Transferring multipartite entanglement among different cavities

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The transfer of quantum entanglement (or quantum coherence) is not only fundamental in quantum mechanics but also important in quantum information processing. We here propose a way to achieve the coherent transfer of W-class entangled states of qubits among different cavities. Because no photon is excited in each cavity, decoherence caused by the photon decay is suppressed during the transfer. In addition, only one coupler qubit and one operational step are needed and no classical pulses are used in this proposal, thus the engineering complexity is much reduced and the operation is greatly simplified. We further give a numerical analysis, showing that high-fidelity transfer of a three-qubit W state is feasible within the present circuit QED technique. The proposal can be applied to a wide range of physical implementation with various qubits such as quantum dots, nitrogen-vacancy centers, atoms, and superconducting qubits.

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I. INTRODUCTION

Quantum entanglement, as a cornerstone of quantum physics, plays an important role in the foundation of quantum theory and has many potential applications in quantum information processing (QIP) and communication. It is known [1] that the two inequivalent and non-converted classes of multipartite entangled states, i.e., Greenberger-Horne-Zeilinger (GHZ) [2] state and W state [1], present quite different behaviors. For example, it has been shown [2] that a three-qubit W state is robust against losses of qubits since it retains bipartite entanglement if any one qubit is traced out, whereas a three-qubit GHZ state is fragile since the remaining two partite states result in separable states. This feature makes W states very useful in various quantum information tasks. A W state can be used as a quantum channel for quantum key distribution [3], entangled-pairs teleportation [4], quantum teleportation [5] and so on. During the past years, many theoretical schemes have been proposed to generate the W state in many physical systems [6-18]. Moreover, the experimental demonstration of W states has been reported with up to eight trapped ions [19], four optical modes [20], three capacitively-coupled superconducting phase qubits [21], two superconducting phase qubits plus a resonant cavity [22], and atomic ensembles in four quantum memories [23].

Instead of generating entangled states, we here focus on transferring quantum entanglement among qubits distributed in different cavities. During the past years, much attention has been paid to quantum entanglement transfer. For instances, many proposals for transferring quantum entanglement via quantum teleportation protocols have been presented [24-28], and schemes for transferring quantum entanglement based on cavity QED or circuit QED have been also proposed [29-31]. Moreover, quantum entanglement transfer has been experimentally demonstrated in linear optics [32,33].

This work is also motivated for the following reason. Large-scale QIP will most likely need a large number of qubits, and placing all of them in a single cavity may cause practical problems such as decreasing the qubit-cavity coupling strength and increasing the cavity decay rate. Hence, future QIP most likely requires quantum networks consisting of many cavities, each hosting and coupled to multiple qubits. In this type of architecture, preparation, transfer, exchange, and manipulation of quantum states (e.g., GHZ states, W states and cluster states, etc.) will not only occur among qubits in the same cavity, but also among qubits distributed in different cavities.

We consider a quantum system consisting of $2n$ cavities each hosting qubits. The qubits can be made to be decoupled from their respective cavities before/after the operation. And, the coupling of qubits with their cavities, which is necessary for quantum operation, can be achieved, by prior adjustment of the level spacings of the qubits or the frequencies of the cavities. In the following, we will present a method to implement the coherent transfer of a W-class entangled state from n qubits in n cavities onto n qubits in another n cavities. As shown below, this proposal has the following advantages: (i) the entanglement transfer is performed without excitation of the cavity photons, and thus decoherence induced by the cavity decay is greatly suppressed during the entire operation; (ii) this proposal needs only one coupler qubit and one operational step and does not require using a classical pulse for the entanglement transfer, hence the engineering complex is much reduced and the operation procedure is greatly simplified. Finally, this proposal is quite general, and can be applied to accomplish the same task with different types of qubits such as quantum dots, atoms, NV centers, various superconducting qubits and so on. To the best of our knowledge, how

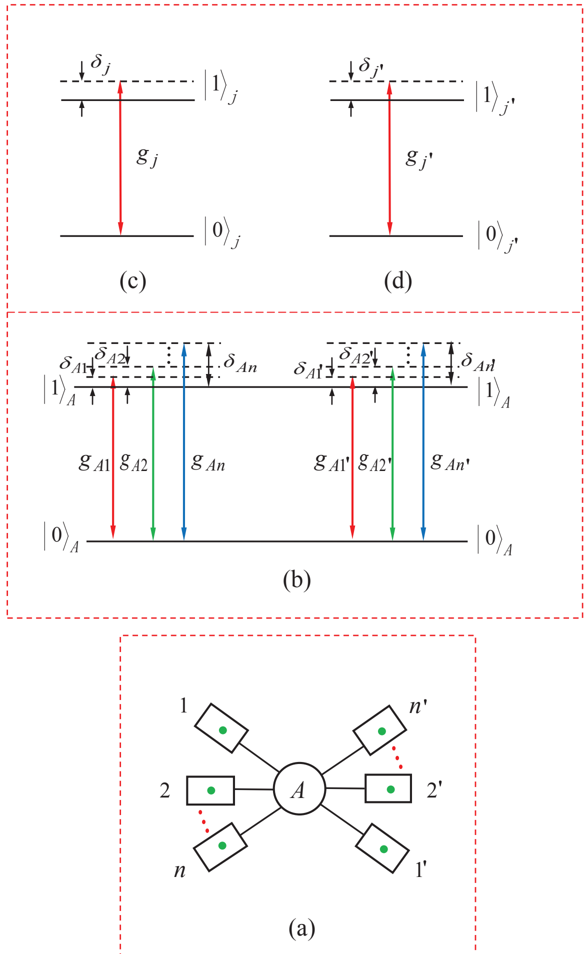

FIG. 1: (color online) (a) Diagram of a coupler qubit A (a circle at the center) and $2n$ cavities $(1, 2, ..., n, 1', 2', ..., n')$ each hosting a qubit. A dark square represents a cavity while a green dot labels a qubit placed in each cavity. (b) Dispersive interaction of the coupler qubit A with $2n$ cavities $(1, 2, ..., n, 1', 2', ..., n')$. Cavity j is coupled to qubit A with coupling constant g_{Aj} and detuning δ_{Aj} $(j = 1, 2, ..., n)$, and Cavity j' is coupled to qubit A with coupling constant $g_{Aj'}$ and detuning $\delta_{Aj'}$ $(j' = 1', 2', ..., n')$. Here, $\delta_j = \delta_{Aj} = \delta_{Aj'} = \delta_{j'}$. (c) Dispersive interaction of qubit j (placed in cavity j) with cavity j $(j = 1, 2, ..., n)$. Here, g_j is the coupling constant while δ_j is detuning. (d) Dispersive interaction of qubit j' (placed in cavity j') with cavity j' $(j' = 1', 2', ..., n')$. Here, $g_{j'}$ is the coupling constant while $\delta_{j'}$ is detuning.

to transfer multipartite entanglement among qubits distributed in different cavities, which are coupled by a single two-level qubit, has not been reported so far.

