $(\alpha|00\rangle+\gamma|01\rangle+\mu|10\rangle+\beta|11\rangle)$. All the possible states obtained after Alice's measurement are orthogonal, so it completes the deterministic teleportation protocol.

Table III: The outcome of the measurement i.e., $\left|\Omega_{i}\right\rangle^{\prime} s(i=0,1,2, \ldots, 15)$ performed by Alice and the state obtained by Bob, where $1,2,3,4,5$ in $\left|\Omega_{i}\right\rangle_{1,2,3,4,5}$ denote the order of the Alice's five particle state from left hand side.

| Outcome of the measurement | State obtained |
| :---: | :---: |
| $\left\|\Omega_{0}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00000\rangle+\|01001\rangle+\|10110\rangle+\|11111\rangle)$ | $\alpha\|00\rangle+\gamma\|01\rangle+\mu\|10\rangle+\beta\|11\rangle$ |
| $\left\|\Omega_{1}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00000\rangle+\|01001\rangle-\|10110\rangle-\|11111\rangle)$ | $\alpha\|00\rangle+\gamma\|01\rangle-\mu\|10\rangle-\beta\|11\rangle$ |
| $\left\|\Omega_{2}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00000\rangle-\|01001\rangle+\|10110\rangle-\|11111\rangle)$ | $\alpha\|00\rangle-\gamma\|01\rangle+\mu\|10\rangle-\beta\|11\rangle$ |
| $\left\|\Omega_{3}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00000\rangle-\|01001\rangle-\|10110\rangle+\|11111\rangle)$ | $\alpha\|00\rangle-\gamma\|01\rangle-\mu\|10\rangle+\beta\|11\rangle$ |
| $\left\|\Omega_{4}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00110\rangle+\|01111\rangle+\|10000\rangle+\|11001\rangle)$ | $\alpha\|10\rangle+\gamma\|11\rangle+\mu\|00\rangle+\beta\|01\rangle$ |
| $\left\|\Omega_{5}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00110\rangle+\|01111\rangle-\|10000\rangle-\|11001\rangle)$ | $\alpha\|10\rangle+\gamma\|11\rangle-\mu\|00\rangle-\beta\|01\rangle$ |
| $\left\|\Omega_{6}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00110\rangle-\|01111\rangle+\|10000\rangle-\|11001\rangle)$ | $\alpha\|10\rangle-\gamma\|11\rangle+\mu\|00\rangle-\beta\|01\rangle$ |
| $\left\|\Omega_{7}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00110\rangle-\|01111\rangle-\|10000\rangle+\|11001\rangle)$ | $\alpha\|10\rangle-\gamma\|11\rangle-\mu\|00\rangle+\beta\|01\rangle$ |
| $\left\|\Omega_{8}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00001\rangle+\|01000\rangle+\|10111\rangle+\|11110\rangle)$ | $\alpha\|01\rangle+\gamma\|00\rangle+\mu\|11\rangle+\beta\|10\rangle$ |
| $\left\|\Omega_{9}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00001\rangle+\|01000\rangle-\|10111\rangle-\|11110\rangle)$ | $\alpha\|01\rangle+\gamma\|00\rangle-\mu\|11\rangle-\beta\|10\rangle$ |
| $\left\|\Omega_{10}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00001\rangle-\|01000\rangle+\|10111\rangle-\|11110\rangle)$ | $\alpha\|01\rangle-\gamma\|00\rangle+\mu\|11\rangle-\beta\|10\rangle$ |
| $\left\|\Omega_{11}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00001\rangle-\|01000\rangle-\|10111\rangle+\|11110\rangle)$ | $\alpha\|01\rangle-\gamma\|00\rangle-\mu\|11\rangle+\beta\|10\rangle$ |
| $\left\|\Omega_{12}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00111\rangle+\|01110\rangle+\|10001\rangle+\|11000\rangle)$ | $\alpha\|11\rangle+\gamma\|10\rangle+\mu\|01\rangle+\beta\|00\rangle$ |
| $\left\|\Omega_{13}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00111\rangle+\|01110\rangle-\|10001\rangle-\|11000\rangle)$ | $\alpha\|11\rangle+\gamma\|10\rangle-\mu\|01\rangle-\beta\|00\rangle$ |
| $\left\|\Omega_{14}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00111\rangle-\|01110\rangle+\|10001\rangle-\|11000\rangle)$ | $\alpha\|11\rangle-\gamma\|10\rangle+\mu\|01\rangle-\beta\|00\rangle$ |
| $\left\|\Omega_{15}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(\|00111\rangle-\|01110\rangle-\|10001\rangle+\|11000\rangle)$ | $\alpha\|11\rangle-\gamma\|10\rangle-\mu\|01\rangle+\beta\|00\rangle$ |

