High-efficiency multipartite entanglement purification of electron-spin states with charge detection^{*}

Tao Li, Bao-Cang Ren, Hai-Rui Wei, Ming Hua, and Fu-Guo Deng[†]

Department of Physics, Applied Optics Beijing Area Major Laboratory,

Beijing Normal University, Beijing 100875, China

(Dated: November 20, 2018)

We present a high-efficiency multipartite entanglement purification protocol (MEPP) for electron-spin systems in a Greenberger-Horne-Zeilinger state based on their spins and their charges. Our MEPP contains two parts. The first part is our normal MEPP with which the parties can obtain a high-fidelity N-electron ensemble directly, similar to the MEPP with controlled-not gates. The second one is our recycling MEPP with entanglement link from N'-electron subsystems (2 < N' < N). It is interesting to show that the N'-electron subsystems can be obtained efficiently by measuring the electrons with potential bit-flip errors from the instances which are useless and are just discarded in all existing conventional MEPPs. Combining these two parts, our MEPP has the advantage of the efficiency higher than other MEPPs largely for electron-spin systems.

Keywords: entanglement purification, electron spin, entanglement link, charge detection, decoherence

PACS numbers: 03.67.Bg, 03.67.Pp, 03.67.Hk

I. INTRODUCTION

Entanglement has been regarded as the essential resource for quantum information processing and quantum communication [1]. From a perspective of applications, the entangled systems shared by the separated parties in distant locations are required to be in a maximally entangled state for the efficiency and the security of quantum communication [2–7]. Especially, multipartite entangled systems have many vital applications in quantum computation [1] and quantum communication, such as controlled teleportation [8, 9], quantum state sharing [10–12], quantum secret sharing [13–15], and so on. However, all these tasks are based on the fact that the quantum channel with multipartite entangled states shared by the legitimate distant participants has been set up beforehand. It is well known that the parties in quantum communication cannot create nonlocal entanglement with local operations and classical communication (LOCC). The distribution of entanglement created locally is inevitable. However, in a practical transmission, the particles propagated away from each other are destined to suffer from channel noises, which will degrade the entanglement or even make the maximally entangled state become a mixed one. Therefore, it will decrease the fidelity of quantum teleportation [5] and quantum dense coding [6, 7], and make the quantum communication insecure [2–4].

Recently, much attention has been drawn to entanglement purification [16–30], a fascinating tool for the parties in quantum communication to extract some high-fidelity entangled states from a set of less entangled systems. The original entanglement purification protocol (EPP) by Bennett et al. [16] and that by Deutch et al. [17] are expressed in terms of the quantum controlled-not (CNOT) logic operations. Subsequently, Pan et al. [18] introduced an EPP with linear optical elements based on the polarization degree of freedom of photons. In 2002, Simon and Pan [19] presented an EPP with a currently available parametric down-conversion (PDC) source. In 2008, Sheng et al. [20] proposed an efficient EPP based on a PDC source with cross-Kerr nonlinearity and it can, in principle, be repeated to obtain a high-fidelity entangled ensemble. In 2011, Wang, Zhang, and Jin [21] proposed an interesting EPP based on cross-Kerr nonlinearity and the measurement on the intensity of coherent beams. In 2010, Sheng and Deng [27] introduced the concept of deterministic entanglement purification and proposed a two-step deterministic entanglement purification protocol (DEPP), the first DEPP in which the parties can obtain a maximally entangled state from each system transmitted, far different from the conventional entanglement purification protocols (CEPPs) [16–26]. Subsequently, a one-step DEPP [28–30] was proposed, only resorting to the spatial entanglement or the frequency entanglement of a practical PDC source and linear optical elements. In essence, both the CEPPs [16–26] and the DEPPs [27–30] are based on entanglement transfer. Taking the EPP for photon systems as an example, the CEPPs are based on the entanglement transfer between different entangled photon systems, while the DEPPs are based on the transfer between different degrees of freedom of the entangled photon system itself. The DEPPs require that at least one

^{*} Published in Quant. Inform. Proc. 12, 855 - 876 (2013). (DOI 10.1007/s11128-012-0427-2)

[†] Corresponding author. E-mail address: fgdeng@bnu.edu.cn.

degree of freedom of photons is stable when the photons are transmitted over a noisy channel, while the CEPPs only require that there is entanglement in the ensemble after transmission.

Up to now, most of EPPs [16–20, 27, 28] are focused on bipartite entangled photon systems and there are only several multipartite entanglement purification protocols, including high-dimension EPPs [22–26]. For instance, in 1998, Murao *et al.* [22] proposed the first multipartite entanglement purification protocol (MEPP) to purify multipartite entangled systems in a Werner-type state, with CNOT gates. In 2007, this protocol was extended to high-dimensional multipartite quantum systems by Cheong *et al.* [24], resorting to some generalized XOR gates, instead of the common CNOT gates. In 2009, with the considerable experimental progress achieved, Sheng *et al.* introduced a feasible MEPP for *N*-photon systems in a Greenberger-Horne-Zeilinger (GHZ) state [25]. In their protocol, quantum nondemolition detector (QND) is exploited to fulfill the functions of the parity-check gate. With QNDs, the parties can obtain some high-fidelity GHZ-state systems from the less entangled ones by performing the protocol iteratively.

A conduction electron can act as a qubit in both the charge degree of freedom and the spin degree of freedom, which are relatively independent on each other. In other words, when we measure the charge of an electron system, its spin state would be kept unaffected, and vice versa. Owing to this fantastic feature, charge detection [31] has been exploited to accomplish many works, such as the CONT gate between electronic qubits [32], the generation of the entangled spins [33], the multipartite entanglement analyzer [34], and so on. Also, some EPPs and an entanglement concentration protocol for electron systems have been proposed [35–38]. For example, in 2005, Feng *et al.* [35] proposed an electronic EPP for purifying two-electron systems in a Werner state with parity-check measurements based on charge detection [32], following some ideas in the original EPP proposed by Bennett *et al.* [16] for photon pairs. In 2011, Sheng *et al.* [36] presented a MEPP for electron-spin states and Wang *et al.* [37] proposed a two-electron EPP by using quantum-dot spins in optical microcavities. Although there are some MEPPs and some two-electron EPPs, the efficiency in these protocols is relatively low.

In this work, we present a high-efficiency MEPP for electron-spin systems in a GHZ state with charge detection. It contains two parts. One is our normal MEPP with which the parties in quantum communication can distill a high-fidelity N-electron ensemble directly, by replacing perfect CNOT gates with the parity- check detectors based on charge detection in Ref. [22], but with a higher efficiency. The other is our recycling MEPP in which the entanglement link based on charge detection is used to produce some N-electron entangled systems from entangled N'-electron subsystems ($2 \le N' < N$). It is interesting to show that the entangled N'-electron subsystems can be obtained efficiently from the cross-combination items, which are useless and are just discarded in all existing conventional EPPs [16–25, 35–37]. With these two parts, the present MEPP for electron-spin states has efficiency higher than all other MEPPs largely. We discuss the detail of our high-yield MEPP for electron-spin states of three-electron systems and its principle is suitable to the N-electron systems in an arbitrary GHZ state.

II. HIGH-EFFICIENCY THREE-ELECTRON ENTANGLEMENT PURIFICATION FOR BIT-FLIP ERRORS WITH ENTANGLEMENT LINK AND CHARGE DETECTION

Our thee-electron EPP contains two parts, which makes it different from others [16–25, 35–37]. One is our normal MEPP with which the parties can obtain a high-fidelity three-electron ensemble directly, similar to all existing MEPPs. The other is our recycling MEPP with entanglement link from subsystems. In the second part, the quantum resources are obtained from the systems with less entanglement which are just discarded in all other MEPPs. We introduce the principles of these two parts independently as follows.

A. Normal three-electron entanglement purification for bit-flip errors

Before we start to explain the principle of our MEPP, we present a detailed description of a parity-check detector (PCD) which is thought to be more feasible as a basic element for EPP than a perfect CNOT gate. In Fig.1, the polarizing beam splitter (PBS) transmits the electrons in the spin-up state $|\uparrow\rangle$ and reflects the ones in the spin-down state $|\downarrow\rangle$. Therefore, for two electrons coming from two different inputs of the first PBS, if they leave through different outputs of the first PBS, the charge detector (C) will get the charge occupation number C = 1; otherwise C = 0 or 2. The charge detector can distinguish the occupation number one from the occupation numbers 0 and 2, but it cannot distinguish the case between 0 and 2. In other words, it can only distinguish the instances that the occupation number is even or odd [32]. With this feature, one can see that the states $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ will lead the charge detector to obtain the charge occupation number C = 1 as the two electrons passing through the first PBS will leave through different modes. However, the states $|\uparrow\downarrow\rangle$ and $|\downarrow\downarrow\rangle$ will lead the charge detector to be C = 0 and C = 2, respectively. As mentioned above, the charge detector cannot distinguish 0 and 2, and it will show the same result, i.e., C = 0 for simplicity. That is, we can distinguish the states $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ from $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$, according

to the different outcomes of the charge detector. The second PBS is used to split the two electrons passing through the charge detector, without destroying their spin states. In essence, the setup shown in Fig.1 is just a parity-check detector (PCD) for the spins of two electrons.

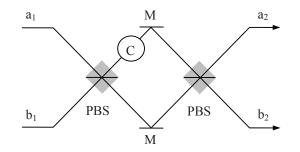


FIG. 1: The principle of a parity-check detector (PCD) based on charge detection. PBS represents a polarizing beam splitter for electron spins, which transmits the electron in the spin-up state $|\uparrow\rangle$ and reflects the electron in the spin-down state $|\downarrow\rangle$, respectively. *M* represents a mirror for the spins of an electron. *C* represents a charge detector and it can distinguish the electron number C = 1 from C = 0.

