Quantum Entanglement Establishment between two Strangers

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Abstract

This paper presents the first quantum entanglement establishment scheme for strangers who neither pre-share any secret nor have any authenticated classical channel between them. The proposed protocol requires only the help of two almost dishonest third parties (TPs) to achieve the goal. The security analyses indicate that the proposed protocol is secure against not only an external eavesdropper's attack, but also the TP's attack.

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1 Introduction

Quantum entanglement, one of the most attractive physical phenomena, has been widely researched in recent years. Quantum entanglement provides a

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"spooky relation at a distance," which allows two or more participants who share entangled quantum states to have correlated information. Based on the concept of quantum entanglement, various quantum cryptographic protocols are possible. For example, quantum key distribution allows two remote participants to share a secure key [1]; quantum teleportation "sends" quanta to a remote location without any physical photon transmission [2]; quantum dense-coding communication allows one to transmit two-bit information via a one-bit quantum transmission; and quantum blind computation [3] allows a user to perform quantum computations with the help of a quantum server, without revealing the intended computations. In addition, quantum secret sharing [4], quantum state sharing [5], quantum remote state preparation [6], quantum signature [7], quantum private comparison [8, 9], etc., are all possible because of shared quantum entanglement states. Research has shown that if shared quantum entanglements are in incorrect states or are interrupted by malicious users during the entanglement establishment process, then incorrect results may occur, and the protocol is considered to be insecure [1, 10-16]. Accordingly, assurance of security and correctness during the establishment of entanglement becomes an imperative issue in quantum cryptography.

The problem with the establishment of entanglement has been most often treated in two ways. The first is to simply assume that the entanglement is preshared by the participants [2]. The second–which is also our focus here–describes the entanglement establishment procedure in detail [1]. For this approach, one often assumes the existence of an **authentication classical channel** between two users, which can be used to discuss the correctness of the shared entangled states. For example, if Alice wants to share an entanglement with Bob, Alice will generate a series of entangled quantum states, which include multiple quantum particles, and transmit the entangled particles to Bob. Then Alice and Bob choose some entangled states for public discussion: they respectively measure the selected entangled states and compare the measurement result via the authenticated classical channel. If the comparison result is accepted, then Alice and Bob believe that the entanglement is well established.

However, to share an authenticated classical channel, the implication is that Alice and Bob should know each other beforehand. What if Alice and Bob are strangers-i.e., they did not meet each other beforehand? In that case, Alice and Bob might have to look for another client-say, Charlie-as a third party (TP) [17–19], who respectively shares a quantum channel and an authenticated classical channel with them and eventually can help them share an entanglement. The trustworthiness of this TP has been an interesting topic in quantum cryptography. The issue surrounds the usefulness of the constructed protocol in practice. The ideal case assumes the existence of a completely trusted TP who always executes the protocol loyally and never reveals the important information of the users. This case is conceptually the same as assuming the existence of an authenticated classical channel between two involved users. In these cases, the TP is assumed to be a **semi-honest** agent who will loyally execute the protocol, but may try to steal Alice and Bob's secret using passive attacks. The semihonest TP will passively collect the classical information exchanged between Alice and Bob and try to reveal their secrets from this information [8, 9].

In the other more practical cases, the TP is assumed to be **almost dishon**est and may deviate from the normal procedure of the protocol to reveal the participants' secret information except acting in collusion with the clients. That is, the TP not only can passively collect useful information but also can actively perform any attack on the protocol except conspiring with the participant. In this case, however, Alice and Bob, who are strangers and thus do not directly share an authenticated channel, may not be able to detect and avoid the TP's attack, and their secret information might thus be leaked to the TP [10]. In this regard, can we also develop a secure protocol for a pair of strangers to share entanglement under the help of almost dishonest TPs? This is the question addressed by this work.

In order to do that, we assume the existence of two non-communicating TPs. That is, Alice and Bob attempt to find two TPs from the group of clients who simultaneously share both the quantum channels and authenticated classical channels with them. Because the chosen TPs are non-communicating, they do not know who will be the counter party–e.g., the other TP–of the current scenario. This also increases the difficulty of collusion between both TPs. That is, the TPs are non-communicating and do not know each other, and hence it is more difficult for them to act in collusion.

If we let the first almost dishonest TP generate entangled quantum states for Alice and Bob, and the second almost dishonest TP helps Alice and Bob check the correctness of the shared entanglement. Moreover, let each TP watch the other TP's malicious behavior; then the entanglement could thus be established between Alice and Bob securely and correctly.

It should be noted here that the power of the conventional TP is now divided into two parts, which is similar to the idea of secret sharing, in which a top secret is divided into several shadows and only when a sufficient amount t of shadows is collected can the top secret be derived. Fewer than t shadows will never reveal the top secret. In our case, owing to this power separation of TPs and due to the assumption of non-colluding TPs, the trustworthiness of TPs in our protocol can be demoted from semi-honest to almost dishonest.

Now, because the TPs are non-communicating and almost dishonest, other clients in the protocol can serve well as TPs as long as they simultaneously share both quantum channels and authenticated classical channels with both clients who want to establish entanglement in the protocol. In other words, within our protocol, the only involved roles are the clients themselves. Among them, some connect with quantum or authenticated classical channels whereas others are strangers who do not have any direct connection. Whoever wants to establish entanglement with the other, has to identify two other clients serving as their TPs, who can directly communicate with both users in both the quantum channel and the authenticated classical channel. (see also Fig. 1)

The rest of this paper is organized as follows: Section 2 describes the proposed environment and the quantum entanglement establishment protocol between strangers; a quantum communication protocol is also presented as an example. Section 3 provides the security analyses of the proposed protocol. Finally, the conclusions are presented in Section 4.

2 Proposed Protocol

This section introduces the proposed protocol. First, the environment with two TPs is introduced in Section 2.1; then, the proposed quantum entanglement establishment protocol is presented in Section 2.2. Finally, as an example, a quantum secure direct communication protocol based on the proposed entanglement establishment is described in Section 2.3.

