Multi-party quantum privacy comparison of size based on d-level GHZ states

Hao Cao^{a,b}, Wenping Ma^a, Liangdong Lyu^a, Yefeng He^c, Ge Liu^a

^aState Key Laboratory of Integrated Services Networks, Xidian University, Xi'an, 710071, China

^bSchool of Information and Network Engineering, Anhui Science and Technology University, Chuzhou, 233100, China

^cSchool of Telecommunications and Information Engineering, Xi'an University of Posts and Telecommunications, Xi'an 710121,

China

Abstract

Quantum privacy comparison(QPC) plays an important role in secret ballot elections, private auctions and so on. To date, many multi-party QPC(MQPC) protocols have been proposed to compare the equality of $k(k \ge 3)$ participants. However, there are few examples of MQPC used to compare the sizes or values of their privacies. In this paper, we propose a MQPC protocol by which any $k(k \ge 3)$ participants can compare the sizes of their privacies with executing the protocol just once. The proposed MQPC protocol takes the d-level GHZ states as quantum resources, and a semi-honest TP is introduced to help the participants to determine the relationship of their privacies. Further more, only single-particle unitary transformations and measurements are involved, and the participants need not to share common secrets with each other beforehand which makes the proposed protocol much more efficient. Analysis shows that our protocol is secure against internal and external attack in theory.

Keywords: Quantum Cryptography; Multi-party Quantum Privacy Comparison(MQPC); Quantum Fourier Transform(QFT); Third Party(TP); GHZ State.

1. Introduction

Privacy comparison originates from the concept of millionaire problem introduced by Yao which can be described as follows: two millionaires want to know who is richer without divulging any information about their wealth, and a novel solution for the problem was proposed by him[1]. Later, many solutions[2, 3] have been proposed, and the millionaire problem, especially privacy comparison became an important topic in classical cryptography. In the other hand, with the revolutionary application, known as BB84, of the quantum mechanics in the cryptography [4], quantum cryptography attracts much more attention from all over the world, and many kinds of cryptography protocols such as quantum key distribution (QKD)[5, 6], quantum secret sharing (QSS)[7, 8, 9], quantum direct communication(QDC)[10], quantum key agreement(QKA)[11], and so on, have been proposed. As an important topic, privacy comparison in the quantum circumstances, i.e., quantum privacy comparison(QPC), has attracted wide attention from many cryptographers.

In 2009, the first two-party QPC protocol for comparing information of equality based on bell states and a

^{*}Hao Cao, caohao2000854@163.com.

hash function was proposed by Yang and Wen [12]. Thereafter, several two-party QPC protocols[13, 14, 15] based on entangled quantum resources, such as GHZ states, χ -type states and so on, were proposed. However, only two parties were involved in the above protocols. Until 2013, the first multi-party QPC(MQPC) was proposed by Chang et al[16]. Since then, various two-party [17, 18, 19, 20, 21] and multi-party [22, 23, 24, 25] structures were proposed. However, the aforementioned protocols are only suitable for comparing the equality of information. When it comes to size comparison, these protocols are powerless.

Fortunately, in 2011, Jia et al presented the first two-party QPC protocol for comparing the sizes of privacies based on d - level three-particle GHZ states [26], in which the information of sizes was encoded into the phase of GHZ states. Later, in 2013, three two-party QPC protocols [27, 28, 29] for comparing the information of sizes based on d - level bell states were proposed. In the same year, Yu et al [30] proposed another one based on d - level single particles. However, the five QPC protocols mentioned above only relate to comparing the size of two parties. Until 2014, the first protocol of size comparison in multi-party circumstance[31] was proposed by Luo et al. However, the participants needed to share a privacy key K beforehand by using QKA protocol which will waste a lot of quantum resources. Besides, each participant and TP need to establish an authenticated classical channel beforehand. Later, Huang et al [32] proposed another MQPC protocol based on GHZ states, which can also be used to compare the sizes of all privacies. However, we found that there exists a serious security flaw in the protocol after close analysis, i.e., an internal participant can get the privacy of any other participant without being found.

In this paper, we will propose a novel MQPC protocol by which any $k(k \ge 3)$ participants can compare the sizes of their privacies with executing the protocol just once. In this protocol, a semi-honest third party(TP)[31] is introduced to help the participants to compare the sizes of their information. The semi-honest means that the TP will always execute the protocol honestly, record the information of the participants and try to extract their privacies from the records, but he will not conspire with any participant or outside eavesdropper. First, TP prepares some $d - level \ k - particle$ GHZ states and distributes them to every participant. Second, each participant encodes them with unitary operations based on a random sequence, and sends them back to TP. Next, each participant encrypts his size by the random sequence and sends it to TP. At last, TP measures the GHZ states on the Z-basis separately, compares them with the encrypted size, and obtains the results of comparison. The proposed MQPC protocol can ensure that

(1) correctness: all participants can get the size relationship of their privacy correctly with the help of TP if they execute the protocol honestly.

(2)security: the semi-honest party TP cannot get any information about the privacies of participants except the size relationship. Besides, each participant cannot deduce privacy of others from the comparison result.

The structure of our paper is as follows. Section 2 devotes to the details and correctness of our proposed protocol, and a novel example is presented. Section 3 analyzes the proposed protocol and compares it to the existed protocols, and a brief conclusion is given in section 4.

2. Results

Before going further, firstly we recall some definitions and quantum resources which will be used in the description of our protocol.