This paper is organized as follows. In Sec. II, we show a way to transfer a n-qubit W state from n qubits in n cavities onto n qubits in another n cavities. In Sec. III, as an example, we analyze the experimental feasibility of transferring a three-qubit W state in circuit QED. A concluding summary is given in Sec. IV.

II. W-STATE TRANSFER

In the following, we first construct a Hamiltonian for the W state transfer and then describe the procedure for implementing the W-state transfer.

A. Hamiltonian

Consider n cavities $(1,2,...,n)$ and another n cavities $(1',2',...,n')$. The $2n$ cavities are connected by a coupler qubit A, as illustrated in Fig. 1(a). The n qubits placed in the n cavities $(1,2,...,n)$ are labelled as qubits $1,2,...,n$ while the n qubits placed in the other n cavities $(1',2',...,n')$ are denoted as qubits $1',2',...,n'$. Assume that the coupling constant of qubit j with cavity j is g_j $(j=1,2,...,n)$ while the coupling constant of qubit j' with cavity j' is $g_{j'}$ $(j'=1',2',...,n')$. The coupling and decoupling of each qubit from its cavity (cavities) can be achieved by prior adjustment of the qubit's level spacings. For superconducting devices, their level spacings can be rapidly adjusted by varying external control parameters (e.g., magnetic flux applied to phase, transmon, or flux qutrits; see, e.g., [34-36]).

Adjust the level spacings of the coupler qubit A such that this qubit interacts with the $2n$ cavities simultaneously. Denote g_{Aj} as the coupling constant of qubit A with cavity j while $g_{Aj'}$ as the coupling constant of qubit A with cavity j'. In the interaction picture under the free Hamiltonian of the whole system and applying the rotating-wave approximation, we have

$$\begin{aligned} H_I \;=\; & \sum_{j=1}^{n} g_j \left(e^{i\delta_j t} a_j \sigma_j^+ + h.c.\right) + \sum_{j=1}^{n} g_{Aj} \left(e^{i\delta_{Aj} t} a_j \sigma_A^+ + h.c.\right) \\ & + \sum_{j'=1'}^{n'} g_{j'} \left(e^{i\delta_{j'} t} a_{j'} \sigma_{j'}^+ + h.c.\right) + \sum_{j'=1'}^{n'} g_{Aj'} \left(e^{i\delta_{Aj'} t} a_{j'} \sigma_A^+ + h.c.\right), \end{aligned} \tag{1}$$

where the first two terms correspond to the subsystem composed of the coupler qubit A, the n cavities $(1,2,...,n)$ and the n qubits $(1,2,...,n)$, while the last two terms correspond to the subsystem composed of the coupler qubit A, the n cavities $(1',2',...,n')$ and the n qubits $(1',2',...,n')$; a_j $(a_{j'})$ is the annihilation operator for the mode of cavity j (j'); $\sigma_j^+ = |1\rangle_j \langle 0|$ $(\sigma_{j'}^+ = |1\rangle_{j'} \langle 0|)$ is the raising operator of qubit j (j'); δ_j, $\delta_{j'}$, δ_{Aj}, and $\delta_{Aj'}$ are the detunings, given by $\delta_j = \omega_{10j} - \omega_{c_j}$, $\delta_{j'} = \omega_{10j'} - \omega_{c_{j'}}$, $\delta_{Aj} = \omega_{10A} - \omega_{c_j}$, and $\delta_{Aj'} = \omega_{10A} - \omega_{c_{j'}}$ [Fig. 1(b,c,d)].

In the case $\delta_j \gg g_j$, $\delta_{j'} \gg g_{j'}$, $\delta_{Aj} \gg g_{Aj}$, and $\delta_{Aj'} \gg g_{Aj'}$, there is no energy exchange between the qubit system and the cavities. Under the condition of

$$\begin{aligned} \frac{\left|\delta_{A(j+1)} - \delta_{Aj}\right|}{\delta_{Aj}^{-1} + \delta_{A(j+1)}^{-1}} & \gg g_{Aj} g_{A(j+1)}, \\ \frac{\left|\delta_{A(j+1)'} - \delta_{Aj'}\right|}{\delta_{Aj'}^{-1} + \delta_{A(j+1)'}^{-1}} & \gg g_{Aj'} g_{A(j+1)'}, \end{aligned} \tag{2}$$

there is no interaction between the n cavities $(1,2,...,n)$ and there is no interaction between another n cavities $(1',2',...,n')$, which are induced by the coupler qubit A. For simplicity, we set

$$\delta_j = \delta_{Aj} = \delta_{Aj'} = \delta_{j'} \;\; (j=1,2,...,n). \tag{3}$$

Hence, we can obtain the following effective Hamiltonian [37,38]

$$
\begin{aligned}
H_{\mathrm{eff}} &= -\sum_{j=1}^{n}\frac{g_j^2}{\delta_j}\left(|0\rangle_j\langle 0|\,a_j^{+}a_j-|1\rangle_j\langle 1|\,a_ja_j^{+}\right)\\
&\quad-\sum_{j=1}^{n}\frac{g_{Aj}^2}{\delta_{Aj}}\left(|0\rangle_A\langle 0|\,a_j^{+}a_j-|1\rangle_A\langle 1|\,a_ja_j^{+}\right)\\
&\quad-\sum_{j'=1'}^{n'}\frac{g_{j'}^2}{\delta_{j'}}\left(|0\rangle_{j'}\langle 0|\,a_{j'}^{+}a_{j'}-|1\rangle_{j'}\langle 1|\,a_{j'}a_{j'}^{+}\right)\\
&\quad-\sum_{j'=1'}^{n'}\frac{g_{Aj'}^2}{\delta_{Aj'}}\left(|0\rangle_A\langle 0|\,a_{j'}^{+}a_{j'}-|1\rangle_A\langle 1|\,a_{j'}a_{j'}^{+}\right)\\
&\quad+\sum_{j=1}^{n}\lambda_j\left(\sigma_j^{+}\sigma_A+\sigma_j\sigma_A^{+}\right)\\
&\quad+\sum_{j'=1'}^{n'}\lambda_{j'}\left(\sigma_{j'}^{+}\sigma_A+\sigma_{j'}\sigma_A^{+}\right)\\
&\quad+\sum_{j=1}^{n}\mu_j\left(a_j^{+}a_{j'}+a_ja_{j'}^{+}\right)\left(|1\rangle_A\langle 1|-|0\rangle_A\langle 0|\right),
\end{aligned}
\tag{4}
$$

where $\lambda_j = g_j g_{Aj}/\delta_j$, $\lambda_{j'} = g_{j'} g_{Aj'}/\delta_j$, and $\mu_j = g_j g_{j'}/\delta_j$ because of the setting described by Eq. (3). The last term of Eq. (4) describes the interaction between cavity j and cavity j' ($j = 1, 2, ..., n$), which is induced by the coupler qubit A.