2.1 Environment

This section describes the proposed environment and its security requirement. The environment, including two participants, Alice and Bob, and two TPs, TP1 and TP2, is described as follows. (see also Fig. 1, in which the dotted lines denote the quantum channels and the solid lines denote the authenticated classical channels.)

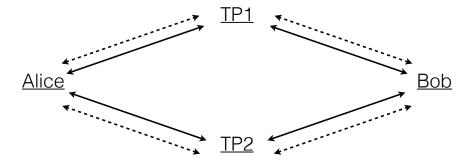


Figure 1: The Proposed Environment

- Alice (Bob) shares authenticated classical channels and quantum channels with two TPs, respectively. Note that Alice and Bob do not have any authenticated channel directly connected to each other.
- 2. The transmitted information on the authenticated classical channel is public, but the receiver can verify its integrity and originality.
- 3. TPs are almost dishonest in the sense that they can perform any possible attacks except conspiring with Alice, Bob, or the other TP.
- 4. In our proposed environment, if a TP cannot successfully attack the shared entanglement, then the TP will not announce the fake results during the public discussion.
- 5. Each TP is designed to prevent the other TP from attacks; hence, both TPs can be almost dishonest.
- 6. An external attacker, Eve, may try to perform any attack to disturb, forge, or eavesdrop on the state of Alice and Bob's shared entanglement.

2.2 Proposed Quantum Entanglement Establishment

This section comprises a quantum entanglement establishment protocol for the proposed environment. The proposed protocol allows the sender, Alice to share an entangled state with the stranger Bob. In the proposed protocol, though the quantum signals are generated by TP1, Alice and Bob can determine whether TP1 performs any attack on the quantum signals with the help of TP2. The proposed protocol proceeds as follows (see Fig. 2):

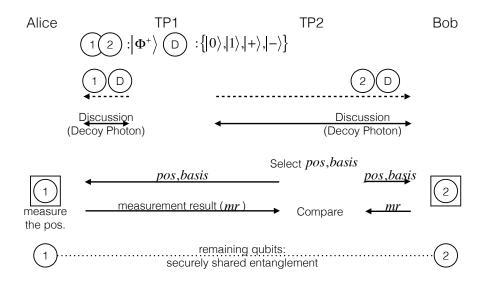


Figure 2: The Proposed Entanglement Establishment.

(Step1) TP1 generates a sequence of EPR entangled states, $|\Phi^+\rangle_{12} = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)_{12}$, where the subscripts 1 and 2 denote respectively the first and the second qubits. Let Q_1 (Q_2) denotes the particle sequence includes all the first (second) qubit of each EPR state in order. TP1 then inserts enough amount of decoy photons [20–22] randomly chosen from the four states: $\{|0\rangle, |1\rangle, |+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle), |-\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \}$ into Q_1 (Q_2) to form a new sequence S_1 (S_2 .) TP1 sends the sequence S_1 to Alice, and S_2 to Bob, respectively.

- (Step2) Once Alice receives the quantum sequence S_1 from TP1, she sends an acknowledgement to TP1 via the authenticated classical channel. TP1 and Alice then will publicly discuss the decoy photons for the eavesdropping detection. TP1 informs the position and the basis of each decoy photon to Alice. Alice measures these decoy photons, and then sends the measurement results to TP1. By comparing the initial states and the measurement results, TP1 can detect the existence of eavesdroppers. Similarly, Bob will also publicly discuss the decoy photons in S_2 with TP1. According to the quantum cryptographic protocol which will be executed after entanglement establishment, if Alice will send the received particles out later, Alice has to use the photon number splitter (PNS) and the wavelength filter to check if Trojan Horse attacks exist in the protocol [23–26]. (The detailed analyses of the Trojan Horse attacks will be given in Section 3.2)
- (Step3) If the quantum transmissions are free from the eavesdroppers and the Trojan Horse attacks, Alice and Bob can remove the decoy photons and recover the sequences Q_1 and Q_2 . They then will discuss the entanglement of the shared EPR states via the help of TP2. TP2 randomly selects the position and basis (X basis or Z basis) for each photon to be checked and announces the positions and bases to Alice and Bob. Alice and Bob then measure the selected particles in Q_1 (Q_2) with the bases chosen by TP2, and sends the measurement results to TP2. Because

$$\begin{aligned} |\Phi^+\rangle &= \frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle \right) \\ &= \frac{1}{\sqrt{2}} \left(|++\rangle + |--\rangle \right), \end{aligned}$$
(1)

TP2 can compare the measurement results from Alice to Bob to determine

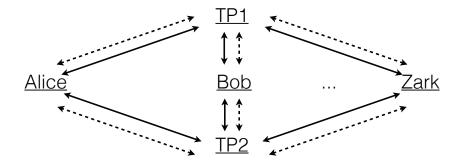


Figure 3: The Multiparty Environment

if Alice and Bob's qubits are in $|\Phi^+\rangle$.

(Step4) If the entanglement correlations between Alice's and Bob's qubits are correct, Alice (Bob) will remove the measured qubits selected by TP2 from Q_1 (Q_2 ,) and have a new sequence, Q'_1 (Q'_2 .) The entanglement establishment between Alice and Bob is completed.

It should be noted that the proposed entanglement establishment scheme can also be extended to a multi-participant scenario. Suppose that the participants– Alice, Bob, Charlie, ..., and Zack, who are strangers to one another–want to share an entanglement. They can look for two almost dishonest TPs who share quantum channels and authenticated classical channels with all participants (see Fig. 3). TP1 can generate the entanglement and securely distribute the particles to them, which are the same as Step 1 and Step 2; then, similar to Step 3, TP2 selects the measurement bases and positions for each participant and compares the measurement results returned from every participant. Hence, TP2 can help the participants check the correctness of the entanglement they shared. Finally, the participants can share an entanglement.

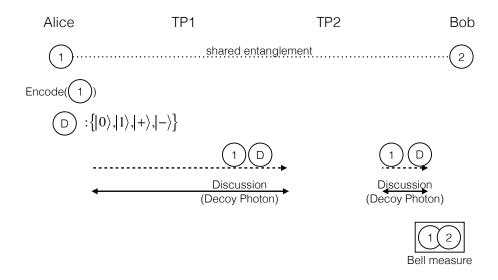


Figure 4: The Quantum Direct Secure Communication based on the Proposed Entanglement Establishment.