By rearranging the order of the particles we can write the same state as, $\left|\Omega_{0}\right\rangle_{1,4,3,2,5}=\frac{1}{2}(|00000\rangle+|00011\rangle+$ $|11100\rangle+|11111\rangle)=\left|\xi^{+}\right\rangle\left|\psi^{+}\right\rangle$. Simillarly, $\left|\Omega_{1}\right\rangle_{1,4,3,2,5}=\left|\zeta^{-}\right\rangle\left|\psi^{+}\right\rangle,\left|\Omega_{2}\right\rangle_{1,4,3,2,5}=\left|\zeta^{+}\right\rangle\left|\psi^{-}\right\rangle,\left|\Omega_{3}\right\rangle_{1,4,3,2,5}=\left|\zeta^{-}\right\rangle\left|\psi^{-}\right\rangle$, $\left|\Omega_{4}\right\rangle_{1,4,3,2,5}=\left|\chi^{+}\right\rangle\left|\psi^{+}\right\rangle$etc.

## Generalized teleportation of arbitrary $N$-qubit state

To achieve the purpose of $N$-qubit state teleportation, we have to start with the $(2 N+1)$-qubit state $\left|\zeta_{0}\right\rangle$ given in the Eq. 3, such that Alice possesses ' $N+1$ ' qubits and Bob possesses ' $N$ ' qubits. The arbitrary $N$-qubit state which Alice wants to teleport to Bob is, $\Sigma_{i=0}^{2^{N}-1} \alpha_{i}\left|a_{i}\right\rangle$ satisfying $\Sigma_{i=0}^{2^{N}-1}\left|\alpha_{i}\right|^{2}=1$. Here $a_{i}$ is binary representation of ' $i$ '. The combined state can be written as the linear combination of $2^{2 N}$ number of states which are decomposed into Alice and Bob system:
$\left(\Sigma_{i=0}^{2^{N}-1} \alpha_{i}\left|a_{i}\right\rangle\right)\left|\zeta_{0}\right\rangle=\Sigma_{j=0}^{2^{2 N}-1}\left|\Omega_{j}\right\rangle_{A}\left|\eta_{j}\right\rangle_{B}$, such that $\left|\Omega_{j}\right\rangle_{A}^{\prime} s\left(j=0,1,2, \ldots, 2^{2 N}-1\right)$ are mutually orthogonal states. We now explicate how to obtain $\left|\Omega_{j}\right\rangle^{\prime} s$ for any value of $N$. For any given $N,\left|\Omega_{0}\right\rangle$ is the product of one $\left|\xi^{+}\right\rangle$and ( $N-1$ ) number of $\left|\psi^{+}\right\rangle$states with suitable rearrangement of the particles:

For even $N,\left|\Omega_{0}\right\rangle_{1,4,3,2,7, \ldots, 2 N, 2 N-3,2 N-2,2 N+1}=\left|\zeta^{+}\right\rangle\left|\psi^{+}\right\rangle \ldots \ldots . .\left|\psi^{+}\right\rangle$and
for odd $N,\left|\Omega_{0}\right\rangle_{1,2,5,4,3, \ldots, 2 N, 2 N-3,2 N-2,2 N+1}=\left|\zeta^{+}\right\rangle\left|\psi^{+}\right\rangle \ldots \ldots . .\left|\psi^{+}\right\rangle$, where $1,2,3, \ldots, 2 N, 2 N+1$ are the order of the particles in the state, $\left|\Omega_{0}\right\rangle_{A}$.
Once, the $\left|\Omega_{0}\right\rangle$ is known we can calculate other $\left|\Omega_{j}\right\rangle$ 's :
$\left|\Omega_{j}\right\rangle_{1,2,3 \ldots, 2 N, 2 N+1}=\otimes_{k=1}^{N}\left(Z_{k}\right)^{b_{k}}\left(X_{k}\right)^{b_{k+N}}\left|\Omega_{0}\right\rangle_{1,2,3 \ldots, 2 N, 2 N+1}$
where $j$ is the decimal representation of a binary bit string $b_{2 N} \ldots \ldots . b_{2} b_{1}$ ( $b_{k}=0$ or 1 ). After the measurement using these orthogonal states, Alice's state evolves into one of the $\left|\Omega_{j}\right\rangle^{\prime} s$, which ensures that this protocol is deterministic. If Alice's state collapses to $\left|\Omega_{j}\right\rangle$, then Bob has to apply $\otimes_{k=1}^{N}\left(Z_{k}\right)^{b_{k}}\left(X_{k}\right)^{b_{k+N}}$ on his $N$-qubit system to get the unknown state, where $j$ is the decimal representation of a binary bit string $b_{2 N} \ldots \ldots . . b_{2} b_{1}$.

For example, if we consider the two-qubit state teleportation, then from Eq. 7 we get, $\left|\Omega_{0}\right\rangle_{1,4,3,2,5}=\left|\zeta^{+}\right\rangle\left|\psi^{+}\right\rangle$ or, $\left|\Omega_{0}\right\rangle_{1,2,3,4,5}=\frac{1}{2}(|00000\rangle+|01001\rangle+|10110\rangle+|11111\rangle)$. Suppose, we want to obtain $\left|\Omega_{12}\right\rangle$. We know that 1100 is the binary representation of 12 , so using Eq. 8, one can get, $\left|\Omega_{12}\right\rangle=\left(X_{1}\right)^{1}\left(Z_{1}\right)^{0}\left(X_{2}\right)^{1}\left(Z_{2}\right)^{0}\left|\Omega_{0}\right\rangle=\left(X_{1}\right)\left(X_{2}\right)\left|\Omega_{0}\right\rangle=$ $\frac{1}{2}(|11000\rangle+|10001\rangle+|01110\rangle+|00111\rangle)$, which exactly matches with the $\left|\Omega_{12}\right\rangle$ in Table III. So after the measurement, if Alice's state collapses to $\left|\Omega_{12}\right\rangle$, then Bob has to apply $\left(X_{1}\right)\left(X_{2}\right)$ on his state to reconstruct the unknown state.

## IV. QUANTUM INFORMATION SPLITTING

It can be shown that, the five-qubit state $|\zeta\rangle$ can be used for single qubit QIS through three different protocols, where as only one protocol is feasible for two-qubit QIS [36].

## QIS of a single qubit state

The distributions of the qubits, within the three parties of the initial state $|\zeta\rangle$ are different for three different protocols of single qubit QIS. Those states are:
(i) $\frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{B} 0_{B}\right\rangle+\left|1_{A} 1_{B} 1_{B}\right\rangle\right) \frac{1}{\sqrt{2}}\left(\left|0_{B} 0_{C}\right\rangle+\left|1_{B} 1_{C}\right\rangle\right)$,
(ii) $\frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{A} 0_{B}\right\rangle+\left|1_{A} 1_{A} 1_{B}\right\rangle\right) \frac{1}{\sqrt{2}}\left(\left|0_{B} 0_{C}\right\rangle+\left|1_{B} 1_{C}\right\rangle\right)$,
(iii) $\frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{A} 0_{B}\right\rangle+\left|1_{A} 1_{A} 1_{B}\right\rangle\right) \frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{C}\right\rangle+\left|1_{A} 1_{C}\right\rangle\right)$.