2.1. Preparation for the protocol

The quantum resource used in our protocol is the d – level k – particle GHZ state which can be represented as

$$|\Phi\rangle = \frac{1}{\sqrt{d}} \underbrace{(|0\rangle|0\rangle\cdots|0\rangle}_{k} + \underbrace{|1\rangle|1\rangle\cdots|1\rangle}_{k} + \cdots + \underbrace{|d-1\rangle|d-1\rangle\cdots|d-1\rangle}_{k}$$
(1)

For a d – level quantum system, there are two indistinguishable orthogonal bases, Z-basis and X-basis :

$$Z = \{|0\rangle, |1\rangle, |2\rangle, \dots, |d-1\rangle\}$$

$$X = \{QFT|0\rangle, QFT|1\rangle, QFT|2\rangle, \dots, QFT|d-1\rangle\}$$
(2)

where $QFT: |x\rangle \to \frac{1}{\sqrt{d}} \sum_{z=0}^{d-1} exp(\frac{2\pi i x z}{d}) |x\rangle$ is the quantum Fourier transform (QFT). Let us introduce an unitary operation (we call it shift operator) as follows:

$$U_r = \sum_{t=0}^{d-1} exp(\frac{2\pi i t (t\oplus r)}{d}) |t\oplus r\rangle\langle t|$$
(3)

Hereafter, the symbols \oplus and \ominus denote modular d addition and subtraction. It is easy to verify that the shift operator is an one-to-one map from Z-basis to itself and X-basis to itself, i.e.,

$$U_r(|s\rangle) = |s \oplus r\rangle$$

$$U_r(QFT|s\rangle) = QFT|s \oplus r\rangle(s = 0, 1, 2, \dots, d-1)$$
(4)

2.2. The MQPC protocol for comparing the sizes of information

Let TP be a semi-honest party and $P_0, P_1, P_2, \ldots, P_{k-1}$ be k participants. Each participant P_i $(i \in \{0, 1, 2, \ldots, k-1\})$ possesses a m-length privacy $p_i = (p_{i,1}, p_{i,2}, \ldots, p_{i,m}) \in \{0, 1, \ldots, l\}^m$ (here d = 2l + 1). They want to compare the size of $p_{0,j}, p_{1,j}, \ldots, p_{k-1,j}$ $(j = 1, 2, \ldots, m)$ without revealing any information. Through executing the following protocol, they could achieve their goals with the help of TP. The detailed description of our MQPC protocol can be seen as follows:

Step 1 Preparation. *TP* prepares *m* identical $d - level \ k - particle$ GHZ states in the form of equation (1), and splits them into *k* particle-sequences: $S_0, S_1, \ldots, S_{k-1}$. The i - th sequence $S_i (i = 0, 1, \ldots, k - 1)$ is consisting of the i - th particles of these GHZ states. Next, he will get a series of new sequence $S'_0, S'_1, \ldots, S'_{k-1}$ by inserting *m* decoy particles which are selected from X-basis or Z-basis (see equation (2)) randomly into each sequence S_i , and sends the resulted sequence $S'_i (i = 0, 1, \ldots, k - 1)$ to the i - th participant P_i .

Step 2 Eavesdropping Checking. After confirming that each participant P_i has received the sequence S'_i , TP publishes the position and measurement basis(X-basis or Z-basis) of each decoy particle in S'_i . P_i and TP execute eavesdropping checking similar to BB84. If the safety of the channel is not acceptable, the protocol goes to **Step 1.** Otherwise, the protocol will continue. After successfully passed the eavesdropping checking, each participant P_i will recover the sequence S_i by deleting the decoy particles from S'_i .

Step 3 Encoding. Each participant P_i selects a m - length random sequence $r_i = (r_{i,1}, r_{i,2}, \ldots, r_{i,m}) \in \{0, 1, \ldots, d-1\}^m$, and performs the shift operator $U_{r_{i,j}}(j = 1, 2, \ldots, m)$ in the form of equation (3) to the j - th particle of the sequence S_i . Then he sends the resulted sequence $\overline{S_i}$ together with k decoy particles (similar to Step 1) to TP.

Step 4 Measurement. Having received the sequence from every participant P_i , TP will execute eavesdropping checking with every P_i separately similar to Step 2. After successfully passed the eavesdropping checking, TP will extracts $\overline{S_i}$ by deleting the decoy particles. Next, he measures each particle in $\overline{S_i}$ on the Z-basis, and the measurement result is denoted by $|w_i\rangle = |w_{i,1}\rangle|w_{i,2}\rangle \cdots |w_{i,m}\rangle$.

Step 5 Transmitting privacy. Each participant P_i encrypts his privacy $p_i = (p_{i,1}, p_{i,2}, \ldots, p_{i,m})$ into $\overline{p_i} = (\overline{p_{i,1}}, \overline{p_{i,2}}, \ldots, \overline{p_{i,m}}) = (p_{i,1} \ominus r_{i,1}, p_{i,2} \ominus r_{i,2}, \ldots, p_{i,m} \ominus r_{i,m})$, and sends it to TP through an authenticated channel. Step 6 Comparison. Having received $\overline{p_i}$ from every participant P_i , TP calculates:

$$t_{i} = (t_{i,1}, t_{i,2}, \dots, t_{i,m})$$

$$= (\overline{p_{i,1}} \oplus w_{i,1}, \overline{p_{i,2}} \oplus w_{i,2}, \dots, \overline{p_{i,m}} \oplus w_{i,m})$$

$$t(i, i') = (t_{i,1} \ominus t_{i',1}, t_{i,2} \ominus t_{i',2}, \dots, t_{i,m} \ominus t_{i',m}))$$

$$s(i, i') = (s(i, i')_{1}, s(i, i')_{2}, \dots, s(i, i')_{m})$$

$$= (Sign[t_{i,1} \ominus t_{i',1}], Sign[t_{i,2} \ominus t_{i',2}], \dots, Sign[t_{i,m} \ominus t_{i',m})])$$
(5)

where $i, i' \in \{0, 1, \dots, k-1\}, i < i'$, i < j and $Sign[\cdot]$ is the signal function which is defined by:

$$Sign[x] = \begin{cases} 1 & x \in \{1, 2, \cdots, l\} \\ 0 & x = 0 \\ -1 & x \in \{l+1, l+2, \cdots, 2l\} \end{cases}$$
(6)