For a three-electron spin system, there are eight GHZ states,

$$\begin{split} |\Phi_{0}^{\pm}\rangle_{ABC} &= \frac{1}{\sqrt{2}} (|\uparrow\uparrow\uparrow\rangle \pm |\downarrow\downarrow\downarrow\rangle)_{ABC}, \\ |\Phi_{1}^{\pm}\rangle_{ABC} &= \frac{1}{\sqrt{2}} (|\downarrow\uparrow\uparrow\rangle \pm |\uparrow\downarrow\downarrow\rangle)_{ABC}, \\ |\Phi_{2}^{\pm}\rangle_{ABC} &= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\uparrow\rangle \pm |\downarrow\uparrow\downarrow\rangle)_{ABC}, \\ |\Phi_{3}^{\pm}\rangle_{ABC} &= \frac{1}{\sqrt{2}} (|\uparrow\uparrow\downarrow\rangle \pm |\downarrow\downarrow\uparrow\rangle)_{ABC}. \end{split}$$
(1)

Here the subscripts A, B, and C represent the three electrons belonging to the three parties, say Alice, Bob, and Charlie, respectively. Suppose that the original GHZ state transmitted is $|\Phi_0^+\rangle_{ABC}$. As we know, the noisy channel will inevitably degrade the entanglement of the state or even make it be a mixed one. In detail, if the initial state $|\Phi_0^+\rangle_{ABC}$ becomes $|\Phi_i^+\rangle_{ABC}$, a bit-flip error takes place on the *i*-th qubit (i = 1, 2, 3). If the state $|\Phi_0^+\rangle_{ABC}$ evolves to $|\Phi_0^-\rangle_{ABC}$, we say that a phase-flip error appears. Sometimes, both a bit-flip error and a phase-flip error will take place on the three-electron system such as the state $|\Phi_i^-\rangle_{ABC}$. In order to purify three-electron entangled systems, we are required to correct both bit-flip errors and phase-flip errors on the quantum system. Usually, an EPP can be divided into two steps [18, 20, 36]. One is used to purify the bit-flip error with a Hadamard operation on each qubit and then the parties purify the bit-flip error with the similar processes to these in the first step. That is, the phase-flip error can, in principle, be purified with the similar processes [18]. We only discuss the principle of the present MEPP for three-electron systems with bit-flip errors below.

Suppose the state of the tripartite electronic systems ρ shared by Alice, Bob, and Charlie is

$$\rho_{ABC} = F_0 |\Phi_0^+\rangle \langle \Phi_0^+| + F_1 |\Phi_1^+\rangle \langle \Phi_1^+| + F_2 |\Phi_2^+\rangle \langle \Phi_2^+| + F_3 |\Phi_3^+\rangle \langle \Phi_3^+|.$$
(2)

Here F_0 is the fidelity of the state $|\Phi_0^+\rangle$ after it is transmitted over a noisy channel. F_i (i = 1, 2, 3) is the probability that the three-electron system is in the state $|\Phi_i^+\rangle$. They satisfy the relation

$$F_0 + F_1 + F_2 + F_3 = 1. (3)$$

For obtaining some high-fidelity three-electron entangled systems, the three parties should operate a pair of threeelectron systems in the state ρ with LOCC. The principle of our normal three-electron EPP is shown in Fig.2. We label the two three-electron systems with $A_1B_1C_1$ and $A_2B_2C_2$, respectively. The state of the six-electron system $A_1B_1C_1A_2B_2C_2$ is $\rho_{A_1B_1C_1} \otimes \rho_{A_2B_2C_2}$. It can be viewed as the mixture of the 16 pure states, i.e., $|\Phi_i^+\rangle \otimes |\Phi_j^+\rangle$ with the probability of F_iF_j (i, j = 0, 1, 2, 3). The three parties make the electron pair they own pass through their PCDs. That is, the electron A_1 entrances the up spatial mode and A_2 the down-spatial mode, for comparing the spin parity

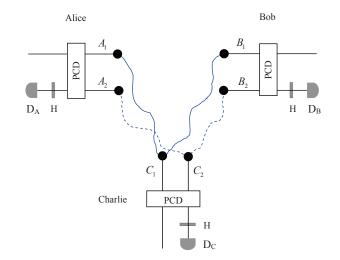


FIG. 2: The principle of our normal three-electron EPP with PCDs based on charge detection. PCD represents a parity-check detector. H represents a Hadamard operation. D_A , D_B , and D_C represent the single-electron measurements with the basis $Z = \{|\uparrow\rangle, |\downarrow\rangle\}$ done by Alice, Bob, and Charlie, respectively.

of their electron pair. After the parity-check measurements, Alice, Bob, and Charlie communicate their outcomes, and they keep the instances in which all the three parties obtain an even parity or an odd parity.

When the parities of the electron pairs obtained by Alice, Bob, and Charlie are all even, the state of the complicated system composed of the six electrons $A_1B_1C_1A_2B_2C_2$ becomes a mixed one $\rho_{t_{even}}$ (without normalization),

$$\rho_{t_{even}} = \frac{1}{2} (F_0^2 |\phi_0\rangle \langle \phi_0| + F_1^2 |\phi_1\rangle \langle \phi_1| + F_2^2 |\phi_2\rangle \langle \phi_2| + F_3^2 |\phi_3\rangle \langle \phi_3|).$$
(4)

where

$$|\phi_0\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\uparrow\rangle_{A_1B_1C_1}|\uparrow\uparrow\uparrow\uparrow\rangle_{A_2B_2C_2} + |\downarrow\downarrow\downarrow\downarrow\rangle_{A_1B_1C_1}|\downarrow\downarrow\downarrow\downarrow\rangle_{A_2B_2C_2}), \tag{5}$$

$$|\phi_1\rangle = \frac{1}{\sqrt{2}} (|\downarrow\uparrow\uparrow\rangle_{A_1B_1C_1}|\downarrow\uparrow\uparrow\rangle_{A_2B_2C_2} + |\uparrow\downarrow\downarrow\rangle_{A_1B_1C_1}|\uparrow\downarrow\downarrow\rangle_{A_2B_2C_2}, \tag{6}$$

$$\phi_2\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\uparrow\rangle_{A_1B_1C_1}|\uparrow\downarrow\uparrow\rangle_{A_2B_2C_2} + |\downarrow\uparrow\downarrow\rangle_{A_1B_1C_1}|\downarrow\uparrow\downarrow\rangle_{A_2B_2C_2}, \tag{7}$$

$$|\phi_{3}\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\downarrow\rangle_{A_{1}B_{1}C_{1}}|\uparrow\uparrow\downarrow\rangle_{A_{2}B_{2}C_{2}} + |\downarrow\downarrow\uparrow\rangle_{A_{1}B_{1}C_{1}}|\downarrow\downarrow\uparrow\rangle_{A_{2}B_{2}C_{2}}.$$
(8)

When the parties all get the odd parity, the state should be $\rho_{t_{odd}}$ (without normalization),

$$\rho_{t_{odd}} = \frac{1}{2} (F_0^2 |\psi_0\rangle \langle \psi_0| + F_1^2 |\psi_1\rangle \langle \psi_1| + F_2^2 |\psi_2\rangle \langle \psi_2| + F_3^2 |\psi_3\rangle \langle \psi_3|).$$
(9)

where

$$|\psi_0\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\uparrow\rangle_{A_1B_1C_1}|\downarrow\downarrow\downarrow\downarrow\rangle_{A_2B_2C_2} + |\downarrow\downarrow\downarrow\downarrow\rangle_{A_1B_1C_1}|\uparrow\uparrow\uparrow\uparrow\rangle_{A_2B_2C_2}, \tag{10}$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}} (|\downarrow\uparrow\uparrow\rangle_{A_1B_1C_1}|\uparrow\downarrow\downarrow\rangle_{A_2B_2C_2} + |\uparrow\downarrow\downarrow\rangle_{A_1B_1C_1}|\downarrow\uparrow\uparrow\rangle_{A_2B_2C_2}, \tag{11}$$

$$\psi_2\rangle = \frac{1}{\sqrt{2}} (\uparrow \downarrow \uparrow \rangle_{A_1 B_1 C_1} |\downarrow \uparrow \downarrow \rangle_{A_2 B_2 C_2} + |\downarrow \uparrow \downarrow \rangle_{A_1 B_1 C_1} |\uparrow \downarrow \uparrow \rangle_{A_2 B_2 C_2}, \tag{12}$$

$$\psi_3\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\downarrow\rangle_{A_1B_1C_1}|\downarrow\downarrow\uparrow\rangle_{A_2B_2C_2} + |\downarrow\downarrow\uparrow\rangle_{A_1B_1C_1}|\uparrow\uparrow\downarrow\rangle_{A_2B_2C_2}.$$
(13)

It is obvious that Alice, Bob, and Charlie obtain the same outcomes as the case in which they all obtain an even parity, after they perform a bit-flip operation on each of the three electrons $A_2B_2C_2$. That is, they can obtain the result that the complicated system composed of the six electrons $A_1B_1C_1A_2B_2C_2$ is in a mixture state ρ'_{teven} (without normalization)

$$\rho_{t_{even}}' = F_0^2 |\phi_0\rangle \langle \phi_0| + F_1^2 |\phi_1\rangle \langle \phi_1| + F_2^2 |\phi_2\rangle \langle \phi_2| + F_3^2 |\phi_3\rangle \langle \phi_3|.$$
(14)

We need only discuss the case that the system is in the states $|\phi_i\rangle$ with the probabilities F_i^2 below.

