

Relations between entanglement, Bell-inequality violation and teleportation fidelity for the two-qubit X states

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Based on the assumption that the receiver Bob can apply any unitary transformation, Horodecki *et al.* [Phys. Lett. A **222**, 21 (1996)] proved that any mixed two spin-1/2 state which violates the Bell-CHSH inequality is useful for teleportation. Here, we further show that any X state which violates the Bell-CHSH inequality can also be used for nonclassical teleportation even if Bob can only perform the identity or the Pauli rotation operations. Moreover, we showed that the maximal difference between the two average fidelities achievable via Bob's arbitrary transformations and via the sole identity or the Pauli rotation is 1/9.

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I. INTRODUCTION

Entanglement and Bell nonlocality are two aspects of quantum correlations that have been the research interests since the early days of quantum mechanics and still have not been completely understood until now [1–3]. A basic trait of the entangled state of a composite system is that it cannot be written as products of states of each subsystem. As a physical resource, entanglement is essential for various quantum information processing (QIP) tasks [1]. One of them is quantum teleportation [4], by which an unknown state can be replicated at a distant location with the help of local operations and classical communication. However, not all the states that are entangled can be used for teleportation with average fidelity (see sections below) better than that achievable via classical communication alone [5, 6], and the average fidelity is even not a monotone function of the degree of entanglement of the channel state. This seemingly counterintuitive phenomenon has been noticed by a number of authors [7–11]. Particularly, there are situations in which the channel states possess greater amount of entanglement with however the average fidelity cannot exceed the classical limiting value of 2/3 [11].

Bell nonlocality corresponds to another quantum correlation that cannot be reproduced by any classical local-hidden variable models, and this nonlocal behavior can be detected by the violation of different Bell-type inequalities [1]. The investigation of Bell-nonlocality violation is significant not only for the fundamental role it plays in better understanding of the subtle aspects of quantum mechanics, but also because these violations are crucial for some practical applications in quantum information processing, such as to guarantee the safety of the device-independent key distribution protocols in quantum cryptography [12, 13].

Due to their close relations, distinction between entanglement and Bell-inequality violation has been studied

extensively. Particularly, it has been demonstrated that for the two-level systems, the inseparability of a bipartite pure state corresponds to the violation of the Bell-CHSH inequality, and vice versa [6, 14]. However, this is not the case for the mixed states. As pointed out initially by Werner [15], there are bipartite mixed states who are entangled but do not violate any Bell-type inequalities, thus one cannot distinguish whether these correlations are produced by a classical local-hidden variable model or not.

Moreover, when considering the issue of quantum teleportation protocol, Popescu [5] noticed that there are bipartite mixed states which do not violate any Bell-type inequalities, but still can be used for teleportation with average fidelity larger than the classical limiting value of 2/3. It is then natural to ask for the generic relations between entanglement, Bell-nonlocality violation and quantum teleportation. In general, this problem is rather complicated and difficult to answer. But for the special case of the bipartite two-level systems, Horodecki *et al.* showed that the question concerning the Bell-CHSH inequality violation and the inseparability of the mixed states can be derived definitely [6]. Based on the assumption that during the teleportation process, the sender Alice uses only the Bell basis in her measurement, while the receiver Bob is allowed to apply any unitary transformation, Horodecki *et al.* demonstrated that any two spin-1/2 state (pure or mixed) which violates the Bell-CHSH inequality is useful for teleportation.

The statement of Horodecki *et al.* [6] relies crucially on Bob's ability to perform any unitary transformation on his qubit. But it is worthwhile to note that in real circumstances the performance of certain unitary transformations, particularly for solid-state construction of qubit systems, may be very difficult [1], thus it is significant to consider the situation in which Bob performs only some easy-realized transformations. For instance, if Bob can only perform the identity (i.e., do nothing) or the Pauli rotation operations, then what will happen to the teleportation process? Can it still enable nonclassical fidelity when the channel state violates the Bell-CHSH inequality [3]. In fact, this is not the case as we will show in

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the following text after introducing the definitions of the corresponding average fidelities.