For the $jth(j = 1, 2, \dots, m)$ elements of all participants' privacies $p_{0,j}, p_{1,j}, \dots, p_{k-1,j}, TP$ can deduces the size relationship of them from the values of $s(i, i')(i, i' = 0, 1, \dots, k-1)$. The rules of judgement are as follows:

$$If \ s(i, i')_{j} = 1, \ then \ p_{i,j} > p_{i',j};$$

$$If \ s(i, i')_{j} = 0, \ then \ p_{i,j} = p_{i',j};$$

$$If \ s(i, i')_{j} = -1, \ then \ p_{i,j} < p_{i',j}.$$
(7)

Next, TP arranges the elements $p_{0,j}, p_{1,j}, \ldots, p_{k-1,j}$ in ascending order together with a relationship symbol < or = between every two elements, and gets a relation expression $p_{i_0^1,j} < p_{i_1^1,j} < \ldots < p_{i_{k-1}^1,j}$, where $i_0^j, i_1^j, \cdots, i_{k-1}^j$ is a permutation of $0, 1, \cdots, k-1$, and < denotes the symbol < or = .

At last, for each $j \in \{1, 2, \dots, m\}$, TP publishes the information $R_j \triangleq i_0^j < i_1^j < \dots < i_{k-1}^j$, which is consisting of subscripts information of the relation expression. So far, all participants can get the comparison results from $R_j (j \in \{1, 2, \dots, m\})$.

2.3. Correctness of the protocol

For the convenience of description, the phase of eavesdropping checking in step 2 is not considered. Next, we will show that our protocol can work efficiently if all participants and TP execute the protocol honestly. Consider the *jth* elements $p_{0,j}, p_{1,j}, \ldots, p_{k-1,j} (j = 1, 2, \cdots, m)$ of all participants.

(a) TP prepares a sequence of d – level k – particle GHZ states:

$$|\Phi\rangle_{0,1,\cdots,k-1} = \frac{1}{\sqrt{d}} (|0\rangle|0\rangle\cdots|0\rangle + |1\rangle|1\rangle\cdots|1\rangle + \cdots + |d-1\rangle|d-1\rangle\cdots|d-1\rangle)_{0,1,\cdots,k-1}$$

He splits it into k single particle sequence $S_0, S_1, \ldots, S_{k-1}$ and sends the *i*th sequence S_i to P_i .

(b) In the step 3, each participant P_i selects a $r_i = (r_{i,1}, r_{i,2}, \ldots, r_{i,m}) \in \{0, 1, \ldots, d-1\}^m$ randomly, and performs the shift operator $U_{r_{i,j}}$ in the form of equation (3) to the *jth* particle in his own hand. Then he sends the resulted sequence to TP.

(c)In the step 4, the final state of the jth GHZ state will be as follows:

$$\begin{split} |\Phi\rangle_{0,1,\cdots,k-1} \\ &= \frac{1}{\sqrt{d}} (|0 \oplus r_{0,j}\rangle |0 \oplus r_{1,j}\rangle \cdots |0 \oplus r_{d-1,j}\rangle + |1 \oplus r_{0,j}\rangle |1 \oplus r_{1,j}\rangle \cdots |1 \oplus r_{d-1,j}\rangle + \\ &\cdots + |(d-1) \oplus r_{0,j}\rangle |(d-1) \oplus r_{1,j}\rangle \cdots |(d-1) \oplus r_{d-1,j}\rangle)_{0,1,\cdots,k-1} \end{split}$$
(8)
$$&= \frac{1}{\sqrt{d}} (|r_{0,j}\rangle |r_{1,j}\rangle \cdots |r_{d-1,j}\rangle + |1 \oplus r_{0,j}\rangle |1 \oplus r_{1,j}\rangle \cdots |1 \oplus r_{d-1,j}\rangle + \cdots + \\ |(d-1) \oplus r_{0,j}\rangle |(d-1) \oplus r_{1,j}\rangle \cdots |(d-1) \oplus r_{d-1,j}\rangle)_{0,1,\cdots,k-1} \end{split}$$

TP measures it in the Z-basis, and the state will collapse into one of the following states:

$$|r_{0,j}\rangle|r_{1,j}\rangle\cdots|r_{d-1,j}\rangle$$

$$|1\oplus r_{0,j}\rangle|1\oplus r_{1,j}\rangle\cdots|1\oplus r_{d-1,j}\rangle$$

$$\cdots$$

$$|(d-1)\oplus r_{0,j}\rangle|(d-1)\oplus r_{1,j}\rangle\cdots|(d-1)\oplus r_{d-1,j}\rangle$$
(9)

Hence, there exists an $c_j \in \{0, 1, \dots, d-1\}$, the GHZ state in the form of equation (8) will collapse into $|c_j \oplus r_{0,j}\rangle|c_j \oplus r_{1,j}\rangle \cdots |c_j \oplus r_{d-1,j}\rangle$, which implies that $|w_{i,j}\rangle = |c_j \oplus r_{i,j}\rangle(i=0,1,\dots,d-1)$.