Assume that each cavity is initially in the vacuum state. The Hamiltonian (4) then reduces to

$$H_{\mathrm{eff}} = H_0 + H_{\mathrm{int}}, \tag{5}$$

with

$$
\begin{aligned}
H_0 &= \sum_{j=1}^{n}\frac{g_j^2}{\delta_j}|1\rangle_j\langle 1| + \sum_{j=1}^{n}\frac{g_{Aj}^2}{\delta_{Aj}}|1\rangle_A\langle 1|\\
&\quad+\sum_{j'=1'}^{n'}\frac{g_{j'}^2}{\delta_{j'}}|1\rangle_{j'}\langle 1| + \sum_{j'=1'}^{n'}\frac{g_{Aj'}^2}{\delta_{Aj'}}|1\rangle_A\langle 1|,
\end{aligned}
\tag{6}
$$

$$H_{\mathrm{int}} = \sum_{j=1}^{n}\lambda_j\left(\sigma_j^{+}\sigma_A+\sigma_j\sigma_A^{+}\right) + \sum_{j'=1'}^{n'}\lambda_{j'}\left(\sigma_{j'}^{+}\sigma_A+\sigma_{j'}\sigma_A^{+}\right). \tag{7}$$

In a new interaction picture under the Hamiltonian H_0 and applying the following conditions

$$\frac{g_1^2}{\delta_1} = \frac{g_2^2}{\delta_2} = \cdots = \frac{g_n^2}{\delta_n} = \frac{g_{1'}^2}{\delta_{1'}} = \frac{g_{2'}^2}{\delta_{2'}} = \cdots = \frac{g_{n'}^2}{\delta_{n'}} = \chi \tag{8}$$

and

$$\frac{g_k^2}{\delta_k} = \frac{g_{k'}^2}{\delta_{k'}} = \sum_{j=1}^{n}\frac{g_{Aj}^2}{\delta_{Aj}} + \sum_{j'=1'}^{n'}\frac{g_{Aj'}^2}{\delta_{Aj'}} \tag{9}$$

where $k \in \{1, 2, ..., n\}$ and $k' \in \{1', 2', ..., n'\}$, we have

$$\widetilde{H}_{\mathrm{int}} = e^{iH_0t}H_{\mathrm{int}}e^{-iH_0t} = H_{\mathrm{int}}, \tag{10}$$

where all phase factors, caused during the Hamiltonian transformation, are cancelled due to the use of conditions (8) and (9).

Eq. (10) shows that the Hamiltonian $\widetilde{H}_{\rm int}$ takes the same form as the Hamiltonian $H_{\rm int}$ given in Eq. (7). We set

$$\begin{aligned} \lambda_1 = \lambda_2 = ... = \lambda_n = \lambda \ , \\ \lambda_{1'} = \lambda_{2'} = ... = \lambda_{n'} = \lambda \ . \end{aligned} \tag{11}$$

In this case, the coupling constants λ_j and $\lambda_{j'}$ involved in the Hamiltonian $H_{\rm int}$ [see Eq. (7)] can be moved out of the summation symbols. Thus, the Hamiltonian $\widetilde{H}_{\rm int}$ can be expressed as

$$\widetilde{H}_{\rm int} = \lambda \left(J_+ \sigma_A + J_- \sigma_A^+ \right) + \lambda \left(J'_+ \sigma_A + J'_- \sigma_A^+ \right), \tag{12}$$

where $J_+ = \sum_{j=1}^n \sigma_j^+$, $J_- = \sum_{j=1}^n \sigma_j$, $J'_+ = \sum_{j'=1'}^{n'} \sigma_{j'}^+$ and $J'_- = \sum_{j'=1'}^{n'} \sigma_{j'}$. In the following, the Hamiltonian (12) will be used to transfer the W state from the n qubits $(1, 2, ..., n)$ to the other n qubits $(1', 2', ..., n')$.

B. W-state transfer

The W state $|W_{n-1,1}\rangle$ of n qubits $(1, 2, ..., n)$ is described by [1]

$$|W_{n-1,1}\rangle = \frac{1}{\sqrt{n}} \sum P_z \left|0\right\rangle^{\otimes (n-1)} |1\rangle , \tag{13}$$

where P_z is the symmetry permutation operator for qubits $(1, 2, ..., n)$, $\sum P_z \left|0\right\rangle^{\otimes (n-1)} |1\rangle$ denotes the totally symmetric state in which $n-1$ of qubits $(1, 2, ..., n)$ are in the state $|0\rangle$ while the remaining qubit is in the state $|1\rangle$. For instance, we have $|W_{2,1}\rangle = \frac{1}{\sqrt{3}} (|001\rangle + |010\rangle + |100\rangle)$ when $n = 3$.

Assume that (i) each cavity is initially in the vacuum state, (ii) the n qubits $(1, 2, ..., n)$ are initially in the W state $|W_{n-1,1}\rangle$ described above, while the n qubits $(1', 2', ..., n')$ are initially in the ground state, i.e., qubit j' is in the state $|0\rangle_{j'}$, (iii) the coupler qubit A is initially in the ground state $|0\rangle_A$, and (iv) all qubits are decoupled from their respective cavities.

To transfer the W state, adjust the level spacings of each qubit (including the coupler qubit A) to have the state of the qubit system undergo the time evolution described by the Hamiltonian (12). Based on this Hamiltonian and after returning to the original interaction picture by performing a unitary transformation $e^{-iH_0 t}$, it is easy to find that the initial state $|W_{n-1,1}\rangle_{12...n} \prod\limits_{j'=1'}^{n'} |0\rangle_{j'} |0\rangle_A$ of the qubit system evolves into

$$\begin{aligned} & N \left[e^{-i\chi t} \left(1 + \cos \Lambda t\right) |W_{n-1,1}\rangle_{12...n} \prod_{j'=1'}^{n'} |0\rangle_{j'} |0\rangle_A \right. \\ & \left. + e^{-i\chi t} \left(\cos \Lambda t - 1\right) \prod_{j=1}^{n} |0\rangle_j |W_{n-1,1}\rangle_{1'2'...n'} |0\rangle_A \right] \\ & -i\sqrt{N} e^{-i2\chi t} \sin \Lambda t \prod_{j=1}^{n} |0\rangle_j \prod_{j'=1'}^{n'} |0\rangle_{j'} |1\rangle_A , \end{aligned} \tag{14}$$

where $N = 1/2$, and $\Lambda = \sqrt{2n} |\lambda|$. Here, the factors $e^{-i\chi t}$ and $e^{-i2\chi t}$ were obtained by performing the unitary transformation $e^{-iH_0 t}$ and applying the conditions (8) and (9).

One can see that for $t = \pi/\Lambda$, the state (14) becomes $\prod\limits_{j=1}^{n} |0\rangle_j |W_{n-1,1}\rangle_{1'2'...n'} |0\rangle_A$, which shows that the n qubits $(1', 2', ..., n')$ are in the state $|W_{n-1,1}\rangle$. Namely, the W state of the qubits $(1, 2, ..., n)$ in n cavities is transferred onto the n qubits $(1', 2', ..., n')$ in the other n cavities after the operation. To maintain the W state unaffected, the level spacings for each intracavity qubit and the coupler qubit A need to be adjusted back to the original configuration after the above operation.