2.3 Quantum Secure Direct Communication

Section 2.2 describes the processes of the proposed quantum entanglement establishment protocol. In this section, we show how a quantum secure direct communication (QSDC) protocol can be constructed based on the entanglement establishment protocol given in Section 2.2. In a QSDC protocol, a sender can send secret messages to a receiver without any pre-shared key between them, and they do not require any transmission of classical information except for the eavesdropping detection. Here we assume that Alice wants to send a two-bit message to Bob (see Fig. 4).

(Step1~4) These steps are the same as those mentioned in Section 2.2.

(Step5) To transmit her secret message, Alice applies dense coding on her photons by performing the unitary operation on each qubit of Q'_1 obtained in Section 2.2 according to her two-bit messages. If the two-bit message is 00, she will perform $I = |0\rangle \langle 0| + |1\rangle \langle 1|$; if the two-bit message is 01, she will perform $\sigma_z = |0\rangle \langle 0| - |1\rangle \langle 1|$; if the two-bit message is 10, she will perform $\sigma_x = |0\rangle \langle 1| + |1\rangle \langle 0|$; otherwise, she will perform $i\sigma_y = |0\rangle \langle 1| - |1\rangle \langle 0|$. After the encoding, Alice generates decoy photons as in Step 1 of Section 2.2 by TP1, and inserts them to Q'_1 to form a new sequence S'_1 , which is then transmitted to TP2. Upon receiving S'_1 , TP2 publicly discusses the decoy photons with Alice as in Step 2. If there are eavesdroppers detected, they will abort the protocol and return to Step 1.

- (Step6) TP2 removes the decoy photons from S'_1 , and inserts new decoy photons into the particle sequence to form S''_1 , which is then transmitted to Bob.
- (Step7) Bob and TP2 again discuss the decoy photons for detecting the eavesdroppers. If the quantum transmission between TP2 and Bob is secure, Bob can remove the decoy photons and obtain the particle sequence Q'_1 . Bob then performs Bell measurement (EPR measurement) on every pair of qubits respectively from Q'_1 and Q'_2 . According to the measurement results, Bob can obtain Alice's secret message. (See Eq. (2))

$$I |\Phi^{+}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) = |\Phi^{+}\rangle$$

$$\sigma_{z} |\Phi^{+}\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle) = |\Phi^{-}\rangle$$

$$\sigma_{x} |\Phi^{+}\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle) = |\Psi^{+}\rangle$$

$$i\sigma_{y} |\Phi^{+}\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle) = |\Psi^{-}\rangle$$
(2)

3 Security Analyses

This session analyzes the security of the proposed protocols. First, in Section 3.1, the security of the entanglement establishment protocol is analyzed. The security of the QSDC protocol is analyzed in Section 3.2. The formal security proof using the random oracle model is described in the appendix.

3.1 Security of the Entanglement Establishment Protocol

For an entanglement establishment protocol, the attacker may try to obtain Alice's entanglement qubits. Hence, the attacker can share a quantum entanglement with Bob. Here, three possible attack strategies will be discussed: the entangle-and-measure attack, the intercept-and-resend attack, and the entanglement swapping attack. The analyses indicate that the external attacker and the TPs in the protocol cannot successfully obtain the entanglement shared between Alice and Bob without being detected.

The Entangle-and-measure Attack

When TP1 sends S_1 to Alice in Step 1, the external eavesdropper, Eve, may perform the entangle-and-measure attack [27–29] to steal the transmitted qubits in S_1 . Because S_1 contains TP1's decoy photons, to avoid being detected, Eve will try to obtain the states of the decoy photons. For each qubit, q_1 , in S_1 , Eve prepares an ancillary qubit in an arbitrary known state $q_e = |E\rangle$, and performs her attack operation U on q_1 and q_e . The result of Eve's operation is as follows:

$$U |0\rangle_{1} |E\rangle_{e} = a |0\rangle_{1} |e_{00}\rangle_{e} + b |1\rangle_{1} |e_{01}\rangle_{e}$$

$$U |1\rangle_{1} |E\rangle_{e} = c |0\rangle_{1} |e_{10}\rangle_{e} + d |1\rangle_{1} |e_{11}\rangle_{e}$$

$$U |+\rangle_{1} |E\rangle_{e} = \frac{1}{2} \begin{bmatrix} |+\rangle_{1} (a |e_{00}\rangle_{e} + b |e_{01}\rangle_{e} + c |e_{10}\rangle_{e} + d |e_{11}\rangle_{e}) + \\ |-\rangle_{1} (a |e_{00}\rangle_{e} - b |e_{01}\rangle_{e} + c |e_{10}\rangle_{e} - d |e_{11}\rangle_{e}) \end{bmatrix}$$
(3)
$$U |-\rangle_{1} |E\rangle_{e} = \frac{1}{2} \begin{bmatrix} |+\rangle_{1} (a |e_{00}\rangle_{e} + b |e_{01}\rangle_{e} - c |e_{10}\rangle_{e} - d |e_{11}\rangle_{e}) + \\ |-\rangle_{1} (a |e_{00}\rangle_{e} - b |e_{01}\rangle_{e} - c |e_{10}\rangle_{e} - d |e_{11}\rangle_{e}) + \\ |-\rangle_{1} (a |e_{00}\rangle_{e} - b |e_{01}\rangle_{e} - c |e_{10}\rangle_{e} + d |e_{11}\rangle_{e}) \end{bmatrix},$$

where $|e_{00}\rangle$, $|e_{01}\rangle$, $|e_{10}\rangle$, and $|e_{11}\rangle$ are four states which Eve can distinguish, and $|a^2| + |b^2| = |c^2| + |d^2| = 1.$ To pass the eavesdropping detection, Eve sets b = c = 0 and $(a |e_{00}\rangle_e + b |e_{01}\rangle_e + c |e_{10}\rangle_e + d |e_{11}\rangle_e) = (a |e_{00}\rangle_e + b |e_{01}\rangle_e - c |e_{10}\rangle_e - d |e_{11}\rangle_e) = \overrightarrow{0}$. Eve's operation thus will not change the state of q_1 , and Eve can successfully pass the eavesdropping detection. However, b = c = 0 implies $(a |e_{00}\rangle_e - d |e_{11}\rangle_e) = \overrightarrow{0}$, that is, $a |e_{00}\rangle_e = d |e_{11}\rangle_e$. In this case, Eve cannot distinguish $|e_{00}\rangle$ and $|e_{11}\rangle$, and she cannot obtain the information in q_1 . If Eve wants to distinguish $a |e_{00}\rangle_e$ from $d |e_{11}\rangle_e$, her operation, U, will change the state of q_1 , which will cause her attack to be detected by TP1 and Alice.