(d) Each participant P_i encodes his privacy $p_i = (p_{i,1}, p_{i,2}, \dots, p_{i,m}) \in \{0, 1, \dots, l\}^m$ into $\overline{p_i} = \overline{p_{i,1}} = (p_{i,1} \oplus r_{i,1}, p_{i,2} \oplus r_{i,2}, \dots, p_{i,m} \oplus r_{i,m}) \in \{0, 1, \dots, l\}^m$, and sends $\overline{p_i}$ to TP. Note that the *jth* elements of all participants' privacies are encoded into $p_{0,j} \oplus r_{0,j}, p_{1,j} \oplus r_{1,j}, \dots, p_{m-1,j} \oplus r_{m-1,j}$.

(e) At last, TP calculates the equation(5). Now, we only consider $s(i, i')_j = Sign[t_{i,j} \ominus t_{i',j})](i, i' \in \{0, 1, \dots, d-1\})$

$$t_{i,j} \oplus t_{i\prime,j}$$

$$= (\overline{p_{i,j}} \oplus w_{i,j}) \oplus (\overline{p_{i\prime,j}} \oplus w_{i\prime,j})$$

$$= [(p_{i,j} \oplus r_{i,j}) \oplus (c_j \oplus r_{i,j})] \oplus [(p_{i\prime,j} \oplus r_{i\prime,j}) \oplus (c_j \oplus r_{i\prime,j})]$$

$$= p_{i,j} \oplus p_{i\prime,j} \begin{cases} \in \{1, 2, \cdots, l\} & p_{i,j} > p_{i\prime,j} \\ = 0 & p_{i,j} = p_{i\prime,j} \\ \in \{l+1, l+2, \cdots, l\} & p_{i,j} < p_{i\prime,j} \end{cases}$$

$$(10)$$

Then TP will get

$$Sign[t(i, i')] = \begin{cases} 1 & p_{i,j} > p_{i',j} \\ 0 & p_{i,j} = p_{i',j} \\ -1 & p_{i,j} < p_{i',j} \end{cases}$$
(11)

From $s(i, i')_j = Sign[t_{i,j} \ominus t_{i',j})](i, i' \in \{0, 1, \dots, d-1\})$, TP can give the size relationship of $p_{0,j}, p_{1,j}, \dots, p_{k-1,j}(j = 1, 2, \dots, m)$ correctly.

2.4. A novel example of the protocol

Let us give a novel example for illustration without considering the eavesdropping checking. Let k = 3, m = 2, l = 4, and d = 2l + 1 = 9. The privacies of P_0, P_1 and P_2 are $p_0 = (1, 4), p_1 = (2, 2), and p_2 = (2, 3)$.

(1) *TP* prepares 2 identical 9 – *level* 3 – *particle* GHZ states $|\Phi\rangle_{0,1,2}^1 = |\Phi\rangle_{0,1,2}^2 = \frac{1}{3}(|0\rangle|0\rangle|0\rangle + |1\rangle|1\rangle|1\rangle + \cdots + |8\rangle|8\rangle|8\rangle_{0,1,2}$, splits them into 3 particle S_0, S_1 and S_2 and sends them to P_0, P_1 and P_2 separately.

(2) P_0 (P_1 , P_2) selects a 2 - length random sequence $r_0 = (4, 6)$ ($r_1 = (2, 5)$, $r_2 = (6, 1)$), performs the shift operator $U_{r_{0,j}}$ ($U_{r_{1,j}}$, $U_{r_{2,j}}$) to the j - th particle of the sequence S_0 (S_1, S_2), where j = 1, 2, and sends the resulted particle sequence to TP.

(3) At this moment, TP possesses the 3 – particle GHZ states $|\Phi\rangle_{0,1,2}^{1}$ and $|\Phi\rangle_{0,1,2}^{2}$ which will be

$$\begin{split} |\Phi\rangle_{0,1,2}^{1} &= \frac{1}{3} (|0 \oplus r_{0,1}\rangle |0 \oplus r_{1,1}\rangle |0 \oplus r_{2,1}\rangle + |1 \oplus r_{0,1}\rangle |1 \oplus r_{1,1}\rangle |1 \oplus r_{2,1}\rangle + \\ &\cdots + |8 \oplus r_{0,1}\rangle |8 \oplus r_{1,1}\rangle |8 \oplus r_{2,1}\rangle)_{0,1,2} \\ &= \frac{1}{3} (|4\rangle |2\rangle |6\rangle + |5\rangle |3\rangle |7\rangle + \cdots + |3\rangle |1\rangle |5\rangle)_{0,1,2} \\ |\Phi\rangle_{0,1,2}^{2} &= \frac{1}{3} (|0 \oplus r_{0,2}\rangle |0 \oplus r_{1,2}\rangle |0 \oplus r_{2,2}\rangle + |1 \oplus r_{0,2}\rangle |1 \oplus r_{1,2}\rangle |1 \oplus r_{2,2}\rangle + \\ &\cdots + |8 \oplus r_{0,2}\rangle |8 \oplus r_{1,2}\rangle |8 \oplus r_{2,2}\rangle)_{0,1,2} \\ &= \frac{1}{3} (|6\rangle |5\rangle |1\rangle + |7\rangle |6\rangle |2\rangle + \cdots + |5\rangle |4\rangle |0\rangle)_{0,1,2} \end{split}$$
(12)