II. BELL-INEQUALITY VIOLATION AND TELEPORTATION VIA THE X STATES

In this paper, we will address the problem concerning possible relations between entanglement, Bell-nonlocality violation and quantum teleportation for the two-qubit X state ρ [16], which has nonzero elements only along the main diagonal and anti-diagonal. We assume that Alice uses the generalized Bell basis $|\Psi^{0,3}\rangle = (|00\rangle \pm e^{-i\alpha}|11\rangle)/\sqrt{2}$ and $|\Psi^{1,2}\rangle = (|01\rangle \pm e^{-i\beta}|10\rangle)/\sqrt{2}$ in her measurement, while Bob performs only the identity or the Pauli rotation operation to his qubit according to the classical information he received from Alice [4]. Here the exponential terms $e^{-i\alpha}$ and $e^{-i\beta}$ are related to the matrix elements of the X state via $w = |w|e^{i\alpha}$ and $z = |z|e^{i\beta}$. The X state can be written in the following form

$$\rho = \begin{pmatrix} a & 0 & 0 & w \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ w^* & 0 & 0 & d \end{pmatrix}. \quad (1)$$

Such X states arise in a wide variety of physical situations and include pure Bell states [1] as well as the well-known Werner-like mixed states [15]. The usual density matrix conditions such as normalization, positive semi-definiteness and Hermiticity require that the diagonal elements a, b, c, d are non-negative and the equality $a + b + c + d = 1$ holds. Moreover, the anti-diagonal elements w and z satisfy

$$|w|^2 \leq ad, \quad |z|^2 \leq bc. \quad (2)$$

If Bob is equipped to perform all kinds of unitary transformations to the qubit at his possession, then the maximal average fidelity achievable can be expressed as [6]

$$F_{\max}^{(1)} = \frac{1}{2} + \frac{1}{6}N(\rho), \quad (3)$$

where $N(\rho) = \text{tr}\sqrt{T^\dagger T}$. Here T is a 3×3 positive matrix with elements $t_{nm} = \text{tr}(\rho\sigma^n \otimes \sigma^m)$, and $\sigma^{1,2,3}$ are the usual Pauli spin operators. $N(\rho)$ can be written explicitly as $N(\rho) = \sum_{i=1}^3 \sqrt{u_i}$, where u_i ($i = 1, 2, 3$) are eigenvalues of the matrix $T^\dagger T$. Particularly, for the X state expressed in Eq. (1), the eigenvalues of $T^\dagger T$ can be obtained straightforwardly as $u_{1,2} = 4(|w| \pm |z|)^2$ and $u_3 = (a + d - b - c)^2$, thus we get $N(\rho) = 2(|w| + |z| + ||w| - |z||) + |a + d - b - c|$.

If Bob can only perform the identity or the Pauli rotation operations, then the maximal average fidelity for quantum teleportation can be obtained explicitly as [9, 17]

$$F_{\max}^{(2)} = \frac{1}{3} + \frac{2}{3}\mathcal{F}(\rho), \quad (4)$$

where $\mathcal{F}(\rho) = \max\{\chi_0, \chi_1, \chi_2, \chi_3\}$ is the fully entangled fraction [18, 19], with the notations $\chi_{0,3} = (a + d \pm 2|w|)/2$ and $\chi_{1,2} = (b + c \pm 2|z|)/2$. Clearly, $F_{\max}^{(2)}$ is in fact determined only by the quantity χ_0 or χ_1 . One can show now that for some channel states that admit $F_{\max}^{(1)} > 2/3$, the average fidelity $F_{\max}^{(2)}$ may be equal to or smaller than $2/3$. A representative example is the maximally entangled state $|\varphi\rangle = (|00\rangle + |01\rangle + |10\rangle - |11\rangle)/2$ (this state can be generated by applying a Hadamard operation [1] to the second qubit of the Bell state $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$) which yields $F_{\max}^{(1)} = 1$ and $F_{\max}^{(2)} = 2/3$.