In above adjusting the qubit level spacings is unnecessary. Alternatively, the coupling or decoupling of the qubits with the cavities can be obtained by adjusting the frequency of each cavity. The rapid tuning of cavity frequencies has been demonstrated in superconducting microwave cavities (e.g., in less than a few nanoseconds for a superconducting transmission line resonator [39]).

C. Discussion

For the approach to work, the following requirements need to be satisfied:

(i) The conditions (2), (3), (8), (9) and (11) need to be met for the protocol to work. The condition (2) can be reached by prior design of cavities with appropriate frequencies. The condition (3) is automatically ensured for the identical qubits and pairs of cavities j and j' with same frequency. Given $\delta_1, \delta_2, ..., \delta_n$ and $\delta_{1'}, \delta_{2'}, ..., \delta_{n'}$, the condition (8) can be met by adjusting the coupling constants $g_1, g_2, ..., g_n$ and $g_1', g_2', ..., g_n'$ (e.g., for solid-state qubits, the qubit-cavity coupling constants can be readily changed by varying the positions of the qubits embedded in their cavities). The condition (9) can be met by setting

$$g_{Aj}/g_j = g_{Aj'}/g_{j'} = 1/\sqrt{2n}, \tag{15}$$

where $j = 1, 2, ..., n$ and $j' = 1', 2', ..., n'$. Given g_j and $g_{j'}$, this requirement (15) can be obtained by adjusting g_{Aj} and $g_{Aj'}$. For a solid-state coupler qubit A, g_{Aj} and $g_{Aj'}$ can be adjusted by changing the qubit-cavity coupler capacitance C_j and $C_{j'}$, respectively (Fig. 2). Finally, note that the condition (11) is automatically satisfied because of the conditions (3), (8), (9) and (15). Overall, all necessary conditions here can be readily met.

(ii) The operation time required for the entanglement transfer needs to be much shorter than T_1 (energy relaxation time) and T_2 (dephasing time) of the level $|1\rangle$, so that the decoherence, caused by energy relaxation and dephasing of the qubits, is negligible for the operation.

(iii) The lifetime of the cavity modes is given by

$$T_{\text{cav}} = \frac{1}{2n} \min\{T_{\text{cav}}^1, T_{\text{cav}}^2, ..., T_{\text{cav}}^n, T_{\text{cav}}^{1'}, T_{\text{cav}}^{2'}, ..., T_{\text{cav}}^{n'}\}, \tag{16}$$

which needs to be much longer than the operation time, such that the effect of cavity decay is negligible for the operation.

(iv) When the coupler qubit A is a solid-state qubit, there may exist an inter-cavity cross coupling during the operation, which should be negligibly small in order to reduce its effect on the operation fidelity. In the present proposal, the unwanted inter-cavity crosstalk may not be a problem because each cavity is virtually excited during the entire operation, as long as the large detuning conditions can be well satisfied.

III. POSSIBLE EXPERIMENTAL IMPLEMENTATION

The physical systems composed of cavities and superconducting qubits have been considered to be one of the most promising candidates for QIP [40-44]. In above a general type of qubit was considered. Let us now consider that each qubit is a superconducting transmon qubit and each cavity is a one-dimensional transmission line resonator (TLR). In addition, assume that the coupler qubit A is connected to each TLR via a capacitance. As an example of experimental implementation, consider a setup in Fig. 2 for transferring the three-qubit W state from three transmon qubits $(1, 2, 3)$ each embedded in a different TLR to the other three transmon qubits $(1', 2', 3')$ each in another different TLR. To be more realistic, a third higher level $|2\rangle$ for each qubit here needs to be considered during the operations described above, since this level $|2\rangle$ may be excited due to the $|1\rangle \leftrightarrow |2\rangle$ transition induced by the cavity mode(s), which will turn out to affect the operation fidelity. Therefore, to quantify how well the proposed protocol works out, an analysis of the operation fidelity will be given for the W-state transfer, by taking this higher level $|2\rangle$ into account. Because of three levels being considered, each qubit is renamed as a qutrit in the following.

When the inter-cavity crosstalk coupling and the unwanted $|1\rangle \leftrightarrow |2\rangle$ transition of each qutrit are considered, the Hamiltonian (1) is modified as follows

$$h_I = H_I + \Theta_I, \tag{17}$$

where H_I is the needed interaction Hamiltonian in Eq. (1) for $n = 3$ and $n' = 3'$, while Θ_I is the unwanted interaction

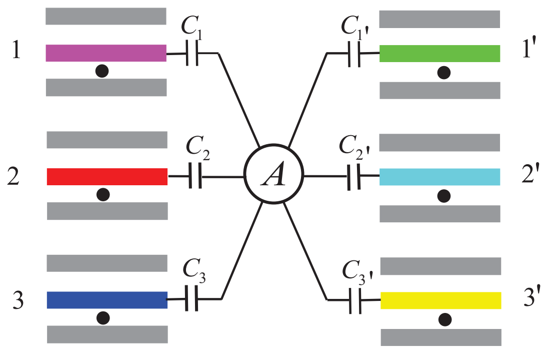

FIG. 2: (color online) Setup for six cavities $(1, 2, 3, 1', 2', 3')$ coupled by a superconducting transmon qubit A. Each cavity here is a one-dimensional coplanar waveguide transmission line resonator. The circle A represents a superconducting transmon qubit, which is capacitively coupled to cavity j via a capacitance C_j $(j = 1, 2, 3)$ and cavity j' via a capacitance $C_{j'}$ $(j' = 1', 2', 3')$. The six dark dots indicate the six superconducting transmon qubits $(1, 2, 3, 1', 2', 3')$ embedded in the six cavities, respectively. The interaction of qubits (1,2,3) with their cavities are respectively illustrated in Fig. 3(a,b,c) while the interaction of qubits $(1', 2', 3')$ with their cavities are respectively illustrated in Fig. 4(a,b,c). In addition, the interaction of the coupler qubit A with three cavities (1,2,3) is illustrated in Fig. 3(d) while the interaction of the coupler qubit A with three cavities $(1', 2', 3')$ is illustrated in Fig. 4(d). Since three levels for each qubit is involved in our analysis, each qubit is renamed a qutrit in Figs. 3 and 4.