 $TP \text{ measures each particle in } |\Phi\rangle_{0,1,2}^1 \text{ and } |\Phi\rangle_{0,1,2}^2 \text{ on the Z-basis, he will get } |w_0\rangle = |w_{0,1}\rangle |w_{0,2}\rangle = |c_1 \oplus r_{0,1}\rangle |c_2 \oplus r_{0,2}\rangle, |w_1\rangle = |w_{1,1}\rangle |w_{1,2}\rangle = |c_1 \oplus r_{1,1}\rangle |c_2 \oplus r_{1,2}\rangle, |w_2\rangle = |w_{2,1}\rangle |w_{2,2}\rangle = |c_1 \oplus r_{2,1}\rangle |c_2 \oplus r_{2,2}\rangle, \text{ where } c_1, c_2 \in \{0, 1, \dots, 8\}.$ For example, if $|w_0\rangle = |4\rangle |7\rangle$, then $c_1 = 0, c_2 = 1, |w_1\rangle = |2\rangle |6\rangle, |w_2\rangle = |6\rangle |2\rangle.$

(4) P_0 (P_1, P_2) encodes his privacy into $\overline{p_0} = (p_{0,1} \ominus r_{0,1}, p_{0,2} \ominus r_{0,2}) = (6,7)$ (similarly, $\overline{p_1} = (0,6), \overline{p_2} = (5,2)$) by $r_0(r_1, r_2)$, and sends it to TP through an authenticated channel.

(5) TP calculates:

$$t_{0} = \overline{p_{0}} \oplus w_{0} = (6,7) \oplus (4,7) = (1,5)$$

$$t_{1} = \overline{p_{1}} \oplus w_{1} = (0,6) \oplus (2,6) = (2,3)$$

$$t_{2} = \overline{p_{2}} \oplus w_{2} = (5,2) \oplus (6,2) = (2,4)$$

$$t(0,1) = t_{0} \oplus t_{1} = (8,2)$$

$$t(0,2) = t_{0} \oplus t_{2} = (8,1)$$

$$t(1,2) = t_{1} \oplus t_{2} = (0,8)$$

$$s(0,1) = (Sign[8], Sign[2]) = (-1,1)$$

$$s(0,2) = (Sign[8], Sign[1]) = (-1,1)$$

$$s(1,2) = (Sign[0], Sign[8]) = (0,-1)$$

From the equation(7) and s(0,1) = (-1,1), *TP* will get $p_{0,1} < p_{1,1}$ and $p_{0,2} > p_{1,2}$. Similarly, *TP* will get $p_{0,1} < p_{2,1}$ and $p_{0,2} > p_{2,2}$, $p_{1,1} = p_{2,1}$ and $p_{1,2} < p_{2,2}$. Hence, *TP* obtains the size relationship of their privacies, i.e., $p_{0,1} < p_{1,1} = p_{2,1}$ and $p_{1,2} < p_{2,2} < p_{0,2}$. At last, he publishes the information $R_1 \triangleq 0 < 1 = 2$ and $R_2 \triangleq 2 < 3 < 0$.

3. Security analysis and efficiency comparison

In this section, we will analyze the security of our protocol from both external and internal attacks. Also, we will analyze the efficiency of our protocol and compare it with other exited protocols.

3.1. Security analysis of the protocol

Case 1 External attack. Suppose that an outsider eavesdropper, Eve, tries to obtain the privacies of participants. From the procession of the protocol, the privacy of each participant P_i is transmitted only once and is encrypted by a random sequence $r_i = (r_{i,0}, r_{i,1}, \dots, r_{i,m})$. Hence, Eve must find a way to intercept the sequence $r_i = (r_{i,0}, r_{i,1}, \dots, r_{i,m})$ in Step 3 and the encrypted sequence $\overline{p_i} = (p_{i,1} \ominus r_{i,1}, p_{i,2} \ominus r_{i,2}, \dots, p_{i,m} \ominus r_{i,m})$ in Step 5. To obtain r_i , he must carry out intercept-resend attack, i.e., he intercepts and takes measurements on the particles of S_i and the particles of $\overline{S_i}$, and resents them to receiver. Let us take the intercept-resend attack on the particles of S_i for example. Due to the existence of the decoy states, Eve need to choose the correct position and measurement-basis of each decoy state in order not to detected by the eavesdropping checking. However,

he does not have any information on the position and measurement-basis of each decoy state. If he chooses the right position and right basis, no error will be introduced; or else, the probability of introducing error will be at least $\frac{d-1}{d}$. Hence, his eavesdropping behavior will be detected with $1 - (\frac{d-1}{2d})^m$, which will approaches to 1 when m is large enough. It is the same with the case of intercept-resend attack on the the particles of $\overline{S_i}$. Therefore, Eve can not obtain the random sequence $r_i = (r_{i,0}, r_{i,1}, \dots, r_{i,m})$. Also, he can not obtain the sequence $\overline{p_i} = (p_{i,1} \ominus r_{i,1}, p_{i,2} \ominus r_{i,2}, \dots, p_{i,m} \ominus r_{i,m})$ in Step 5 because the channels between the TP and participants are authenticated. From the analysis above, the protocol is immune to external attack.

Case 2 Internal attack from participants. Suppose that a participant, P_0 , is a dishonest participant who tries to obtain the privacies of other participants, and TP is the semi-honest party who will not collude with anyone. If P_0 wants to steal the privacy of a certain participant $P_i(i \in \{1, 2, \dots, d-1\})$, he could firstly measures the particles in the sequence of S_0 on the Z-basis before performing the random shift operators on them, and the measurement results are identical to the particles in S_i . Next, to obtain the random sequence r_i , P_0 needs to measure the particles in the sequence $\overline{S_i}$ by using the intercept-resend attack. In this environment, P_0 can be considered as an outside attacker, and his interception behavior will be caught by P_i and TP similar to the case of external attack. Also, P_0 can not obtain the sequence $\overline{p_i}$ in Step 5 because the channel between TP and P_i is authenticated. The collusion attack from multiple participant is the same.