Generally, if Eve want to pass the eavesdropping check, she cannot get any information. If Eve tries to reveal the whole information from a qubit, she will change the state of the qubit, and eventually be detected in the public discussion.

The Intercept-and-resend Attack

TP2 may perform the intercept-and-resend attack when TP1 sends S_1 to Alice. TP2 intercepts all qubits in S_1 , and generates a sequence of fake photons, which are sent to Alice. If TP2 can pass the eavesdropping detection process, TP2 then could successfully share an entanglement with Bob. However, the decoy photons inserted in S_1 are generated by TP1. Because the positions and the bases of the decoy photons are unknown to TP2, TP2 is unable to exactly generate the same decoy photons as TP1 did. Hence, TP2's fake photons will cause errors in the public discussion between Alice and TP1 with the probability $1 - (75\%)^n$ [20–22], where n is the number of decoy photons. If n is large enough, the probability will be close to 1.

The Entanglement Swapping Attack

TP1, who generates the EPR states for Alice and Bob, may also try to perform the entanglement swapping attack [10] to obtain Alice's secret message. In Step 1, instead of generating one EPR pair and distributing these two particles to Alice and Bob, respectively, TP1 generates two EPR pairs, namely $|\Phi^+\rangle_{T1,T2}$ and $|\Phi^+\rangle_{T3,T4}$. TP then distributes q_{T1} , the first particle of the first EPR state, to Alice, and q_{T3} , the first particle of the second EPR state, to Bob. Because all the decoy photons are generated by TP1, TP1 can successfully pass the eavesdropping check of decoy photons in Step 2. If the entanglement correlation check in Step 3 can be passed, in Step 5, Alice will send the encoded particle, q_{T1} to TP2. TP1 can intercept them, remove the decoy photons according to Alice and TP2's public communication, and perform an EPR measurement on the qubit pair q_{T1} and q_{T2} . According to the measurement result, TP1 can obtain Alice's secret message.

But the fact is , when Alice, TP2, and Bob discuss the entanglement of the shared EPR states in Step 3, for each discussed position, TP1 can measure the qubit pair q_{T2} and q_{T4} . The qubits q_{T1} and q_{T3} then will be in one of four EPR states, $\{|\Phi^+\rangle, |\Phi^-\rangle, |\Psi^+\rangle, |\Psi^-\rangle\}$, which is also known by TP1. Because these two particles held respectively by Alice and Bob are still in EPR state, they cannot detect that TP1 generated two EPRs rather than one. However, the above situation happens only when TP1 is allowed to generate variable EPR states. In the proposed protocol, however, TP1 is only allowed to generate $|\Phi^+\rangle$, if he/she performs the above attack, the EPR state shared by Alice and Bob in public discussion will be in one of $\{|\Phi^+\rangle, |\Phi^-\rangle, |\Psi^+\rangle, |\Psi^-\rangle\}$, rather than in $|\Phi^+\rangle$ as in normal situation. For example, if the shared state is $|\Psi^-\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)$, Alice's and Bob's Z-basis measurement will be $|0\rangle$ and $|1\rangle (|1\rangle$ and $|0\rangle$,) whereas the legal measurement results are $|0\rangle$ and $|0\rangle (|1\rangle$ and $|1\rangle)$.

3.2 Security of QSDC Protocol

The above analyses denote the security of the proposed entanglement establishment scheme. The following analyses focus on the security of the QSDC protocol. Four special attacks—the Trojan Horse attacks, the correlation-elicitation (CE) attack, the dense coding attack, and the modification attack—will be respectively analyzed. We also indicate that the QSDC protocol satisfies the Deng-Long criteria, a security requirement for quantum communication protocols.

The Trojan Horse Attacks

Eve (TP1, TP2) may perform the Trojan Horse attacks to reveal Alice's message. When TP1 sends S_1 to Alice, she can insert her own particle sequence into S_1 by adopting the invisible-photon attack [24] or the delay-photon attack [23] strategy. When Alice sends out S'_1 to TP2 after her encoding, Eve can retrieve her particles, and then obtains Alice's secret. However, because Alice has set a wavelength filter and PNS, Eve's particles can be detected by these devices. If illegal particles are detected, Alice and TP1 will drop these transmitted particles and restart the protocol. Hence, Eve cannot obtain any information about Alice's secret.

The Correlation-elicitation Attack

The almost dishonest TP2 may try to steal Alice's secret by performing the correlation-elicitation (CE) attack [13–16]. When the second qubit of the EPR state generated by TP1 (denoted as q_2) is transmitted to Bob in Step 1, TP2 intercepts it, and generates an ancillary photon $q_e = |0\rangle$. TP2 then performs the first controlled-NOT (CNOT) operation on q_2 and q_e , where q_2 is the control bit, and q_e is the target bit. As a result, the two-particle EPR state and the

ancillary photon can be described as follows:

$$CNOT_{2e} \left| \Phi^+ \right\rangle_{12} \otimes \left| 0 \right\rangle_e = \frac{1}{\sqrt{2}} \left(\left| 000 \right\rangle + \left| 111 \right\rangle \right), \tag{4}$$