For 2×2 systems, the nonlocality of a quantum state can be detected by the violation of the Bell-CHSH inequality proposed by Clauser, Horne, Shimony and Holt [3], which is given by

$$|\langle B_{\text{CHSH}} \rangle_\rho| \leq 2, \quad (5)$$

where $\langle B_{\text{CHSH}} \rangle_\rho = \text{tr}(\rho B_{\text{CHSH}})$, and B_{CHSH} is the Bell operator associated with the quantum CHSH inequality. It has been demonstrated [20] that $B_{\max}(\rho) = \max|\langle B_{\text{CHSH}} \rangle_\rho|$ is related to a quantity $M(\rho)$ via $B_{\max}(\rho) = 2\sqrt{M(\rho)}$, where $M(\rho) = \max_{i < j} (u_i + u_j)$, with u_i ($i = 1, 2, 3$) being the eigenvalues of the matrix $T^\dagger T$. For the X state expressed in Eq. (1), we can obtain

$$M(\rho) = \max\{8(|w|^2 + |z|^2), 4(|w| + |z|)^2 + (a + d - b - c)^2\}, \quad (6)$$

Clearly, the inequality (5) is violated if and only if $M(\rho) > 1$, and the quantity $M(\rho)$ can also be used to measure the degree of violation of the Bell nonlocality for a bipartite state.

We now begin our discussion about possible relations between Bell-nonlocality violation and average fidelity $F_{\max}^{(2)}$ for the situation in which Alice performs her joint measurement using the generalized Bell operators while Bob is only allowed to perform the identity or the Pauli rotation operation. We will show that if the Bell-CHSH inequality in Eq. (5) is violated, i.e., $B_{\max}(\rho) > 2$ or $M(\rho) > 1$, then all the X states will yield $F_{\max}^{(2)} > 2/3$. Since $F_{\max}^{(2)}$ is determined only by χ_0 or χ_1 , and it is easy to prove that the two inequalities $\chi_0 > 1/2$ and $\chi_1 > 1/2$ cannot be satisfied simultaneously. This is because if they are satisfied simultaneously, then from the normalization of ρ one can obtain $\chi_0 + \chi_1 = (1 + 2|w| + 2|z|)/2 > 1$, which gives rise to $|w| + |z| > 1/2$. Clearly, this is in contradiction with the fact that $|w| + |z| \leq \sqrt{ad} + \sqrt{bc} \leq (a + d + b + c)/2 = 1/2$, which can be derived directly from Eq. (2). Thus in the following we only need to prove that the two inequalities $\chi_0 < 1/2$ and $\chi_1 < 1/2$ cannot be satisfied simultaneously if $M(\rho) > 1$.

The relative magnitude of $M(\rho)$ is determined by the maximum of $M_1(\rho) = 8(|w|^2 + |z|^2)$ and $M_2(\rho) = 4(|w| + |z|)^2 + (a + d - b - c)^2$. First, for the case of $M_1(\rho) \geq M_2(\rho)$, we have $M(\rho) = 8(|w|^2 + |z|^2)$. If $\chi_0 < 1/2$ and $\chi_1 < 1/2$ are satisfied simultaneously, then we get

$$2|w| < b + c, \quad 2|z| < a + d, \quad (7)$$

where we have used the normalization condition $a + b + c + d = 1$ in deriving the above equations. Moreover, from Eq. (2) and the positive semi-definiteness of the density matrix it is direct to show that the following inequalities hold

$$a + d \geq 2\sqrt{ad} \geq 2|w|, \quad b + c \geq 2\sqrt{bc} \geq 2|z|. \quad (8)$$

By combination of Eqs. (7) and (8) one can obtain

$$|w| < \frac{1}{4}, \quad |z| < \frac{1}{4}, \quad (9)$$

which yields $M(\rho) = 8(|w|^2 + |z|^2) < 1$. Thus one see that for the case of $M_1(\rho) \geq M_2(\rho)$, the Bell-CHSH inequality (5) cannot be violated if $\chi_0 < 1/2$ and $\chi_1 < 1/2$.