Case 3 Internal attack from the semi-honest third party TP. Obviously, the dishonest third party TP is the one who can get the most information during the execution of the protocol. However, due to his semi-honesty, he will prepare the k - particle d - level GHZ states rather than other types of particles such as single particles(even if he prepared other quantum states, his dishonest behavior would be discovered by participants in the following way. Before step 3, all participants consult to select some positions of particles randomly, and measure each particle of these positions using either X-basis or Z-basis. They can verify whether these quantum states are GHZ states or not by publishing the measurement results). Next, TP will execute the protocol honestly. The only way to derive the privacy of P_i relies on the analysis of information received from P_i . Firstly, he can obtain $\overline{p_i} = (p_{i,1} \ominus r_{i,1}, p_{i,2} \ominus r_{i,2}, \ldots, p_{i,m} \ominus r_{i,m})$ legally in Step 5. So he needs to get the random sequence $r_i = (r_{i,1}, r_{i,2}, \ldots, r_{i,m}) \in \{0, 1, \ldots, d - 1\}^m$ and nextly extracts the privacy of P_i . Apparently, the random sequence $r_i = (r_{i,1}, r_{i,2}, \ldots, r_{i,m}) \in \{0, 1, \ldots, d - 1\}^m$ is encoded into the sequence $\overline{S_i}$ which is entangle with $\overline{S_j}$ s. When it comes to measure the particles in the sequence $\overline{S_i}$, TP will randomly get one of the following states: $|r_{i,1}\rangle|r_{i,2} \oplus (-1)\rangle \cdots |r_{i,m} \oplus (-1)\rangle \cdots |r_{i,m} \oplus 1\rangle, \cdots, |r_{i,1} \oplus (d-2)\rangle|r_{i,2} \oplus (d-2)\rangle \cdots |r_{i,m} \oplus (d-2)\rangle$ and $|r_{i,1} \oplus (d-1)\rangle|r_{i,2} \oplus (d-1)\rangle \cdots |r_{i,m} \oplus (d-1)\rangle$. Hence, TP can not obtain $r_i = (r_{i,1}, r_{i,2}, \ldots, r_{i,m}) \in \{0, 1, \ldots, d-1\}^m$ accurately, and can not derive the privacy of P_i .

3.2. Efficiency comparison with existed protocols

Here, we will compare the protocol with four existed MQPC protocols in the following five aspects: quantum resources used, the category of MQPC (size or equality comparison), the qubit or qudit efficiency which is defined as $\eta = \frac{c}{q+b}$ (here c is the length of privacies of participants, q and b are the numbers of qudits and classical

QPC Protocol	quantum resources	Category of QPC	Efficiency η	Need to share privacy key	Security
CTH2013[16]	2 - level GHZ class states	Equality	$\frac{1}{3k}$	No	Secure
HHH2017[25]	2 - level Bell states	Equality	$\frac{1}{8k}$	No	Secure
LYS2014 protocol $[31]$	d-level entangled states	Size	$\frac{1}{3k}$	Yes	secure
HHG2015 protocol [32]	d - level GHZ and entangled particles	Size	$\frac{1}{6k}$	No	Insecure
Ours	d - level GHZstates	Size	$\frac{1}{3k}$	No	Secure

Table 1: Comparison between the existed four QPC protocols with ours

bits used in transmission and eavesdropping checking, whether participants need to share privacy common key beforehand, and security. For the sake of discussion, it is assumed that the length of the privacies is m, and the number of decoy particles is equal to the number of quantum particles transmitted in each MQPC protocol. The four existing MQPC protocols are CTH2013 protocol [16], HHH2017 protocol [25], LYS2014 protocol [31], and HHG2015 protocol [32]. Now, we will show the comparison result as follows (see table 1).

(1) CTH2013 protocol. The authors proposed a 4-party QPC protocol, and a multiparty (say k-party hereafter) QPC protocol which are used to compare the equality of the privacies. We only consider the case of k-party. The quantum resources used in this protocol are $2-level \ k-particle$ GHZ-class states. The transmission of information includes two stages. First, TP prepares $m \ k-particle$ GHZ-class states. Then, he splits them into k particle-sequence and sends every sequence to the corresponding participant with m decoy particles. Second, each participant sends his encoded privacy which is m bits to TP. Hence, the efficiency $\eta = \frac{m}{mk+mk+mk} = \frac{1}{3k}$. Besides, the participants need not to share privacy common key beforehand, and the protocol is secure at present because there is no efficient attack for it.

(2) HHH2017 protocol. The authors proposed a k - party QPC protocol of comparing the equality in which two TPs are introduced to deal with the comparison in a strange environment. The quantum resources used in this protocol are $2 - level \ k - particle$ GHZ-class states. The transmission of information includes three stages. First, TP_1 prepares $2m \ k - particle$ GHZ-class states. Then, he splits them into k particle-sequence and sends every sequence to the corresponding participant with 2m decoy particles. Second, TP_1 sends the information of the GHZ states to TP_2 using quantum secure direct communication and the quantum resource used here is at least 2mk qubits. Third, each participant sends his encoded privacy which is m bits to TP_1 and TP_2 . Hence, the efficiency $\eta = \frac{m}{2mk+2mk+mk+mk} = \frac{1}{8k}$. Besides, the participants also need not to share privacy common key beforehand, and the protocol is secure at present.