where \otimes denotes the tensor product operation. TP2 then resends q_2 to Bob. When Alice sends the encoded first qubit, q_1 , of the EPR state to TP2 in Step 5, TP2 performs the second CNOT operation on q_1 and q_e , where q_1 is the control bit, and q_e is the target bit. Due to Alice's encoding operation, the state of q_1 and q_2 becomes one of $|\Phi^+\rangle$, $|\Phi^-\rangle$, $|\Psi^+\rangle$, and $|\Psi^-\rangle$ (see Eq. (2).) The four possible states after the second CNOT operation are as follows:

$$CNOT_{1e}CNOT_{2e} |\Phi^{+}\rangle_{12} \otimes |0\rangle_{e} = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)_{12} \otimes |0\rangle_{e}$$

$$CNOT_{1e}CNOT_{2e} |\Phi^{-}\rangle_{12} \otimes |0\rangle_{e} = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle)_{12} \otimes |0\rangle_{e}$$

$$CNOT_{1e}CNOT_{2e} |\Psi^{+}\rangle_{12} \otimes |0\rangle_{e} = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle)_{12} \otimes |1\rangle_{e}$$

$$CNOT_{1e}CNOT_{2e} |\Psi^{-}\rangle_{12} \otimes |0\rangle_{e} = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)_{12} \otimes |1\rangle_{e}$$

$$(5)$$

TP2 is now able to obtain Alice's partial secret according to the Z-basis measurement result of q_e . According to Eq. (5,) if the measurement result of q_e is $|0\rangle$, TP2 knows that the state of q_1 and q_2 is either $|\Phi^+\rangle$ or $|\Phi^-\rangle$; otherwise, the state is $|\Psi^+\rangle$ or $|\Psi^-\rangle$. TP2 can thus obtain partial information about Alice's secret message. However, when TP1 sends the sequence S_2 , which includes q_2 of each EPR pair, to Bob, S_2 also contains TP1's decoy photons, where the positions and bases of these decoy photons are unknown to TP2. If TP2's first CNOT operation is performed on an X-basis decoy photon, for example, $q_d = |+\rangle$, the result is as follows:

$$CNOT_{de} \left| + \right\rangle \otimes \left| 0 \right\rangle = \frac{1}{\sqrt{2}} \left(\left| + + \right\rangle + \left| - - \right\rangle \right)_{de} \tag{6}$$

It can be seen that if Bob measures the decoy photon in X basis, the measurement result will be $|+\rangle$ or $|-\rangle$ with an equal probability of 50%. Hence, if the decoy photon is in X basis, TP2's attack may disturb the state of decoy photon. Eventually, it causes TP2 to be detected with a probability of 50%. However, if the decoy photon is in Z basis, TP2's first CNOT operation will not disturb the state. Assume that TP1 selects the basis of each decoy photon with equal probability in Z basis or X basis. TP2's attack will be detected with the following probability: $50\% \times 50\% + 50\% \times 0 = 25\%$. Consequently, if there are n decoy photons, the detection rate of TP2's attack is $1 - (75\%)^n$. If n is large enough, the probability will be close to 1.

The Dense Coding Attack

The external attacker, Eve, may try to perform the dense coding attack [30] to reveal Alice's secret message. When TP1 transmits S_1 to Alice in Step 2, Eve intercepts it, and prepares a sequence of EPR states $|\Phi^+\rangle_{e1,e2}$, where e1and e^2 respectively denote the first and the second particles of the EPR states generated by Eve. Eve sends all q_{e1} , the first particle of each EPR state, to Alice in hope that she successfully passes the eavesdropping detection, and thus Alice's encoding operations will be performed on Eve's q_{e1} . Consequently, when Alice sends out the encoded qubits to TP2 in Step 5, Eve can retrieve her q_{e1} and performs EPR measurement on every pair of q_{e1} and q_{e2} . That is, according to the measurement results (see Eq. (2),) Eve can reveal Alice's secret message. However, S_1 contains decoy photons. According to Eq. (1,) it can be seen that the first particle has two measurement results in both two basis. If the original decoy photon is $|1\rangle$, and Alice measures the fake photon, q_{e1} , in Z basis, then the measurement result will be $|0\rangle$ or $|1\rangle$ with equal probability. If the measurement result is $|0\rangle$, Eve's attack will be detected. For each decoy photon, Alice will get an illegal measurement result on Eve's fake photon with the probability of 50%. Let n be the number of decoy photons, Eve's attack will be detected with the probability of $1 - (50\%)^n$. If n is large enough, the probability will be close to 1.

The above analyses denote that the proposed protocol is not only secure against the general attack, but also secure against some special attacks. If TP1 attacks the protocol, he/she will be detected in the public discussion held by TP2 in Step 3. Similarly, if TP2 attacks the protocol, he/she will be detected in Step 2, the public discussion of the decoy photons generated by TP1. Two almost dishonest TPs, TP1 and TP2, share duty to watch each other and as a result, two strangers, Alice and Bob can have a secure communication between each other.

The Modification Attack

Eve (TP1) may perform random unitary operations on the encoded qubits when these qubits are sent from Alice to Bob via TP2. Hence Alice's message could be modified [31, 32]. However, the decoy photons are inserted in the quantum transmission, and Eve does not know the positions of the decoy photons. Eve's random operations will cause she being detected in the public discussion between Alice and TP2 (or TP2 and Bob.)

Considering the following situations: (1) Eve (TP1) might perform only one operation in hope that the selected position is the encoded qubit rather than the decoy photon; (2) TP2 performs the modification attack, where TP2 knows all the positions of the decoy photons. Alice and Bob can simplify use the message authenticate code to protect the integrity of the transmitted secret message. The modification thus can be detected.

The Deng-Long Criteria

The Deng-Long criteria [27] defines the requirements for a secure quantum communication protocol. The requirements are listed as follows:

- 1. A QSDC protocol does not require any additional classical information transmissions except for the eavesdropping check. The receiver can directly read the secret information after the quantum transmissions.
- 2. Eve, an eavesdropper, cannot obtain any useful information about the secret message.
- 3. The sender and the receiver can detect Eve before they encode the secret message on the quantum states.
- 4. The quantum states are transmitted in a block by block way.

The following analyses respectively indicate the proposed QSDC protocol satisfies the Deng-Long criteria.