Second, for the case of $M_1(\rho) < M_2(\rho)$ we have $M(\rho) = 4(|w| + |z|)^2 + (a + d - b - c)^2$. Still one can prove that the relations $\chi_0 < 1/2$ and $\chi_1 < 1/2$ cannot be satisfied simultaneously. This is because we always have

$$|w| + |z| < b + c, \quad |w| + |z| < a + d, \quad (10)$$

if $\chi_0 < 1/2$ and $\chi_1 < 1/2$, where the first inequality in Eq. (10) can be obtained by combination of the first inequality in Eq. (7) and the second inequality in Eq. (8), while the second inequality in Eq. (10) can be obtained by combination of the second inequality in Eq. (7) and the first inequality in Eq. (8). Because the parameters appeared both in the left-hand side and the right-hand side of the inequalities of Eq. (10) is non-negative, we get

$$(|w| + |z|)^2 < (a + d)(b + c). \quad (11)$$

On the other hand, violation of the Bell-CHSH inequality $|\langle B_{\text{CHSN}} \rangle_\rho| \leq 2$ for the X state ρ requires $M(\rho) = 4(|w| + |z|)^2 + (a + d - b - c)^2 > 1$. It is easy to check that this inequality can also be expressed equivalently as $(|w| + |z|)^2 > (a + d)(b + c)$, which is obviously contradicts the result of Eq. (11). Thus by using apagogic reasoning we demonstrated again that for the case of $M_1(\rho) < M_2(\rho)$, the Bell-CHSH inequality still cannot be violated if $\chi_0 < 1/2$ and $\chi_1 < 1/2$.

Based on the above discussions, we came to the following proposition about possible relations between Bell-nonlocality violation and quantum teleportation.

Proposition 1. All the X states that violate the Bell-CHSH inequality can be used for teleportation with average fidelity $F_{\text{max}}^{(2)}$ greater than the classical limiting value of $2/3$.

However, one should note that the inequality $B_{\text{max}}(\rho) > 2$ or $M(\rho) > 1$ is only a sufficient condition for nonclassical teleportation fidelity, because there are states which do not violate the Bell-CHSH inequality, but still give rise to $F_{\text{max}}^{(2)} > 2/3$. One such example is the Werner mixed state [15] described by the density matrix

$\rho_{\text{W}} = p|\Psi^-\rangle\langle\Psi^-| + (1-p)\mathbb{I}_4/4$, where p is a real parameter ranges from 0 to 1, $|\Psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$ is the Bell singlet state, and \mathbb{I}_4 denotes the 4×4 identity operator. For ρ_{W} , from the above formulae one can obtain $M(\rho) = 2p^2$ and $F_{\text{max}}^{(1)} = F_{\text{max}}^{(2)} = (1 + p)/2$. Clearly, The Werner mixed state yields $F_{\text{max}}^{(2)} > 2/3$ for $p > 1/3$, while it violates the CHSH inequality only when $p > 1/\sqrt{2}$. Thus in the region of $p \in (1/3, 1/\sqrt{2}]$ we have the state ρ_{W} which are suitable for nonclassical quantum teleportation but do not violate the Bell-CHSH inequality. Moreover, it is straightforward to check that entanglement of the Werner state ρ_{W} measured by the concurrence [21] is given by $C = \max\{0, (3p - 1)/2\}$. This indicates that all the entangled Werner states are useful for teleportation. But it should be note that this is not the case for general forms of entangled X states [10, 11].