(3) LYS2014 protocol. The authors proposed a k - party QPC protocol of comparing the sizes of privacies. The quantum resources used in this protocol are d-level entangled states, and the participants need to share a privacy common key K beforehand through a secure QKA protocol. The transmission of information contains three step. First, TP prepares $m \ k - particle \ d - level$ entangled states. Then, he splits them into k particle-sequence and

sends each sequence to the corresponding participant with m decoy particles. Second, each participant measures the received particle-sequence which will be transformed into a classical m - bit sequence, and he encrypts his privacy by the classical bit-sequence and the privacy common key K using one-time pad. At last, each participant sends his encrypted privacy information(m - bit sequence) to TP through an authenticated channel. Hence, the efficiency $\eta = \frac{m}{mk+mk+mk} = \frac{1}{3k}$. However, the actual efficiency is lower than $\frac{1}{3k}$ because the participants need to share a privacy common key K beforehand through a QKA protocol which will waste a lot of quantum resource. This protocol is secure at present because there is no efficient attack for it.

(4)HHG2015 protocol. The authors proposed a k - party QPC protocol of comparing the sizes of privacies. The quantum resources used in this protocol are d - level GHZ states and d - level entangled states. The transmission of information includes two stages. First, TP prepares $m \ k - particle \ d - level$ GHZ states and $m \ k - particle \ d - level$ GHZ states into k particle-sequences and sends them to the corresponding participant with m decoy particles. Also, he splits the $m \ k - particle \ d - level$ entangled states into k particle-sequences and sends them to the corresponding participant with m decoy particles. Also, he splits the $m \ k - particle \ d - level$ entangled states into k particle-sequences and sends them to the corresponding participant measures the first k - particle sequence. Then he performs the unitary operations, which are decided by the measurement results and his privacy, on the second k - particle sequence and sends the resulted k - particle sequence with m decoy particles to TP. The efficiency $\eta = \frac{m}{2mk+2mk+mk} = \frac{1}{6k}$. Besides, the participants need not to share a privacy common key beforehand. Hence, the HHG2015 protocol is much more efficient than LYS2014 protocol.

However, there is a serious bug in the HHG2015 protocol. From step 4 and step 6, We can easily get that $p_i = p_j$ and $q_i = q_j$ for each *i* and *j*. If a dishonest participant(say P_1), wants to steal the privacy of another one(say P_2), he will firstly intercept the particles sent from P_2 and resents forged particles to TP in step 4. Secondly, he deletes the decoy particles and measures the remaining particles after P_2 published the positions of decoy states. Therefore, P_1 will get the value of $MR_2 = (s_2 + p_2 + q_2) \mod d$ and $s_2 = (MR_2 - p_2 - q_2) \mod d$ which is the privacy of P_2 . Although this attack will be discovered by TP and P_2 , but they did not know the identity of the attacker. Hence, P_1 succeeded in obtaining the privacy of P_2 . Similarly, he can get the privacy of any other participant without being found. So, this protocol is insecure.

(5) Our protocol. We proposed a k - party QPC protocol of comparing the sizes of privacies by using k - particle d - level GHZ states, and the participants need not to share privacy common key beforehand. The transmission of information includes three stages. First, TP prepares $m \ k - particle$ GHZ-class states. Then, he splits them into k particle-sequence and sends each sequence to the corresponding participant with m decoy particles. Second, After encoding the received sequence by a series of random unitary operations, each participant inserts m decoy particles into it and sends it back to TP. Third, every participant transmits m classical bits to TPseparately. Hence, the efficiency $\eta = \frac{m}{mk+mk+mk} = \frac{1}{3k}$ which is as good as that of the LYS2014 protocol and CTH2013 protocol, and our protocol is secure against external and internal attacks. However, owing to the waste of quantum resource in the sharing the common key beforehand through a QKA protocol in the LYS2014 protocol, our protocol is more efficient than it because the participants need not to share private common key beforehand in our protocol. Besides, the CTH2013 protocol only solves the problem of equality comparison. Therefore, our protocol is better than the LYS2014 protocol and CTH2013 protocol.

4. Conclusion

We presented a MQPC protocol with k - particle d - level GHZ states. In the protocol, all participants can compare the size of their privacy with the help of a semi-honest party TP. Besides, we gave a novel example of the proposed protocol. Security analysis shows that it is immune to both external attack and internal attack in theory, and efficiency comparison shows that it is prior to all existing protocols of the same type. However, our protocol is only suitable for scenarios in an ideal environment. How to improve the agreement to adapt to a more complicated environment is our main work in the future.

Acknowledgments

This work was supported in part by the National Key R&D Program of China under Grant 2017YFB0802400, the National Science Foundation of China under Grant 61373171, 61702007 and 11801564, the 111 Project under Grant B08038, and the Key Project of Science Research of Anhui Province under Grant KJ2017A519, the Natural Science Foundation of Shaanxi Province under Grant No.2017JQ1032, the Basic Research Project of Natural Science of Shaanxi Province under Grant 2017JM6037.