- 1. In the proposed QSDC protocol, Bob can directly reveal Alice's secret message according to his measurement results (see Step 7.) Alice sends classical information in Step 2, Step 3, and Step 5, and they are all for detecting the eavesdropping. Alice does not send any classical information except for the eavesdropping check.
- 2. As shown in the above security analyses, Eve cannot obtain the secret information sent by Alice.
- 3. In the proposed QSDC protocol, if Eve (TP1, or TP2) performs attacks in the entanglement establishment process, Alice and Bob can detect the attacks in the public discuss process in Step 2 and Step 3. After confirming the security of the quantum transmission, Alice will encode her message in Step 5.

4. The quantum transmissions in the proposed schemes, i.e., the entanglement establishment and the QSDC, are sending a sequence of particles including all the first (second) qubits of the EPR states, which is the "block by block way."

4 Conclusions

This paper presents a new method in quantum cryptography that allows multiple strangers to establish an entanglement with the help of two almost dishonest TPs. Each TP is designed to prevent the other TP from acting maliciously; hence, both TPs can be almost dishonest. The proposed protocol can also be easily transformed into a quantum communication, a quantum teleportation, a quantum key distribution, a quantum private comparison, etc., between two strangers. It is indeed a challenging task to provide a scenario, secure entanglement establishment between two strangers using other approaches. It would be an interesting future research to have a secure entanglement establishment for strangers, who cannot find two common TPs to help them.

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Appendix - Formal Security Model and Analysis

In this section, we define the adversarial model of the two public discussions in Step 2 and Step 3 of the proposed scheme. The security of the first public discussion (Step 2) is analyzed in Section A.1 and then, the second public discussion (Step 3) is analyzed in Section A.2.

A.1 The First Public Discussion

In the following analyses, the public discussion between Alice and TP1 is analyzed. Note that the security of the public discussion between Bob and TP1 is the same as the one between Alice and TP1, hence we omit that in the following description.

Formal Security Model

The security model of the interactions between an adversary and the protocol participants occurs only via oracle queries which model the adversary's capabilities in a real attack. Let A denote Alice, TP1 denote TP1, and P1is the public discussion they participate. The participants of P1 can launch more than one instance. Here we allow a probabilistic polynomial time (PPT) adversary \mathscr{A} to potentially control all the communication in the network via accessing to a set of oracles as defined below. Let A^i denotes the instance i of A. $TP1^j$ is the instance j of TP1.

- **Execute** $(A^i, TP1^j)$: The query models the passive attack. An adversary can obtain all messages exchanged between A^i and $TP1^j$.
- **Reveal** (A^i) : In this query model, if the oracle has accepted, it returns the secret quantum state between A^i and $TP1^j$ to the adversary; otherwise, it returns the *null* value to the adversary.
- **Send** $(A^i/TP1^j,m)$: This query models an active attack. It returns the information corresponded to an input m that A^i or $TP1^j$ would send to each other.
- **Corrupt** (A^{i},a) : This query models corruption capability of the adversary. If a = 0, it returns a *null* value; otherwise, it returns the secret quantum

states between A^i and $TP1^j$.

Test $(TP1^{j})$: This query measures whether the public discussion is secure or not. By throwing an unbiased coin, b, if b = 1, it returns a random bit sequence with the same length as A^{i} 's measurement result. The query can only be called once.

In this model, we consider two kinds of adversaries. A passive adversary is allowed to issue the **Execute** and **Test** queries and an active adversary is additionally allowed for sending the **Send** query.

Definitions of Security

To demonstrate the security of the first public discussion, we will give the security definition as follows.

Definition 1 (Partnering): A^i and $TP1^j$ are partnered, if they mutually authenticate each other.

Definition 2 (Freshness): An entity A^i with the partner $TP1^j$ is freshness if the following two conditions hold:

(1) If it has accepted an measurement result $MR \neq null$ and both the entity and its partner have not been sent a **Reveal** query.

(2) There is no **Corrupt** query has been asked before the query **Send** has been asked.

The advantage of the adversary \mathscr{A} is measured by the ability of distinguish a legal measurement result from a random value. We define **Succ** to be an event that \mathscr{A} correctly guesses the bit b, which is chosen in the **Test** query. Hence, the advantage of \mathscr{A} in the attacked scheme P1 is defined as: $Adv_{P1}(\mathscr{A}) = |2 \times Pr[Succ] - 1|$. We argue that the public discussion P1 is secure, as $Adv_{P1}(\mathscr{A})$ is negligible. Precisely, the adversary \mathscr{A} does not have any advantage to obtain the correct measurement result between the participants.

Security Analysis

In the following description, we show that the public discussion, P1, holds several security properties, which are required for a secure quantum cryptographic public discussion. Let the maximum advantage of the adversary with running time Tm be for a certain task denoted as Adv_{Task} (Tm). The following advantages will be used in the analyses.

 $Adv_{Qubit}^{Clone}(Tm)$: The advantage for cloning a qubit.

 $Adv_{A}^{Forge}\left(Tm\right)$: The advantage for impersonate himself/herself as Alice (A).

Lemma1 The advantage for cloning a qubit, $Adv_{Qubit}^{Clone}(Tm)$, is negligible.

Proof The quantum no-cloning theory has already been well-proven in several researches [20], here, we briefly describe the proof.

Assume that for an input qubit q_i with an arbitrary state, there exists a clone operation U. The clone operation can be defined as follows:

$$U |0\rangle_{i} |e\rangle_{o} = |0\rangle_{i} |0\rangle_{o}$$

$$U |1\rangle_{i} |e\rangle_{o} = |1\rangle_{i} |1\rangle_{o}$$

$$U |+\rangle_{i} |e\rangle_{o} = |+\rangle_{i} |+\rangle_{o},$$
(7)

where $|e\rangle_o$ denotes the output qubit, and $|e\rangle$ is an arbitrary initial state.