It is noted that Alice possesses one, two and three qubits in three different initial states and Charlie possesses one qubit in all those states. We first explicitly demostrate this protocol using the state $\frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{B} 0_{B}\right\rangle+\right.$ $\left.\left|1_{A} 1_{B} 1_{B}\right\rangle\right) \frac{1}{\sqrt{2}}\left(\left|0_{B} 0_{C}\right\rangle+\left|1_{B} 1_{C}\right\rangle\right)$ and then we discuss about the other two protocols. The combined state of $\frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{B} 0_{B}\right\rangle+\left|1_{A} 1_{B} 1_{B}\right\rangle\right) \frac{1}{\sqrt{2}}\left(\left|0_{B} 0_{C}\right\rangle+\left|1_{B} 1_{C}\right\rangle\right)$ with the unknown qubit $(\alpha|0\rangle+\beta|1\rangle)$ where $|\alpha|^{2}+|\beta|^{2}=1$, can be written as,

$$
\begin{align*}
& (\alpha|0\rangle+\beta|1\rangle) \frac{1}{2}(|00000\rangle+|00011\rangle+|11100\rangle+|11111\rangle) \\
& =\frac{1}{2}\{|00\rangle \alpha(|0000\rangle+|0011\rangle)+|01\rangle \alpha(|1100\rangle+|1111\rangle)+|10\rangle \beta(|0000\rangle+|0011\rangle)+|11\rangle \beta(|1100\rangle+|1111\rangle)\} \tag{9}
\end{align*}
$$

Alice first performs a measurement using Bell pairs such that Bob and Charlie evolve into an entangled state given in Table IV and conveys her outcome to Charlie by two cbits.

Table IV: The outcome of the measurement performed by Alice and the entangled state obtained by Bob and Charlie (the coefficients are removed for convenience).

| Outcome of the measurement | Entangled state obtained by Bob and Charlie |
| :---: | :---: |
| $\|00\rangle+\|11\rangle$ | $\alpha(\|0000\rangle+\|0011\rangle)+\beta(\|1100\rangle+\|1111\rangle)$ |
| $\|01\rangle+\|10\rangle$ | $\alpha(\|1100\rangle+\|1111\rangle)+\beta(\|0000\rangle+\|0011\rangle)$ |
| $\|01\rangle-\|10\rangle$ | $\alpha(\|1100\rangle+\|1111\rangle)-\beta(\|0000\rangle+\|0011\rangle)$ |
| $\|00\rangle-\|11\rangle$ | $\alpha(\|0000\rangle+\|0011\rangle)-\beta(\|1100\rangle+\|1111\rangle)$ |

Bob then performs a three-particle measurement and conveys his outcome to Charlie via two cbits of information. Having known the outcomes of both their measurements, Charlie can obtain the unknown qubit, by performing a suitable unitary operation on his qubit. Hence, the QIS protocol is satisfied. Suppose, after the measurement of Alice, the Bob-Charlie system evolves into the state $\alpha(|0000\rangle+|0011\rangle)+\beta(|1100\rangle+|1111\rangle)$, then the outcome of the measurement performed by Bob and the state obtained by Charlie are shown in Table V .

Table V: The possible outcomes of the measurement performed by Bob and the state obtained by Charlie.

| Outcome of the measurement | State obtained |
| :---: | :---: |
| $\|000\rangle+\|111\rangle$ | $\alpha\|0\rangle+\beta\|1\rangle$ |
| $\|001\rangle+\|110\rangle$ | $\alpha\|1\rangle+\beta\|0\rangle$ |
| $\|000\rangle-\|111\rangle$ | $\alpha\|0\rangle-\beta\|1\rangle$ |
| $\|001\rangle-\|110\rangle$ | $\alpha\|1\rangle-\beta\|0\rangle$ |

For the another protocol Alice, Bob and Charlie share a state, $\frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{A} 0_{B}\right\rangle+\left|1_{A} 1_{A} 1_{B}\right\rangle\right) \frac{1}{\sqrt{2}}\left(\left|0_{B} 0_{C}\right\rangle+\left|1_{B} 1_{C}\right\rangle\right)$, so the combined state containing the unknown qubit will be,

$$
\begin{align*}
& (\alpha|0\rangle+\beta|1\rangle) \frac{1}{2}(|00000\rangle+|00011\rangle+|11100\rangle+|11111\rangle) \\
& =|000\rangle \alpha(|000\rangle+|011\rangle)+|011\rangle \alpha(|100\rangle+|111\rangle)+|100\rangle \beta(|000\rangle+|011\rangle)+|111\rangle \beta(|100\rangle+|111\rangle) . \tag{10}
\end{align*}
$$

