



## Correction to: Entanglement-assisted quantum error-correcting codes over arbitrary finite fields

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### Correction to: Quantum Inf Process

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The parameters of the codes in Section 4 of our article [1] are not correctly stated. The inequality in Proposition 5 should be with respect to the dual code  $C^{\perp_s}$ . The correct statement of Proposition 5 is

**Proposition 5** *Assume that a positive integer  $c$  satisfies  $2c \leq d_H(C^{\perp_s} \setminus \{0\}) - 1$ , then*

$$\dim_{\mathbb{F}_q} P(C) = \dim_{\mathbb{F}_q} C, \text{ and}$$

$$\dim_{\mathbb{F}_q} P(C) \cap P(C)^{\perp_s} = \dim_{\mathbb{F}_q} S(C) = \dim_{\mathbb{F}_q} C - 2c.$$

The original article can be found online at <https://doi.org/10.1007/s11128-019-2234-5>.

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This affects the corresponding inequalities in the statement of Theorem 7, Theorem 8 and Theorem 9. Moreover, the dimension of the codes was erroneously displayed. The correct statements of the above theorems are the following:

**Theorem 7** Let  $C \subseteq \mathbb{F}_q^{2n}$  be an  $\mathbb{F}_q$ -linear code with  $\dim_{\mathbb{F}_q} C = n - k$  and  $C \subseteq C^{\perp_s}$ . Assume that a positive integer  $c$  satisfies  $2c \leq d_H(C^{\perp_s} \setminus \{\mathbf{0}\}) - 1$ , then the punctured code  $P(C)$  provides an

$$[[n - c, k, \geq d_s(C^{\perp_s} \setminus C); c]]_q$$

entanglement-assisted code.

**Theorem 8** Let  $C \subseteq \mathbb{F}_{q^2}^n$  be an  $\mathbb{F}_{q^2}$ -linear code with  $\dim_{\mathbb{F}_{q^2}} C = (n - k)/2$ , and suppose that  $C \subseteq C^{\perp_h}$ . Let  $c$  be a positive integer such that  $c \leq d_H(C^{\perp_h} \setminus \{\mathbf{0}\}) - 1$ , then the punctured code  $P_h(C)$  provides an

$$[[n - c, k, \geq d_H(C^{\perp_h} \setminus C); c]]_q$$

entanglement-assisted code.

**Theorem 9** Let  $C_2 \subseteq C_1 \subseteq \mathbb{F}_q^n$  be two  $\mathbb{F}_q$ -linear codes such that  $\dim C_i = k_i$ ,  $1 \leq i \leq 2$ . The standard construction of CSS codes uses  $C_2 \times C_1^{\perp}$  as the stabilizer. Assume that  $c$  is a positive integer such that

$$c \leq \min \left\{ d_H(C_2^{\perp} \setminus \{\mathbf{0}\}), d_H(C_1 \setminus \{\mathbf{0}\}) \right\} - 1,$$

then the punctured code  $P_h(C_2) \times P_h(C_1^{\perp})$  provides an

$$[[n - c, k_1 - k_2, \geq \min \left\{ d_H(C_1 \setminus C_2), d_H(C_2^{\perp} \setminus C_1^{\perp}) \right\}; c]]_q$$

entanglement-assisted code.

Proofs are the same with the exception that  $c \leq d_H(C \setminus \{\mathbf{0}\}) - 1$  should be replaced by  $c \leq d_H(C^{\perp_h} \setminus \{\mathbf{0}\}) - 1$ , in the proof of Theorem 8 and

$$c \leq \min \{ d_H(C_2 \setminus \{\mathbf{0}\}), d_H(C_1^{\perp} \setminus \{\mathbf{0}\}) \} - 1$$

should be replaced by

$$c \leq \min \left\{ d_H(C_2^{\perp} \setminus \{\mathbf{0}\}), d_H(C_1 \setminus \{\mathbf{0}\}) \right\} - 1,$$

in the first line of the proof of Theorem 9.

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## Reference

1. Galindo, C., Hernando, F., Matsumoto, R., Ruano, D.: Entanglement-assisted quantum error-correcting codes over arbitrary finite fields. *Quantum Inf. Process.* **18**, 116 (2019)

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