In the three down-spatial modes, a Hadamard (H) operation is performed on each of the electrons A_2 , B_2 , and C_2 , which will lead to the transformation

$$|\uparrow\rangle \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle), \qquad |\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle).$$
 (15)

That is, the states $|\phi_i\rangle$ and $|\psi_j\rangle$ (i, j = 0, 1, 2, 3) will be changed into some other states. Alice, Bob, and Charlie measure the spin states of the down-spatial modes $A_2B_2C_2$. It is not difficult to find that the outcomes can be divided into two groups, by considering the number of the spin-down electrons N_{\downarrow} . In the first case where N_{\downarrow} is even, that is, if they obtain the outcomes of measurements on their electrons through the down-spatial modes $|\uparrow\uparrow\uparrow\rangle A_2B_2C_2$, $|\downarrow\downarrow\uparrow\rangle A_2B_2C_2$, $|\downarrow\downarrow\downarrow\rangle A_2B_2C_2$ or $|\downarrow\uparrow\downarrow\rangle A_2B_2C_2$ or $|\downarrow\uparrow\downarrow\rangle A_2B_2C_2$, Alice, Bob, and Charlie will get the three-electron GHZ states $|\Phi_0^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$, $|\Phi_1^+\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\uparrow\uparrow\rangle + |\uparrow\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$, $|\Phi_2^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$, or $|\Phi_3^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\downarrow\downarrow\rangle + |\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$ with the probabilities of $\frac{1}{2}F_0^2$, $\frac{1}{2}F_1^2$, $\frac{1}{2}F_2^2$, $|\downarrow\downarrow\uparrow\rangle_{A_2B_2C_2}$ or $|\downarrow\downarrow\downarrow\rangle_{A_2B_2C_2}$, they will get the other three-electron GHZ states $|\Phi_0^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$, or $|\Phi_3^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$, or $|\Phi_2^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$, or $|\Phi_2^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$, $|\Phi_1^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$, $|\Phi_2^-\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$, $|\Phi_2^-\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$, $|\Phi_2^-\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$, $|\Phi_2^-\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$, $|\Phi_2^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$, $|\Phi_2^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\downarrow\rangle)A_1B_1C_1$, $|\Phi_2^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\downarrow\rangle)A_1B_$

Up to now, by keeping the instances in which all the parties obtain the same parity, and then measuring the electron spins from the down-spatial modes after the H operations, the quantum state of the three-electron system $A_1B_1C_1$ becomes ρ' . Here

$$\rho' = F_0^{'} |\Phi_0^+\rangle \langle \Phi_0^+| + F_1^{'} |\Phi_1^+\rangle \langle \Phi_1^+| + F_2^{'} |\Phi_2^+\rangle \langle \Phi_2^+| + F_3^{'} |\Phi_3^+\rangle \langle \Phi_3^+|.$$
(16)

where

$$F_{0}^{'} = \frac{F_{0}^{2}}{F_{0}^{2} + F_{1}^{2} + F_{2}^{2} + (1 - F_{0} - F_{1} - F_{2})^{2}},$$

$$F_{1}^{'} = \frac{F_{1}^{2}}{F_{0}^{2} + F_{1}^{2} + F_{2}^{2} + (1 - F_{0} - F_{1} - F_{2})^{2}},$$

$$F_{2}^{'} = \frac{F_{2}^{2}}{F_{0}^{2} + F_{1}^{2} + F_{2}^{2} + (1 - F_{0} - F_{1} - F_{2})^{2}},$$

$$F_{3}^{'} = \frac{(1 - F_{0} - F_{1} - F_{2})^{2}}{F_{0}^{2} + F_{1}^{2} + F_{2}^{2} + (1 - F_{0} - F_{1} - F_{2})^{2}}.$$
(17)

The fidelity of the new ensemble $F'_0 > F_0$ when the initial fidelity F_0 satisfies the relation

$$F_0 > \frac{1}{4} \left\{ 3 - 2F_1 - 2F_2 - \sqrt{1 + 4(F_1 + F_2) - 12(F_1^2 + F_2^2) - 8F_1F_2} \right\}.$$
(18)

For more distinct, we take the case $F_1 = F_2 = F_3$ (a symmetric noise model) as an example, and find that the fidelity of the state $|\Phi_0^+\rangle$ will be improved by our normal MEPP just when $F_0 > \frac{1}{4}$. That is, the initial fidelity before EPP is required to be $F_0 > \frac{1}{4}$, not the case in other MEPPs [22–25, 36] in which it is required to be $F_0 > \frac{1}{2}$.

We have fully discussed our normal MEPP for general bit-flip errors in three-electron systems. We use the PCDs based on charge detection, instead of the perfect CNOT gates [22], to fulfill the purification of bit-flip errors. Moreover, we give a general form for the purification of bit-flip errors in three-electron systems, not a Werner-type state [22] or a simplified mixed entangled state [36], which makes our normal MEPP have a higher efficiency than others [22, 36]. Especially, it doubles the efficiency of the MEPP for three-electron systems in Ref. [36] as the three parties not only consider the case in which they all obtain an even parity but also the case they all obtain an odd parity.

B. Recycling three-electron entanglement purification for bit-flip errors from subsystems

In our normal three-electron EPP for bit-flip errors, the three parties do not take the cross-combination items $|\Phi_i^+\rangle_{A_1B_1C_1} \otimes |\Phi_j^+\rangle_{A_2B_2C_2}$ $(i \neq j \in \{0, 1, 2, 3\})$ into account for obtaining high-fidelity three-electron systems because they obtain different parties. This is just the flaw in all existing CEPPs [16–25, 35–37]. For these cross-combination items, when the three participants perform some operations on their electrons A_2 , B_2 and C_2 , respectively, they cannot determine the state of the remaining three electrons $A_1B_1C_1$ from the up-spatial modes because the item $|\Phi_j^+\rangle_{A_1B_1C_1} \otimes |\Phi_i^+\rangle_{A_2B_2C_2}$ has the same probability F_iF_j as the item $|\Phi_i^+\rangle_{A_1B_1C_1} \otimes |\Phi_j^+\rangle_{A_2B_2C_2}$ ($i \neq j \in \{0, 1, 2, 3\}$). That is, Alice, Bob, and Charlie will obtain the state $|\Phi_i^+\rangle_{A_1B_1C_1}$ and $|\Phi_j^+\rangle_{A_1B_1C_1}$ with the same probability. These instances will decrease the fidelity of the state $|\Phi_0^+\rangle_{A_1B_1C_1}$ in the three-electron systems kept. This is just the reason that all existing CEPPs discard the cross-combination items. However, we cannot come to a simple conclusion that the cross-combination items are useless, because they can be used to distill some high-fidelity two-electron entangled subsystems, Alice, Bob, and Charlie can produce a subset of high-fidelity three-electron entangled subsystems, Alice, Bob, and Charlie can produce a subset of high-fidelity three-electron entangled subsystems, Alice, Bob, and Charlie can produce a subset of high-fidelity three-electron entangled subsystems, Alice, Bob, and Charlie can produce a subset of high-fidelity three-electron entangled subsystems, Alice, Bob, and Charlie can produce a subset of high-fidelity three-electron entangled subsystems, Alice, Bob, and Charlie can produce a subset of high-fidelity three-electron entangled subsystems with entanglement link based on charge detection, which is far different from the existing CEPPs [16–25, 35–37], including the MEPPs. We call this part of our MEPP the recy

In detail, our recycling EPP for three-electron systems includes three steps. One is to distill a set of high-fidelity entangled two-electron systems from the cross-combination items $|\Phi_i^+\rangle_{A_1B_1C_1} \otimes |\Phi_j^+\rangle_{A_2B_2C_2}$ $(i \neq j \in \{0, 1, 2, 3\})$. The second step is to improve the fidelity of subsystems with a two-electron EPP. The third step is to produce entangled three-electron systems from subspaces with entanglement link based on a PCD.

1. Two-electron entanglement distillation from the cross-combination items of three-electron systems

We take the two cross-combination items $|\varphi\rangle_1 = |\Phi_0^+\rangle_{A_1B_1C_1} \otimes |\Phi_1^+\rangle_{A_2B_2C_2}$ and $|\varphi\rangle_2 = |\Phi_1^+\rangle_{A_1B_1C_1} \otimes |\Phi_0^+\rangle_{A_2B_2C_2}$ as an example to demonstrate the principle of our two-electron entanglement distillation from three-electron systems with bit-flip errors. As for the other cross-combination items, we could deal with them in the same way with or without a little modification. It is interesting to point out that whether the cross-combination items is $|\varphi\rangle_1$ or $|\varphi\rangle_2$, the three parties will obtain the maximally entangled two-electron state $|\phi^+\rangle_{B_1C_1} \equiv \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)_{B_1C_1}$, by measuring the electron spins with potential errors.