Alice first performs measurement using $\{|000\rangle \pm|111\rangle,|011\rangle \pm|100\rangle,|001\rangle \pm|110\rangle,|010\rangle \pm|101\rangle\}$ and subsequently Bob performs a Bell measurement. Then both of them convey their outcomes to Charlie. It can be shown that after Bob's measurement, Charlie's state collapses into $(\alpha|0\rangle \pm \beta|1\rangle)$ or ( $\alpha|1\rangle \pm \beta|0\rangle$ ). Knowing the outcomes of those measurements classically, Charlie can reconstruct the information.

Simillarly, in the third protocol, where Alice, Bob and Charlie are sharing the state $\frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{A} 0_{B}\right\rangle+\left|1_{A} 1_{A} 1_{B}\right\rangle\right)$ $\frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{C}\right\rangle+\left|1_{A} 1_{C}\right\rangle\right)$, Alice has to perform four particle measurement using the orthogonal states $(|0000\rangle \pm|1111\rangle)$, $(|0110\rangle \pm|1001\rangle),(|0001\rangle \pm|1110\rangle),(|1000\rangle \pm|0111\rangle)$ and then Bob has to perform one particle measurement using $\left\{\frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}}\right\}$. After that, Charlie will be able to get the unknown qubit by applying unitary operations according to the outcomes of those measurements.

## QIS of two-qubit state

We now procced to show QIS of two-qubit state where Alice, Bob and Charlie are sharing the state:
$|\zeta\rangle=\frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{B} 0_{C}\right\rangle+\left|1_{A} 1_{B} 1_{C}\right\rangle\right) \frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{C}\right\rangle+\left|1_{A} 1_{C}\right\rangle\right)$.
Alice combines the unknown qubit $(\alpha|00\rangle+\gamma|01\rangle+\mu|10\rangle+\beta|11\rangle),\left(|\alpha|^{2}+|\beta|^{2}+|\gamma|^{2}+|\mu|^{2}=1\right)$ with the state and performs a four-particle measurement using the orthonormal states given in the first column of Table VI. All the orthonormal states which form the basis of Alice's measurement can be broken down into two Bell pairs using suitable rearrangement of particles. Depending on the outcome of the measurement, Bob and Charlie evolve to an entangled state. After that, Bob performs a single particle measurement using Hadamard basis and sends his outcome to Charlie.

Having known the outcomes of both their measurements, Charlie can obtain the unknown two qubit state, by performing a suitable unitary operation on his qubit. For instance, after the measurement of Alice, the BobCharlie system collapses to the state $(\alpha|000\rangle+\gamma|001\rangle+\mu|110\rangle+\beta|111\rangle)$. Then the possible outcomes of Bob's measurement are $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ or $\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$ and the states obtained by Charlie are $(\alpha|00\rangle+\gamma|01\rangle+\mu|10\rangle+\beta|11\rangle)$ or $(\alpha|00\rangle+\gamma|01\rangle-\mu|10\rangle-\beta|11\rangle)$ respectively, from which the original state can be recovered.

Table VI: The outcome of the measurement performed by Alice and the entangled state obtained by Bob and Charlie.