Hamiltonian, given by

$$\begin{aligned} \Theta_I \;=\; & \sum_{j=1}^{3} \widetilde{g}_j \left(e^{i\widetilde{\delta}_j t} a_j \sigma_{21j}^+ + h.c. \right) + \sum_{j=1}^{3} \widetilde{g}_{Aj} \left(e^{i\widetilde{\delta}_{Aj} t} a_j \sigma_{21A}^+ + h.c. \right) \\ & + \sum_{j'=1'}^{3'} \widetilde{g}_{j'} \left(e^{i\widetilde{\delta}_{j'} t} a_{j'} \sigma_{21j'}^+ + h.c. \right) + \sum_{j'=1'}^{3'} \widetilde{g}_{Aj'} \left(e^{i\widetilde{\delta}_{Aj'} t} a_{j'} \sigma_{21A}^+ + h.c. \right) \\ & + \sum_{k \neq l} g_{kl} \left(e^{-i\Delta_{kl} t} a_k a_l^+ + h.c. \right), \end{aligned} \tag{18}$$

where $k, l \in \{1, 2, 3, 1', 2', 3'\}, \sigma_{21j}^+ = |2\rangle_j \langle 1|, \sigma_{21j'}^+ = |2\rangle_{j'} \langle 1|$, and $\sigma_{21A}^+ = |2\rangle_A \langle 1|$. The first term represents the unwanted off-resonant coupling between the mode of cavity j and the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit j, with coupling constant $\widetilde{g}_j$ and detuning $\widetilde{\delta}_j = \omega_{21j} - \omega_{c_j}$ [Fig. 3(a,b,c)], while the second term indicates the unwanted off-resonant coupling between the mode of cavity j and the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit A, with coupling constant $\widetilde{g}_{Aj}$ and detuning $\widetilde{\delta}_{Aj} = \omega_{21A} - \omega_{c_j}$ [Fig. 3(d)]. The third term represents the unwanted off-resonant coupling between the mode of cavity j' and the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit j', with coupling constant $\widetilde{g}_{j'}$ and detuning $\widetilde{\delta}_{j'} = \omega_{21j'} - \omega_{c_{j'}}$ [Fig. 4(a,b,c)], while the fourth term indicates the unwanted off-resonant coupling between the mode of cavity j' and the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit A, with coupling constant $\widetilde{g}_{Aj'}$ and detuning $\widetilde{\delta}_{Aj'} = \omega_{21A} - \omega_{c_{j'}}$ [Fig. 4(d)]. The last term describes the inter-cavity crosstalk between any two cavities, with $\Delta_{kl} = \omega_{c_k} - \omega_{c_l} = \delta_l - \delta_k$ (the frequency difference between two cavities k and l) and g_{kl} (the inter-cavity coupling constant between two cavities k and l).

The dynamics of the lossy system, with finite qutrit relaxation and dephasing and photon lifetime included, is determined by the following master equation

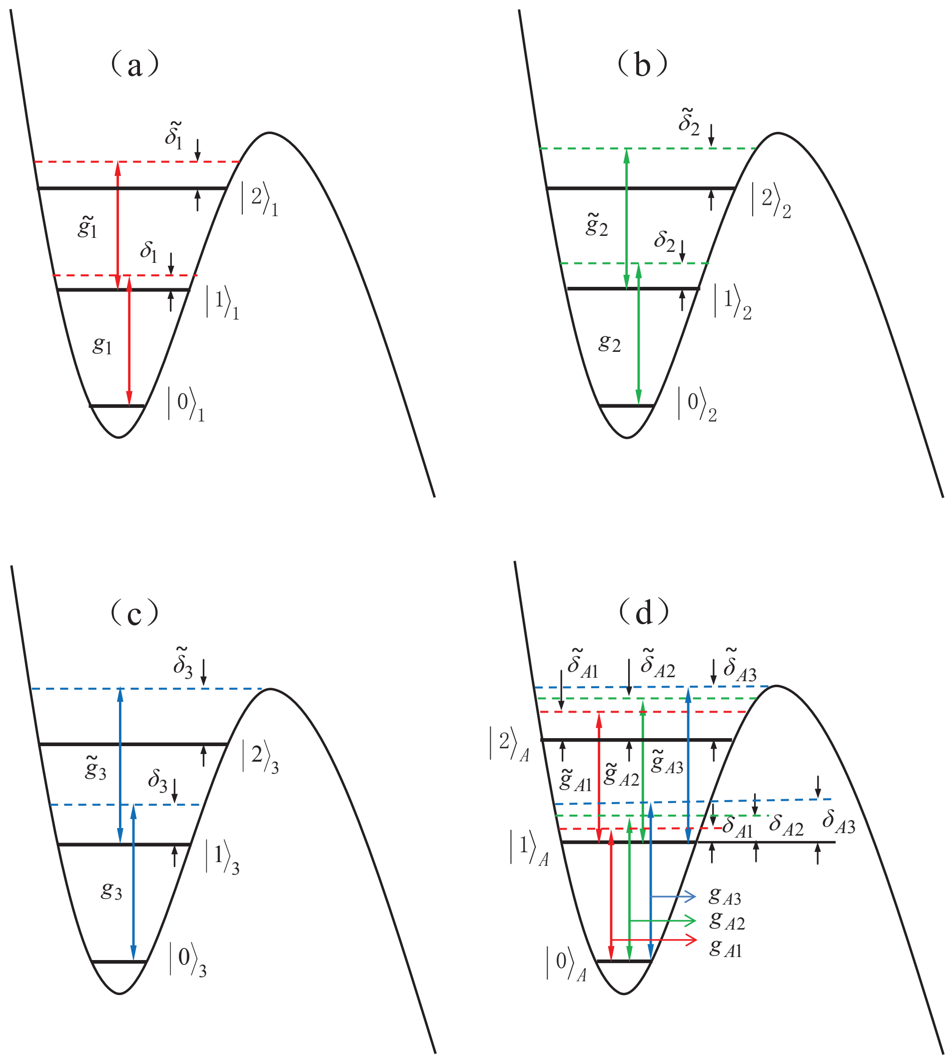

FIG. 3: (Color online) Illustration of the interaction between qutrits (1,2,3,A) and three cavities (1,2,3). (a) Cavity 1 is dispersively coupled to the $|0\rangle \leftrightarrow |1\rangle$ transition with coupling constant g_1 and detuning δ_1, but far-off resonant (i.e., more detuned) with the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit 1 with coupling constant $\widetilde{g}_1$ and detuning $\widetilde{\delta}_1$. (b) [and (c)] corresponds to the case that cavity 2 (3) is dispersively coupled to the $|0\rangle \leftrightarrow |1\rangle$ transition but far-off resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit 2 (3). (d) Cavities $(1, 2, 3)$ dispersively interact with the $|0\rangle \leftrightarrow |1\rangle$ transition of qutrit A with coupling constants (g_{A1}, g_{A2}, g_{A3}) and detunings $(\delta_{A1}, \delta_{A2}, \delta_{A3})$, respectively; but they are far-off resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit A with coupling constants $(\widetilde{g}_{A1}, \widetilde{g}_{A2}, \widetilde{g}_{A3})$ and detunings $(\widetilde{\delta}_{A1}, \widetilde{\delta}_{A2}, \widetilde{\delta}_{A3})$, respectively. Here, $\delta_j = \omega_{10j} - \omega_{cj}, \widetilde{\delta}_j = \omega_{21j} - \omega_{cj}, \delta_{Aj} = \omega_{10A} - \omega_{cj}$, and $\widetilde{\delta}_{Aj} = \omega_{21A} - \omega_{cj}$ $(j = 1, 2, 3)$, where ω_{10j} (ω_{21j}) is the $|0\rangle \leftrightarrow |1\rangle$ $(|1\rangle \leftrightarrow |2\rangle)$ transition frequency of qutrit j, ω_{10A} (ω_{21A}) is the $|0\rangle \leftrightarrow |1\rangle$ $(|1\rangle \leftrightarrow |2\rangle)$ transition frequency of qutrit A, and ω_{cj} is the frequency of cavity j.