To write the states $|\varphi\rangle_1$ and $|\varphi\rangle_2$ in a detail way, they can be described as

$$\begin{aligned} |\varphi\rangle_1 &= \frac{1}{2} (|\uparrow\uparrow\uparrow\rangle_{A_1B_1C_1}|\uparrow\downarrow\downarrow\rangle_{A_2B_2C_2} + |\downarrow\downarrow\downarrow\downarrow\rangle_{A_1B_1C_1}|\downarrow\uparrow\uparrow\rangle_{A_2B_2C_2} \\ &+ |\uparrow\uparrow\uparrow\uparrow\rangle_{A_1B_1C_1}|\downarrow\uparrow\uparrow\rangle_{A_2B_2C_2} + |\downarrow\downarrow\downarrow\downarrow\rangle_{A_1B_1C_1}|\uparrow\downarrow\downarrow\rangle_{A_2B_2C_2}), \end{aligned}$$
(19)

$$\begin{aligned} |\varphi\rangle_2 &= \frac{1}{2} (|\uparrow\downarrow\downarrow\rangle_{A_1B_1C_1}|\uparrow\uparrow\uparrow\uparrow\rangle_{A_2B_2C_2} + |\downarrow\uparrow\uparrow\rangle_{A_1B_1C_1}|\downarrow\downarrow\downarrow\rangle_{A_2B_2C_2} \\ &+ |\downarrow\uparrow\uparrow\rangle_{A_1B_1C_1}|\uparrow\uparrow\uparrow\uparrow\rangle_{A_2B_2C_2} + |\uparrow\downarrow\downarrow\rangle_{A_1B_1C_1}|\downarrow\downarrow\downarrow\rangle_{A_2B_2C_2}). \end{aligned}$$
(20)

One can see that if the outcomes of parity-check measurements done by Alice, Bob and Charlie are even, odd, and odd, respectively, the six-electron system is in the state

$$|\zeta\rangle_1 \equiv \frac{1}{\sqrt{2}} (|\uparrow\uparrow\uparrow\rangle_{A_1B_1C_1}|\uparrow\downarrow\downarrow\rangle_{A_2B_2C_2} + |\downarrow\downarrow\downarrow\rangle_{A_1B_1C_1}|\downarrow\uparrow\uparrow\rangle_{A_2B_2C_2})$$
(21)

which comes from the state $|\varphi\rangle_1$, or

$$|\zeta\rangle_2 \equiv \frac{1}{\sqrt{2}} (|\uparrow\downarrow\downarrow\rangle_{A_1B_1C_1}|\uparrow\uparrow\uparrow\uparrow\rangle_{A_2B_2C_2} + |\downarrow\uparrow\uparrow\rangle_{A_1B_1C_1}|\downarrow\downarrow\downarrow\downarrow\rangle_{A_2B_2C_2})$$
(22)

from $|\varphi\rangle_2$ with the same probability of $\frac{1}{2}F_0F_1$. If the outcomes are odd, even, and even, the whole state of the system is

$$\zeta\rangle_3 \equiv \frac{1}{\sqrt{2}} (|\uparrow\uparrow\uparrow\rangle_{A_1B_1C_1}|\downarrow\uparrow\uparrow\rangle_{A_2B_2C_2} + |\downarrow\downarrow\downarrow\rangle_{A_1B_1C_1}|\uparrow\downarrow\downarrow\rangle_{A_2B_2C_2})$$
(23)

from $|\varphi\rangle_1$, or

$$\zeta\rangle_4 \equiv \frac{1}{\sqrt{2}} (|\downarrow\uparrow\uparrow\rangle\rangle_{A_1B_1C_1}|\uparrow\uparrow\uparrow\uparrow\rangle_{A_2B_2C_2} + |\uparrow\downarrow\downarrow\rangle\rangle_{A_1B_1C_1}|\downarrow\downarrow\downarrow\downarrow\rangle\rangle_{A_2B_2C_2})$$
(24)

from $|\varphi\rangle_2$ with the same probability of $\frac{1}{2}F_0F_1$ too. With an *H* operation on each of the four electrons A_1 , A_2 , B_2 , and C_2 , the state $|\zeta\rangle_1$ will be transformed into

$$\begin{aligned} |\zeta\rangle_{1}^{H} &= \frac{1}{2} \{ (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)_{B_{1}C_{1}} (|\uparrow\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\uparrow\uparrow\rangle + |\uparrow\uparrow\downarrow\downarrow\rangle + |\downarrow\downarrow\downarrow\downarrow\rangle - |\downarrow\uparrow\downarrow\uparrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle \\ &- |\uparrow\downarrow\uparrow\downarrow\rangle)_{A_{1}A_{2}B_{2}C_{2}} + (|\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle)_{B_{1}C_{1}} (|\downarrow\uparrow\uparrow\uparrow\uparrow\rangle + |\uparrow\downarrow\downarrow\downarrow\downarrow\rangle + |\downarrow\downarrow\downarrow\downarrow\rangle + |\downarrow\downarrow\downarrow\uparrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle \\ &- |\downarrow\downarrow\uparrow\downarrow\downarrow\rangle - |\downarrow\uparrow\uparrow\downarrow\downarrow\rangle - |\uparrow\uparrow\uparrow\downarrow\downarrow\rangle - |\uparrow\uparrow\downarrow\downarrow\uparrow\rangle)_{A_{1}A_{2}B_{2}C_{2}} \}. \end{aligned}$$

$$(25)$$

In order to distill a two-electron entangled state, Bob and Charlie detect their electrons B_2 and C_2 , and Alice detects her two electrons A_1 and A_2 with the basis $\sigma_z \equiv \{|\uparrow\rangle, |\downarrow\rangle\}$, respectively. When the occupation number of $|\downarrow\rangle$ in the outcomes is even (such as $|\uparrow\uparrow\uparrow\uparrow\uparrow\rangle$, $|\downarrow\downarrow\uparrow\uparrow\downarrow\rangle$, $|\downarrow\uparrow\uparrow\downarrow\rangle$, and so on), Bob and Charlie obtain the two-electron entangled state $|\phi^+\rangle_{B_1C_1}$ with the probability of $\frac{1}{4}F_0F_1$. When it is odd, they can obtain the state $|\phi^-\rangle_{B_1C_1} \equiv \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)$ with the same probability of $\frac{1}{4}F_0F_1$. The state $|\phi^-\rangle_{B_1C_1}$ can be transformed into the state $|\phi^+\rangle$ by flipping its relative phase. As for the cases $|\zeta\rangle_2$, $|\zeta\rangle_3$, and $|\zeta\rangle_4$, the same conclusions can be drawn by simple calculations. Thus, the total probability of obtaining $|\phi^+\rangle_{B_1C_1}$ from the cross-combination items $|\varphi\rangle_1$ and $|\varphi\rangle_2$ is $2F_0F_1$.

Up to now, the principle of distilling two-electron entangled states from the cross-combination items $|\Phi_0^+\rangle_{A_1B_1C_1} \otimes |\Phi_1^+\rangle_{A_2B_2C_2}$ and $|\Phi_1^+\rangle_{A_1B_1C_1} \otimes |\Phi_0^+\rangle_{A_2B_2C_2}$ in which only one bit-flip error takes place on Alice's electrons, has been completely discussed. As for the other cross-combination items with one bit-flip error, using the same process described above, the three parties can obtain the two-electron entangled states $|\phi^+\rangle_{A_1C_1} \equiv \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)_{A_1C_1}$ and $|\phi^+\rangle_{A_1B_1} \equiv \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)_{A_1B_1}$ with the probabilities of $2F_0F_2$ and $2F_0F_3$, respectively. When we come to the cases that there is a bit-flip error on both the systems $A_1B_1C_1$ and $A_2B_2C_2$ (the two electrons with a bit-flip error belong to different participants), that is, $|\Phi_i^+\rangle_{A_1B_1} \equiv \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)_{A_1C_1} \equiv \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\downarrow\rangle)_{A_1C_1}, |\psi^+\rangle_{A_1B_1} \equiv \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)_{A_1B_1}, and |\psi^+\rangle_{B_1C_1} \equiv \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)_{B_1C_1}$ with the probabilities $2F_1F_3$, $2F_1F_2$, and $2F_2F_3$, respectively. That is, the states of the two-electron systems kept can be described as (without normalization)

$$\rho_{AB} = 2F_0F_3|\phi^+\rangle_{AB}\langle\phi^+| + 2F_1F_2|\psi^+\rangle_{AB}\langle\psi^+|,
\rho_{AC} = 2F_0F_2|\phi^+\rangle_{AC}\langle\phi^+| + 2F_1F_3|\psi^+\rangle_{AC}\langle\psi^+|,
\rho_{BC} = 2F_0F_1|\phi^+\rangle_{BC}\langle\phi^+| + 2F_2F_3|\psi^+\rangle_{BC}\langle\psi^+|.$$
(26)

The fidelity $F_i^b \equiv \frac{F_0 F_i}{F_0 F_i + F_j F_k}$ $(i, j \text{ and } k \text{ are different from each other, } i, j, k \in \{1, 2, 3\})$ of the two-electron subsystems in the state $|\phi^+\rangle$ is larger than that of the initial three-electron systems F_0 when the relation $F_0 < 1 - \frac{F_j F_k}{F_i}$ is satisfied. Let us take the symmetric noise model $F_1 = F_2 = F_3$ and $F_0 > F_1$ as an example to show the relation between F_i^b and F_0 . The inequality equation can be simplified to be $F_0 + F_1 < 1$ and the fidelity of the distilled two-electron subsystems $\frac{F_0}{F_0 + F_1}$ is unconditionally larger than that of the transmitted three-electron systems F_0 . That is to say, the parties can distill a two-electron spin subsystems with a fidelity higher than F_0 from each cross-combination item discarded in all conventional MEPPs.

2. Two-electron entanglement purification based on charge detection

After obtaining a set of two-electron entangled subsystems, Alice, Bob, and Charlie can first improve the fidelity of their two-electron ensembles and then produce a subset of three-electron systems with entanglement link based on charge detection. The principle of our EPP for two-electron subsystems is similar to all existing conventional two-qubit EPPs [16–21, 35, 37]. Let us use the two parties Alice and Bob to describe its principle clearly. The principle of two-electron EPP for any other two parties is the same as this one.

Suppose that the two-electron ensemble obtained by Alice and Bob is in the state

$$\rho_0^{AB} = f_0 |\phi^+\rangle_{AB} \langle \phi^+| + f_1 |\psi^+\rangle_{AB} \langle \psi^+|. \tag{27}$$

For each pair of two-electron subsystems, say A_1B_1 and A_2B_2 , their state is $\rho_0^{AB} \otimes \rho_0^{AB}$. It can be viewed as the mixture of the 4 pure entangled states, that is, $|\phi^+\rangle_{A_1B_1} \otimes |\phi^+\rangle_{A_2B_2}$, $|\phi^+\rangle_{A_1B_1} \otimes |\psi^+\rangle_{A_2B_2}$, $|\psi^+\rangle_{A_1B_1} \otimes |\psi^+\rangle_{A_2B_2}$, $|\psi^+\rangle_{A_1B_1} \otimes |\psi^+\rangle_{A_2B_2}$, and $|\psi^+\rangle_{A_1B_1} \otimes |\psi^+\rangle_{A_2B_2}$ with the probabilities of f_0f_0 , f_0f_1 , f_1f_0 , and f_1f_1 , respectively. Alice let her two electrons A_1 and A_2 pass through her PCD, shown in Fig.2. So does Bob. Alice and Bob keep the instances in which they both obtain an even parity or an odd parity.