| Outcome of the measurement | Entangled state obtained by Bob and Charlie |
| :---: | :---: |
| $\frac{1}{2}(\|0000\rangle+\|0101\rangle+\|1010\rangle+\|1111\rangle)$ | $\alpha\|000\rangle+\gamma\|001\rangle+\mu\|110\rangle+\beta\|111\rangle$ |
| $\frac{1}{2}(\|0000\rangle-\|0101\rangle+\|1010\rangle-\|1111\rangle)$ | $\alpha\|000\rangle-\gamma\|001\rangle+\mu\|110\rangle-\beta\|111\rangle$ |
| $\frac{1}{2}(\|0000\rangle+\|0101\rangle-\|1010\rangle-\|1111\rangle)$ | $\alpha\|000\rangle+\gamma\|001\rangle-\mu\|110\rangle-\beta\|111\rangle$ |
| $\frac{1}{2}(\|0000\rangle-\|0101\rangle-\|1010\rangle+\|1111\rangle)$ | $\alpha\|000\rangle-\gamma\|001\rangle-\mu\|110\rangle+\beta\|111\rangle$ |
| $\frac{1}{2}(\|0001\rangle+\|0100\rangle+\|1011\rangle+\|1110\rangle)$ | $\alpha\|001\rangle+\gamma\|000\rangle+\mu\|110\rangle+\beta\|110\rangle$ |
| $\frac{1}{2}(\|0001\rangle-\|0100\rangle+\|1011\rangle-\|1110\rangle)$ | $\alpha\|001\rangle-\gamma\|000\rangle+\mu\|110\rangle-\beta\|110\rangle$ |
| $\frac{1}{2}(\|0001\rangle+\|0100\rangle-\|1011\rangle-\|1110\rangle)$ | $\alpha\|001\rangle+\gamma\|000\rangle-\mu\|110\rangle-\beta\|110\rangle$ |
| $\frac{1}{2}(\|0001\rangle-\|0100\rangle-\|1011\rangle+\|1110\rangle)$ | $\alpha\|001\rangle-\gamma\|000\rangle-\mu\|110\rangle+\beta\|110\rangle$ |
| $\frac{1}{2}(\|0010\rangle+\|0111\rangle+\|1000\rangle+\|1101\rangle)$ | $\alpha\|110\rangle+\gamma\|111\rangle+\mu\|000\rangle+\beta\|001\rangle$ |
| $\frac{1}{2}(\|0010\rangle-\|0111\rangle+\|1000\rangle-\|1101\rangle)$ | $\alpha\|110\rangle-\gamma\|111\rangle+\mu\|000\rangle-\beta\|001\rangle$ |
| $\frac{1}{2}(\|0010\rangle+\|0111\rangle-\|1000\rangle-\|1101\rangle)$ | $\alpha\|110\rangle+\gamma\|111\rangle-\mu\|000\rangle-\beta\|001\rangle$ |
| $\frac{1}{2}(\|0010\rangle-\|0111\rangle-\|1000\rangle+\|1101\rangle)$ | $\alpha\|110\rangle-\gamma\|111\rangle-\mu\|000\rangle+\beta\|001\rangle$ |
| $\frac{1}{2}(\|0011\rangle+\|0110\rangle+\|1001\rangle+\|1100\rangle)$ | $\alpha\|111\rangle+\gamma\|110\rangle+\mu\|001\rangle+\beta\|000\rangle$ |
| $\frac{1}{2}(\|0011\rangle-\|0110\rangle+\|1001\rangle-\|1100\rangle)$ | $\alpha\|111\rangle-\gamma\|110\rangle+\mu\|001\rangle-\beta\|000\rangle$ |
| $\frac{1}{2}(\|0011\rangle+\|0110\rangle-\|1001\rangle-\|1100\rangle)$ | $\alpha\|111\rangle+\gamma\|110\rangle-\mu\|001\rangle-\beta\|000\rangle$ |
| $\frac{1}{2}(\|0011\rangle-\|0110\rangle-\|1001\rangle+\|1100\rangle)$ | $\alpha\|111\rangle-\gamma\|110\rangle-\mu\|001\rangle+\beta\|000\rangle$ |