$$
\begin{aligned}
\frac{d\rho}{dt} = & -i\left[h_I, \rho\right] + \sum_{j=1}^{3} \kappa_j \mathcal{L}\left[a_j\right] + \sum_{j'=1'}^{3} \kappa_{j'} \mathcal{L}\left[a_{j'}\right] \\
& + \sum_l \left\{\gamma_l \mathcal{L}\left[\sigma_l^-\right] + \gamma_{21l} \mathcal{L}\left[\sigma_{21l}^-\right] + \gamma_{20l} \mathcal{L}\left[\sigma_{20l}^-\right]\right\} \\
& + \sum_l \left\{\gamma_{l,\varphi 1}\left(\sigma_{11l}\rho\sigma_{11l} - \sigma_{11l}\rho/2 - \rho\sigma_{11l}/2\right)\right\} \\
& + \sum_l \left\{\gamma_{l,\varphi 2}\left(\sigma_{22l}\rho\sigma_{22l} - \sigma_{22l}\rho/2 - \rho\sigma_{22l}/2\right)\right\}
\end{aligned} \tag{19}
$$

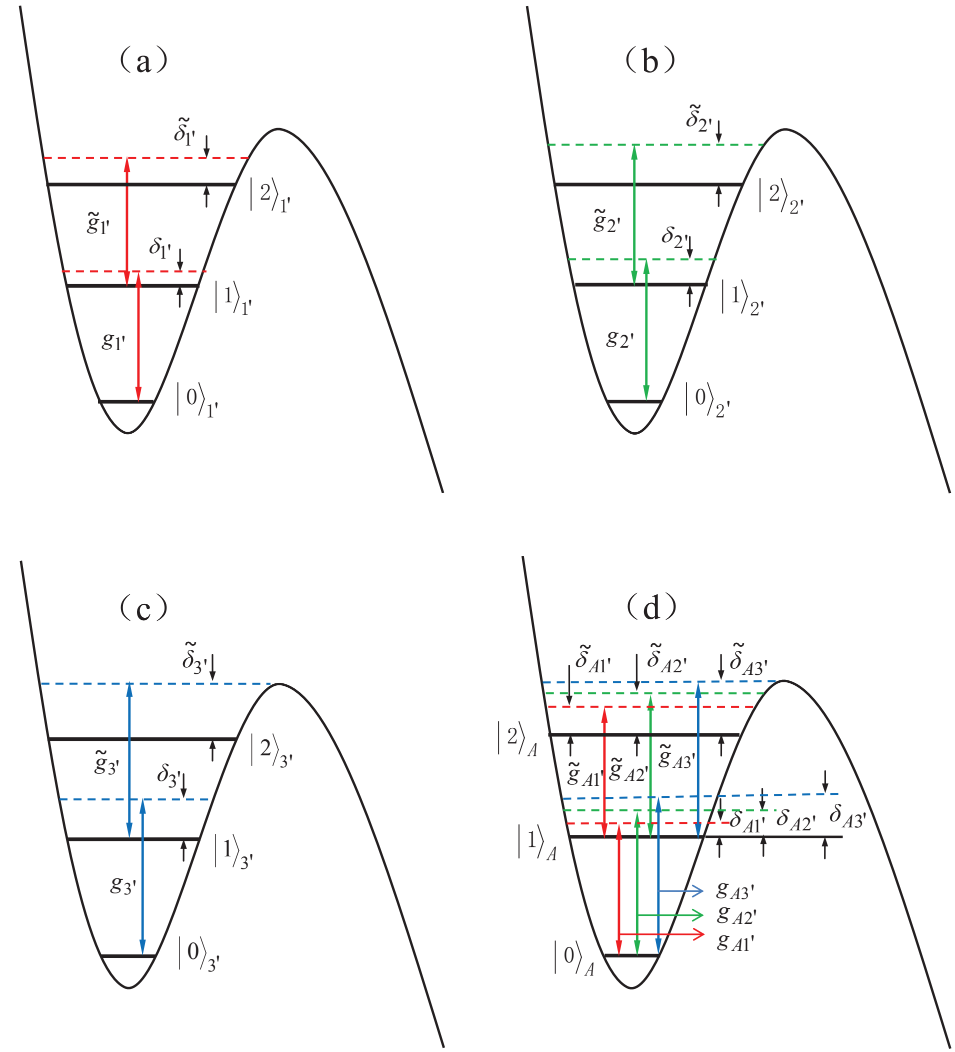

FIG. 4: (Color online) Illustration of the interaction between qutrits $(1', 2', 3', A)$ and three cavities $(1', 2', 3')$. (a) Cavity $1'$ is dispersively coupled to the $|0\rangle \leftrightarrow |1\rangle$ transition with coupling constant $g_{1'}$ and detuning $\delta_{1'}$, but far-off resonant (i.e., more detuned) with the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit $1'$ with coupling constant $\widetilde{g}_{1'}$ and detuning $\widetilde{\delta}_{1'}$. (b) [and (c)] corresponds to the case that cavity $2'$ $(3')$ is dispersively coupled to the $|0\rangle \leftrightarrow |1\rangle$ transition but far-off resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit $2'$ $(3')$. (d) Cavities $(1', 2', 3')$ dispersively interact with the $|0\rangle \leftrightarrow |1\rangle$ transition of qutrit A with coupling constants $(g_{A1'}, g_{A2'}, g_{A3'})$ and detunings $(\delta_{A1'}, \delta_{A2'}, \delta_{A3'})$, respectively; but they are far-off resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit A with coupling constants $(\widetilde{g}_{A1'}, \widetilde{g}_{A2'}, \widetilde{g}_{A3'})$ and detunings $(\widetilde{\delta}_{A1'}, \widetilde{\delta}_{A2'}, \widetilde{\delta}_{A3'})$, respectively. Here, $\delta_{j'} = \omega_{10j'} - \omega_{cj'}, \widetilde{\delta}_{j'} = \omega_{21j'} - \omega_{cj'}, \delta_{Aj'} = \omega_{10A} - \omega_{cj'}$, and $\widetilde{\delta}_{Aj'} = \omega_{21A} - \omega_{cj'}$ $(j' = 1', 2', 3')$, where $\omega_{10j'}$ $(\omega_{21j'})$ is the $|0\rangle \leftrightarrow |1\rangle$ $(|1\rangle \leftrightarrow |2\rangle)$ transition frequency of qutrit j', and $\omega_{cj'}$ is the frequency of cavity j'.

where $l \in \{1, 2, 3, 1', 2', 3', A\}, \sigma_{20l}^{-} = |0\rangle_l \langle 2|, \sigma_{11l} = |1\rangle_l \langle 1|, \sigma_{22l} = |2\rangle_l \langle 2|$; and $\mathcal{L}[\Lambda] = \Lambda\rho\Lambda^{+} - \Lambda^{+}\Lambda\rho/2 - \rho\Lambda^{+}\Lambda/2$, with $\Lambda = a_j, a_{j'}, \sigma_l^{-}, \sigma_{21l}^{-}, \sigma_{20l}^{-}$. Here, κ_j is the photon decay rate of cavity a_j while $\kappa_{j'}$ is the photon decay rate of cavity $a_{j'}$. In addition, γ_l is the energy relaxation rate of the level $|1\rangle$ of qutrit l, γ_{21l} (γ_{20l}) is the energy relaxation rate of the level $|2\rangle$ of qutrit l for the decay path $|2\rangle \to |1\rangle$ $(|0\rangle)$, and $\gamma_{l,\varphi 1}$ $(\gamma_{l,\varphi 2})$ is the dephasing rate of the level $|1\rangle$ $(|2\rangle)$ of qutrit l.