When both Alice and Bob obtain an even parity, the four-electron system $A_1B_1A_2B_2$ is in the states

$$|\lambda_1\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\rangle_{A_1B_1}|\uparrow\uparrow\rangle_{A_2B_2} + |\downarrow\downarrow\rangle_{A_1B_1}|\downarrow\downarrow\rangle_{A_2B_2})$$

and

$$|\lambda_2\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle_{A_1B_1}|\uparrow\downarrow\rangle_{A_2B_2} + |\downarrow\uparrow\rangle_{A_1B_1}|\downarrow\uparrow\rangle_{A_2B_2})$$

with the probabilities of $\frac{1}{2}f_0f_0$ and $\frac{1}{2}f_1f_1$, respectively. When they both obtain an odd parity, the four-electron system is in the states

$$|\lambda_{3}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle_{A_{1}B_{1}}|\downarrow\downarrow\rangle_{A_{2}B_{2}} + |\downarrow\downarrow\rangle_{A_{1}B_{1}}|\uparrow\uparrow\rangle_{A_{2}B_{2}})$$

and

$$|\lambda_4\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle_{A_1B_1}|\downarrow\uparrow\rangle_{A_2B_2} + |\downarrow\uparrow\rangle_{A_1B_1}|\uparrow\downarrow\rangle_{A_2B_2})$$

with the probabilities of $\frac{1}{2}f_0f_0$ and $\frac{1}{2}f_1f_1$, respectively. With a bit-flip operation $\sigma_x = |\uparrow\rangle\langle\downarrow|+|\downarrow\rangle\langle\uparrow|$ on each of the two electrons A_2B_2 , the states $|\lambda_3\rangle$ and $|\lambda_4\rangle$ are transformed into the states $|\lambda_1\rangle$ and $|\lambda_2\rangle$, respectively. That is, Alice and Bob can obtain the states $|\lambda_1\rangle$ and $|\lambda_2\rangle$ with the probabilities of f_0f_0 and f_1f_1 , respectively, when they obtain the same parity.

After an H operation on each of the two electrons A_2 and B_2 , Alice and Bob measure the spins of these two electrons with the basis σ_z . If they obtain the same spin states (that is, both the spin-up state or the spin-down state), the two-electron subsystem A_1B_1 kept by Alice and Bob is in the states $|\phi^+\rangle_{A_1B_1}$ and $|\psi^+\rangle_{A_1B_1}$ with the probabilities of $\frac{1}{2}f_0f_0$ and $\frac{1}{2}f_1f_1$, respectively. If they obtain two different spin states (that is, one is the spin-up state and the other is the spin-down state), the two-electron subsystem A_1B_1 is in the states $|\phi^-\rangle_{A_1B_1}$ and $|\psi^-\rangle_{A_1B_1}$ and $|\psi^-\rangle_{A_1B_1}$ with the probabilities of $\frac{1}{2}f_0f_0$ and $\frac{1}{2}f_1f_1$, respectively. In this time, Alice and Bob can completely transform the states $|\phi^-\rangle_{A_1B_1}$ and $|\psi^-\rangle_{A_1B_1}$ into the states $|\phi^+\rangle_{A_1B_1}$ and $|\psi^+\rangle_{A_1B_1}$, respectively. That is to say, the two-electron ensemble after a round of purification is in the state (without normalization)

$$\rho_1^{AB} = f_0^2 |\phi^+\rangle_{AB} \langle \phi^+| + f_1^2 |\psi^+\rangle_{AB} \langle \psi^+|.$$
(28)

After Alice and Bob perform n times of this two-electron EPP, the two-electron ensemble kept is in the state (without normalization)

$$\rho_n^{AB} = f_0^{2^n} |\phi^+\rangle_{AB} \langle \phi^+| + f_1^{2^n} |\psi^+\rangle_{AB} \langle \psi^+|.$$
⁽²⁹⁾

The fidelity of the state $|\phi^+\rangle$ is

$$f_n' = \frac{f_0^{2^n}}{f_0^{2^n} + f_1^{2^n}}.$$
(30)

9

3. Three-electron entanglement production from two-electron subsystems with entanglement link

With a set of high-fidelity two-electron entangled subsystems, the three parties can create a subset of high-fidelity three-electron entangled systems nonlocally with entanglement link. As an example, we can use the case with three symmetric channels $F_1 = F_2 = F_3 = \frac{1-F_0}{3}$ to show the principle of three-electron entanglement production from two-electron subsystems below.

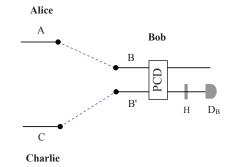


FIG. 3: The principle of the entanglement production of a three-electron system from two two-electron entangled subsystems with entanglement link based on a PCD.

Without the two-electron entanglement purification process, the density matrices in Eq.(26) are reduced to be (without normalization)

$$\rho_{AB}^{s} = 2F_{0}F_{1}|\phi^{+}\rangle_{AB}\langle\phi^{+}| + 2F_{1}^{2}|\psi^{+}\rangle_{AB}\langle\psi^{+}|,
\rho_{AC}^{s} = 2F_{0}F_{1}|\phi^{+}\rangle_{AC}\langle\phi^{+}| + 2F_{1}^{2}|\psi^{+}\rangle_{AC}\langle\psi^{+}|,
\rho_{BC}^{s} = 2F_{0}F_{1}|\phi^{+}\rangle_{BC}\langle\phi^{+}| + 2F_{1}^{2}|\psi^{+}\rangle_{BC}\langle\psi^{+}|.$$
(31)

After n times of the two-electron EPP are performed, the fidelity of the two-electron state $|\phi^+\rangle$ is improved largely and the density matrices in Eq.(31) become (without normalization)

$$\rho_{AB}^{sn} = (2F_0F_1)^{2^n} |\phi^+\rangle_{AB} \langle \phi^+| + (2F_1^2)^{2^n} |\psi^+\rangle_{AB} \langle \psi^+|,
\rho_{AC}^{sn} = (2F_0F_1)^{2^n} |\phi^+\rangle_{AC} \langle \phi^+| + (2F_1^2)^{2^n} |\psi^+\rangle_{AC} \langle \psi^+|,
\rho_{BC}^{sn} = (2F_0F_1)^{2^n} |\phi^+\rangle_{BC} \langle \phi^+| + (2F_1^2)^{2^n} |\psi^+\rangle_{BC} \langle \psi^+|.$$
(32)

Let us assume that $F_0^s \equiv (2F_0F_1)^{2^n}$ and $F_1^s \equiv (2F_1^2)^{2^n}$.

In principle, Alice, Bob, and Charlie can produce a three-electron entangled system from two two-electron subsystems with a high fidelity. Let us take the two-electron subsystems AB and B'C as an example to describe the principle of entanglement production from two-electron subsystems, shown in Fig.3. The state of the complicated quantum system composed of four electrons A, B, B', and C can be viewed as the mixture of four pure states. That is, it is in the states $|\phi^+\rangle_{AB} \otimes |\phi^+\rangle_{B'C}$, $|\phi^+\rangle_{AB} \otimes |\psi^+\rangle_{B'C}$, $|\psi^+\rangle_{AB} \otimes |\phi^+\rangle_{B'C}$, and $|\psi^+\rangle_{AB} \otimes |\psi^+\rangle_{B'C}$ with the probabilities of $F_0^s F_0^s$, $F_0^s F_1^s$, $F_1^s F_0^s$, and $F_1^s F_1^s$, respectively. Bob performs a parity-check detection on his two electrons B and B', and then he divides the four-electron system into two cases according to the outcomes obtained, i.e., an even-parity case and an odd-parity case. When Bob obtains the even parity, the four-electron system is in the states

$$|\Lambda_1\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\uparrow\uparrow\rangle_{ABB'C} + |\downarrow\downarrow\downarrow\downarrow\rangle_{ABB'C}), \tag{33}$$

$$|\Lambda_2\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\uparrow\downarrow\rangle_{ABB'C} + |\downarrow\downarrow\downarrow\uparrow\rangle_{ABB'C}), \qquad (34)$$

$$|\Lambda_3\rangle = \frac{1}{\sqrt{2}} (|\downarrow\uparrow\uparrow\uparrow\rangle_{ABB'C} + |\uparrow\downarrow\downarrow\downarrow\rangle_{ABB'C}), \tag{35}$$

and

$$|\Lambda_4\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\downarrow\uparrow\rangle_{ABB'C} + |\downarrow\uparrow\uparrow\downarrow\rangle_{ABB'C})$$
(36)