 $\begin{array}{l} \text{Because } |+\rangle_i = \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle \right)_i, \text{it implies that } U \left|+\rangle_i \left|e\right\rangle_o = \frac{1}{\sqrt{2}} \left(U \left|0\right\rangle_i \left|e\right\rangle_o + U \left|1\right\rangle_i \left|e\right\rangle_o \right) = \frac{1}{\sqrt{2}} \left(|0\rangle_i \left|0\right\rangle_o + |1\rangle_i \left|1\right\rangle_o \right). \text{ However, } U \left|+\rangle_i \left|e\right\rangle_o = |+\rangle_i \left|+\rangle_o = 1 \end{array}$

 $\frac{1}{\sqrt{2}} \left(|0\rangle_i |0\rangle_o + |0\rangle_i |1\rangle_o + |1\rangle_i |0\rangle_o + |1\rangle_i |1\rangle_o \right), \text{ which is not equal to } \frac{1}{\sqrt{2}} \left(|0\rangle_i |0\rangle_o + |1\rangle_i |1\rangle_o \right).$ The contradiction shows that the qubit cannot be cloned. $Adv_{Qubit}^{Clone} (Tm)$ is negligible.

Lemma2 Suppose that there exists an attacker \mathscr{A} , who impersonates as Alice (A) with the running time Tm in the public discussion. Then the advantage of \mathscr{A} , $Adv_A^{Forge}(Tm) = Adv_{Qubit}^{Clone}(Tm)$.

Proof Suppose that \mathscr{A} impersonates as Alice. In Step 1 of the proposed

scheme, TP1 sends a quantum sequence to Alice, and discusses the decoy photons with Alice in Step 2. If \mathscr{A} can successfully impersonate as Alice, then she can send her fake photon to Alice, and TP1 cannot detect the problem.

When TP1 sends the qubit sequence S_1 to Alice, \mathscr{A} constructs an attack β to clone every qubit in S_1 . The sequence of the cloning outputs is denoted as \hat{S}_1 . Then, β sends the original sequence S_1 to Alice. Alice will acknowledge TP1 that she has received the qubits. Then TP1 will announce the bases and positions of the decoy photons to Alice. Alice will select the corresponding qubits from S_1 and measure them in the bases TP1 announced. Alice then transmits all the measurement results to TP1 and TP1 can compare the measurement results and his/her initial states of decoy photons to detect the existence of the eavesdroppers. Because these public classical informations are transmitted via the authenticated channel shared between Alice and TP1, β cannot forge or modify them. Here, β 's goal is to successfully clone the qubits from S_1 to \hat{S}_1 . β runs a subroutine and simulates its attack environment, and gives all the required public parameters to \mathscr{A} . Without losing the generality, assume that \mathscr{A} does not ask queries on the same message more than once. β maintains a list $L_{CloneQubit}$ to ensure identical responding and avoid collision of the queries. β simulates the oracle queries of \mathscr{A} as follows:

Send-query: The send query is classified into the following types:

- Send(TP1^j, S₁): β clones every qubits in the quantum sequence S₁, and forms the output qubits as a new sequence Ŝ₁. β returns Ŝ₁ to A.
- Send(Aⁱ, ok): Alice sends the acknowledgement to TP1 for receiving qubits. β direct pass the collected information to A.

- Send(TP1^j, pos&bases): TP1 announces the positions and bases of the decoy photons to Alice. β direct pass the collected information to A.
- Send (A^i, mr) : Alice sends the measurement results to TP1. β stores these results for the test query.
- **Execute-query:** When \mathscr{A} asks for an $\mathbf{Execute}(A^i, TP1^j)$ query, β returns the transcript $\langle \hat{S}_1, \text{Send}(A^i, ok), \text{Send}(TP1^j, pos\&bases) \rangle$ to \mathscr{A} by using the simulation of send query.
- **Test-query:** When \mathscr{A} makes the test query, if the query is not asked in the first session, then β will abort it; otherwise, β randomly chooses a bit b. If b = 0, β returns the value of Send (A^i, mr) ; otherwise, β returns a random string to \mathscr{A} . The adversary has to distinguish the random string from a legal measurement result. In order to do that, if the quantum could be cloned, \mathscr{A} can measure the qubits from \hat{S}_1 by using the positions and bases obtained from the query Send $(TP1^j, pos\&bases)$. Then, the adversary can successfully get the legal measurement results, hence the random string and the legal measurement results can be distinguished. Hence, the adversary's advantage, $Adv_{Alice}^{Forge}(Tm) = Adv_{Qubit}^{Clone}(Tm)$.

A.2 The Second Public Discussion

In the following analyses, the public discussion between Alice, Bob and TP2 is analyzed.

Formal Security Model

Let A denotes Alice, B denotes Bob, TP2 denotes TP2, and P2 is the public discussion they participate. To describe the multiple instances of the

participants, let A^i denote the instance *i* of *A*, B^j denote the instance *j* of Bob, and $TP2^k$ is the instance *k* of TP2.

- **Execute** $(A^i, B^j, TP2^k)$: The query models the passive attack. An adversary can obtain all messages exchanged between A^i, B^j and $TP2^k$.
- **Reveal** (A^i, B^j) : In this query model, if the oracle has accepted, it returns the secret quantum state between A^i and B^j to the adversary; otherwise, it returns the *null* value to the adversary.
- **Send** $(A^i/B^j/TP2^k,m)$: This query models an active attack. It returns the information corresponded to an input m that A^i , B^j , or $TP2^k$ would send to each others.
- **Corrupt** (A^i, B^j, a) : This query models corruption capability of the adversary. If a = 0, it returns a *null* value; otherwise, it returns the secret quantum states between A^i and Bs^j .
- **Test** $(TP2^k)$: This query measures whether the public discussion is secure or not. By throwing an unbiased coin, b, if b = 1, it returns a random bit sequence with the same length as A^i and B^j 's measurement results. The query can only be called once.

Similar as the previous model, we consider two kinds of adversaries. A passive adversary is allowed to issue the **Execute** and **Test** queries and an active adversary is additionally allowed for sending the **Send** query.

Definitions of Security

To demonstrate the security of the first public discussion, we will give the security definition as follows.

Definition 1 (Partnering): A^i , B^j and $TP2^k$ are partnered, if they mutually authenticate each other.