The fidelity of the operation is given by

$$\mathcal{F} = \sqrt{\langle \psi_{\mathrm{id}} | \rho | \psi_{\mathrm{id}} \rangle}, \tag{20}$$

where $|\psi_{\mathrm{id}}\rangle$ is the output state $\prod_{j=1}^{3} |0\rangle_j |W_{2,1}\rangle_{1'2'3'} |0\rangle_A \prod_{j=1}^{3} |0\rangle_{c_j} \prod_{j'=1'}^{3'} |0\rangle_{c_{j'}}$ of an ideal system (i.e., without dissipation, dephasing, and crosstalk) as discussed in the previous section; and ρ is the final density operator of the system when the operation is performed in a realistic physical system.

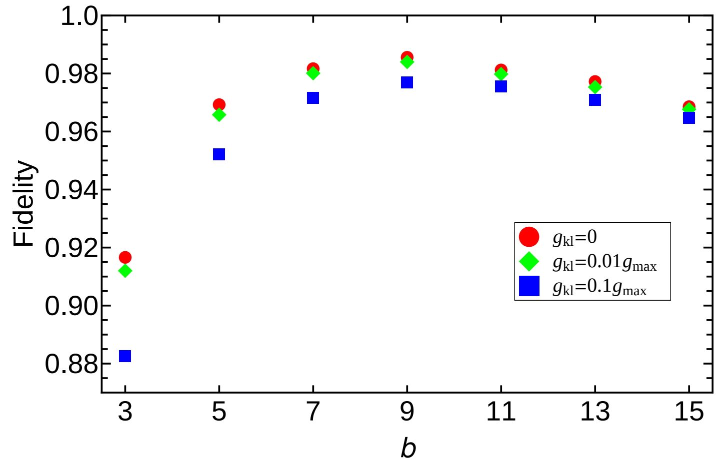

FIG. 5: (Color online) Fidelity of the W-state transfer versus the normalized detuning $b = |\delta_1|/g_1 = \delta_{1'}/g_{1'}$. Refer to the text for the parameters used in the numerical calculation. Here, g_{kl} are the coupling strengths between cavities k and l ($k \neq l$; $k, l \in \{1, 2, 3, 1', 2', 3'\}$), which are taken to be the same for simplicity.

Without loss of generality, consider six identical superconducting transmon qutrits. According to the condition (3), set $\delta_1 = \delta_{A1} = \delta_{A1'} = \delta_{1'} = -2\pi \times 0.5$ GHz, $\delta_2 = \delta_{A2} = \delta_{A2'} = \delta_{2'} = -2\pi \times 1.0$ GHz, $\delta_3 = \delta_{A3} = \delta_{A3'} = \delta_{3'} = -2\pi \times 1.5$ GHz. Set $\widetilde{\delta}_j = \delta_j - 2\pi \times 400$ MHz, $\widetilde{\delta}_{Aj} = \delta_{Aj} - 2\pi \times 400$ MHz, $\widetilde{\delta}_{j'} = \delta_{j'} - 2\pi \times 400$ MHz, and $\widetilde{\delta}_{Aj'} = \delta_{Aj'} - 2\pi \times 400$ MHz (an anharmonicity readily achieved in experiments [45]). For transmon qutrits, the typical transition frequency between two neighbor levels is between 4 and 10 GHz. Thus, choose $\omega_{10A}, \omega_{10j}, \omega_{10j} \sim 2\pi \times 6.5$ GHz. Given $\{\delta_1, \delta_2, \delta_3, \delta_{1'}, \delta_{2'}, \delta_{3'}, g_1\}$, the coupling constants g_2, g_3, $g_{2'}$, and $g_{3'}$ are determined based on Eq. (8). In addition, g_{Aj} and $g_{Aj'}$ are determined by Eq. (15), given g_j and $g_{j'}$ ($j = 1, 2, 3; j' = 1', 2', 3'$). For the present case, $n = 3$. Next, one has $\widetilde{g}_j \sim \sqrt{2} g_j$, $\widetilde{g}_{j'} \sim \sqrt{2} g_{j'}$, $\widetilde{g}_{Aj} \sim \sqrt{2} g_{Aj}$, and $\widetilde{g}_{Aj'} \sim \sqrt{2} g_{Aj'}$ for the transmon qutrit here. Choose $\kappa_j^{-1} = \kappa_{j'}^{-1} = 5$ μs, $\gamma_{l,\varphi 1}^{-1} = \gamma_{l,\varphi 2}^{-1} = 5$ μs, $\gamma_l^{-1} = 10$ μs, $\gamma_{21l}^{-1} = 5$ μs, and $\gamma_{20l}^{-1} = 25$ μs (a conservative consideration), which are available in experiment because T_1 and T_2 can be made to be on the order of $20 - 60$ μs for state-of-the-art superconducting transmon devices at the present time [46-48]. Note that for a transmon qutrit with the three levels considered here, the $|0\rangle \leftrightarrow |2\rangle$ dipole matrix element is much smaller than that of the $|0\rangle \leftrightarrow |1\rangle$ and $|1\rangle \leftrightarrow |2\rangle$ transitions. Thus, $\gamma_{20l}^{-1} \gg \gamma_l^{-1}, \gamma_{21l}^{-1}$.

According to Eqs. (3) and (8), it is easy to see $g_j = g_{j'}$ ($j = 1, 2, ..., n$). For the parameters chosen above, the fidelity versus $b = |\delta_1|/g_1 = |\delta_{1'}|/g_{1'}$ is plotted in Fig. 5 for $g_{kl} = 0$, $0.01 g_{\max}$, and $0.1 g_{\max}$, where $g_{\max} = \max\{g_{A1}, g_{A2}, g_{A3}, g_{A1'},$
$g_{A2'}, g_{A3'}\}$. Fig. 5 shows that for $g_{kl} \leq 0.01 g_{\max}$, the effect of intercavity cross coupling between the cavities on the fidelity of the operation is negligible, which can be seen by comparing the top two curves. It can be seen from Fig. 5 that for $b \sim 9$ and $g_{kl} = 0.01 g_{\max}$ ($0.1 g_{\max}$), a high fidelity $\sim 98.4\%$ (97.7%) is available. For b=9, the operational time is only $0.081 \mu s$, which is much less than decoherence and dephasing times of the system. Moreover, the time averaged photon number in each cavity is ~ 0.006, which means the assumption of no excitation of cavity photons can be guaranteed safely.