with the probabilities $\frac{1}{2}F_0^s F_0^s$, $\frac{1}{2}F_0^s F_1^s$, $\frac{1}{2}F_1^s F_0^s$, and $\frac{1}{2}F_1^s F_1^s$, respectively. To obtain the three-electron entangled state, Bob detects the spin of his electron B' with the basis $\sigma_z \equiv \{| \uparrow \rangle, | \downarrow \rangle\}$ after an H operation is performed on it. When he obtains $|\uparrow\rangle_{B'}$, the three electrons ABC are in a mixed state by mixing four pure states $|\Phi_0^+\rangle$, $|\Phi_3^+\rangle$, $|\Phi_1^+\rangle$, and $|\Phi_2^+\rangle$ with the probabilities of $\frac{1}{4}F_0^s F_0^s$, $\frac{1}{4}F_0^s F_1^s$, $\frac{1}{4}F_1^s F_0^s$, and $\frac{1}{4}F_1^s F_1^s$, respectively. When the state of the electron B' is $|\downarrow\rangle_{B'}$, they can obtain the same result with a phase-flip operation on the electron A. Therefore, the total probabilities that Alice, Bob, and Charlie obtain the states $|\Phi_0^+\rangle$, $|\Phi_1^+\rangle$, $|\Phi_2^+\rangle$, and $|\Phi_3^+\rangle$ are doubled eventually. As for the odd-parity case, the process utilized is nearly the same as the even-parity case but with an additional bit-flip operation σ_x performed on electron C. That is, the state of the three electrons ABC produced with entanglement link can be written as

$$\rho_{T} = (F_{0}^{s})^{2} |\Phi_{0}^{+}\rangle \langle \Phi_{0}^{+}| + F_{1}^{s} F_{0}^{s} |\Phi_{1}^{+}\rangle \langle \Phi_{1}^{+}| + (F_{1}^{s})^{2} |\Phi_{2}^{+}\rangle \langle \Phi_{2}^{+}| + F_{0}^{s} F_{1}^{s} |\Phi_{3}^{+}\rangle \langle \Phi_{3}^{+}| \\
= (2F_{0}F_{1})^{2^{n+1}} |\Phi_{0}^{+}\rangle \langle \Phi_{0}^{+}| + (2F_{0}F_{1})^{2^{n}} (2F_{1}^{2})^{2^{n}} |\Phi_{1}^{+}\rangle \langle \Phi_{1}^{+}| + (2F_{1}^{2})^{2^{n+1}} |\Phi_{2}^{+}\rangle \langle \Phi_{2}^{+}| \\
+ (2F_{0}F_{1})^{2^{n}} (2F_{1}^{2})^{2^{n}} |\Phi_{3}^{+}\rangle \langle \Phi_{3}^{+}|.$$
(37)

The fidelity of the three-electron state $|\Phi_0^+\rangle_{ABC}$ is

$$F_0^T = \frac{F_0^{2^{n+1}}}{(F_0^{2^n} + F_1^{2^n})^2}.$$
(38)

 $F_0^T > F_0$ when $F_0 > \frac{1}{4}$.

C. Numerical comparisons of efficiency, fidelity, and yield between our MEPP and conventional MEPPs

Usually, the efficiency of an EPP E is defined as the probability that the parties can obtain a high-fidelity entangled three-electron system (or entangled two-electron subsystems) from a pair of low-fidelity systems transmitted over a noisy channel without loss after the parties perform a round of the EPP. The yield of an EPP Y is defined as the probability that the parties can obtain an entangled three-electron system with the fidelity higher than a threshold value F_{thr} , from a pair of low-fidelity systems transmitted after some rounds of the EPP are performed. It is obvious that the efficiency and the yield of our three-electron EPP E_o and Y_o depends on the three parameters F_1 , F_2 , and F_3 . For simplicity, we only discuss the case with the parameters $F_1 = F_2 = F_3 = \frac{1-F_0}{3}$ below to show the difference between our MEPP and conventional ones clearly. Let us assume that the threshold value of the final fidelity after purification is $F_{thr} = 0.95$, which means that the parties should improve the fidelity of the three-electron systems kept to be $F' \geq F_{thr}$ by repeating the MEPP some times. For comparison, we use the efficiency obtained with our normal EPP E_n to represent that in conventional MEPPs as it is just the maximal efficiency that the parties can obtain from the identity-combination items.

By running our normal three-photon EPP only once, the efficiency of the three-photon EPP $E_n^{(1)}$ is

$$E_n^{(1)} = F_0^2 + F_1^2 + F_2^2 + F_3^2 = \frac{1 - 2F_0 + 4F_0^2}{3}.$$
(39)

It is just the probability that the pair of three-electron systems are in the identity-combination items $|\Phi_i^+\rangle_{A_1B_1C_1} \otimes |\Phi_i^+\rangle_{A_2B_2C_2}$ (i = 0, 1, 2, 3). After a round of entanglement purification, the fidelity of the three-electron systems kept (i.e., the relative probability of the state $|\Phi_0^+\rangle_{ABC}$) becomes

$$F_n^{(1)} = \frac{F_0^2}{F_0^2 + F_1^2 + F_2^2 + F_3^2} = \frac{3F_0^2}{1 - 2F_0 + 4F_0^2}.$$
(40)

As each cross-combination item $|\Phi_i^+\rangle \otimes |\Phi_j^+\rangle$ $(i \neq j \in \{0, 1, 2, 3\})$ will lead the three parties to obtain an entangled two-electron subsystem, the probability $P_{3\to 2}^{(1)}$ that the three parties obtain two-electron subsystems from a pair of three-electron systems in the cross-combination items is

$$P_{3\to2}^{(1)} = \sum_{\substack{j\neq l=0\\ F_2(F_0+F_1+F_3)+F_3(F_0+F_1+F_2)\\ = \frac{2+2F_0-4F_0^2}{3}}.$$
(41)

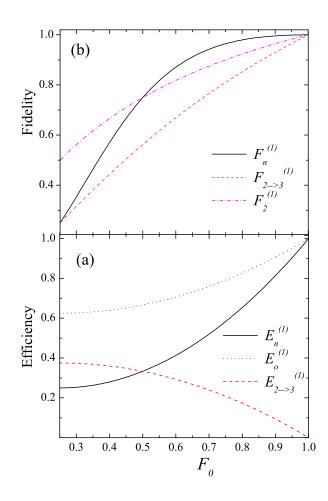


FIG. 4: Numerical comparison for the efficiency and fidelity between our MEPP and others. (a) The efficiency of our EPP $E_o^{(1)}$ and the maximal value of efficiency from the conventional MEPPs $E_n^{(1)}$ for three-electron systems under a symmetric noise $(F_1 = F_2 = F_3 = \frac{1-F_0}{3})$ are shown with a blue dot line and a black solid line, respectively. Here $E_{2\rightarrow3}^{(1)}$ is the efficiency that the three parties can obtain three-electron systems from two-electron systems with entanglement link based on charge detection. (b) The fidelity of the present MEPP F_e and that of the conventional MEPP F_n . Here F_2 and $F_{2\rightarrow3}$ is the fidelities of the two-photon systems obtained from the cross-combinations and that of the three-photon systems obtained directly from two-photon systems with entanglement link, respectively. F_0 is just the original fidelity of three-photon systems before entanglement purification.

Because Alice, Bob, and Charlie can in principle obtain a three-electron system from a pair of two-electron subsystems with entanglement link based on charge detection if they do not improve the fidelity of their two-electron subsystems before entanglement link, the efficiency that the three parties obtain three-electron entangled systems from two-electron entangled subsystems $E_{2\to3}^{(1)}$ is a half of $P_{3\to2}^{(1)}$, that is,

$$E_{2\to3}^{(1)} = \frac{1}{2}P_{3\to2}^{(1)} = \frac{1+F_0 - 2F_0^2}{3}.$$
(42)

Taking three-electron entanglement production with entanglement link into account, the efficiency of our MEPP $E_o^{(1)}$ for three-electron systems after the three parties accomplish a round of entanglement purification for bit-flip errors is

$$E_o^{(1)} = E_n^{(1)} + E_{2 \to 3}^{(1)} = \frac{2 - F_0 + 2F_0^2}{3}.$$
(43)

The efficiency of our MEPP $E_o^{(1)}$ and the maximal value of that from the identity-combination items for three-qubit systems $E_n^{(1)}$ are shown in Fig.4(a). Also, we give the efficiency that the three parties obtain three-electron entangled systems from two-electron entangled subsystems $E_{2\rightarrow 3}^{(1)}$ in Fig.4(a).

The fidelity of the two-electron subsystems obtained from cross-combinations without two-electron entanglement purification is

$$F_2^{(1)} = \frac{2F_0F_1}{2F_0F_1 + 2F_1^2} = \frac{F_0}{F_0 + F_1} = \frac{3F_0}{1 + 2F_0}.$$
(44)

The fidelity of the three-electron systems obtained from two-electron subsystems with entanglement link is

$$F_{2\to3}^{(1)} = \frac{F_0^2}{(F_0 + F_1)^2} = \frac{9F_0^2}{1 + 4F_0 + 4F_0^2}.$$
(45)

The relation of the three fidelities $F_n^{(1)}$, $F_2^{(1)}$, and $F_{2\to3}^{(1)}$ is shown in Fig.4(b).

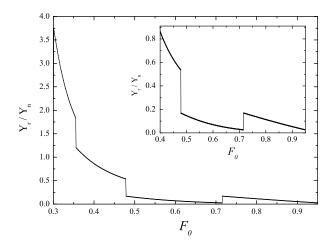


FIG. 5: The rate of the yield from our recycling thee-electron EPP Y_r to that from our normal three-electron EPP Y_n with the threshold value $F_{thr} = 0.95$. In order to see the contribution of our recycling EPP in a clear way, we use an insert to show the rate for the original fidelity F_0 from 0.4 to 0.95.

From Fig.4, one can see that our MEPP is more efficient than the conventional MEPPs, especially in the case that the original fidelity F_0 is not big. On the other hand, the fidelity $F_{2\to3}^{(1)}$ is smaller than $F_n^{(1)}$ although they both are larger than the original fidelity F_0 when $F_0 > \frac{1}{4}$. $F_2^{(1)}$ is larger than $F_n^{(1)}$ when $F_0 < \frac{1}{2}$ and it is smaller than $F_n^{(1)}$ when $F_0 > \frac{1}{2}$. If three parties first run the two-electron EPP *n* times and then produce some three-electron systems with entanglement link from high-fidelity two-electron subsystems, the fidelity $F_{2\to3}^{(n)} = \frac{F_0^{2^{n+1}}}{(F_0^{2^n} + F_1^{2^n})^2}$ can be improved to be larger than $F_n^{(1)}$.