References

References

- Yao A C. Protocols for secure computations[C]//Foundations of Computer Science, 1982. SFCS'08. 23rd Annual Symposium on. IEEE, 1982: 160-164.
- [2] Ioannidis I, Grama A. An efficient protocol for Yao's millionaires' problem[C]//System Sciences, 2003.
 Proceedings of the 36th Annual Hawaii International Conference on. IEEE, 2003: 6 pp.
- [3] Lin H Y, Tzeng W G. An efficient solution to the millionaires problem based on homomorphic encryption[C]//International Conference on Applied Cryptography and Network Security. Springer, Berlin, Heidelberg, 2005: 456-466.
- [4] Bennett C H, Brassard G. Quantum cryptography: public-key distribution and coin tossing [C]. Proceedings of IEEE International Conference on Computer System and Signal Processing, 175-179 (1984).
- [5] Lo H K, Chau H F. Unconditional security of quantum key distribution over arbitrarily long distances[J]. Science, 1999(283): 2050-2056.

- [6] Lo H K, Ma X, Chen K. Decoy state quantum key distribution[J]. Physical Review Letters, 2005(94): 230504.
- [7] Hillery M, Buzek V, Berthiaume A. Quantum secret sharing[J]. Physical Review A, 1999(59): 1829-1834.
- [8] Bai C M, Li Z H, Liu C J, et al. Quantum secret sharing using orthogonal multiqudit entangled states[J]. Quantum Information Processing, 2017, 16(12): 304.
- Cao H, and Ma W. (t, n) Threshold Quantum State Sharing Scheme Based on Linear Equations and Unitary Operation[J]. IEEE Photonics Journal 9.1 (2017): 1-7.
- [10] Zhang W, Ding D S, Sheng Y B, et al. Quantum secure direct communication with quantum memory[J].
 Physical Review Letters, 2017, 118(22): 220501.
- [11] Cao H, Ma W. Multiparty Quantum Key Agreement Based on Quantum Search Algorithm[J]. Scientific Reports, 2017, 7: 45046.
- [12] Yang Y G, Wen Q Y. An efficient two-party quantum privacy comparison protocol with decoy photons and two-photon entanglement[J]. Journal of Physics A: Mathematical and Theoretical, 2009, 42(5): 055305.
- [13] Chen X B, Xu G, Niu X X, et al. An efficient protocol for the privacy comparison of equal information based on the triplet entangled state and single-particle measurement[J]. Optics communications, 2010, 283(7): 1561-1565.
- [14] Xu G A, Chen X B, Wei Z H, et al. An efficient protocol for the quantum privacy comparison of equality with a four-qubit cluster state[J]. International Journal of Quantum Information, 2012, 10(04): 1250045.
- [15] Tseng H Y, Lin J, Hwang T. New quantum privacy comparison protocol using EPR pairs[J]. Quantum Information Processing, 2012, 11(2): 373-384.
- [16] Chang Y J, Tsai C W, Hwang T. Multi-user privacy comparison protocol using GHZ class states[J]. Quantum information processing, 2013: 1-12.
- [17] Zhang B, Liu X, Wang J, et al. Cryptanalysis and improvement of quantum privacy comparison of equality protocol without a third party[J]. Quantum Information Processing, 2015, 14(12): 4593-4600.
- [18] Sun Z, Yu J, Wang P, et al. Quantum privacy comparison with a malicious third party[J]. Quantum Information Processing, 2015, 14(6): 2125-2133.
- [19] Liu B, Xiao D, Huang W, et al. Quantum privacy comparison employing single-photon interference[J]. Quantum Information Processing, 2017, 16(7): 180.
- [20] He G P. Quantum privacy comparison protocol without a third party[J]. International Journal of Quantum Information, 2017, 15(02): 1750014.

- [21] Xu L, Zhao Z. Quantum privacy comparison protocol based on the entanglement swapping between χ^+ state and W-Class state[J]. Quantum Information Processing, 2017, 16(12): 302.
- [22] Wang Q L, Sun H X, Huang W. Multi-party quantum privacy comparison protocol with n-level entangled states[J]. Quantum Information Processing, 2014(13): 2370-2389.
- [23] Huang S L, Hwang T, Gope P. Multi-party quantum privacy comparison protocol with an almost-dishonest third party using GHZ states. International Journal of Theoretical Physics, 2016, 55(6): 2969-2976.
- [24] Ye T Y. Multi-Party Quantum Privacy Comparison Protocol Based on Entanglement Swapping of Bell Entangled States[J]. Communications in Theoretical Physics, 2016, 66(3): 280.
- [25] Hung S M, Hwang S L, Hwang T, et al. Multiparty quantum privacy comparison with almost dishonest third parties for strangers[J]. Quantum Information Processing, 2017, 16(2): 36.
- [26] Jia H Y, Wen Q Y, Song T T, et al. Quantum protocol for millionaire problem[J]. Optics communications, 2011, 284(1): 545-549.
- [27] Lin S, Sun Y, Liu X F, et al. Quantum privacy comparison protocol with d-dimensional Bell states[J]. Quantum information processing, 2013, 12(1): 559-568.
- [28] Guo F Z, Gao F, Qin S J, et al. Quantum privacy comparison protocol based on entanglement swapping of d-level Bell states[J]. Quantum information processing, 2013, 12(8): 2793-2802.
- [29] Zhang W W, Li D, Zhang K J, et al. A quantum protocol for millionaire problem with Bell states[J]. Quantum information processing, 2013, 12(6): 2241-2249.
- [30] Yu C H, Guo G D, Lin S. Quantum privacy comparison with d-level single-particle states[J]. Physica Scripta, 2013, 88(6): 065013.
- [31] Luo Q, Yang G, She K, et al. Multi-party quantum privacy comparison protocol based on d-dimensional entangled states[J]. Quantum information processing, 2014, 13(10): 2343-2352.
- [32] Huang S L, Hwang T, Gope P. Multi-party quantum privacy comparison with an almost-dishonest third party[J]. Quantum Information Processing, 2015, 14(11): 4225-4235.