When considering relations between entanglement and Bell-nonlocality violation, it has been shown by Verstraete and Wolf [22] that for states with a given concurrence C , there exists an exact bound for $B_{\text{max}}(\rho)$: $2\sqrt{2}C \leq B_{\text{max}}(\rho) \leq 2\sqrt{1 + C^2}$, which shows clearly that the violation of the Bell-CHSH inequality is guaranteed when $C > 1/\sqrt{2}$. This relation also applies to the X states considered here, namely, both the lower bound $2\sqrt{2}C$ and the upper bound $2\sqrt{1 + C^2}$ for $B_{\text{max}}(\rho)$ remain unchanged for the X state. Thus we immediately came to the conclusion that any X state with concurrence larger than $1/\sqrt{2}$ is always useful for nonclassical teleportation, even if Bob can only perform the identity (i.e., do nothing) or the Pauli rotation operation. But it should be note that for the case of $C < 1/\sqrt{2}$, it is also possible to achieve nonclassical teleportation fidelity (see, for example, the case of the Werner state ρ_{W}).

Now we turn to discuss possible relations between the average fidelities $F_{\text{max}}^{(1)}$ and $F_{\text{max}}^{(2)}$. Since the former represents the situation in which Bob is equipped to apply any unitary transformation, while the latter corresponds to the situation in which Bob can only perform the identity or the Pauli rotation operation, we have $F_{\text{max}}^{(1)} \geq F_{\text{max}}^{(2)}$ in general. What we concern in the following is the extent to which $F_{\text{max}}^{(2)}$ can be improved via Bob's arbitrary transformations. For this purpose, we consider the difference between $F_{\text{max}}^{(1)}$ and $F_{\text{max}}^{(2)}$, i.e., $\delta F_{\text{max}} = F_{\text{max}}^{(1)} - F_{\text{max}}^{(2)}$, which can be derived straightforwardly as

$$\delta F_{\text{max}} = \frac{N(\rho) - 4\mathcal{F}(\rho) + 1}{6}. \quad (12)$$

Since both the anti-diagonal elements w and z of ρ contribute to $F_{\text{max}}^{(1)}$ and $F_{\text{max}}^{(2)}$ only in the form of $|w|$ and $|z|$ (see section above), it suffice to consider the special case of $\{w, z\} \in \mathbb{R}$, $w \geq 0$ and $z \geq 0$, and the conclusion obtained can be generalized directly to the cases for general X states with negative or complex anti-diagonal elements.

For $w \geq z$ and $\chi_0 \geq \chi_1$, one can obtain $N(\rho) - 4\mathcal{F}(\rho) + 1 = |2(a + d) - 1| - 2(a + d) + 1$, the relative magnitude

of which depends on the parameters a and d involved. If $a + d \geq 1/2$, then we have $N(\rho) - 4\mathcal{F}(\rho) + 1 = 0$ and $\delta F_{\max} = 0$, i.e., during this parameter region both $F_{\max}^{(1)}$ and $F_{\max}^{(2)}$ yield completely the same value. If $a + d < 1/2$, however, $N(\rho) - 4\mathcal{F}(\rho) + 1 = 2 - 4(a + d)$, which increases with decreasing value of $a + d$. Because the assumed condition $\chi_0 \geq \chi_1$ requires $1 + 2z \leq 2(a + d) + 2w$, and from Eq. (2) one can obtain that $2w \leq 2\sqrt{ad} \leq a + d$, thus we get $a + d \geq (1 + 2z)/3 \geq 1/3$. This, together with the assumed condition $a + d < 1/2$ gives rise to $N(\rho) - 4\mathcal{F}(\rho) + 1 \in (0, 2/3]$ and $\delta F_{\max} \in (0, 1/9]$.