In order to show the contribution of the part from our recycling EPP clearly, we calculate the rate of its yield Y_r to that from our normal EPP Y_n for three-electron systems with the threshold value $F_{thr} = 0.95$, shown in Fig.5. From this figure, one can see that the contribution of our recycling three-electron EPP is larger than that of our normal EPP if the original fidelity F_0 is smaller than 0.38. When F_0 is no more than 0.478, the contribution of our recycling EPP is considerable. When F_0 is larger than 0.716, the number that the parties need to repeat their EPP for obtaining three-electron systems with the fidelity larger than the threshold value $F_{thr} = 0.95$ from two-electron subsystems is reduced to one, which increases the rate of the contribution of our recycling EPP. As our three-electron EPP and our recycling EPP, no matter what the original fidelity F_0 is, the yield of our three-electron EPP is larger than that from conventional MEPPs as the latter is just the part discarded in all conventional MEPPs [22–25, 36].

III. DISCUSSION AND SUMMARY

We have fully described the principle of our efficient three-electron EPP for GHZ states. It is not difficult to prove that our efficient EPP works for N-electron systems in a GHZ state. The GHZ state of a multipartite entangled system composed of N electrons can be described as

$$|\Phi_0^+\rangle_N = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\uparrow\cdots\uparrow\rangle + |\downarrow\downarrow\cdots\downarrow\rangle)_{A,B,\cdots,Z}.$$
(46)

Here the subscripts A, B, \cdots , and Z represent the electrons belonging to the parties Alice, Bob, \cdots , and Zach, respectively. Certainly, there are another $2^N - 1$ GHZ states for an N-qubit system and can be written as

$$|\Phi_{ij\cdots k}^{+}\rangle_{N} = \frac{1}{\sqrt{2}} (|ij\cdots k\rangle + |\bar{i}\bar{j}\cdots\bar{k}\rangle)_{AB\cdots Z}$$

$$\tag{47}$$

and

$$|\Phi_{ij\cdots k}^{-}\rangle_{N} = \frac{1}{\sqrt{2}} (|ij\cdots k\rangle - |\bar{ij}\cdots \bar{k}\rangle)_{AB\cdots Z}.$$
(48)

Here $\overline{i} = 1 - i$, $\overline{j} = 1 - j$, $\overline{k} = 1 - k$, and $i, j, k \in \{0, 1\}$. $|0\rangle \equiv |\uparrow\rangle$ and $|1\rangle \equiv |\downarrow\rangle$. For correcting the bit-flip errors in N-electron entangled quantum systems, we can also divide the whole entanglement purification into two parts. One is our normal N-electron entanglement purification and the other is our recycling entanglement purification with entanglement link from subsystems. Our normal entanglement purification for N-electron entangled quantum systems with bit-flip errors is similar to that for three-electron entangled quantum systems. We should only increase the number of the PCDs and the Hadamard operations shown in Fig. 2. Let us assume that the ensemble of N-electron systems after the transmission over a noisy channel is in the state

$$\rho_N = f'_0 |\Phi^+_0\rangle_N \langle\Phi^+_0| + \dots + f'_{ij\dots k} |\Phi^+_{ij\dots k}\rangle_N \langle\Phi^+_{ij\dots k}| + \dots + f'_{2^{N-1}-1} |\Phi^+_{2^{N-1}-1}\rangle_N \langle\Phi^+_{2^{N-1}-1}|.$$
(49)

Here $f_{ij\cdots k}^{'}$ presents the probability that an N-electron system is in the state $|\Phi_{ij\cdots k}^{+}\rangle_{N}$ and

$$f_0' + \dots + f_{ij\dots k}' + \dots + f_{2^{N-1}-1}' = 1.$$
(50)

In our normal N-electron entanglement purification for a pair of systems, the parties will keep the identity-combination items $|\Phi_0^+\rangle_N \otimes |\Phi_0^+\rangle_N, \cdots, |\Phi_{ij\cdots k}^+\rangle_N \otimes |\Phi_{ij\cdots k}^+\rangle_N, \cdots,$ and $|\Phi_{2^{N-1}-1}^+\rangle_N \otimes |\Phi_{2^{N-1}-1}^+\rangle_N$ with the probabilities $f_0'^2, \cdots, f_{ij\cdots k}'^2, \cdots,$ and $f_{2^{N-1}-1}'^2$, respectively. That is, they keep the instances in which they all obtain the even parity and those in which they all obtain the odd parity with their PCDs. With the similar process to the case for three-electron systems, the parties can obtain a new N-electron system which is in the states $|\Phi_0^+\rangle_N = \frac{1}{\sqrt{2}}(|HH\cdots H\rangle + |VV\cdots V\rangle)_{A_1,B_1,\cdots,Z_1}, \cdots, |\Phi_{ij\cdots k}^+\rangle_N = \frac{1}{\sqrt{2}}(|ij\cdots k\rangle + |ij\cdots k\rangle_{A_1,B_1,\cdots,Z_1}, \cdots, \text{ and } |\Phi_{2^{N-1}-1}^+\rangle_N = \frac{1}{\sqrt{2}}(|HV\cdots V\rangle + |VH\cdots H\rangle)_{A_1,B_1,\cdots,Z_1}, \cdots, |\Phi_{ij\cdots k}^+\rangle_N = \frac{1}{\sqrt{2}}(|ij\cdots k\rangle + |ij\cdots k\rangle_{A_1,B_1,\cdots,Z_1}, \cdots, \text{ and } |\Phi_{2^{N-1}-1}^+\rangle_N = \frac{1}{\sqrt{2}}(|HV\cdots V\rangle + |VH\cdots H\rangle)_{A_1,B_1,\cdots,Z_1}$ with the probabilities $f_0'^2, \cdots, f_{ij\cdots k}'^2, \cdots,$ and $f_{2^{N-1}-1}'$, respectively. That is, the parties can obtain a new ensemble of N-electron systems ρ'_N with the fidelity $f_0'' = \frac{f_0'^2}{f_0'^2+\cdots+f_{ij\cdots k}'+f_{2^{N-1}-1}'}$ from the original ensemble in the state ρ_N . Our recycling EPP is used to distill some N'-electron subsystems ($2 \leq N' < N$) from the cross-combination items $|\Phi_{lr\cdots q}^+\rangle_N \otimes |\Phi_{ij\cdots k}^+\rangle_N \otimes |\Phi_{lr\cdots q}^+\rangle_N (l, r, \cdots, q \in \{0, 1\}$ and $l \neq i, r \neq j, \cdots,$ or $q \neq k$). Its process is also similar to the case with three-electron systems. The more the number of the electrons in each system, the more the kinds of the entanglement purification with entanglement link.

Compared with the conventional MEPPs [22–25, 36], the present MEPP contains two parts. One is our normal MEPP which is similar to the conventional MEPPs as the high-fidelity N-electron systems are directly obtained from the identity-combination items $|\Phi_0^+\rangle_N \otimes |\Phi_0^+\rangle_N, \cdots, |\Phi_{ij\cdots k}^+\rangle_N \otimes |\Phi_{ij\cdots k}^+\rangle_N, \cdots$, and $|\Phi_{2N-1-1}^+\rangle_N \otimes |\Phi_{2N-1-1}^+\rangle_N$. However, as the cross-combination items $|\Phi_{lr\cdots q}^+\rangle_N \otimes |\Phi_{ij\cdots k}^+\rangle_N$ and $|\Phi_{ij\cdots k}^+\rangle_N \otimes |\Phi_{lr\cdots q}^+\rangle_N \otimes |\Phi_{2N-1-1}^+\rangle_N$. However, or $q \neq k$) can not be used to obtain high-fidelity N-electron systems directly, they are discarded in the conventional MEPPs. In our recycling EPP, the second part of our efficient MEPPs, the parties distill some subsystems with a high fidelity from the cross-combination items. With entanglement purification on the subsystems and the entanglement production based on local entanglement link, the parties can obtain some additional yield of N-electron systems with the fidelity higher than the threshold value, which makes our MEPP more efficient than the conventional MEPPs.

The PCD is the key element in our high-yield MEPP and charge detection plays a crucial role in constructing the PCD for the spins of two electrons. Charge detection has been realized by means of point contacts in a two-dimensional electron gas. For instance, Field *et al.* [31] used the effect of the electric field of the charge on the conductance of an adjacent point contact to realize the charge detection in 1993. Elzerma *et al.* [39] reported their experimental results that the current achievable time resolution for charge detection is μs in 2004. Trauzettel *et al.* [40] also proposed a realization of a charge-parity meter which is based on two double quantum dots alongside a quantum point contact in 2006. Their realization of such a device can be seen as a specific example of the general class of mesoscopic quadratic quantum measurement detectors which is investigated by Mao *et al.* [41]. Moreover, recent studies showed that the interaction between the polarizations of photons and the electron spins of quantum dots in optical cavities can also be used to construct the PCD, as shown in Refs.[42–47]

In summary, we have proposed a high-efficiency MEPP for N-electron systems in a GHZ state, resorting to the PCD based on charge detection. It contains two parts. One is our normal MEPP with which the parties can obtain a high-fidelity N-electron ensemble directly. This part comes from the identity-combination items of a pair of N-electron systems, similar to conventional MEPPs but with a higher efficiency. The cross-combination items, which are discarded in all existing conventional MEPPs, can be used to distill some N'-electron subsystems ($2 \le N' < N$) by measuring the electrons with potential errors in our recycling MEPP, the second part of our high-yield MEPP. Our normal MEPP has a higher efficiency than the MEPP for a Werner-type state with perfect CNOT gates [22]. Especially, it doubles the efficiency of the MEPP with QNDs based on cross-Kerr nonlinearity in Ref. [25] and the MEPP for electronic systems [36]. In our recycling MEPP, the parties in quantum communication can produce some high-fidelity N-electron systems with entanglement link based on the parity-check detection. Combining the second part of our MEPP with the first one, the present MEPP has a higher efficiency and yield than the conventional MEPPs largely.