Definition 2 (Freshness): The entities A^i and B^j with the partner $TP2^k$ is freshness if the following two conditions hold:

(1) If it has accepted an measurement result $MR \neq null$ and both the entity and its partner have not been sent a **Reveal** query.

(2) There is no **Corrupt** query has been asked before the query **Send** has been asked.

The advantage of the adversary \mathscr{A} is also measured by the ability of distinguish a legal measurement result from a random value. Hence, the advantage of \mathscr{A} in the attacked scheme P2 is defined as: $Adv_{P2}(\mathscr{A}) = |2 \times Pr[Succ] - 1|$. We argue that the public discussion P1 is secure, as $Adv_{P2}(\mathscr{A})$ is negligible. Precisely, the adversary \mathscr{A} does not have any advantage to obtain the correct measurement result between the participants.

Security Analysis

In the following description, we show that the public discussion, P2, holds several security properties, which are required for a secure quantum cryptographic public discussion.

 $Adv_{FakeState}^{Gen}(Tm)$: The advantage for generating a fake entangled state without being detected in the second public discussion.

 $Adv_{P2}^{Attack}(Tm)$: The advantage for attacking P2 successfully.

- **Lemma3** The advantage for generating a fake entangled state $|\psi\rangle$ that can be written as $|\phi\rangle |\Phi^+\rangle$, $Adv_{FakeState}^{Gen}(Tm)$ is negligible.
- **Proof** Let $|\phi\rangle$ be an special entangled state that an adversary can generate. The adversary will try share this state with the legal users, Alice and Bob, before the second public discussion. During the second public discussion, because Alice, Bob, and TP2 will check if the state shared by Alice and Bob is $|\Phi^+\rangle$. In this case, the adversary should make $|\phi\rangle = |\Phi^+\rangle |\psi\rangle$, where $|\psi\rangle$ is the state held by the adversary, and $|\Phi^+\rangle$ is shared among

Alice and Bob.

However, $|\phi\rangle = |\Phi^+\rangle |\psi\rangle$ implies that $|\phi\rangle$ is a product state (i.e., it is the product of $|\psi\rangle$ and $|\Phi^+\rangle$), which is not entangled. The contradiction shows that the advantage to generate such special quantum state, $Adv_{FakeState}^{Gen}(Tm)$ is negligible.

- **Lemma4** Suppose that there exists an attacker \mathscr{A} , who wants to successfully attack P2 with running time Tm in the public discussion. Then $Adv_{P2}^{Attack}(Tm) = 2 \times Adv_{Qubit}^{Clone}(Tm) + Adv_{FakeState}^{Gen}(Tm).$
- **Proof** Suppose that \mathscr{A} wants to attack P2 procedure. The adversary hopes that he/she could share an entangled state with Alice, Bob, and himself/herself. \mathscr{A} constructs an attack γ to help him/her. γ will generate a special quantum state and distribute them to Alice, Bob, and the adversary. To send fake qubits to Alice and Bob without being detected, γ has to pass the first public discussions between Alice and TP1 (Bob and TP1.) Then, when the second public discussion is started, the fake qubits held by Alice and Bob can be converted to $|\Phi^+\rangle$, then the second public discussion will be success.

Here, γ 's goal is to successfully generate a fake entangled state, and pass the first public discussions. γ runs a subroutine and simulates its attack environment, and gives all the required public parameters to \mathscr{A} . Without losing the generality, assume that \mathscr{A} does not ask queries on the same message more than once. γ maintains a list $L_{GenFakeState}$ to ensure identical responding and avoid collision of the queries. γ simulates the oracle queries of \mathscr{A} as follows:

Send-query: the send query is defined as follows:

• Send $(TP1^k, S_1/S_2)$: when $TP1^k$ sends S_1 (S_2) to A^i $(B^j), \gamma$ gener-

ates a sequence of a n-qubit fake state $|\phi\rangle_{123...n}$, sends all the first qubits q_1 to Alice and all the second qubit q_2 to Bob. The remained qubits $q_{3...n}$ of all the fake states are denoted as $\hat{S_{3...n}}$ to \mathscr{A} .

- Send(Aⁱ/B^j, ok): Alice and Bob will notify TP2 the first public discussion has been success. γ direct pass the collected information to A.
- Send(TP2^k, pos&bases): TP2 announces the positions and bases to Alice and Bob. γ direct pass the collected information to A.
- Send(Aⁱ/B^j, mr): Alice and Bob sends the measurement results to TP1. γ stores these results for the test query.
- **Execute-query:** When \mathscr{A} asks for an $\mathbf{Execute}(A^i, B^j TP2^k)$ query, γ returns the transcript $\langle \hat{S_{3...n}}, \text{Send}(A^i/B^j, ok), \text{Send}(TP2^k, pos\&bases) \rangle$ to \mathscr{A} by using the simulation of send query.
- **Test-query:** When \mathscr{A} makes the test query, if the query is not asked in the first session, then γ will abort it; otherwise, γ randomly chooses a bit b. If b = 0, γ returns the value of Send $(A^i/B^j, mr)$; otherwise, β returns a random string to \mathscr{A} . The adversary has to distinguish the random string from the legal measurement results. In order to do that, if the special entangled state $|\phi\rangle_{123...n}$ can be converted to $|\phi\rangle_{123...n} = |\Phi^+\rangle_{12} |\psi\rangle_{3...n}$, Alice and Bob can generate a legal pair of measurement results, and \mathscr{A} can obtain their measurement result from $|\psi\rangle_{3...n}$. To success such attack, γ has to impersonate as Alice and Bob to respectively pass the two public discussions (i.e., the first public discussion between Alice and TP1 and between Bob and TP1.) Hence, the adversary's advantage can be derived as $Adv_{P2}^{Attack}(Tm) = Adv_{Alice}^{Forge}(Tm) + Adv_{Bob}^{Forge}(Tm) +$

 $Adv_{FakeState}^{Gen}(Tm)$. According to Lemma2, $Adv_{P2}^{Attack}(Tm) = 2 \times Adv_{Qubit}^{Clone}(Tm) + Adv_{FakeState}^{Gen}(Tm)$.

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