## Generalized QIS of arbitrary $N$-qubit state

For ' $N$ ' qubit QIS Alice, Bob and Charlie need to share a multipartite entangled state, which is the product of one $\left|\xi^{+}\right\rangle$and $(N-1)$ number of $\left|\psi^{+}\right\rangle$states. The state is given by, $\left|\zeta^{\prime}\right\rangle=\left|\xi^{+}\right\rangle_{A B C}\left|\psi^{+}\right\rangle_{A C} \ldots \ldots .\left|\psi^{+}\right\rangle_{A C}$, where $\left|\xi^{+}\right\rangle_{A B C}=\frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{B} 0_{C}\right\rangle+\left|1_{A} 1_{B} 1_{C}\right\rangle\right)$ and $\left|\psi^{+}\right\rangle_{A C}=\frac{1}{\sqrt{2}}\left(\left|0_{A} 0_{C}\right\rangle+\left|1_{A} 1_{C}\right\rangle\right)$. The combined system, contaning the unknown state $\Sigma_{i=0}^{2^{N}-1} \alpha_{i}\left|a_{i}\right\rangle$ (where $a_{i}$ is ' $N$ ' classical bit string which represents the binary form of ' $i$ ') can be written as the linear combination of $2^{2 N}$ number of states which are decomposed into Alice's system and Bob-Charlie combined system:
$\left(\Sigma_{i=0}^{2^{N}-1} \alpha_{i}\left|a_{i}\right\rangle\right)\left|\zeta^{\prime}\right\rangle=\Sigma_{j=0}^{2^{2 N}-1}\left|\Omega_{j}\right\rangle_{A}\left|\eta_{j}\right\rangle_{B C}$, where $\left|\Omega_{j}\right\rangle_{A}^{\prime} s\left(j=0,1,2, \ldots, 2^{2 N}-1\right)$ are mutually orthogonal states. We now explicate how to obtain $\left|\Omega_{j}\right\rangle^{\prime} s$. For any given $N,\left|\Omega_{0}\right\rangle$ is the products of $N$ number of $\left|\psi^{+}\right\rangle$states with suitable rearrangement of the particles: $\left|\Omega_{0}\right\rangle_{1, N+1,2, N+2, \ldots, N, 2 N}=\left|\psi^{+}\right\rangle\left|\psi^{+}\right\rangle \ldots \ldots .\left|\psi^{+}\right\rangle$ where $1,2,3, \ldots, 2 N, 2 N+1$ are the order of the particles in the state, $\left|\Omega_{0}\right\rangle_{A}$. Using $\left|\Omega_{0}\right\rangle$ we can evaluate other $\left|\Omega_{j}\right\rangle$ 's from the equation:

$$
\begin{equation*}
\left|\Omega_{j}\right\rangle_{1,2,3 \ldots, 2 N}=\otimes_{k=1}^{N}\left(Z_{k}\right)^{b_{k}}\left(X_{k}\right)^{b_{k+N}}\left|\Omega_{0}\right\rangle_{1,2,3 \ldots, 2 N} \tag{12}
\end{equation*}
$$

where $j$ is the decimal representation of a binary bit string $b_{2 N} \ldots \ldots . b_{2} b_{1}$ ( $b_{k}=0$ or 1 ). Measurement by Alice using $\left|\Omega_{j}\right\rangle^{\prime} s$ leads to an entangled state shared by Bob and Charlie, where Bob possesses the first particle and Charlie the others ' $N$ ' particles. After that, Bob has to perform a one-particle measurement using $\left\{\frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}}\right\}$ and convey his outcome to Charlie. Then Charlie will be able to obtain the unknown $N$-qubit state, by performing a suitable unitary operation on his qubits. Hence, QIS of arbitrary $N$-qubit state is achieved using this channel.

## V. CONCLUSION

We have demonstrated a number of useful applications in quantum communication using a general odd particle state which can be obtained by the combination of one GHZ and many Bell pairs. We have shown that, the five-qubit state has maximum entanglement between two sub-parties. Arbitrary one and two qubit states teleportation and quantum information splitting have been carried out explicitly using the five-qubit state. Three different protocols are exploited for single qubit QIS. We have also shown that, teleportation as well as QIS can be generalized using this state. This state works perfectly for superdense coding and the capacity of superdense coding reaches the 'Holevo bound' for ' $2 N+1$ ' classical bits transfer via ' $N+1$ ' qubits. The fact that, Bell and GHZ states are realized in laboratory conditions makes our protocol experimentally achievable. The decoherance properties and NDD of this state can also be investigated [37-40].

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