ACKNOWLEDGEMENTS

This work is supported by the National Natural Science Foundation of China under Grant Nos. 10974020 and 11174039, NCET-11-0031, and the Fundamental Research Funds for the Central Universities.

- Nielsen, M.A., Chuang, I.L.: Quantum Computation and Quantum Information. Cambridge University Press, Cambridge (2000)
- [2] Ekert, A.K.: Quantum cryptography based on Bells theorem. Phys. Rev. Lett. 67, 661 (1991)
- [3] Bennett, C.H., Brassard, G., Mermin, N.D.: Quantum cryptography without Bells theorem. Phys. Rev. Lett. 68, 557 (1992)
- [4] Long, G.L., Liu, X.S.: Theoretically efficient high-capacity quantum-key-distribution scheme. Phys. Rev. A 65, 032302 (2002)
- [5] Bennett, C. H., Brassard, G., Crepeau, C., Jozsa, R., Peres, A., Wootters, W. K.: Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Phys. Rev. Lett. 70, 1895 (1993)
- Bennett, C.H., Wiesner, S.J.: Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states. Phys. Rev. Lett. 69, 2881 (1992)
- [7] Liu, X.S., Long, G.L., Tong, D.M., Feng, L.: General scheme for superdense coding between multiparties. Phys. Rev. A 65, 022304 (2002)
- [8] Karlsson, A., Bourennane, M.: Quantum teleportation using three-particle entanglement. Phys. Rev. A 58, 4394 (1998)
- [9] F.G. Deng, Li, C.Y., Li,Y.S., Zhou, H.Y., Wang, Y.: Symmetric multiparty-controlled teleportation of an arbitrary twoparticle entanglement. Phys. Rev. A **72**, 022338 (2005)
- [10] Cleve, R., Gottesman, D., Lo, H.K., How to Share a Quantum Secret. Phys. Rev. Lett. 83, 648 (1999)
- [11] Lance, A.M., Symul, T., Bowen1, W.P., Sanders B.C., Lam, P.K.: Tripartite quantum state sharing. Phys. Rev. Lett. 92, 177903 (2004)
- [12] F.G. Deng, Li, X.H., Li, C.Y., Zhou, P., Zhou, H.Y.: Multiparty quantum-state sharing of an arbitrary two-particle state with Einstein-Podolsky-Rosen pairs. Phys. Rev. A 72, 044301 (2005)
- [13] Hillery, M., Bužek, V., Berthiaume, A.: Quantum secret sharing. Phys. Rev. A 59, 1829 (1999)
- [14] Karlsson, A., Koashi, M., Imoto, N.: Quantum entanglement for secret sharing and secret splitting. Phys. Rev. A 59, 162 (1999)
- [15] Xiao, L., Long, G.L., Deng, F.G., Pan J.W.: Efficient multiparty quantum-secret-sharing schemes. Phys. Rev. A 69, 052307 (2004)
- [16] Bennett, C.H., Brassard, G., Popescu, S., Schumacher, B., Smolin, J.A., Wootters, W.K.: Purification of noisy entanglement and faithful teleportation via noisy channels. Phys. Rev. Lett. 76, 722 (1996)
- [17] Deutsch, D., Ekert, A., Jozsa, R., Macchiavello, C., Popescu, S., Sanpera, A. : Quantum privacy amplification and the security of quantum cryptography over noisy channels. Phys. Rev. Lett. 77, 2818 (1996)
- [18] Pan, J.W., Simon, C., Zellinger, A.: Entanglement purification for quantum communi- cation. Nature (London) 410, 1067 (2001)
- [19] Simon, C., Pan, J.W.: Polarization entanglement purification using spatial entanglement. Phys. Rev. Lett. 89, 257901 (2002)
- [20] Sheng, Y.B., Deng, F.G., Zhou, H.Y.: Efficient polarization-entanglement purification based on parametric down-conversion sources with cross-Kerr nonlinearity. Phys. Rev. A 77, 042308 (2008)
- [21] Wang, C., Zhang, Y., Jin, G.S.: Polarization-entanglement purification and concentration using cross-kerr nonlinearity. Quantum Inf. Comput. 11, 988 (2011)
- [22] Murao, M., Plenio, M.B., Popescu, S., Vedral, V., Knight, P.L.: Multiparticle entanglement purification protocols. Phys. Rev. A 57, R4075 (1998)

- [23] Horodecki, M., Horodecki, P.: Reduction criterion of separability and limits for a class of distillation protocols. Phys. Rev. A 59, 4206 (1999)
- [24] Cheong, Y.W., Lee, S.W., Lee, J., Lee, H.W.: Entanglement purification for high-dimensional multipartite systems. Phys. Rev. A 76, 042314 (2007)
- [25] Sheng, Y.B., Deng, F.G., Zhao, B.K., Wang, T.J., Zhou, H.Y.: Multipartite entanglement purification with quantum nondemolition detectors. Eur. Phys. J. D 55, 235 (2009)
- [26] Deng, F.G.: Efficient multipartite entanglement purification with the entanglement link from a subspace. Phys. Rev. A 84, 052312 (2011)
- [27] Sheng, Y.B., Deng, F.G.: Deterministic entanglement purification and complete nonlocal Bell-state analysis with hyperentanglement. Phys. Rev. A 81, 032307 (2010)
- [28] Li, X.H.: Deterministic polarization-entanglement purification using spatial entanglement. Phys. Rev. A 82, 044304 (2010)
- [29] Sheng, Y.B., Deng, F.G.: One-step deterministic polarization-entanglement purification using spatial entanglement. Phys. Rev. A 82, 044305 (2010)
- [30] Deng, F.G.: One-step error correction for multipartite polarization entanglement. Phys. Rev. A 83, 062316 (2011)
- [31] Field, M., Smith, C.G., Pepper, M., Ritchie, D.A., Frost, J.E.F., Jones, G.A.C., Hasko, D.G.: Measurements of Coulomb blockade with a noninvasive voltage probe. Phys. Rev. Lett. 70, 1311 (1993)
- [32] Beenakker, C.W.J., DiVincenzo, D.P., Tmary, C., Kindermann, M.: Charge detection enables free-electron quantum computation. Phys. Rev. Lett. 93, 020501 (2004)
- [33] Ionicioiu, R.: Entangling spins by measuring charge: a parity-gate toolbox. Phys. Rev. A 75, 032339 (2007)
- [34] Zhang, X.L., Feng, M., Gao, K. L.: Cluster-state preparation and multipartite entanglement analyzer with fermions. Phys. Rev. A 73, 014301 (2006)
- [35] Feng, X.L., Kwek, L.C., Oh, C.H.: Electronic entanglement purification scheme enhanced by charge detections. Phys. Rev. A 71, 064301 (2005)
- [36] Sheng, Y.B., Deng, F.G., Long, G.L.: Multipartite electronic entanglement purification with charge detection. Phys. Lett. A 375, 396 (2010)
- [37] Wang, C., Zhang, Y., Jin, G.S.: Entanglement purification and concentration of electron-spin entangled states using quantum-dot spins in optical microcavities. Phys. Rev. A 84, 032307 (2011)
- [38] Sheng, Y.B., Deng, F.G., Zhou, H.Y.: Efficient polarization entanglement concentration for electrons with charge detection. Phys. Lett. A 373, 1823 (2009)
- [39] Elzerman, J.M., Hanson, R., Willems van Beveren, L.H., Vandersypen, L.M.K., Kouwenhoven, L.P.: Excited-state spectroscopy on a nearly closed quantum dot via charge detection. Appl. Phys. Lett. 84, 4617 (2004)
- [40] Trauzettel, B., Jordan, A.N., Beenakker, C.W.J., Büttiker, M.: Parity meter for charge qubits: an efficient quantum entangler. Phys. Rev. B 73, 235331 (2006)
- [41] Mao, W., Averin, D.V., Ruskov, R., Korotkov, A.N.: Mesoscopic quadratic quantum measurements. Phys. Rev. Lett. 93, 056803 (2004)
- [42] Waks, E., Vuckovic, J., Dipole induced transparency in drop-filter cavity-waveguide systems. Phys. Rev. Lett. 96, 153601 (2006)
- [43] Hu, C.Y., Young, A., O'Brien, J.L., Munro, W.J., Rarity, J.G. : Giant optical Faraday rotation induced by a single-electron spin in a quantum dot: applications to entangling remote spins via a single photon. Phys. Rev. B 78, 085307 (2008).
- [44] Hu, C.Y., Munro, W.J., Rarity, J.G.: Deterministic photon entangler using a charged quantum dot inside a microcavity. Phys. Rev. B **78**, 125318 (2008)
- [45] Hu, C.Y., Munro, W.J., O'Brien, J.L., Rarity, J.G.: Proposed entanglement beam splitter using a quantum-dot spin in a double-sided optical microcavity. Phys. Rev. B 80, 205326 (2009)
- [46] Bonato, C., Haupt, F., Oemrawsingh, S.S.R., Gudat, J., Ding, D.P., van Exter, M.P., Bouwmeester, D.: CNOT and Bell-state analysis in the weak-coupling cavity QED regime. Phys. Rev. Lett. 104, 160503 (2010)
- [47] Hu, C.Y., Rarity, J.G.: Loss-resistant state teleportation and entanglement swapping using a quantum-dot spin in an optical microcavity. Phys. Rev. B 83, 115303 (2011)