The condition $g_{kl} \leq 0.01 g_{\max}$ is not difficult to satisfy with the typical capacitive cavity-qutrit coupling illustrated in Fig. 2. As discussed in [49], as long as the cavities are physically well separated, the inter-cavity cross-talk coupling strength is $g_{kl} \sim g_{Ak} C_l / C_\Sigma, g_{Al} C_k / C_\Sigma$, where $C_\Sigma = \sum_{j=1}^{3} C_j + \sum_{j'=1'}^{3'} C_{j'} + C_q$ with the qutrit's self capacitance C_q. For $C_1, C_2, C_3, C_{1'}, C_{2'}, C_{3'} \sim 1$ fF and $C_\Sigma \sim 10^2$ fF (the typical values in experiment [39]), one has $g_{kl} \sim 0.01 g_{Ak}, 0.01 g_{Al}$ ($k, l \in \{1, 2, 3, 1', 2', 3'\}$). Because of $g_{A1}, g_{A2}, g_{A3}, g_{A1'}, g_{A2'}, g_{A3'} \leq g_{\max}$, the condition $g_{kl} \leq 0.01 g_{\max}$ can be readily met.

For $b \sim 9$, the coupling strengths are $\{g_1, g_2, g_3, g_{A1}, g_{A2}, g_{A3}\} \sim \{55.6, 78.6, 96.2,$
$22.7, 32.1, 39.3\}$ MHz . The same values apply to $\{g_{1'}, g_{2'}, g_{3'}, g_{A1'}, g_{A2'}, g_{A3'}\}$, respectively. Note that the coupling strengths with this value are readily achievable in experiment because $g/(2\pi) \sim 360$ MHz has been reported for a superconducting transmon qubit coupled to a one-dimensional standing-wave CPW (coplanar waveguide) resonator [50,51]. For the transmon qutrits with frequency $\omega_{10}/(2\pi) \sim 6.5$ GHz chosen above, we have $\omega_{c1}/2\pi, \omega_{c1'}/2\pi \sim 7.0$ GHz, $\omega_{c2}/2\pi, \omega_{c2'}/2\pi \sim 7.5$ GHz, and $\omega_{c3}/2\pi, \omega_{c3'}/2\pi \sim 8.0$ GHZ. For these cavity frequencies and the values of κ_j^{-1} and $\kappa_{j'}^{-1}$ used in the numerical calculation, the required quality factors for the six cavities are $Q_1, Q_{1'} \sim 2.2 \times 10^5$,

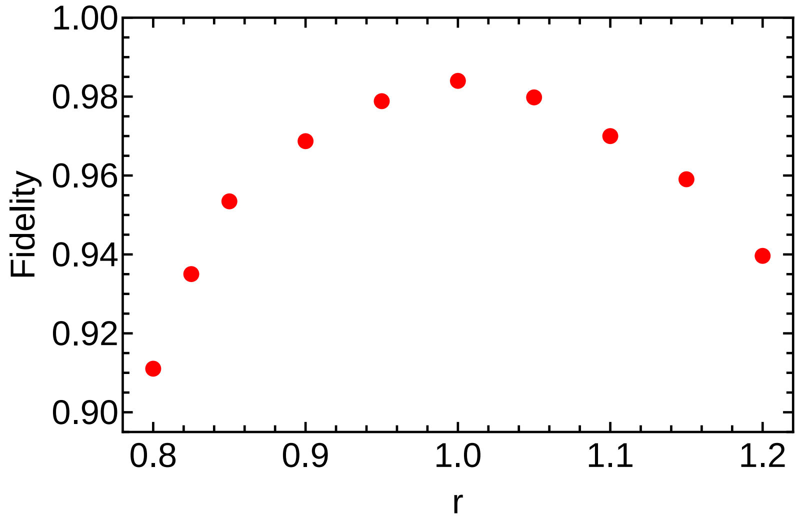

FIG. 6: (Color online) Fidelity of the W-state transfer versus the ratio $r = \delta_{j'}/\delta_j$, plotted for $b = 9$ and $g_{kl} = 0.01g_{max}$.

$Q_2, Q_{2'} \sim 2.4 \times 10^5$, and $Q_{3,3'} \sim 2.5 \times 10^5$, respectively. It should be mentioned that superconducting CPW resonators with a loaded quality factor $Q \sim 10^6$ have been experimentally demonstrated [52,53]. The analysis given here demonstrates that high-fidelity transfer of the three-qubit W state by using this proposal is feasible within present-day circuit QED technique. It should be remarked that further investigation is needed for each particular experimental setup. However, it requires a rather lengthy and complex analysis, which is beyond the scope of this theoretical work.

It is necessary to test whether high-fidelity transfer of the W state can still be obtained if conditions (3), (8), (9) and (11) are not fully satisfied. We assume that Eq. (3) is broken as:

$$\delta_j = \delta_{Aj} \neq \delta_{Aj'} = \delta_{j'} \ (j = 1, 2, 3). \tag{21}$$

We set $\delta_1 = \delta_{A1} = -2\pi \times 0.5$ GHz, $\delta_2 = \delta_{A2} = -2\pi \times 1.0$ GHz, $\delta_3 = \delta_{A3} = -2\pi \times 1.5$ GHz, but set $\delta_{Aj'} = \delta_{j'} = r\delta_j$ $(j = 1, 2, 3)$, where r is a new parameter describing the degree of breakage. We adopt the previous values of $g_j, g_{j'}, g_{Aj}, g_{Aj'}$ used in Fig. 5. For $r \neq 1$, the values taken by $\delta_{Aj'}$ and $\delta_{j'}$ are not equal to the previous ones used in Fig. 5. Thus, it is obvious that the conditions given in Eqs. (3), (8), (9) and (11) are broken simultaneously. Fig. 6 shows the change of fidelity versus r, which is plotted for $b = 9$ and $g_{kl} = 0.01g_{max}$. From Fig. 6, one can see that a high fidelity $\mathcal{F} \gtrsim 0.969$ can be maintained for $0.9 < r < 1.1$.

IV. CONCLUSION

We have shown that transferring the W-class entangled states of multiple qubits among different cavities can be realized by using a single coupler qubit. As shown above, this proposal offers some advantages and features. The entanglement transfer does not employ cavity photons as quantum buses, thus decoherence caused due to the cavity decay is greatly suppressed during the operation. Only one coupler qubit is needed to connect with all cavities such that the circuit complex is greatly reduced. Moreover, only one step of operation is required and no classical pulse is need, so that the operation is much simplified. The numerical simulation shows that high-fidelity transfer of the three-qubit W state is feasible for the current circuit QED technology. The method presented here is quite general, and can be applied to accomplish the same task with different types of qubits such as quantum dots, superconducting qubits (e.g., phase, flux and charge qubits), NV centers, and atoms.

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Conflict of Interest: The authors declare that they have no conflict of interest.

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