For $w \geq z$ and $\chi_0 < \chi_1$, it is straightforward to check that $2(b + c) > 1 + 2(w - z) \geq 1$, which gives rise to $N(\rho) - 4\mathcal{F}(\rho) + 1 = 4(w - z) \leq 4w$. On the other hand, from $\chi_0 < \chi_1$ and Eq. (2) one can derive $1 + 2z > 2(a + d) + 2w$ and $a + d \geq 2\sqrt{ad} \geq 2w$, thus we get $w < (1 + z)/6 \leq 1/6$, which gives rise to the upper bound of $N(\rho) - 4\mathcal{F}(\rho) + 1$ as $2/3$. Moreover, the lower bound of $N(\rho) - 4\mathcal{F}(\rho) + 1$ is 0 because we have assumed $w \geq z$. Thus by combining these results we get $N(\rho) - 4\mathcal{F}(\rho) + 1 \in [0, 2/3)$ and $\delta F_{\max} \in [0, 1/9)$.

From the above analysis one can see that during the parameter region $w \geq z$, the difference between $F_{\max}^{(1)}$ and $F_{\max}^{(2)}$ ranges from 0 to $1/9$, i.e., $\delta F_{\max} \in [0, 1/9]$. The maximal difference $\delta F_{\max} = 1/9$ occurs only when the involved matrix elements satisfying $a + d = 1/3$, $b + c = 2/3$, $w = 1/6$, and $z = 0$, which corresponds to $F_{\max}^{(1)} = 2/3$ and $F_{\max}^{(2)} = 5/9$. Since $F_{\max}^{(2)} > 2/3$ ($\neq 5/9$) is guaranteed if $B_{\max}(\rho) > 2$, we can also conclude that for X states which violate the Bell-CHSH inequality, the difference between $F_{\max}^{(1)}$ and $F_{\max}^{(2)}$ must be smaller than $1/9$.

Moreover, for the case of $w < z$, one can still obtain $\delta F_{\max} \in [0, 1/9]$ after a similar analysis as performed for that of $w \geq z$, with however the maximal difference $\delta F_{\max} = 1/9$ occurs when $a + d = 2/3$, $b + c = 1/3$, $w = 0$, and $z = 1/6$. Thus in light of the above results, we can draw the following proposition.

Proposition 2. For all the X states, the maximal difference between $F_{\max}^{(1)}$ and $F_{\max}^{(2)}$ is $1/9$.

This Proposition represents the extent to which the average fidelity can be improved by Bob's arbitrary transformations. Particularly, for the case of $F_{\max}^{(2)} \in (5/9, 2/3]$, if Bob is equipped to perform some unitary transformations other than that of the identity or the Pauli rotation operation, then the average fidelity can be enhanced to over its classical limiting value of $2/3$.

Before ending this paper, we would also like to see fractions of different types of X states over the ensemble of the X states. Since all the X states which violate the Bell-CHSH inequality are entangled and useful for teleportation, while there also exist X states which satisfy the Bell-CHSH inequality but still can be used for teleportation, we can draw the conclusion that $P_E > P_T > P_B$, where P_E , P_T and P_B denote, respectively, fraction of the entangled X states, fraction of the X states that are useful for nonclassical teleportation and fraction of the X states that violate the Bell-CHSH inequality.

III. SUMMARY

In summary, we've studied possible relations between entanglement, Bell-CHSH inequality violation and quantum teleportation for the X states. As a generalization of the work [6] in which the authors proved that any two spin-1/2 state which violates the Bell-CHSH inequality is useful for teleportation if Bob is equipped to perform all types of the unitary transformations, here we further demonstrated that for the X states, nonclassical teleportation is also guaranteed by violation of the Bell-CHSH inequality even if Bob can only perform the identity or the Pauli rotation operations. Since the X states occur in many contexts [1, 15] and experimental realization of the Pauli rotation is comparatively simple (see [1] and references therein), we hope our results will be relevant to the practical teleportation process. Moreover, we also compared the maximal average fidelities $F_{\max}^{(1)}$ and $F_{\max}^{(2)}$, which associate to the situations in which Bob is allowed to perform any unitary transformation and Bob can only perform the identity or the Pauli rotation operations on his qubit, respectively. Our results revealed that the difference between them ranges from 0 to $1/9$, where the upper bound $1/9$ represents the greatest extent to which the average fidelity can be improved via Bob's arbitrary transformations.

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